KU LEUVEN

Absolute radii of chlorine and potassium:

A heavyweight solution to a small problem

Michael Heines
Supervisor: Thomas Cocolios
On behalf of the muX collaboration

Contents

- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Different physics cases

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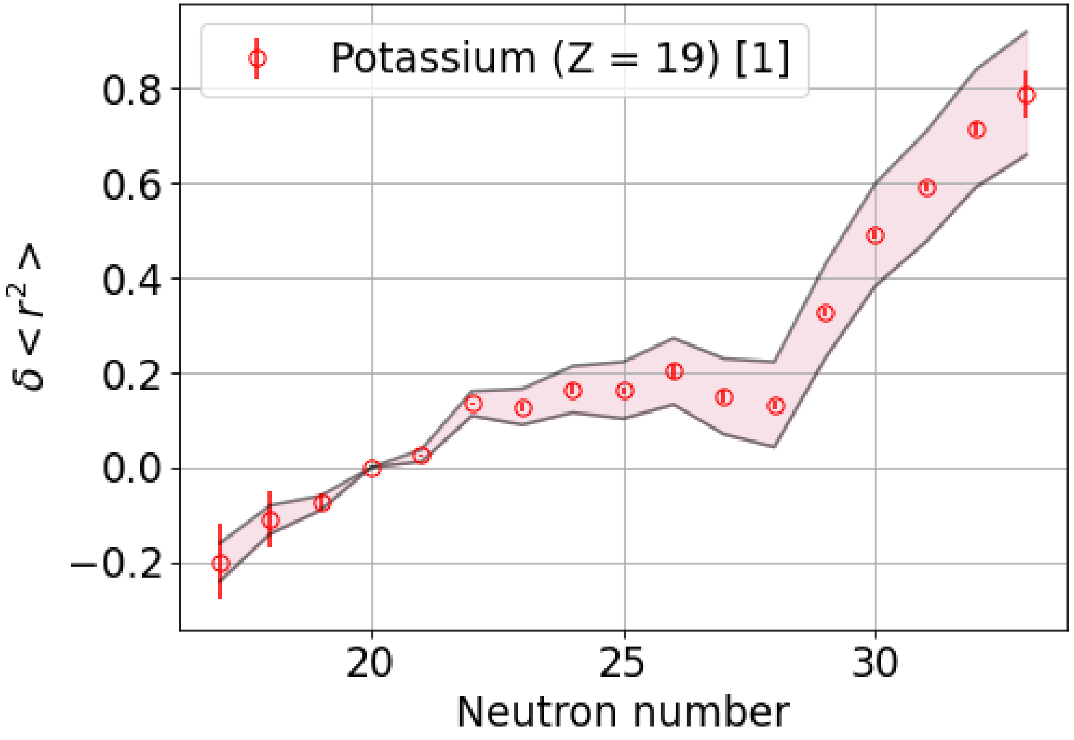
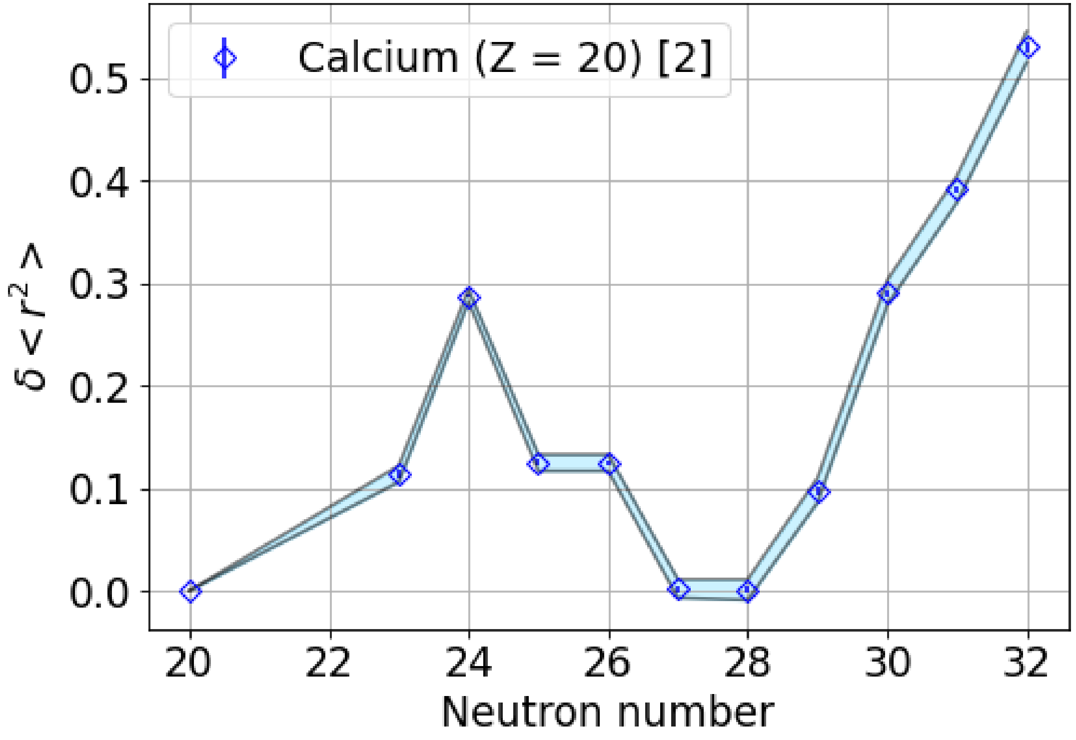
Measurements of $\delta < r^2 >$

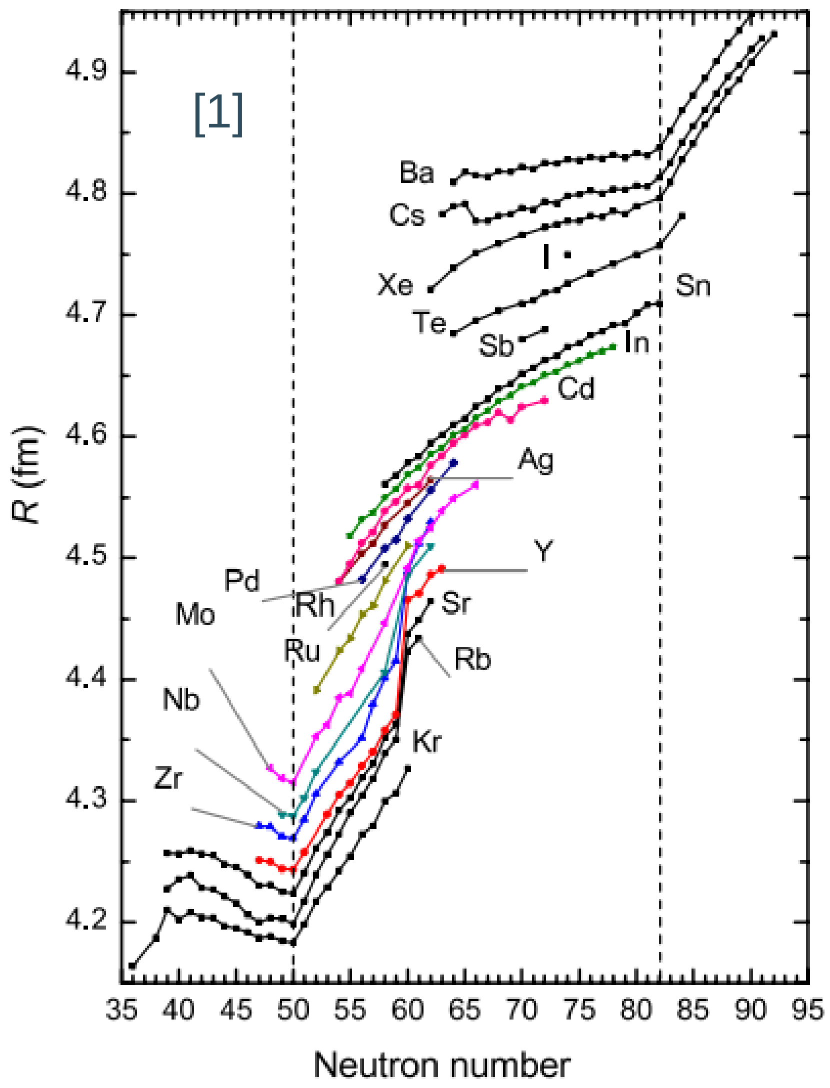




[1] Koszorús, Á., et al. "Charge radii of exotic potassium isotopes challenge nuclear theory and the magic character of N= 32." *Nature Physics* 17.4 (2021): 439-443.
[2] Garcia Ruiz, R. , et al. "Unexpectedly large charge radii of neutron-rich calcium isotopes." *Nature Physics* 12.6 (2016): 594-598.

Benefit of absolute radii

- Visualizing global trends
- Input for other experiments
- Isotone shifts
- Mirror nuclei



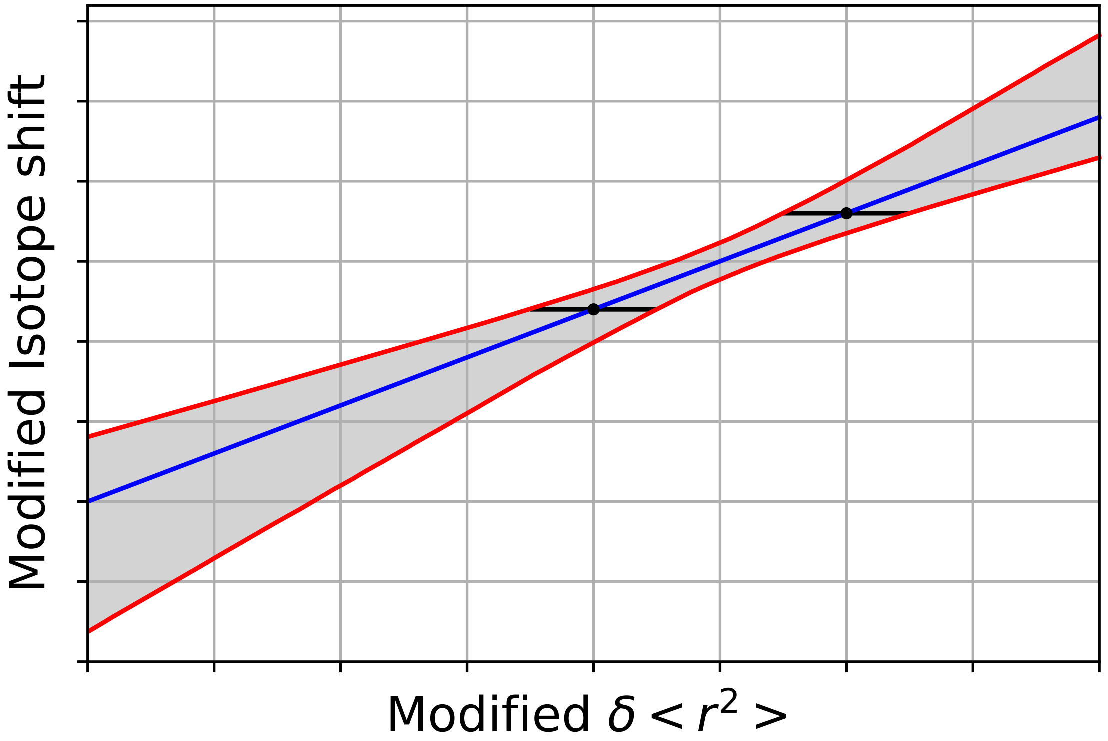
[1] Angeli, István, and Krassimira Petrova Marinova. "Table of experimental nuclear ground state charge radii: An update." *Atomic Data and Nuclear Data Tables* 99.1 (2013): 69-95.

King plot

$$\delta\langle r^2\rangle^{A,A'} = \frac{1}{F_i}\left(\delta\nu_i^{A,A'} - \frac{A - A'}{A\,A'}M_i\right)$$

- M_i : Mass shift factor
- F_i : Field shift factor

$$\frac{A\,A'}{A - A'}\,\delta\nu_i^{A,A'} = M_i + F_i\,\frac{A\,A'}{A - A'}\delta\langle r^2\rangle^{A,A'}$$



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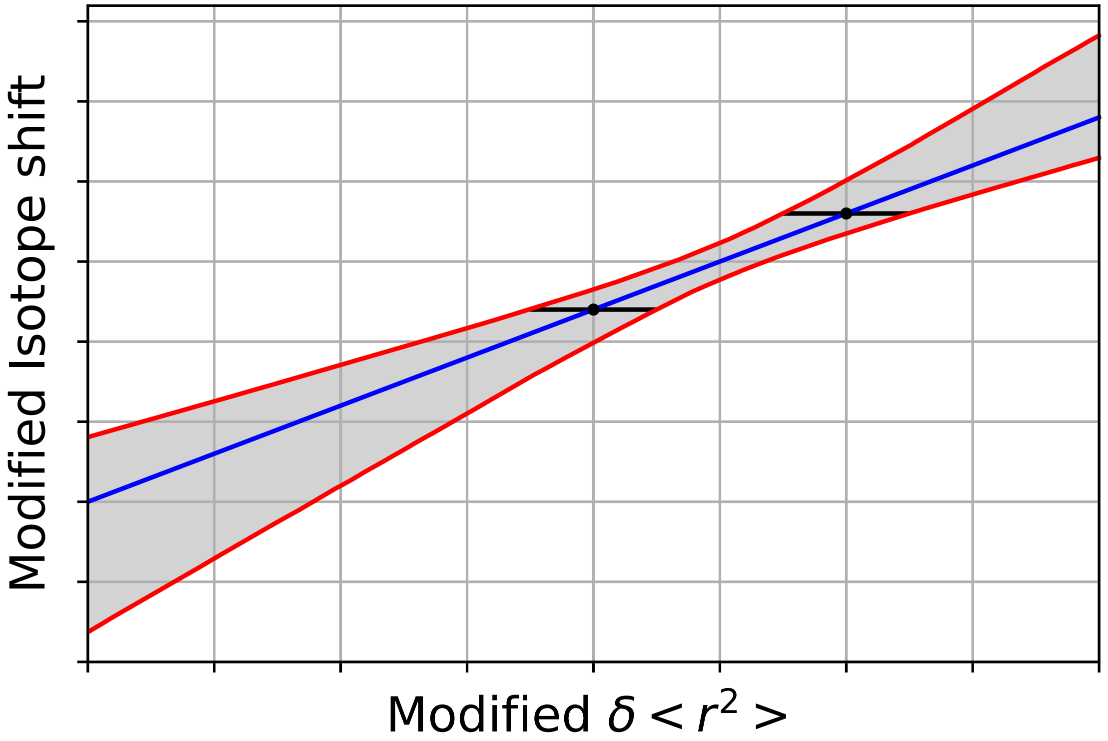

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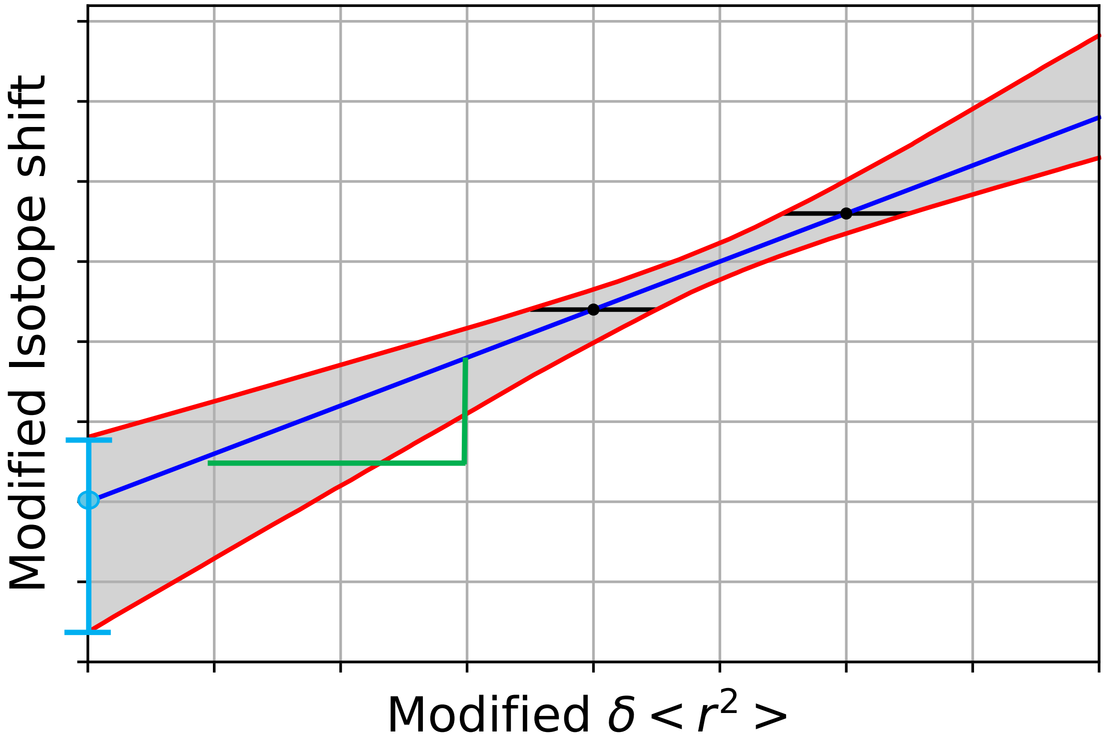
No odd-Z element with 3 stable isotopes!



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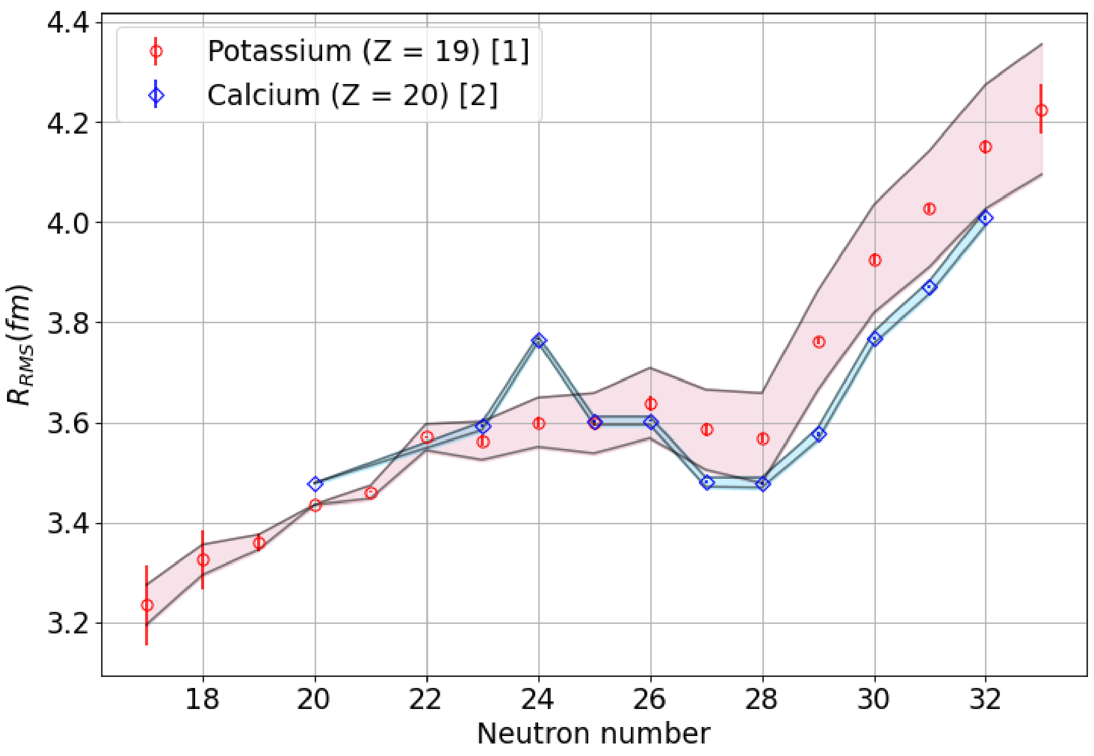

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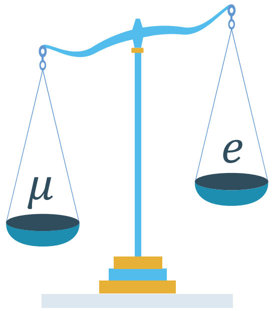
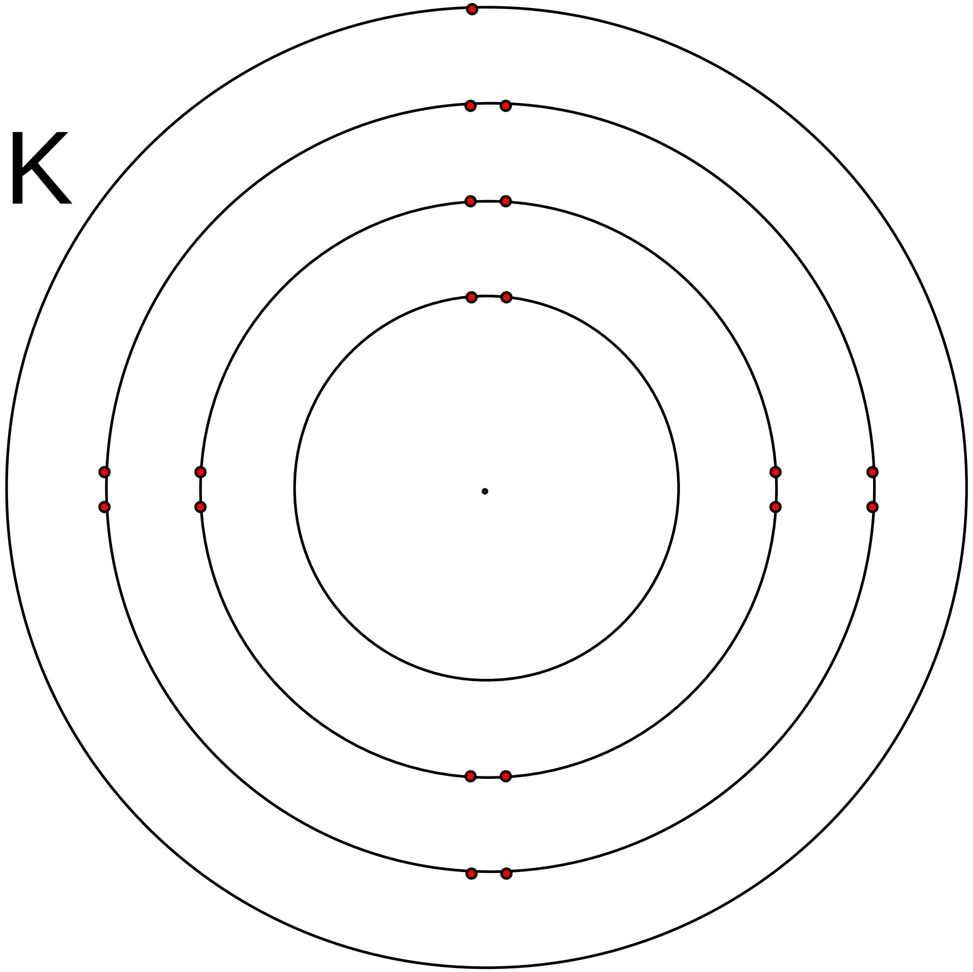
Muonic atoms

- Bohr model
  - $E_n \propto \frac{mZ^2}{n^2}$
  - $r_n \propto \frac{n^2}{mZ}$
- Muons:
  - $m_\mu \approx 207\ m_e$
  - $\tau_\mu \approx 2.2\ \mu s$





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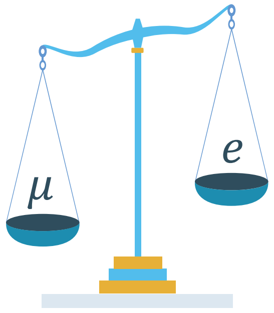

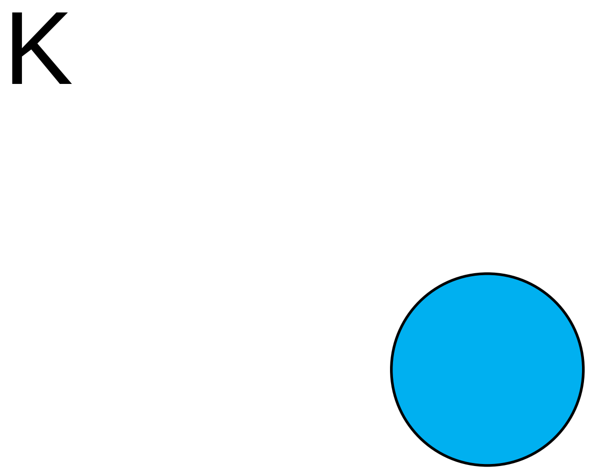

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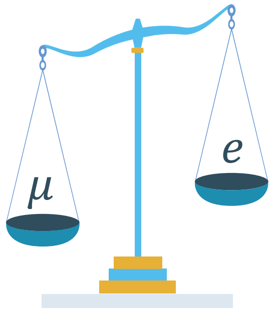
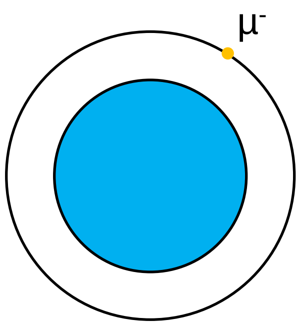


- Effect:
  - Enhanced binding energy
  - Closer to the nucleus → More sensitive to nuclear effects

μK



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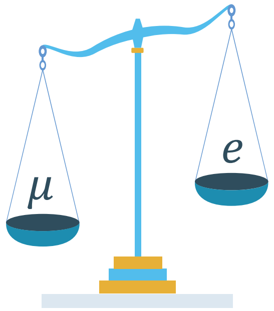

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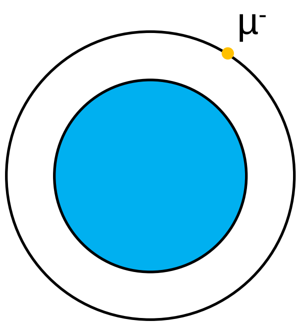
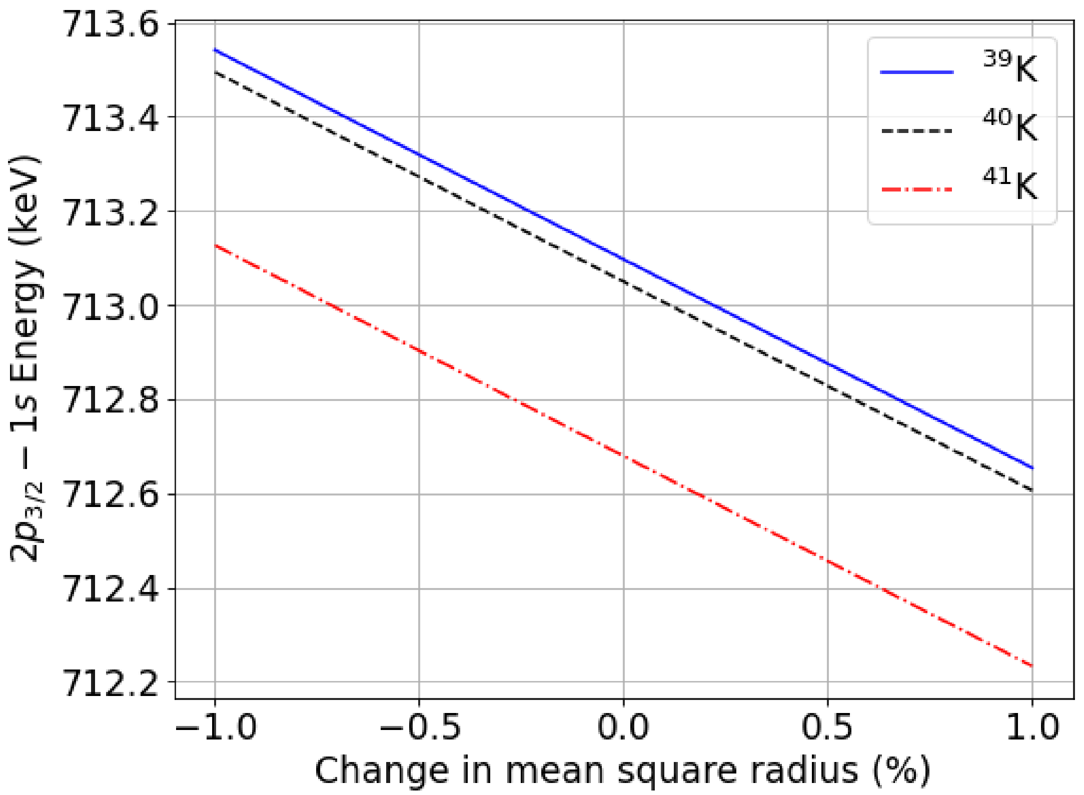


Muon n = 14 orbital is inside electron n = 1 orbital
→ electron correlation is negligible

Extracting radii

- Finite size correction scales with $\frac{1}{r^3} \approx 10^7$
- Calculate transition energy for many radii
  → Compare with experiment
- Typical limitations:
  - Nuclear polarization (theory)
  - Nuclear shape (electron scattering)
  - Energy calibration



Simple calculations with mudirac code [1]

[1] Sturniolo, Simone, and Adrian Hillier. "Mudirac: A Dirac equation solver for elemental analysis with muonic X-rays." *X-Ray Spectrometry* 50.3 (2021): 180-196.

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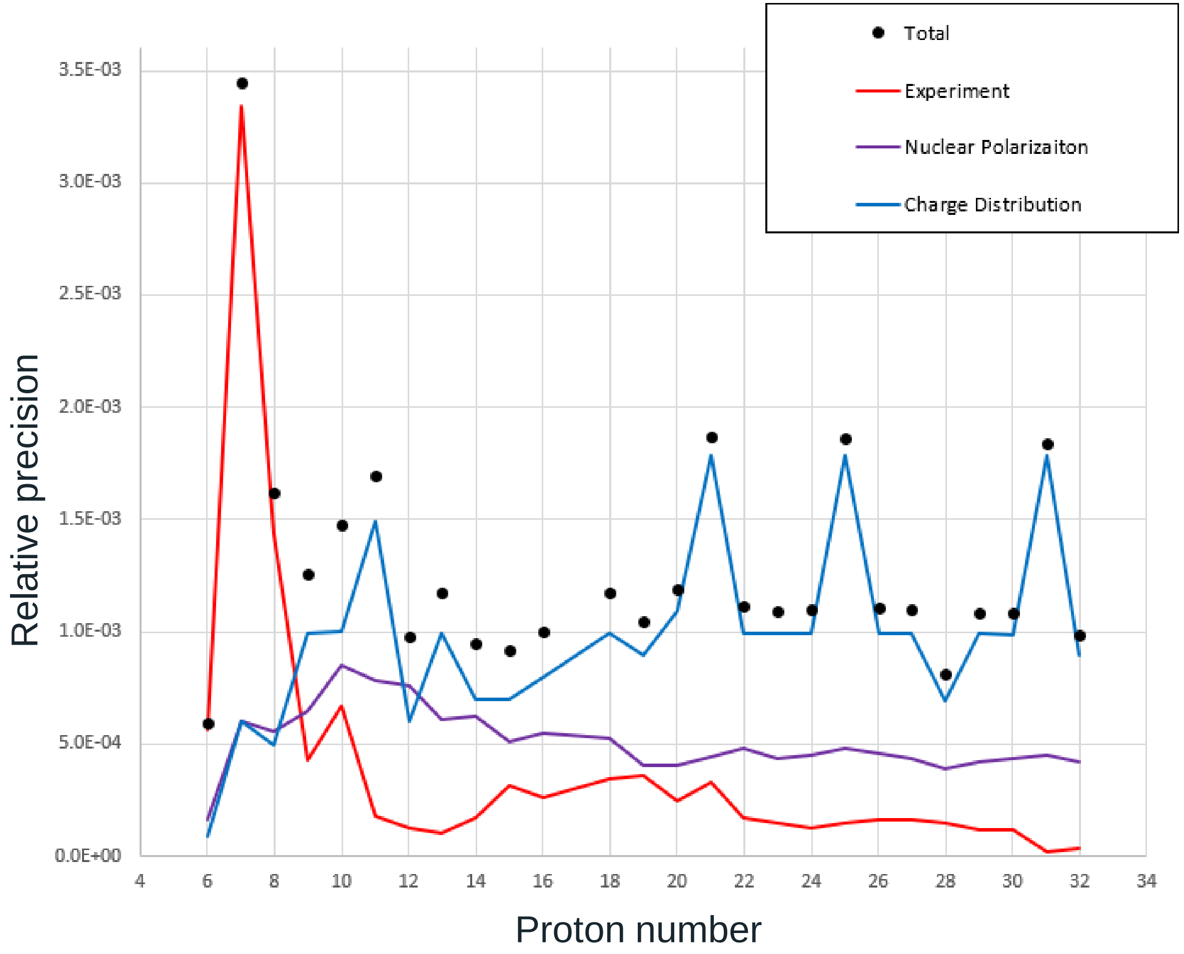

Image courtesy: Ben Ohayon

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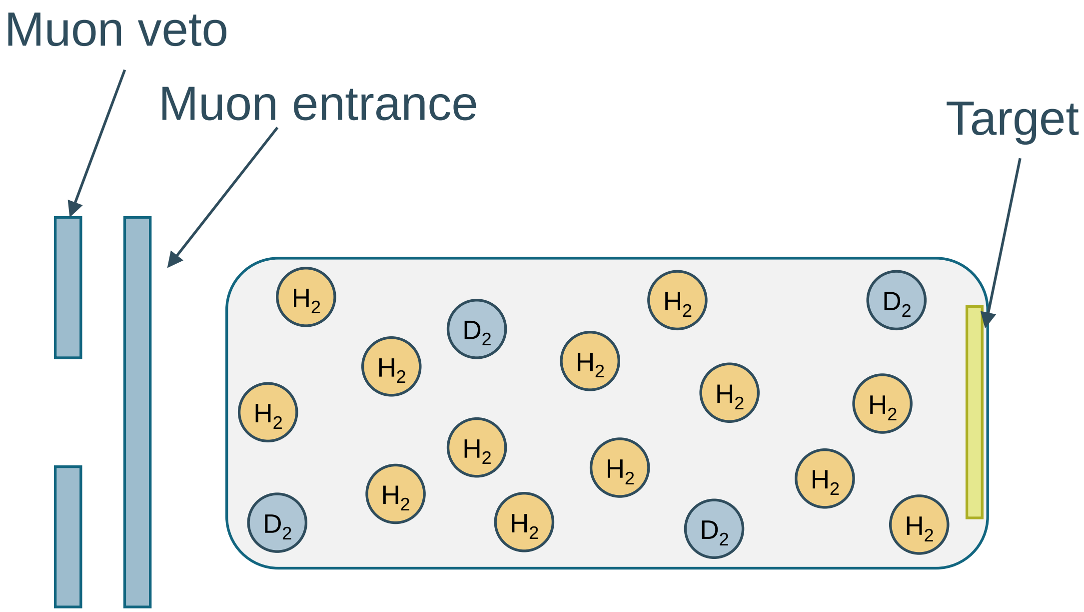
Measuring microgram materials

- Traditionally: Limited to target mass O(10-100 mg)

- Hydrogen gas cell (100 bars; 0.25% deuterium)
  - ➢Limited to O(5 µg)
  - ➢Down to 20 year half-life (radioprotection)



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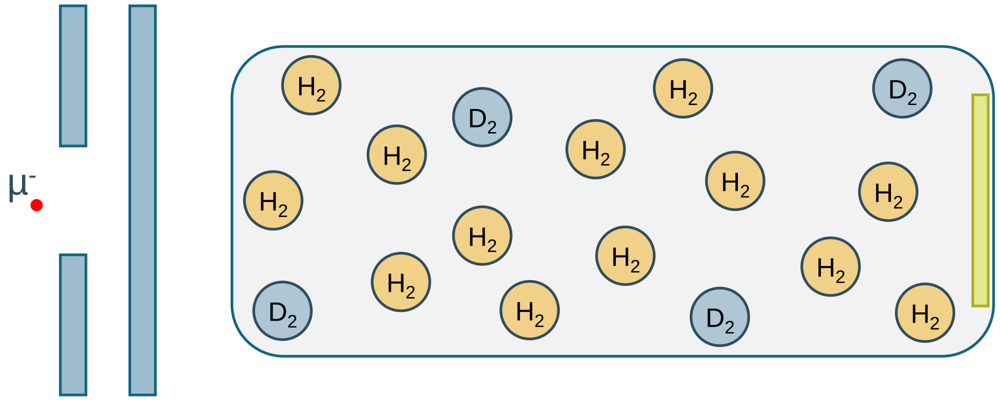

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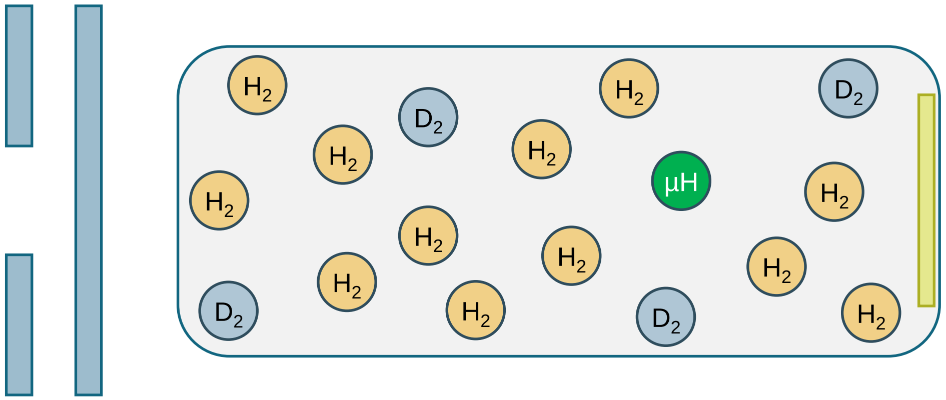

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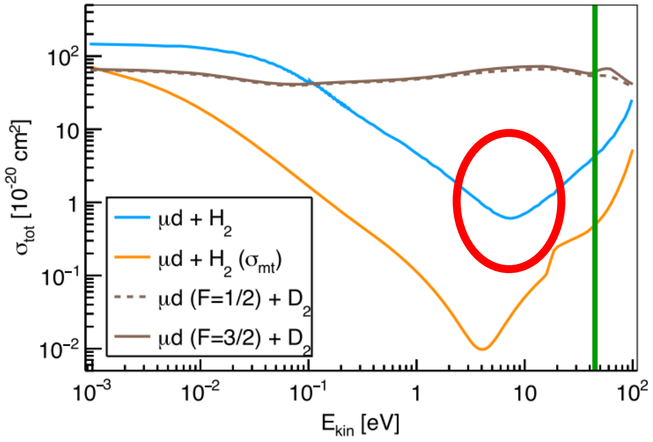

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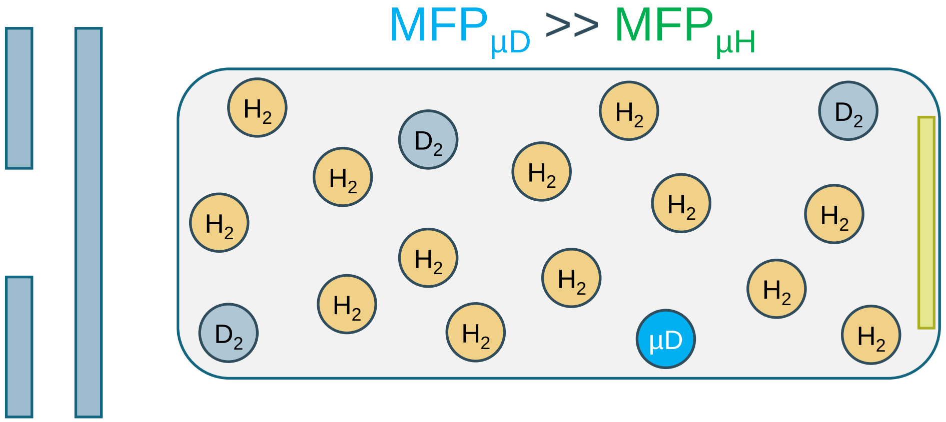

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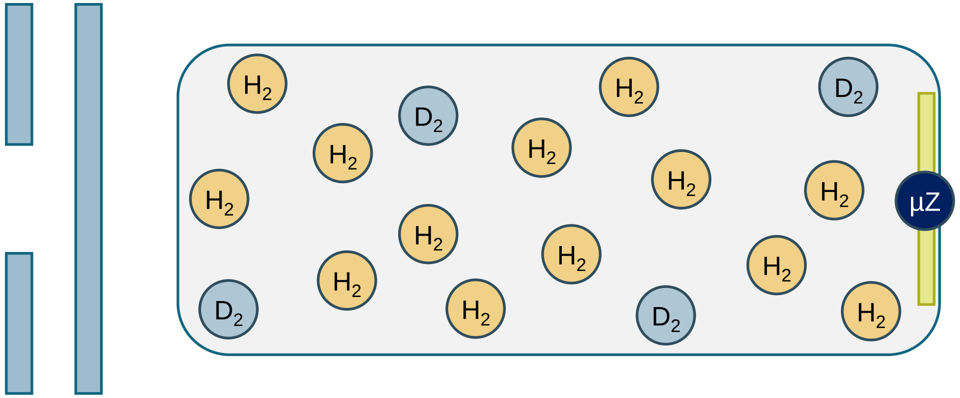

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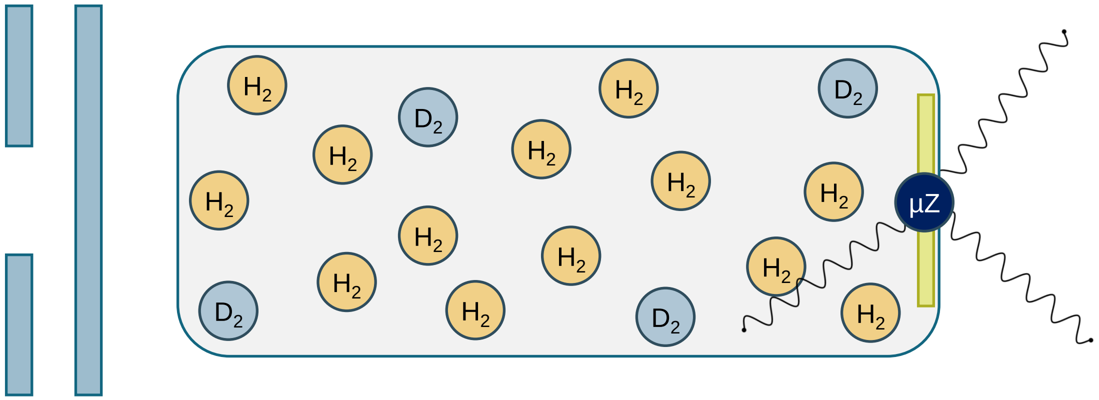

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Primary goals

- Remeasurement of $^{39,\,41}$K (macroscopic target)

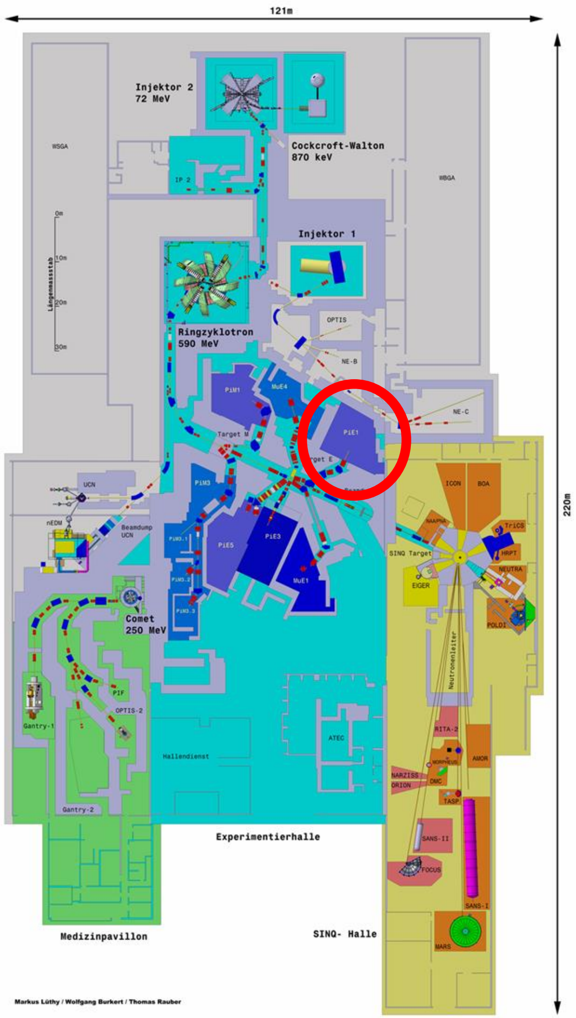
- First measurement of ^{40}K (microscopic implanted target)

NRF National Research Foundation | iThemba LABS Laboratory for Accelerator Based Sciences

- First measurement of isotopically pure $^{35,\,37}$Cl (macroscopic target)

ILL NEUTRONS FOR SCIENCE®   OAK RIDGE National Laboratory

PSI – High intensity proton accelerator facility (HIPA)



Setup

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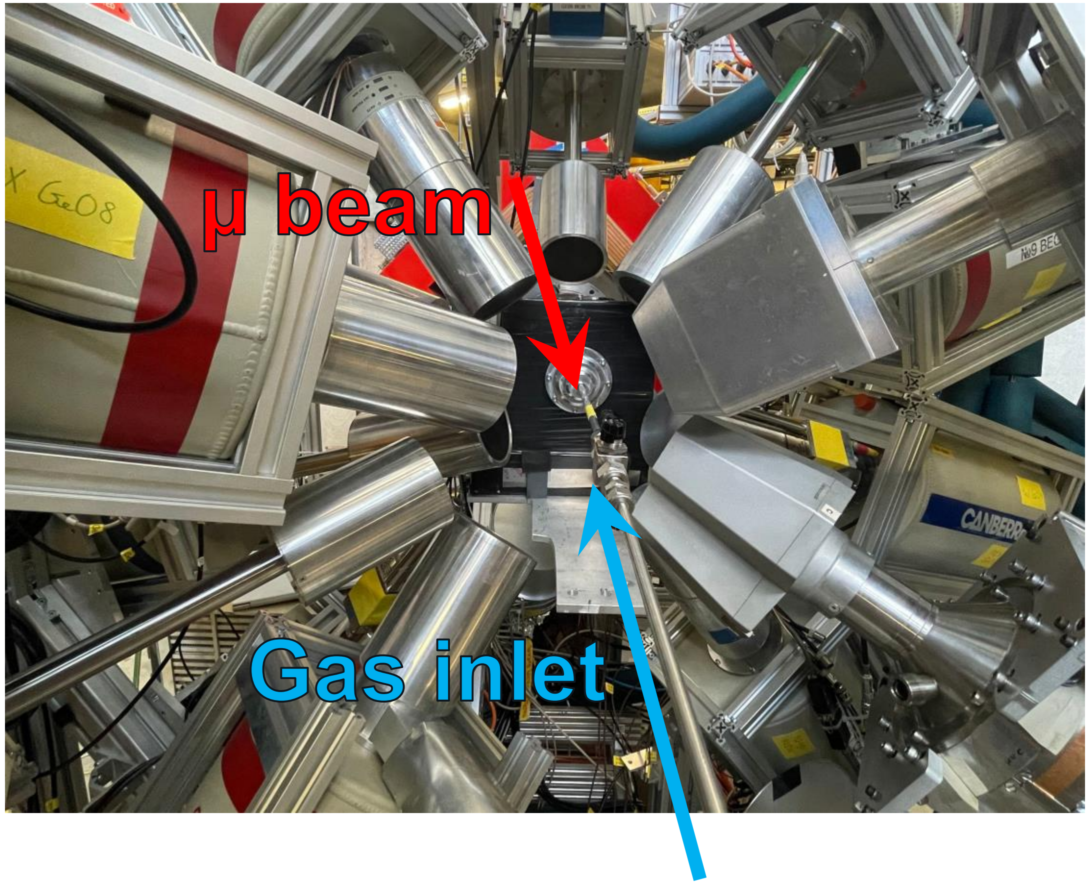
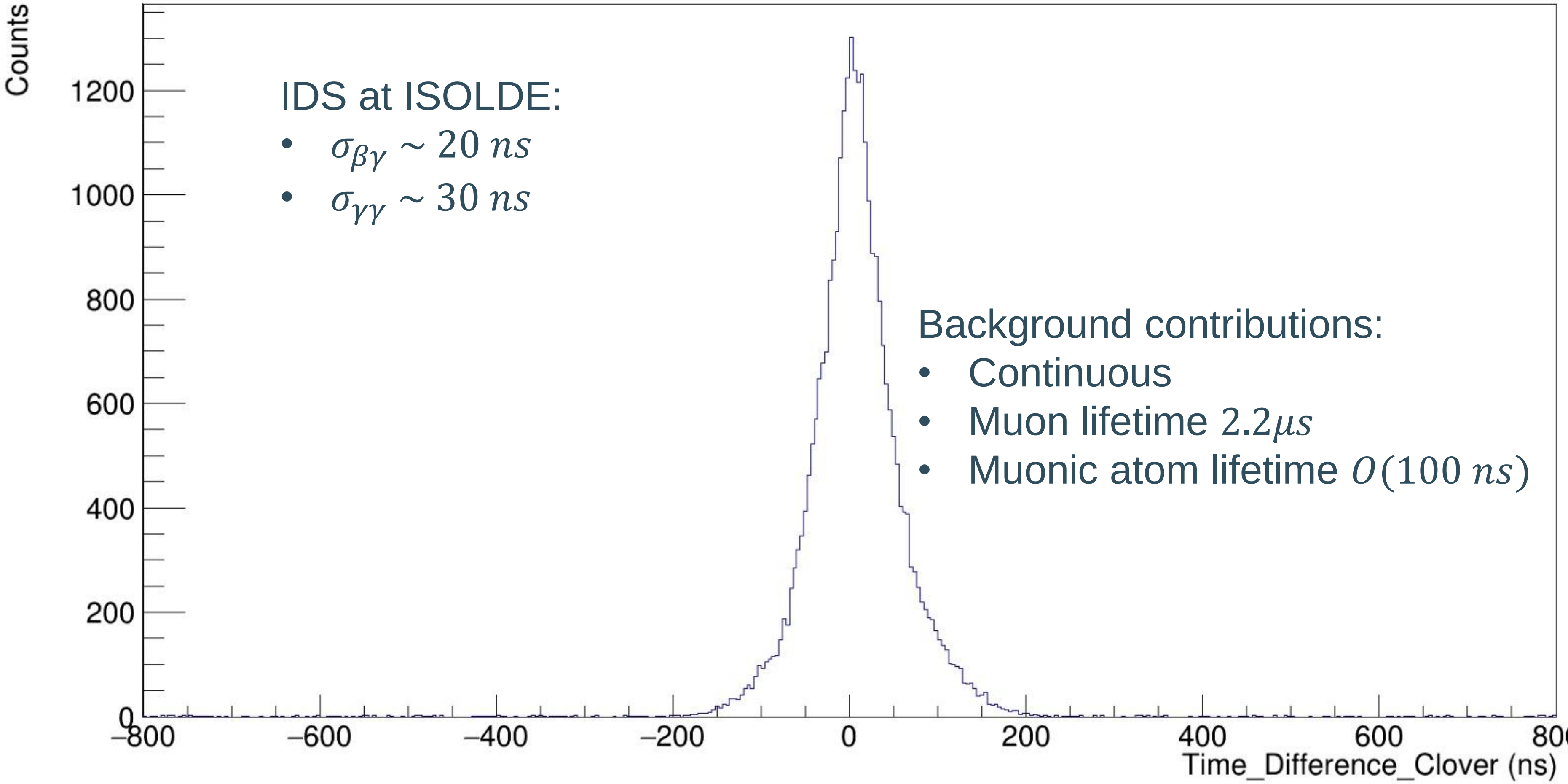


Data filtering: Timing optimization



IDS at ISOLDE:
- $\sigma_{\beta\gamma} \sim 20\ ns$
- $\sigma_{\gamma\gamma} \sim 30\ ns$

Background contributions:
- Continuous
- Muon lifetime $2.2\mu s$
- Muonic atom lifetime $O(100\ ns)$

Image and IDS numbers from R. Lica and Z. Yue

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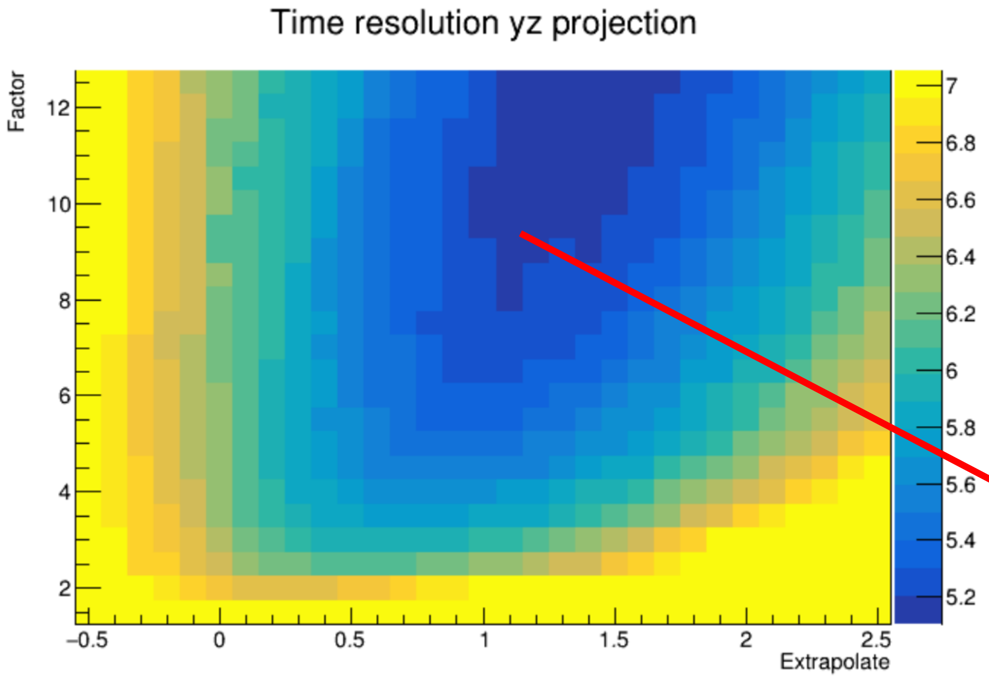
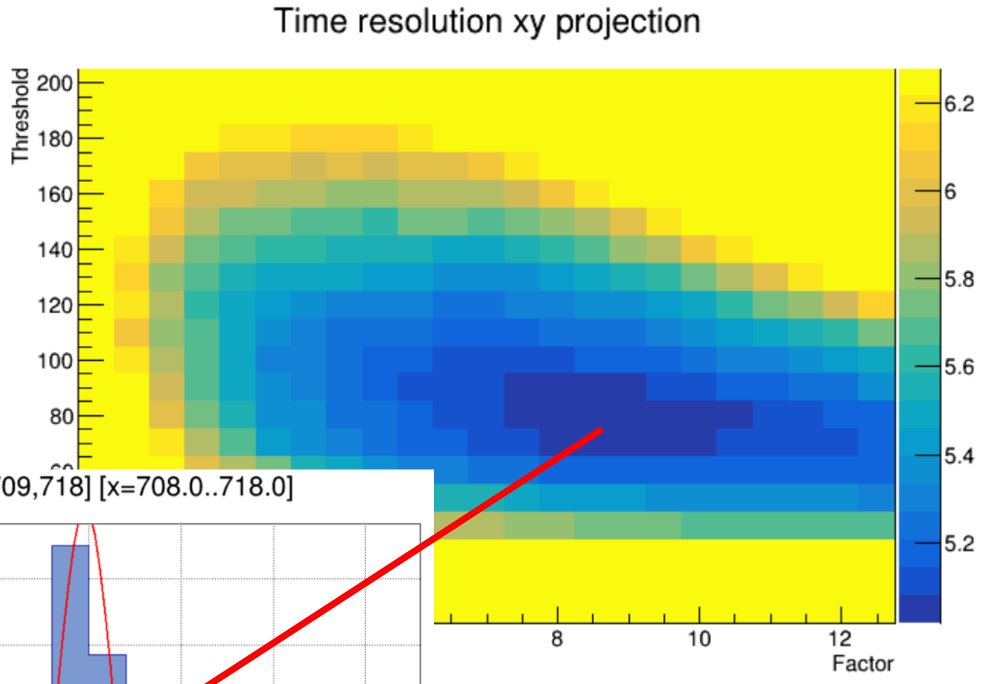
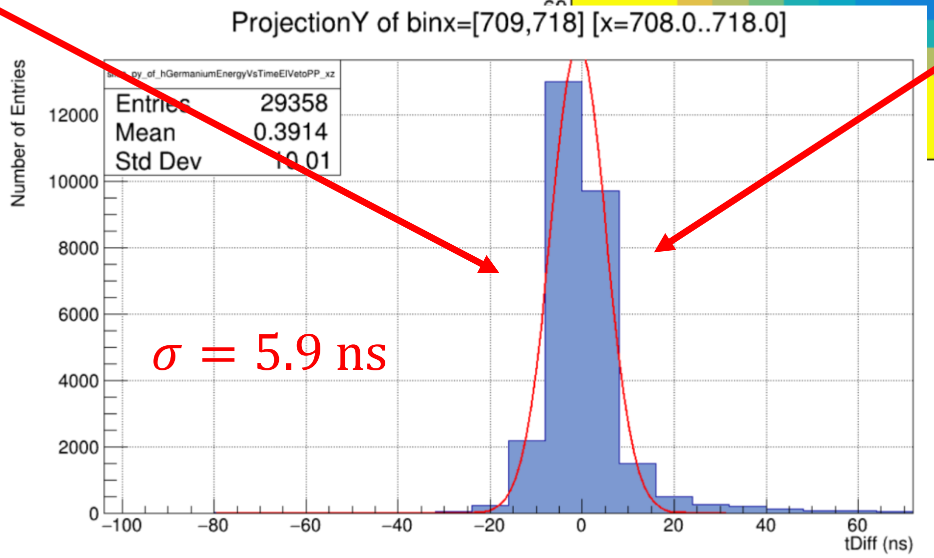


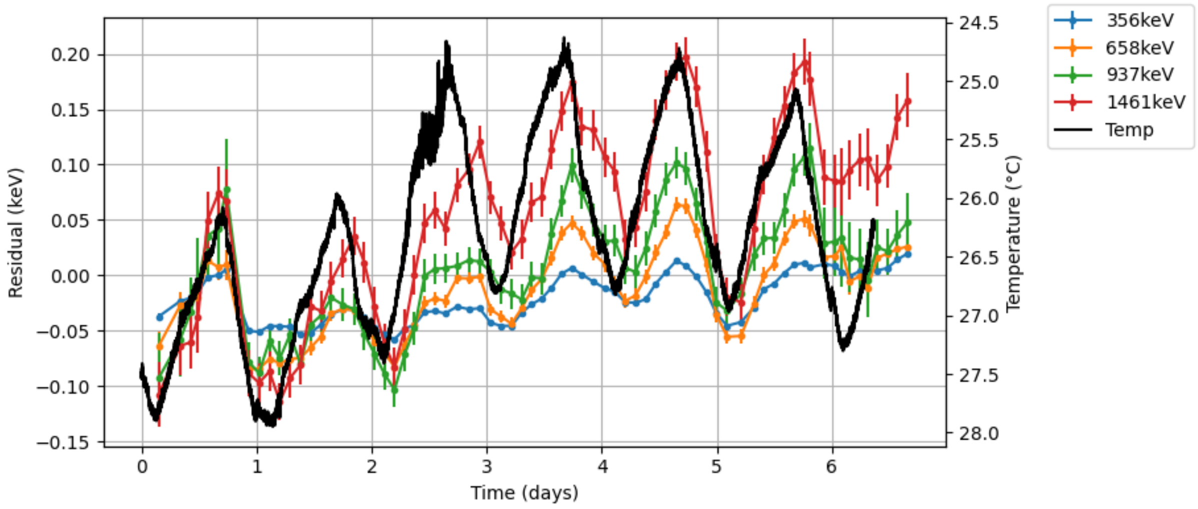




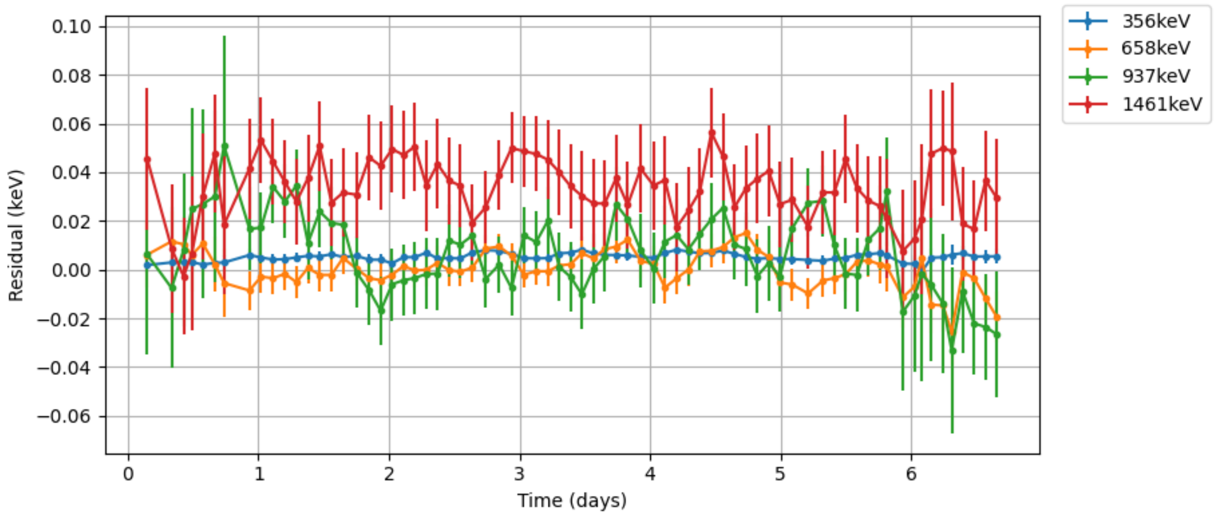
Best detectors: $\sigma = 3.5\ ns$
→ 5 times better timing

Data filtering: Gain drift correction

- Before correcting



- After correcting



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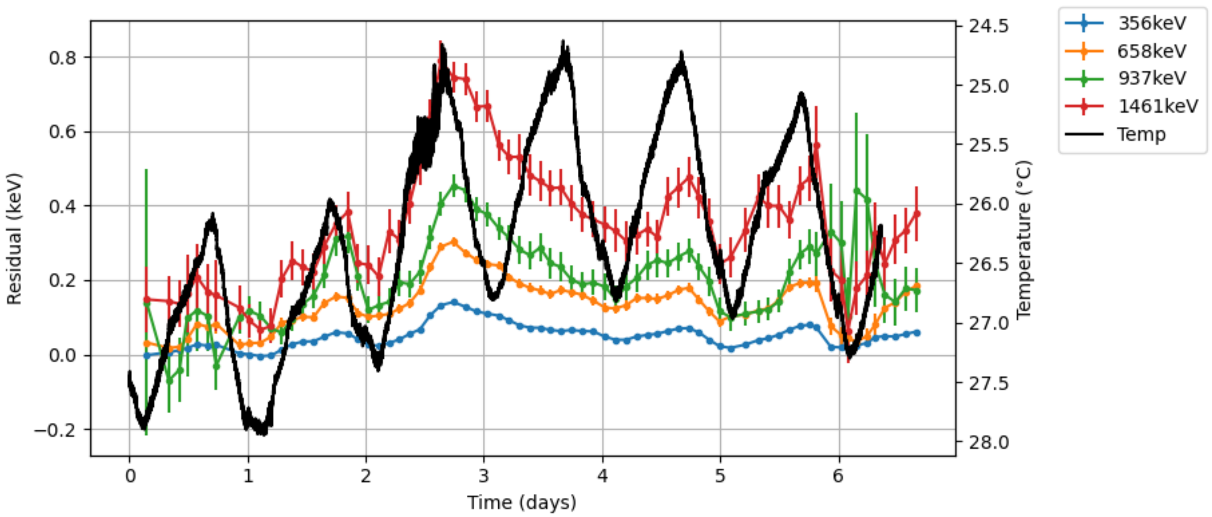

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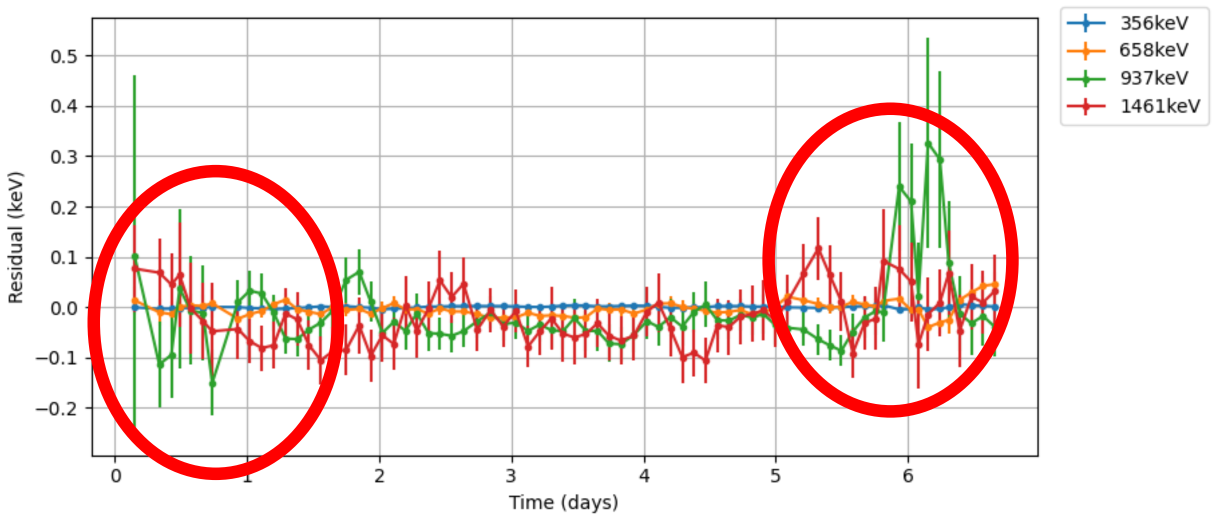
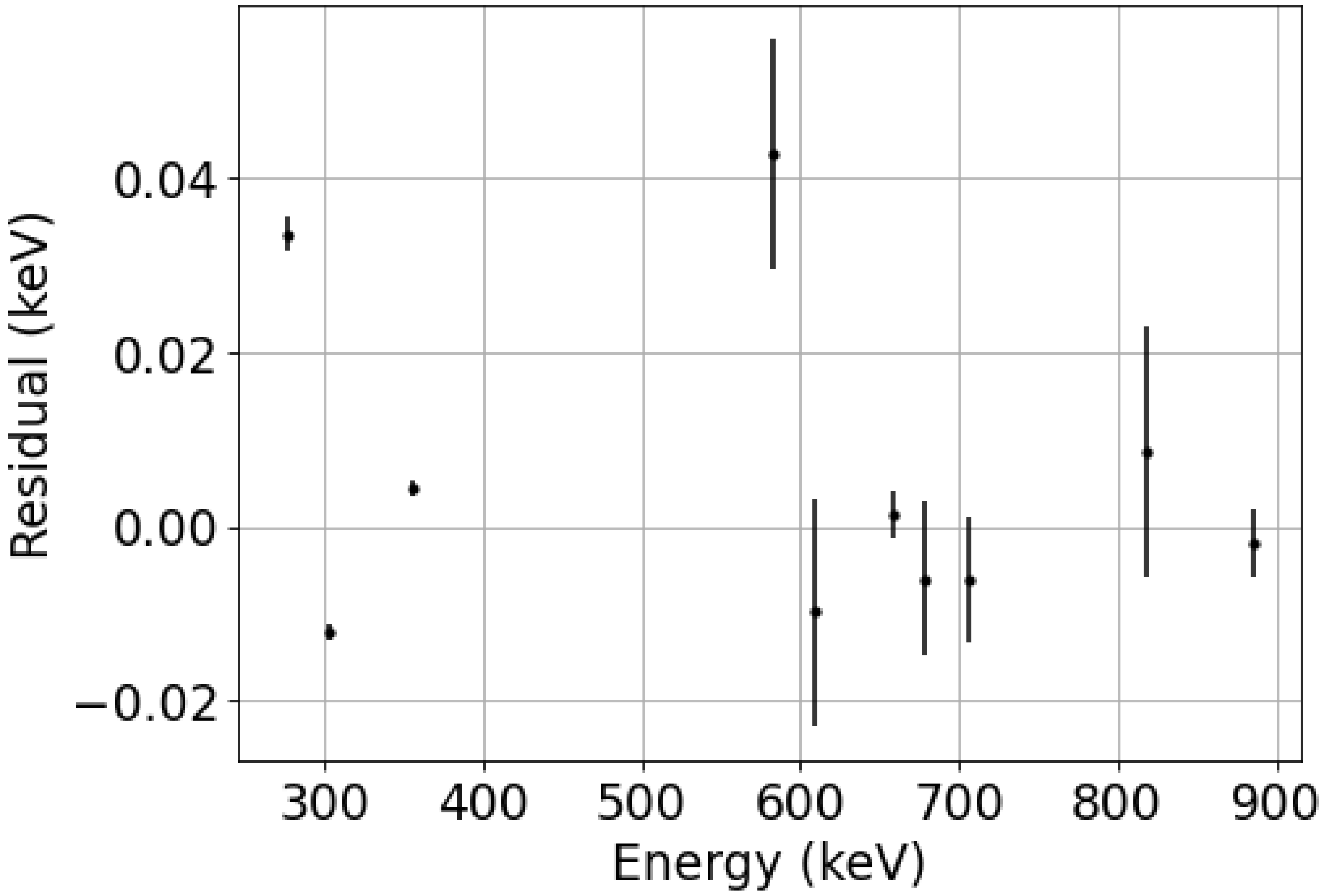
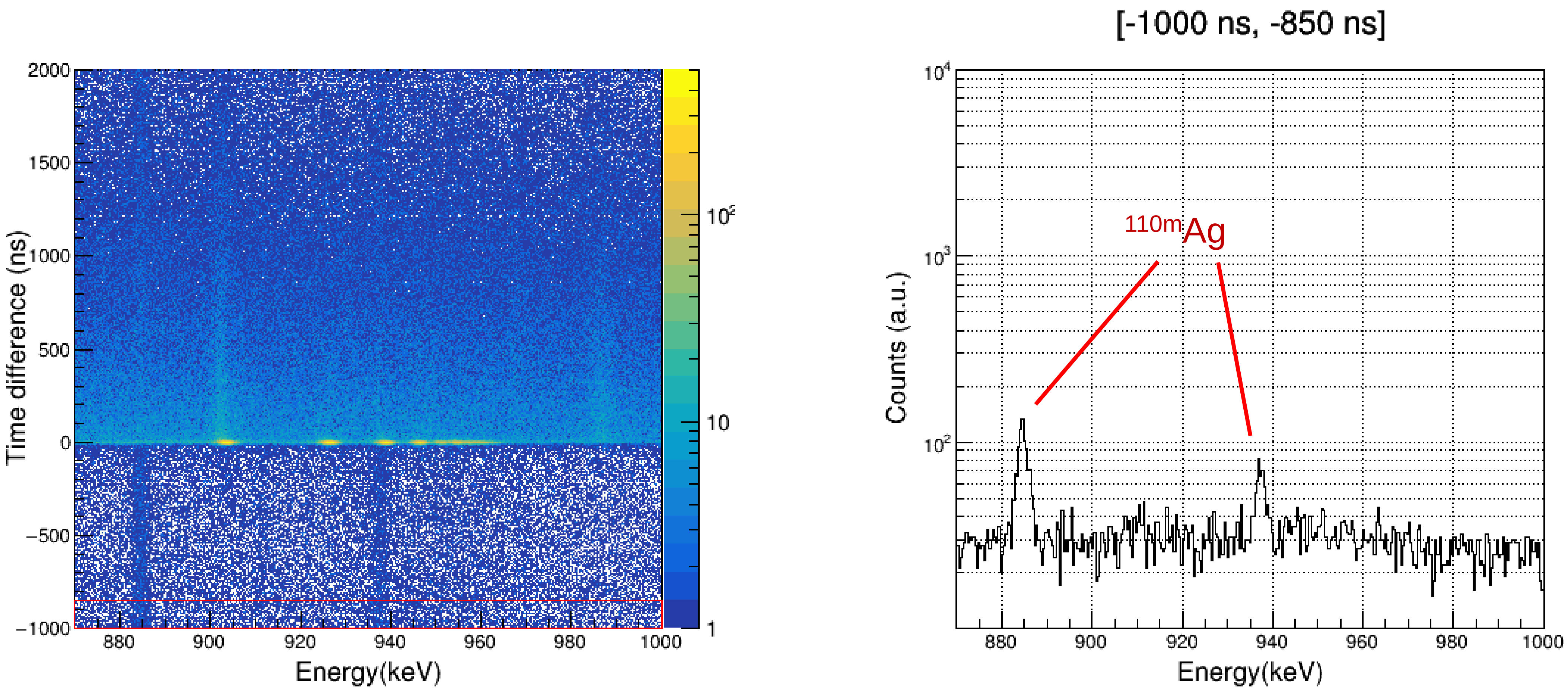


Energy calibration



- Long statistics
- One of the most linear detectors
- Linear background
  (possible reason for deviations)
- Still trying to improve

Interpreting energy Vs time plots



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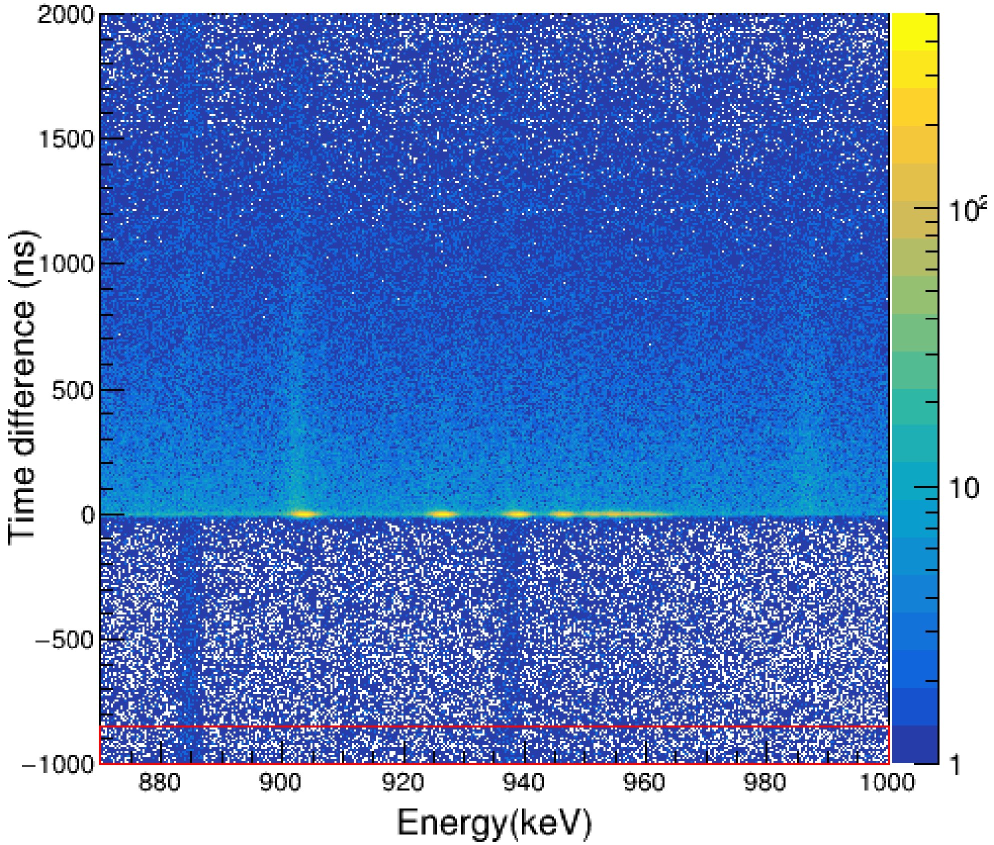

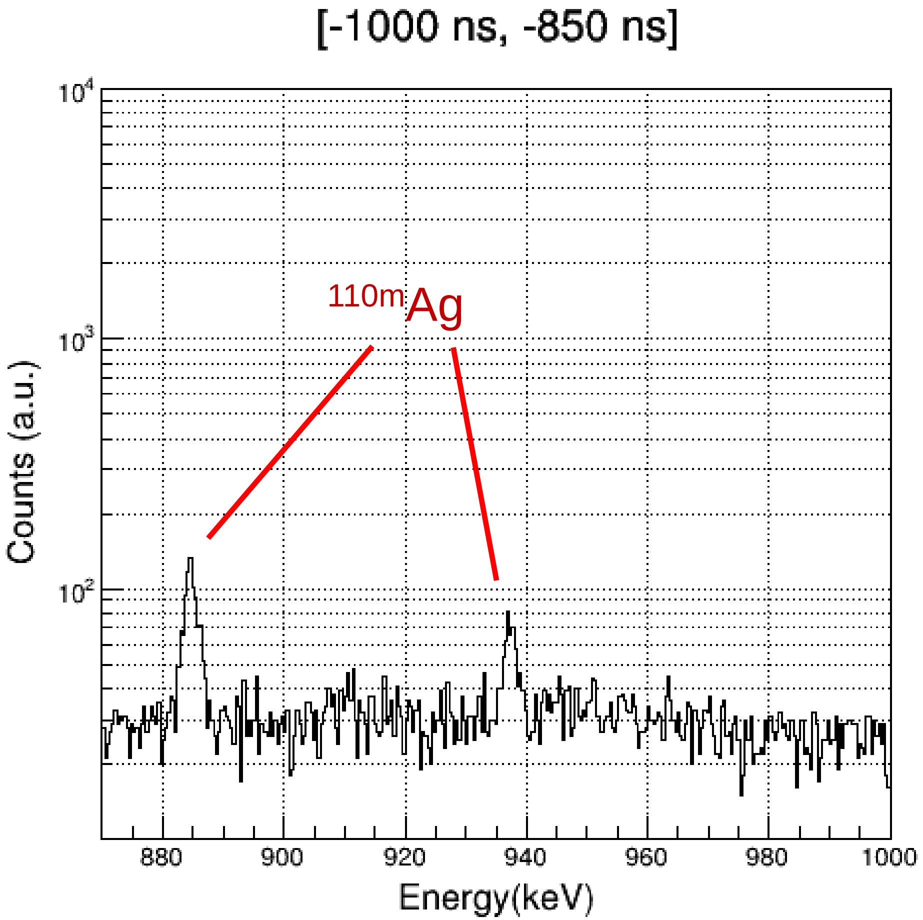

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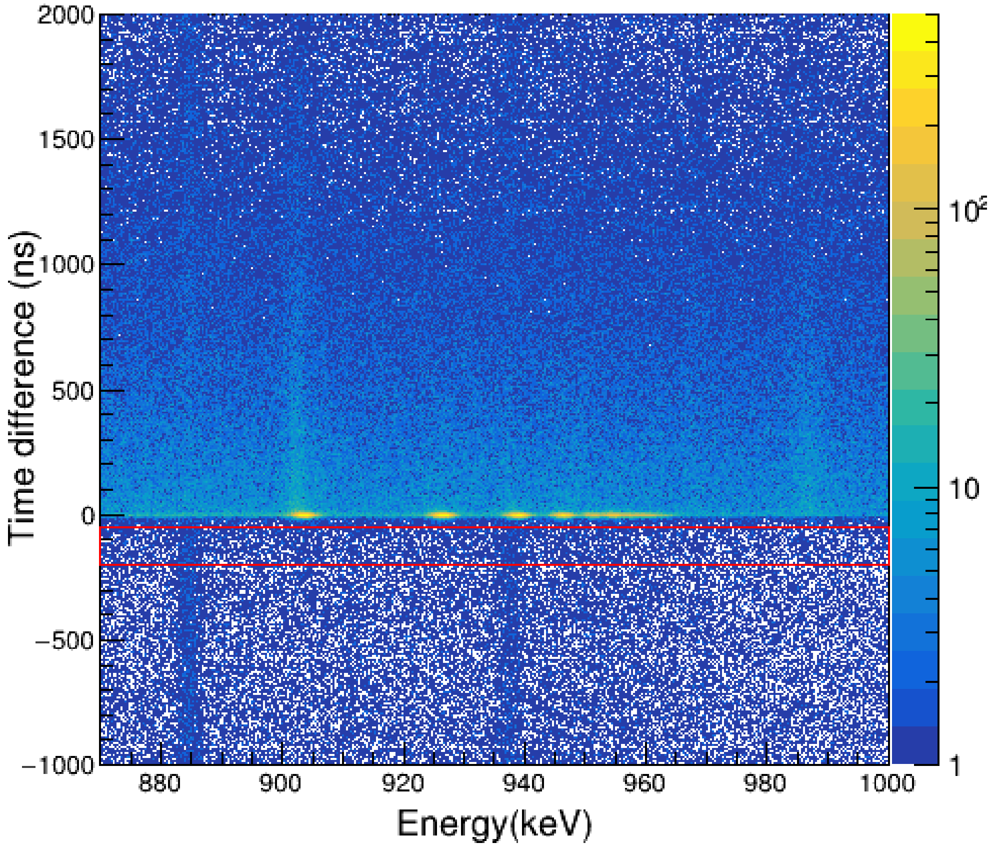
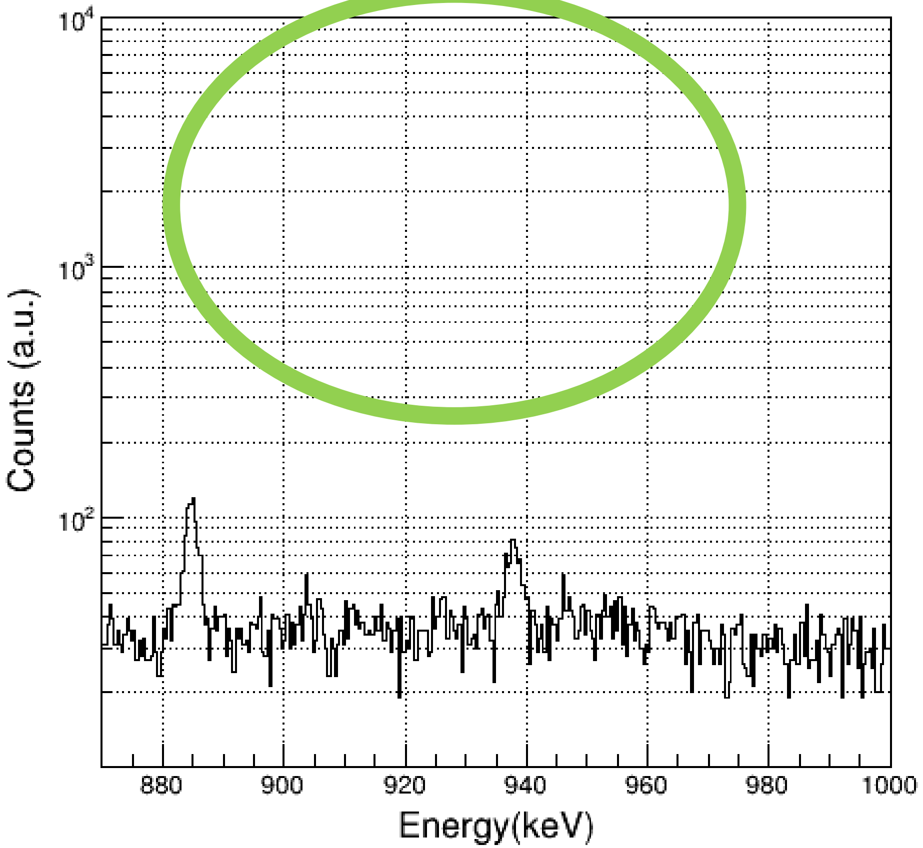


[-200 ns, -50 ns] ^{39}K np-1s



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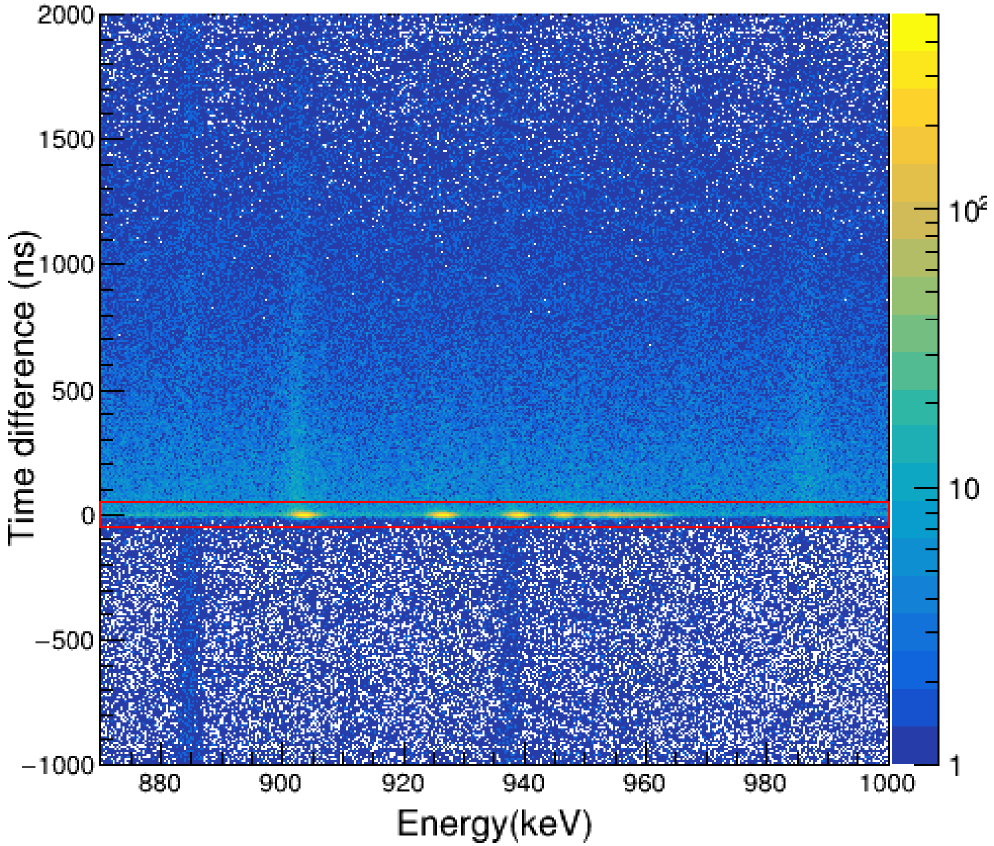
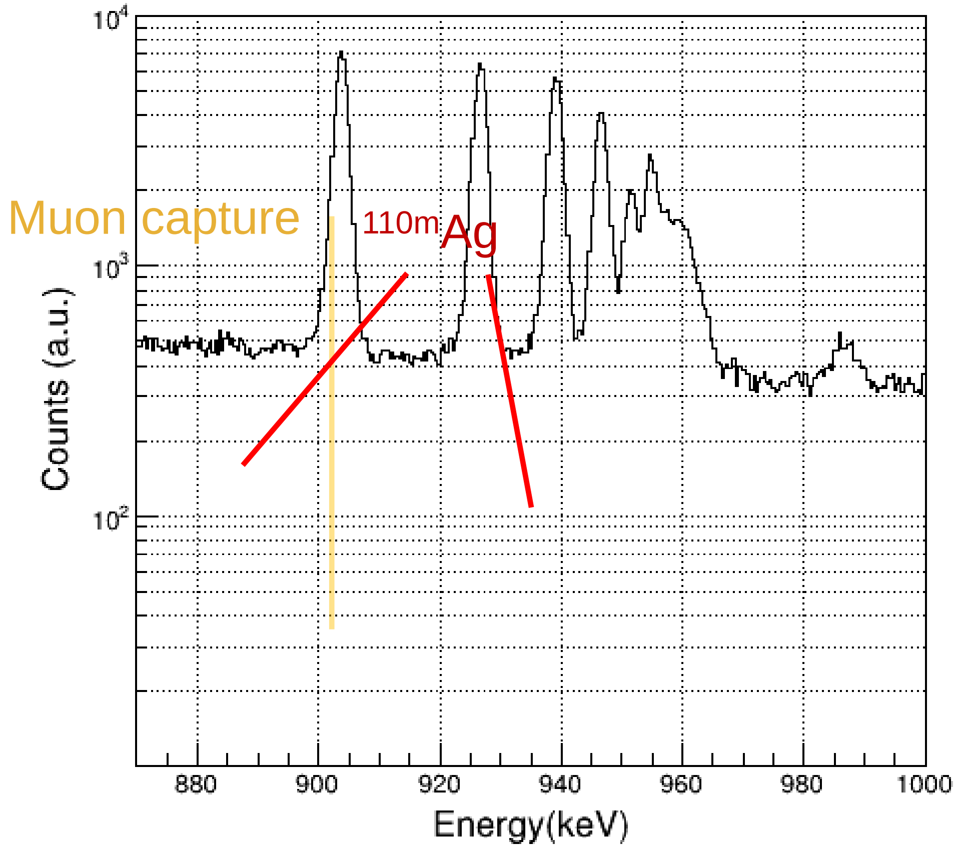
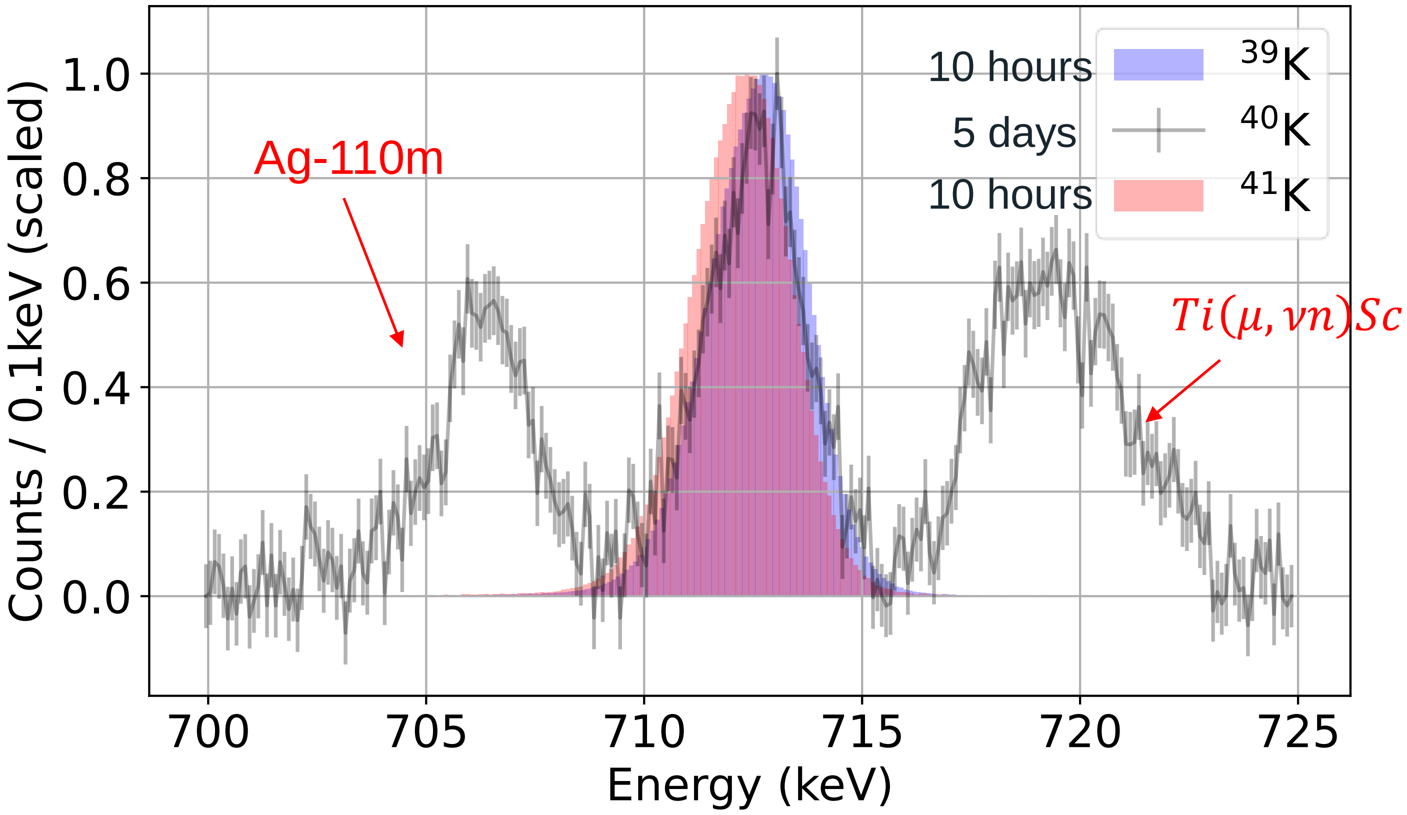
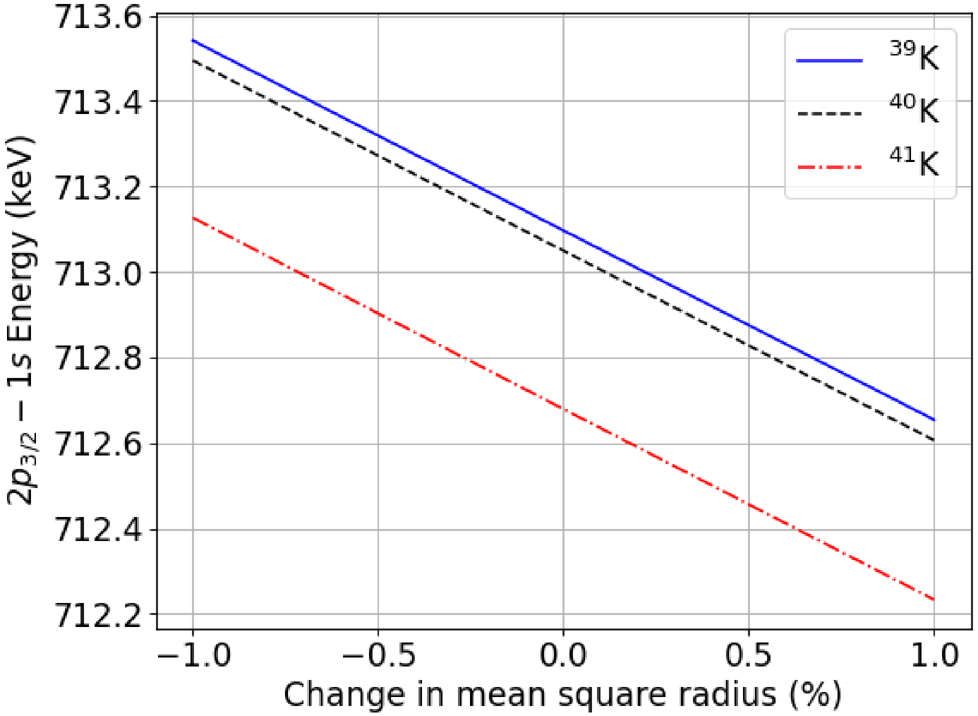


[-50 ns, 50 ns]



Potassium muonic isotope shift (preliminary)





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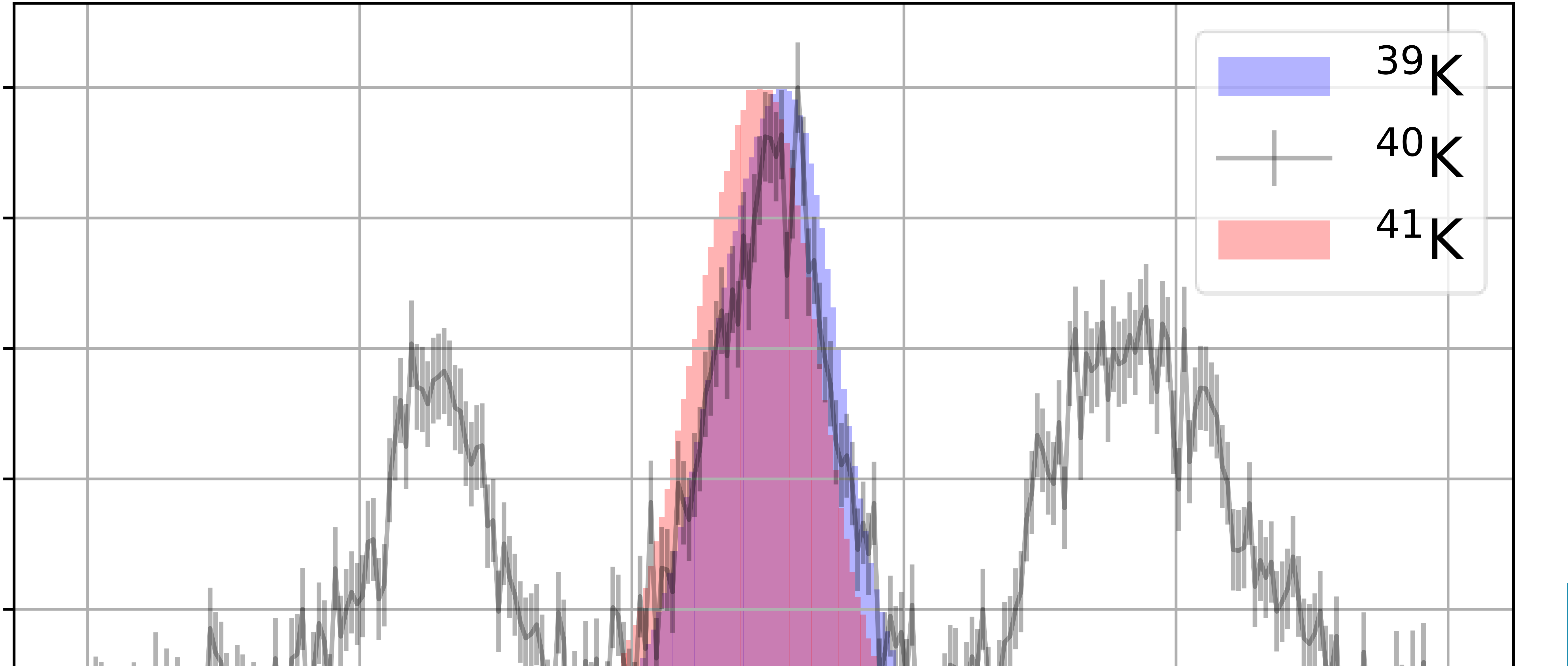


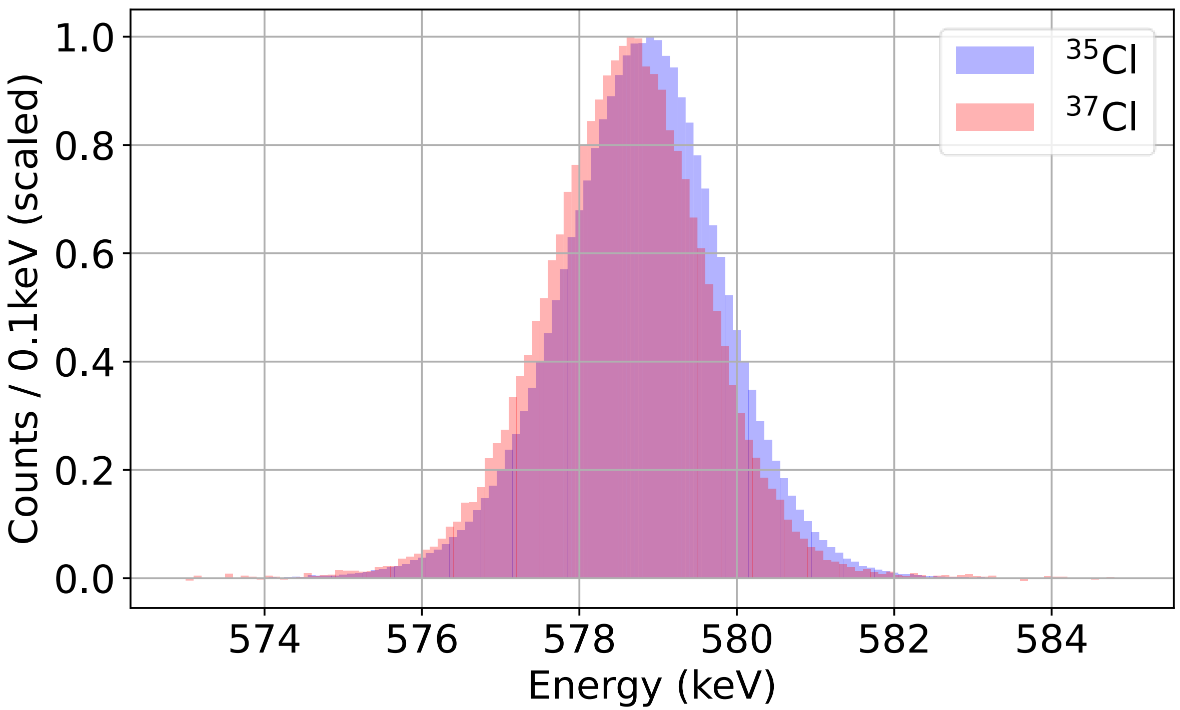
Chlorine measurement (preliminary)

- Muonic 2p-1s energy:
  ^{nat}Cl: 578.56(30) keV

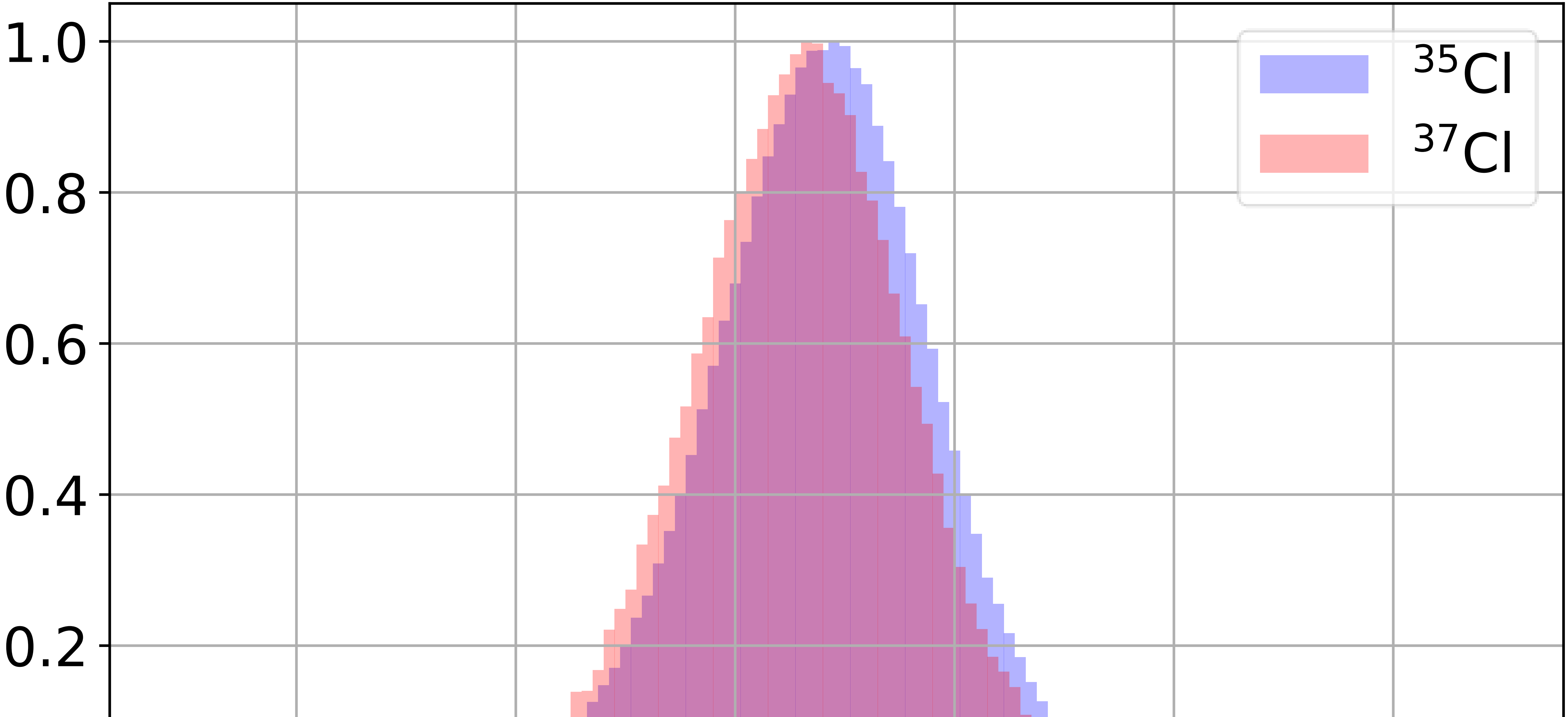
- Expected improvement on 2p-1s transition energy:
  300 eV → Most likely < 30 eV

- Expected improvement on radii:
  0.45% → ~0.10-0.15 % (including systematics)

Literature $\delta < r^2 >^{35,37} = 0.03(16)$



Chlorine measurement (preliminary)

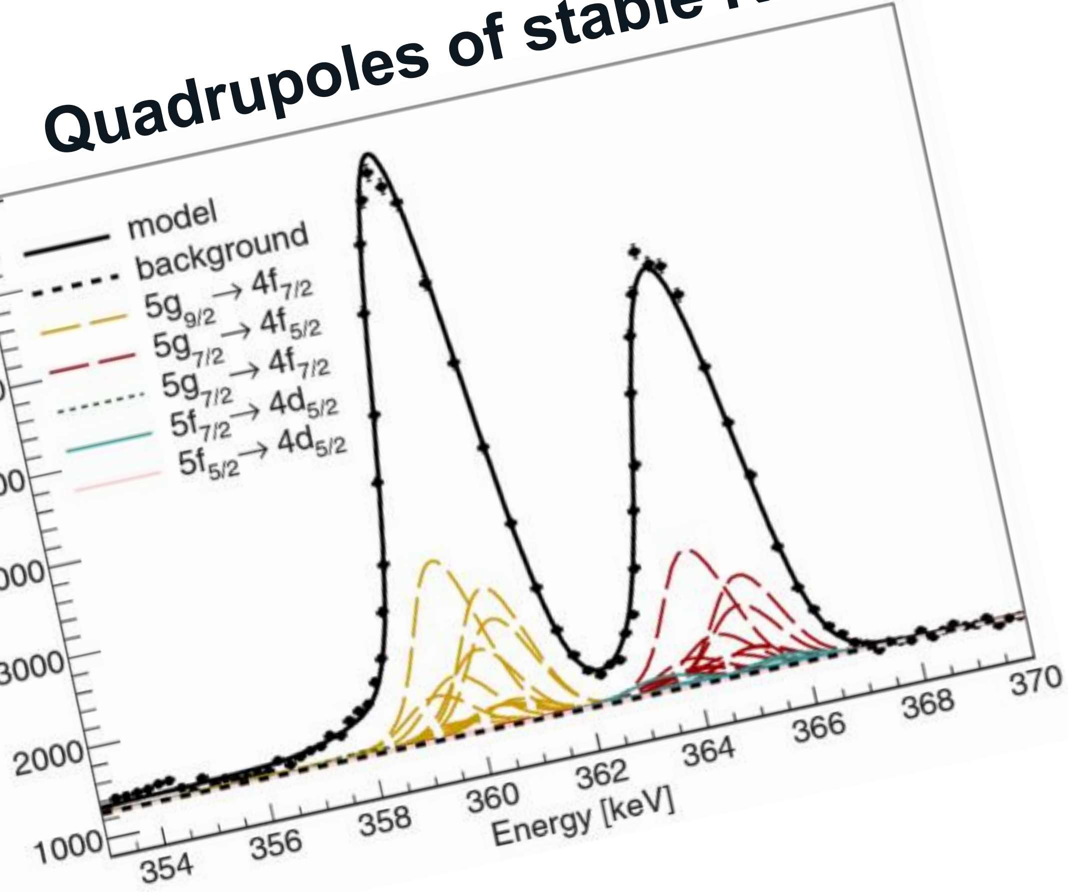


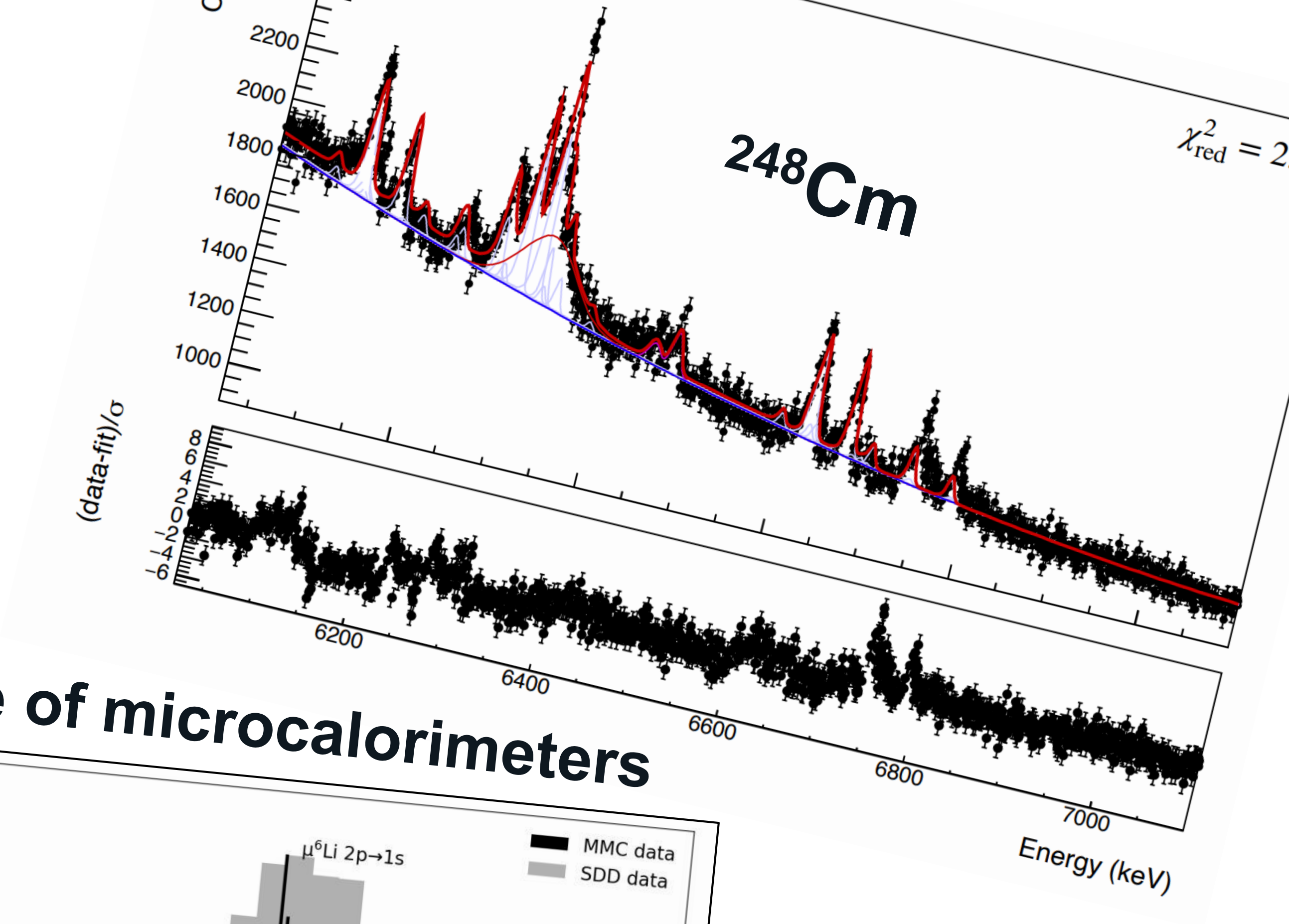
Contents

- Why do we need absolute charge radii?
- Measuring charge radii with muons
- Microgram targets
- Experimental campaign on potassium and chlorine
- Different physics cases

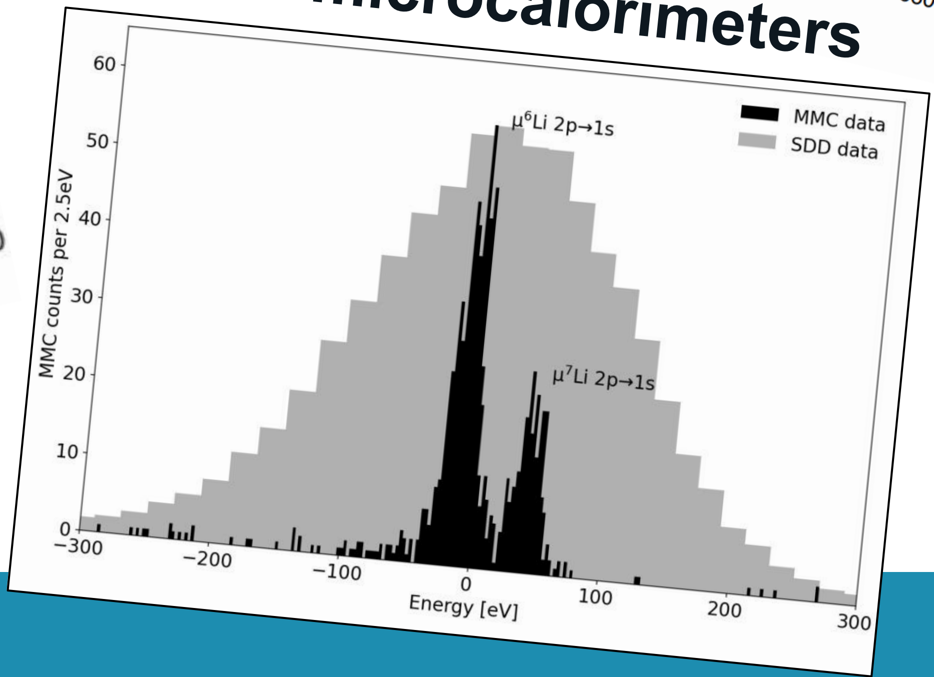
Other stuff we've done

Quadrupoles of stable Re



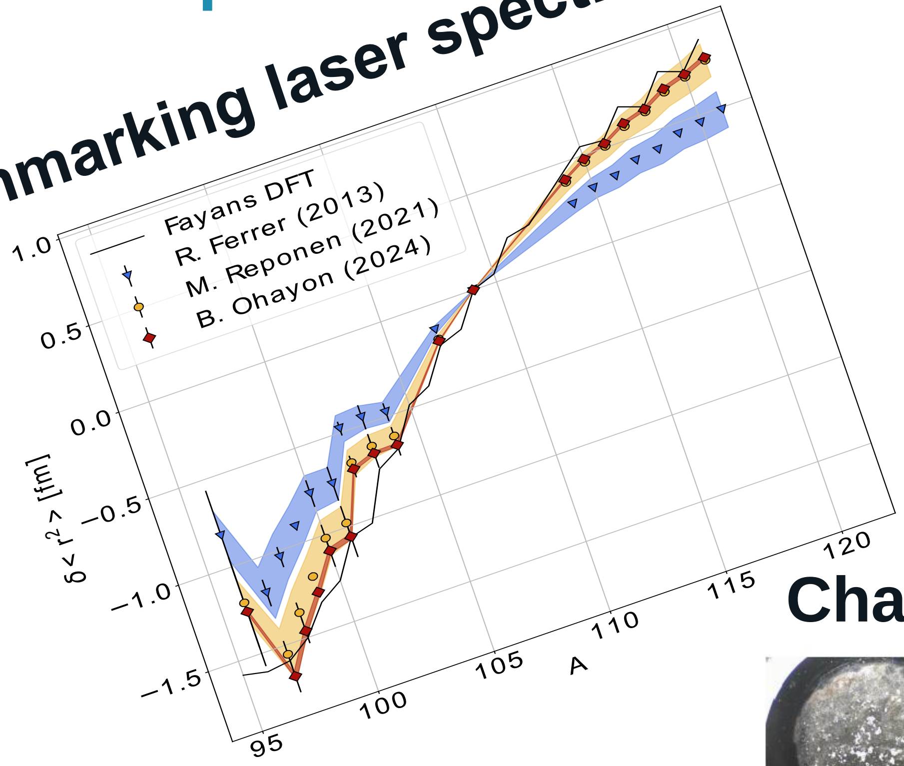


Use of microcalorimeters

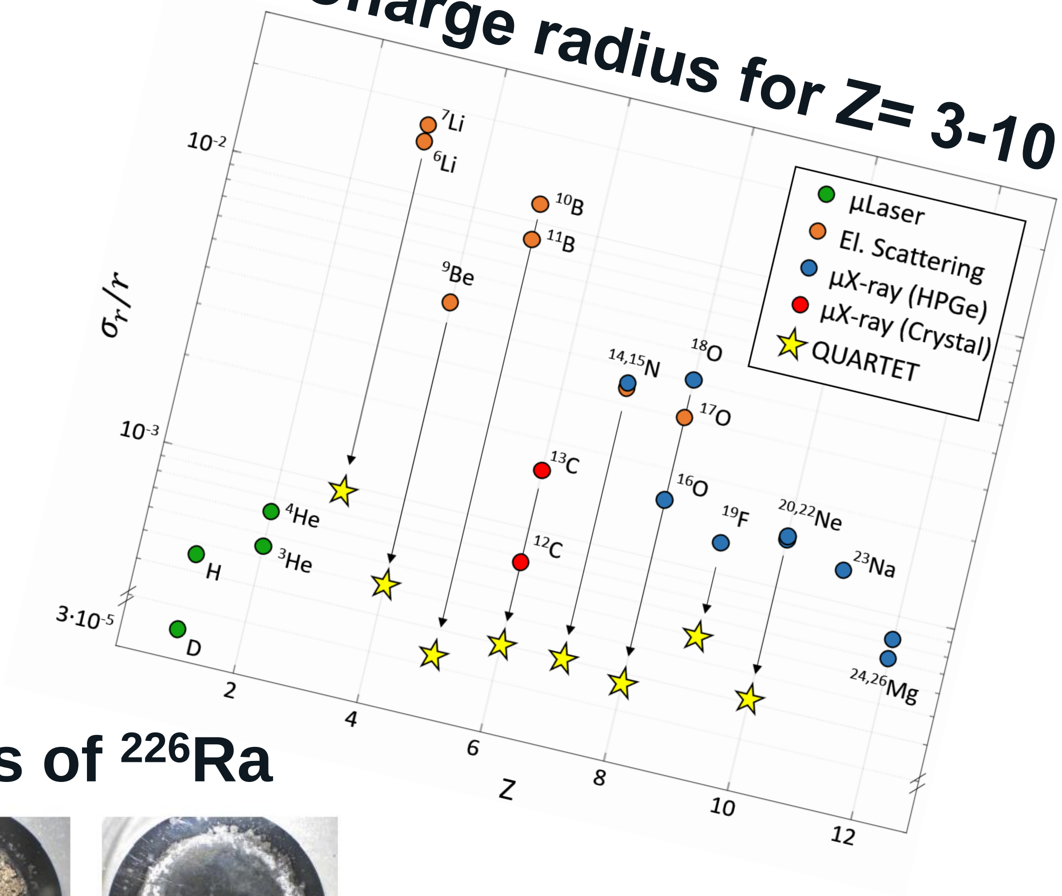


Future plans

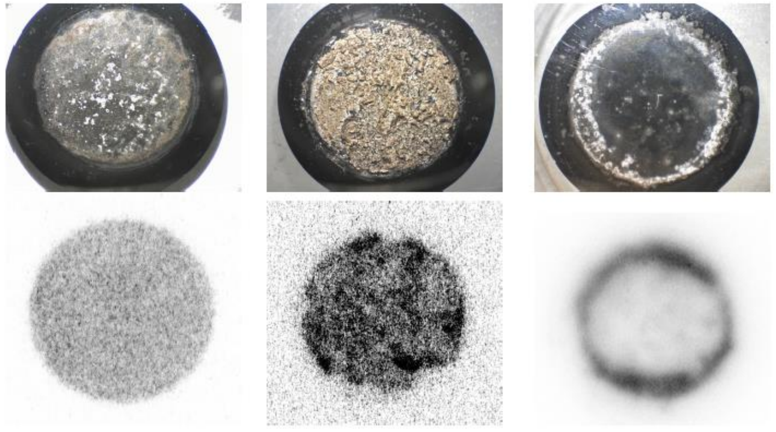
Benchmarking laser spectroscopy



Charge radius for Z= 3-10



Charge radius of ^{226}Ra



(a) Ra-MP1 (b) Ra-MP2 (c) Ra-MP3

Conclusion

- Muonic atoms can be used as precise probes for the nucleus
  - ➢ Giving input for laser spectroscopy
  - ➢ Inputs for other experiments
  - ➢ Radii comparison across elements

- Measured transition energies of Cl and K
  - ➢ Theory calculations have been initiated
  - ➢ In-depth analysis ongoing (ideal precision < 20 eV)

Thank you for your attention!

Thanks to the muX collaboration and the QUARTET collaboration:

Backup slides

Field-theoretical approach

Slide courtesy: Igor Valuev

$$\hat{J}^{\mu}_{\mathrm{N,\,total}}(x) = J^{\mu}_{\mathrm{N,\,stat}}(\mathbf{x}) + \hat{J}^{\mu}_{\mathrm{N,\,fluc}}(x)$$

$$\hat{A}^{\mu}_{\mathrm{total}}(x) = \mathcal{A}^{\mu}_{\mathrm{stat}}(\mathbf{x}) + \hat{A}^{\mu}_{\mathrm{fluc}}(x) + \hat{A}^{\mu}_{\mathrm{free}}(x)$$

$$iD_{\mu\nu}(x - x')$$

$$= \langle 0 | T[\hat{A}^{\mathrm{free}}_{\mu}(x)\hat{A}^{\mathrm{free}}_{\nu}(x')] | 0 \rangle$$

dressed muon propagator

$$:= \hat{A}^{\mu}_{\mathrm{rad}}(x)$$

modified photon propagator:

$$i\mathcal{D}_{\mu\nu}(x, x') = \langle 0 | T[\hat{A}^{\mathrm{rad}}_{\mu}(x)\hat{A}^{\mathrm{rad}}_{\nu}(x')] | 0 \rangle$$

$$= iD_{\mu\nu}(x - x') + \langle 0 | T[\hat{A}^{\mathrm{fluc}}_{\mu}(x)\hat{A}^{\mathrm{fluc}}_{\nu}(x')] | 0 \rangle$$

Modified photon propagator

Slide courtesy: Igor Valuev

$$\mathcal{D}_{\mu\nu}(x,x') = D_{\mu\nu}(x-x') + D^{\mathrm{NP}}_{\mu\nu}(x,x')$$

$$D^{\mathrm{NP}}_{\mu\nu}(x,x') = \int d^4x_1\, d^4x_2\, D_{\mu\xi}(x-x_1) \Big[\overbrace{\Pi^{\xi\zeta}_{\mathrm{N}}(x_1,x_2)}^{\text{NP tensor}} + \overbrace{S^{\xi\zeta}_{\mathrm{N}}(x_1,x_2)}^{\text{“seagull” term}} \Big] D_{\zeta\nu}(x_2-x')$$

$\mathcal{D}_{\mu\nu}(x,x') =$ 

$$i\Pi^{\xi\zeta}_{\mathrm{N}}(x_1,x_2) = \langle 0 | T[\hat{J}^{\xi}_{\mathrm{N,\,fluc}}(x_1)\hat{J}^{\zeta}_{\mathrm{N,\,fluc}}(x_2)] | 0 \rangle$$

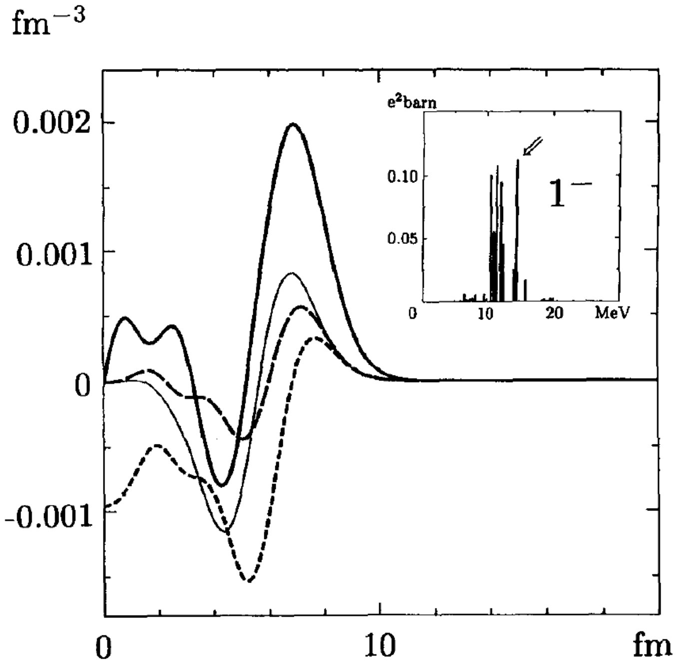
What is needed from the nuclear side

Slide courtesy: Igor Valuev

$$\mathrm{NP} \to \sum_{|\lambda\rangle} [\text{the entire nuclear spectrum}]$$

- excitation energies $\omega_\lambda = E_\lambda - E_0$
- reduced matrix elements:
  - transition (charge) densities
    $\varrho_J^\lambda(\mathrm{x}) = \langle\lambda|| \int d\Omega_{\mathbf{x}}\, Y_J(\Omega_{\mathbf{x}}) \hat{\rho}_{\mathrm{N}}(\mathbf{x}) ||0\rangle$
  - transition current densities
    $\mathcal{J}_{JL}^\lambda(\mathrm{x}) = \langle\lambda|| \int d\Omega_{\mathbf{x}}\, \mathbf{Y}_{JL}(\Omega_{\mathbf{x}}) \cdot \hat{\mathbf{J}}_{\mathrm{N}}(\mathbf{x}) ||0\rangle$

  for different excitation modes:
  $0^+,\ 1^-,\ 2^+,\ 3^-,\ (4^+,\ 5^-,\ 1^+)$
  in the laboratory frame



Y. Tanaka and Y. Horikawa, Nucl. Phys. A580, 291 (1994).

*simplifications are possible in terms of transition probabilities $B(EL)$

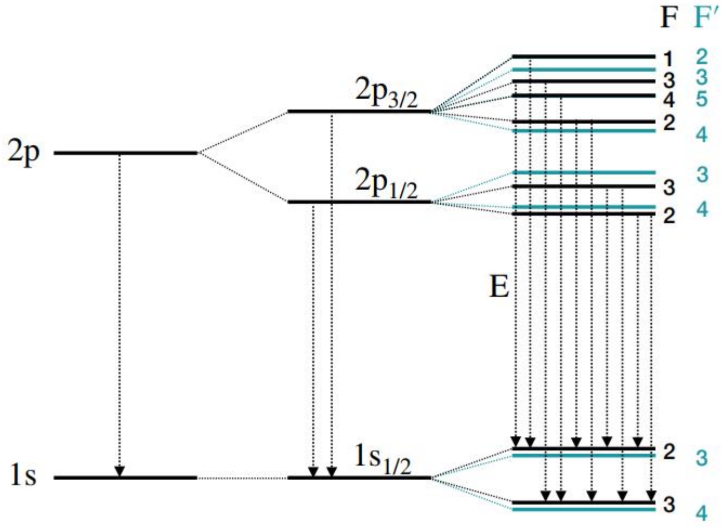
KU LEUVEN

Dynamic hyperfine splitting

Slide courtesy:
Stella Vogiatzi

Fine splitting (FS): $\vec{J} = \vec{l} + \vec{s}$

Static hyperfine splitting (HFS): $\vec{F} = \vec{I} + \vec{J}$



$I_{ground} = \frac{5}{2}^{+}$ $I_{excited} = \frac{7}{2}^{+}$

- Energy shift of hyperfine states due to the electric quadrupole (E2) and magnetic dipole (M1) interaction

Dynamic hyperfine splitting

- The hyperfine levels from ground and excited nuclear states are mixed due to the high energy of muonic transitions
- HFS also observed in even-even nuclei with zero spin in the ground state

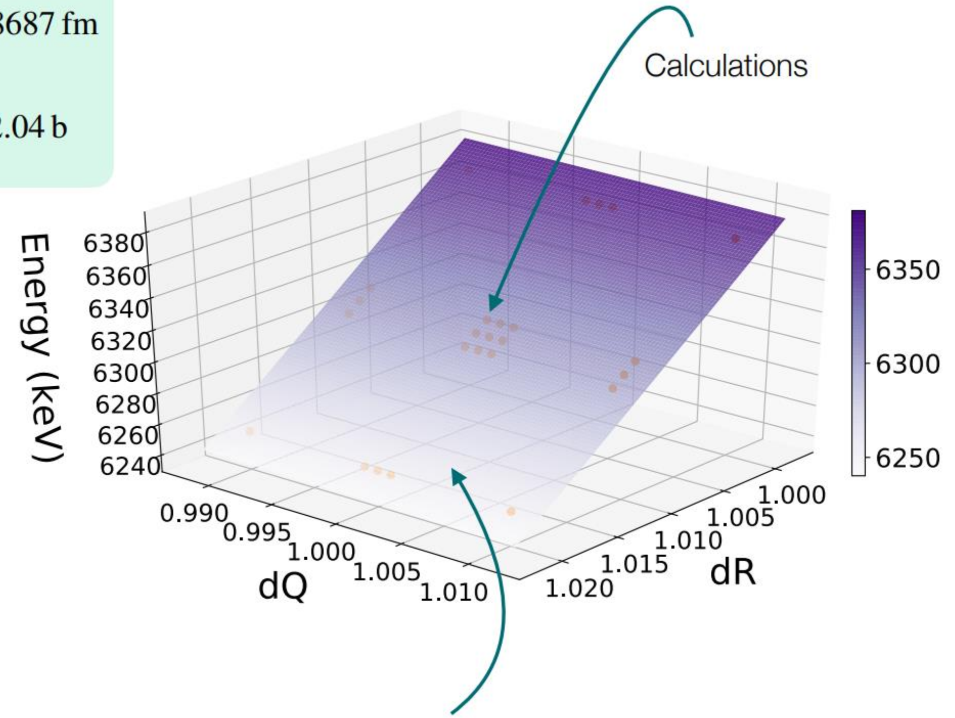
Including the quadrupole moment (^{248}Cm)

Slide courtesy:
Stella Vogiatzi

Theoretical calculations including estimates of the nuclear polarisation corrections are performed by N. Oreshkina & I. Valuev, MPIK, Heidelberg

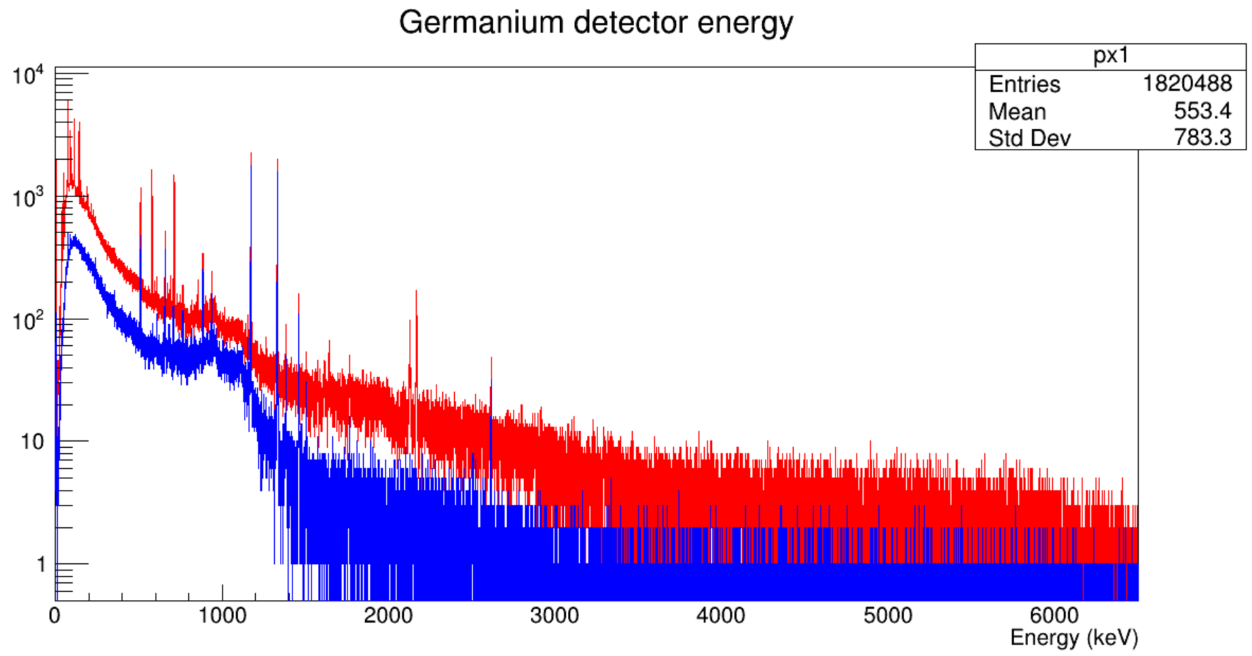
$$dR = \frac{R}{R_N}, \text{ where } R_N = 5.8687\ \text{fm}$$

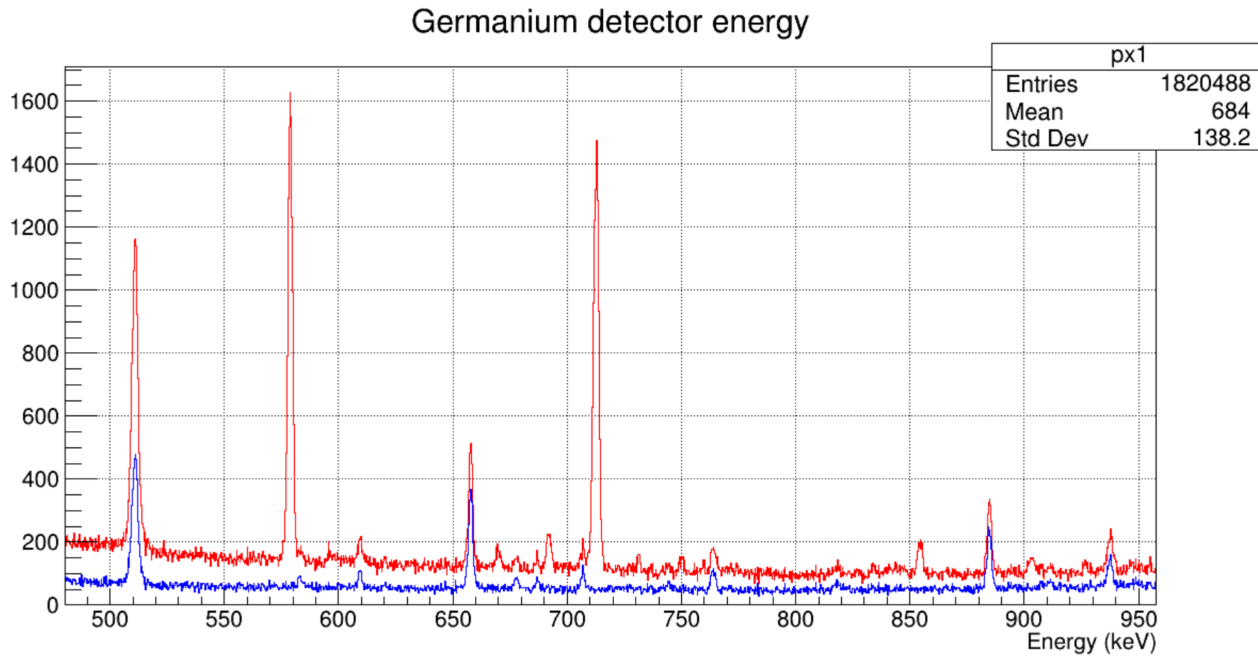
$$dQ = \frac{Q}{Q_N}, \text{ where } Q_N = 12.04\ \text{b}$$



$$\text{Energy}(dR, dQ) = c_0 \cdot 1 + c_1 \cdot dR + c_2 \cdot dQ + c_3 \cdot dR^2 + c_4 \cdot dR^2 \cdot dQ + c_5 \cdot dR^2 \cdot dQ^2 + c_6 \cdot dQ^2 + c_7 \cdot dR \cdot dQ^2 + c_8 \cdot dR \cdot dQ$$

Development – anticoincidence spectrum



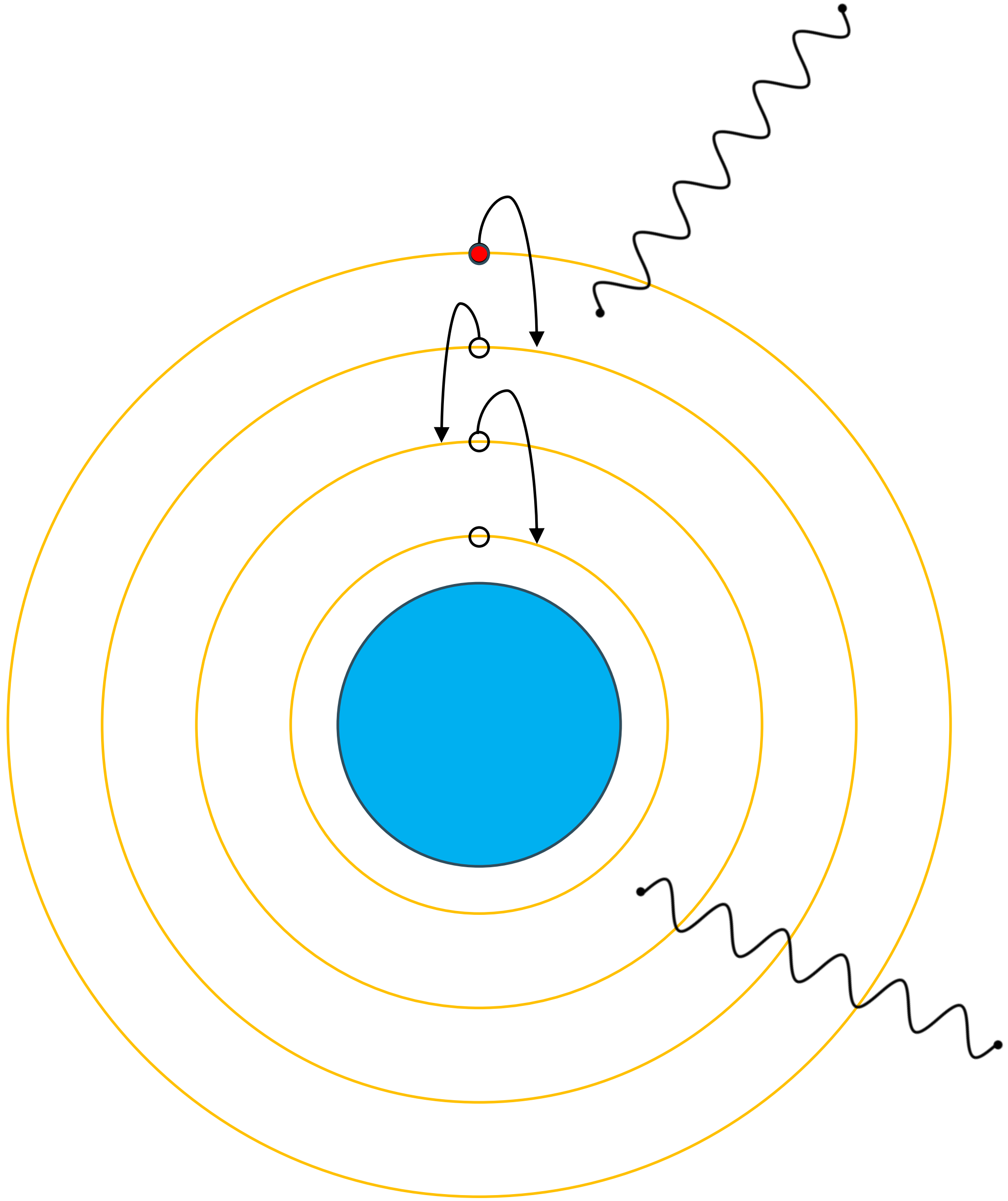


Muonic x rays

- Captured in high-n state → Cascade down

- X rays emitted in atomic transition
  - Electronic atoms: < 100 keV
  - Muonic atoms: Up to 10 MeV

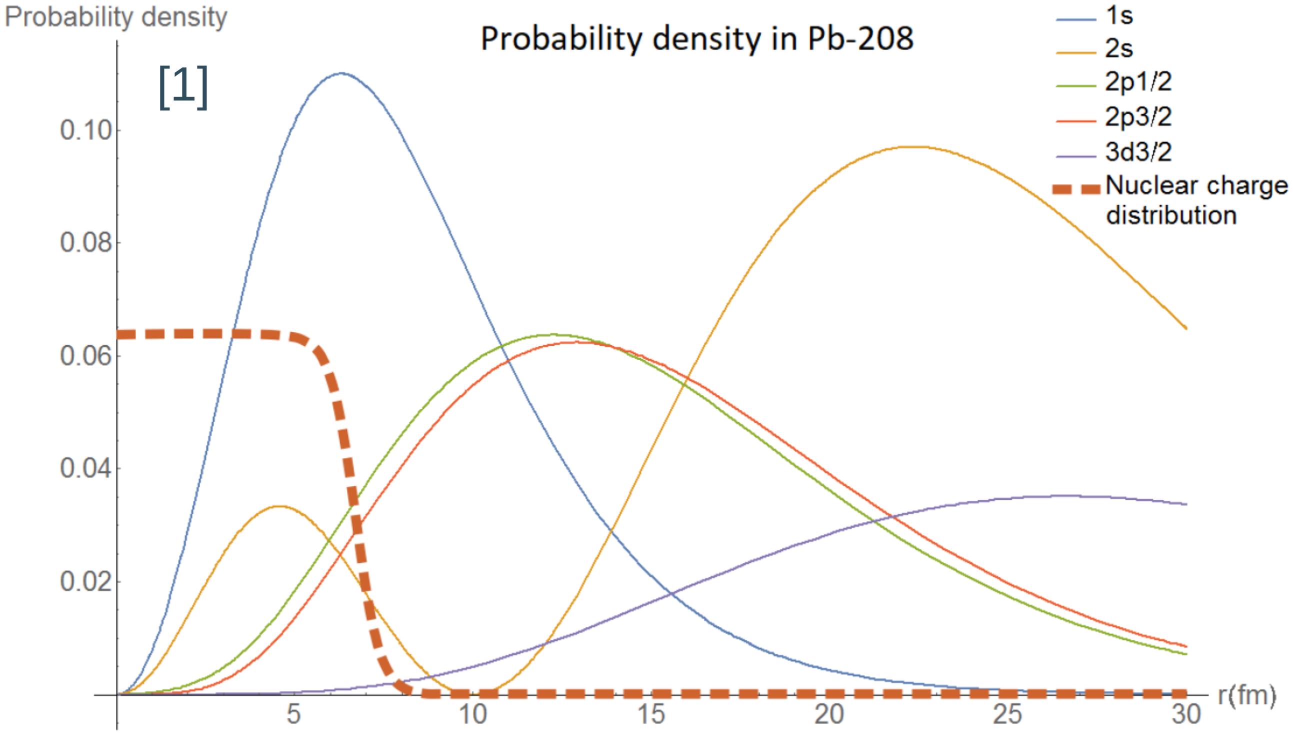
- Information about energy levels → Extract nuclear properties



How sensitive are we?

- Groundstate wavefunction has sizeable overlap with the nucleus

- Sensitivity increase:
  - Nuclear size: $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$
  - Quadrupole: $\left(\frac{m_\mu}{m_e}\right)^2 \approx 5 \times 10^4$
  - Octupole: $\left(\frac{m_\mu}{m_e}\right)^3 \approx 10^7$



[1] S. M. Vogiatzi. The fitting of the hyperfine splitting of the 5 -> 4 transitions in muonic Re-185. Master's thesis, ETH Zurich, 2018.

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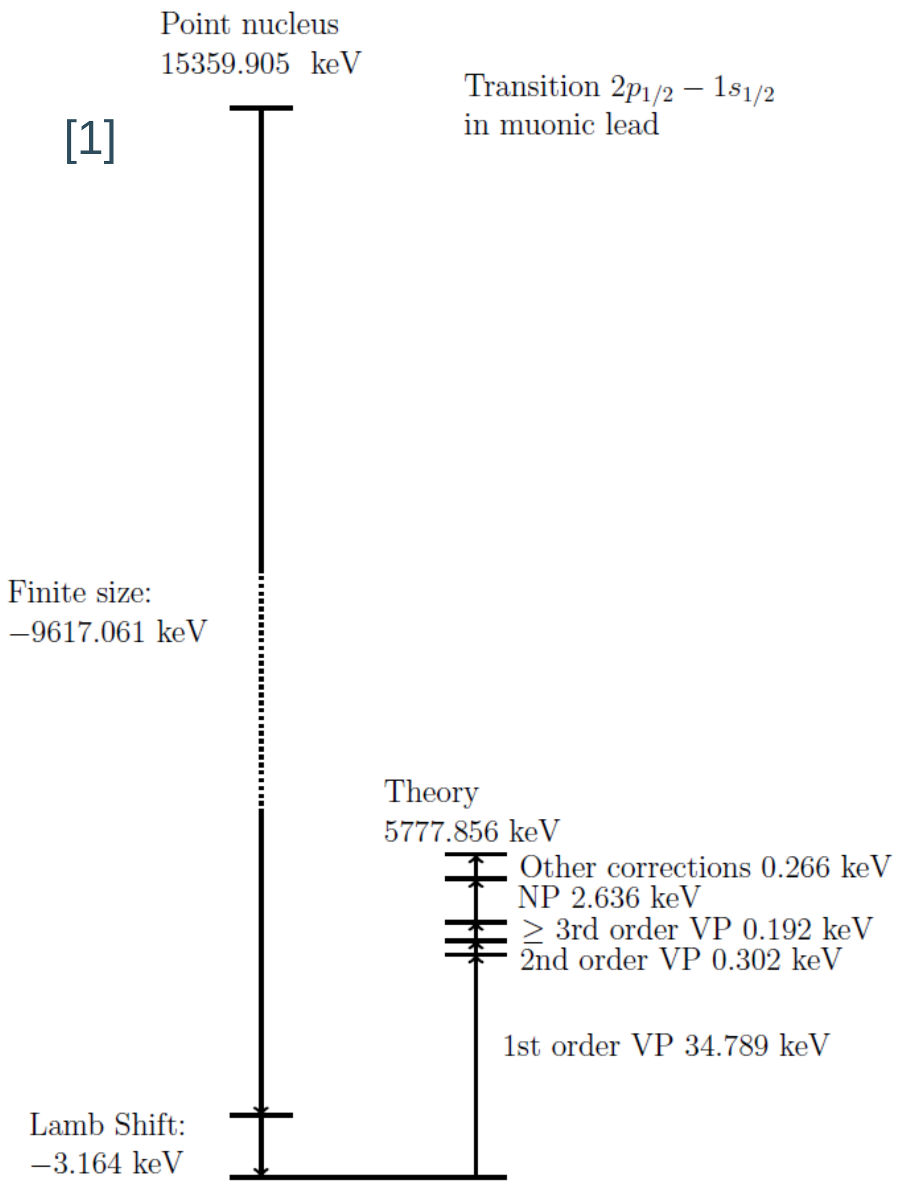
[1]



[1] S. M. Vogiatzi. The fitting of the hyperfine splitting of the 5 -> 4 transitions in muonic Re-185. Master's thesis, ETH Zurich, 2018.

Background – Producing muons

- Protons on a graphite target

n(p, p π^-)p

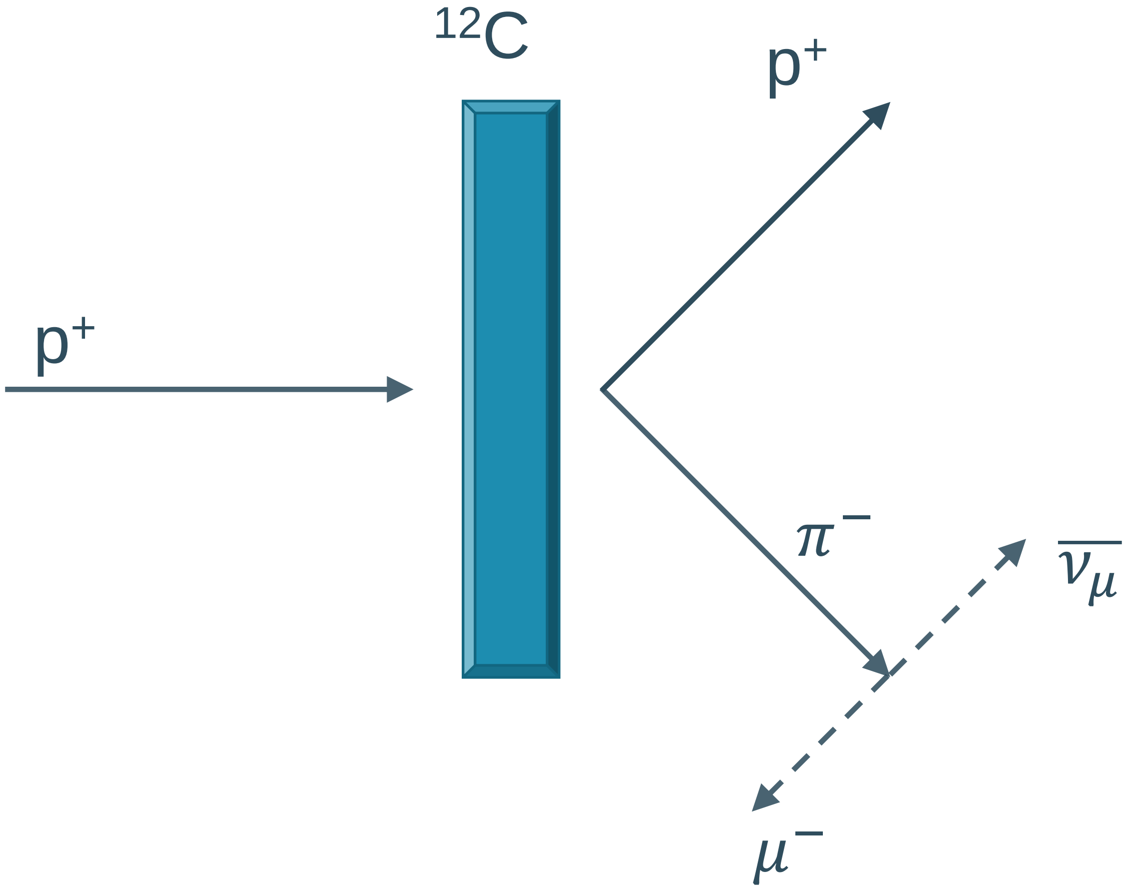
$\pi^- \rightarrow \mu^- + \overline{\nu_\mu}$

$7 \times 10^6 \frac{\mu^-}{s}$

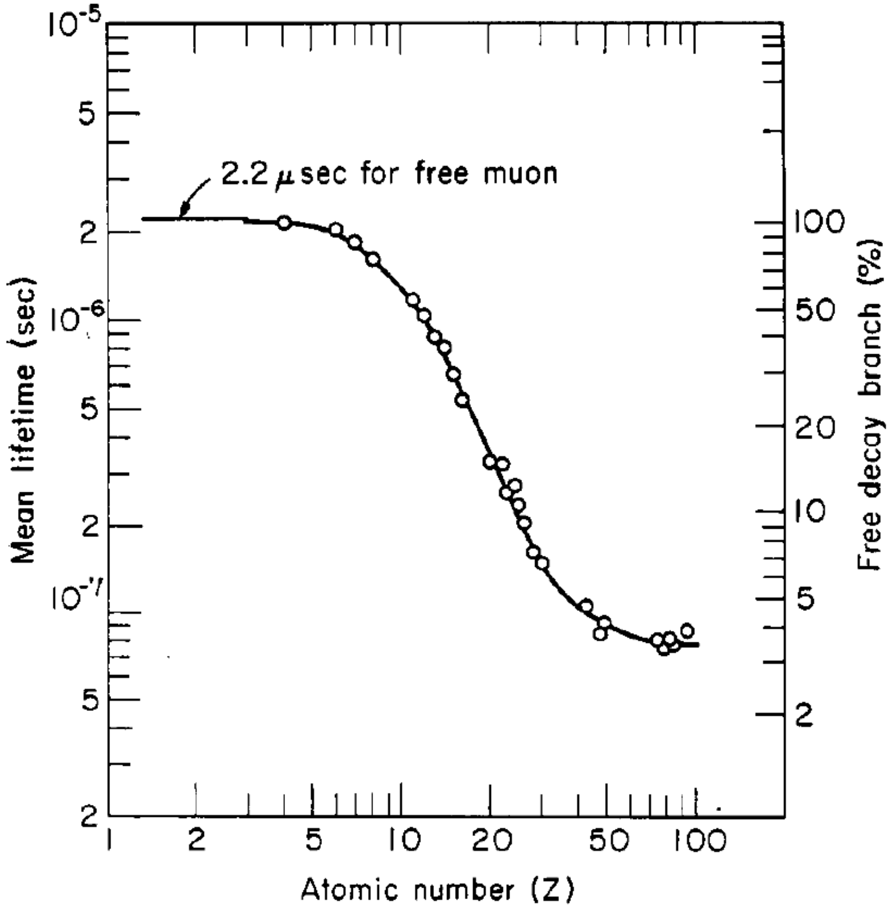
p(p, p π^+) n

$\pi^+ \rightarrow \mu^+ + \nu_\mu$

$5 \times 10^8 \frac{\mu^+}{s}$



Muon half-life

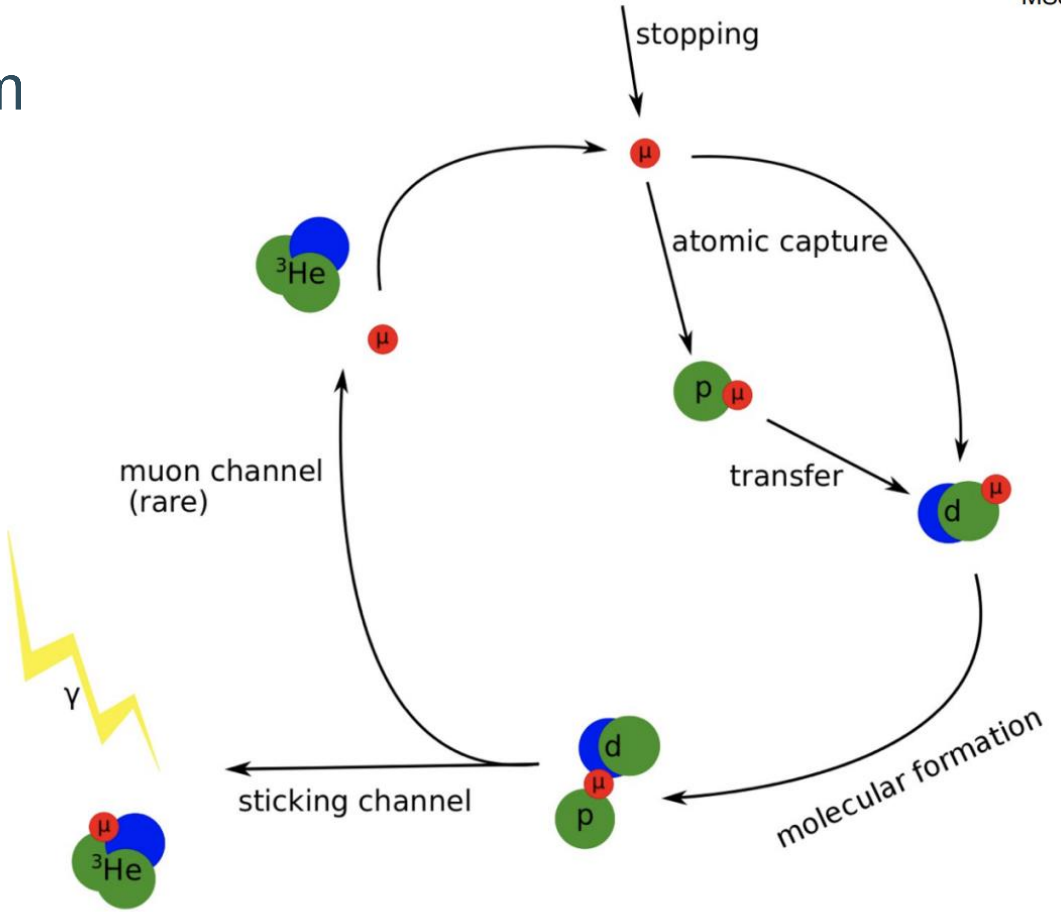


Yamazaki, T., et al. "Negative moun spin rotation." *Physica Scripta* 11.3-4 (1975): 133.

Muon-catalyzed fusion

- Muonic deuterium + hydrogen form a molecule
- Interatomic distance ~200 times smaller
- Thermal vibrations break through fusion barrier (down to 1K)
- Not sufficient for net gain, but still very cool

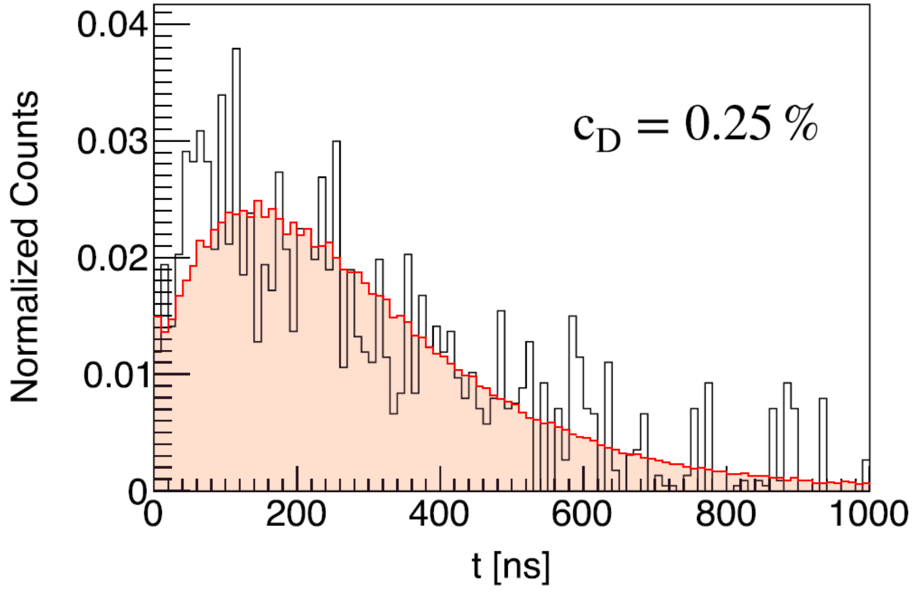
MSc thesis of J. Nuber

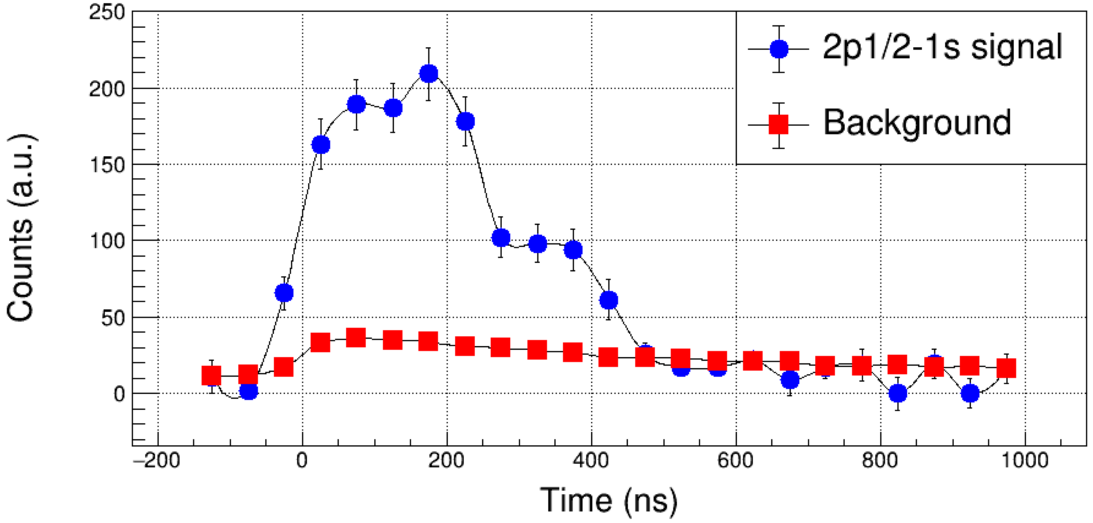


General time cut

- Suboptimal time window:
    - Too small → Miss significant signal
    - Too large → Include additional background

- Imperfect timing → [-50ns; 500ns]





- Adamczak, A., et al. "Muonic atom spectroscopy with microgram target material." *The European Physical Journal A* 59.2 (2023): 15.

Fitting with Hypermet

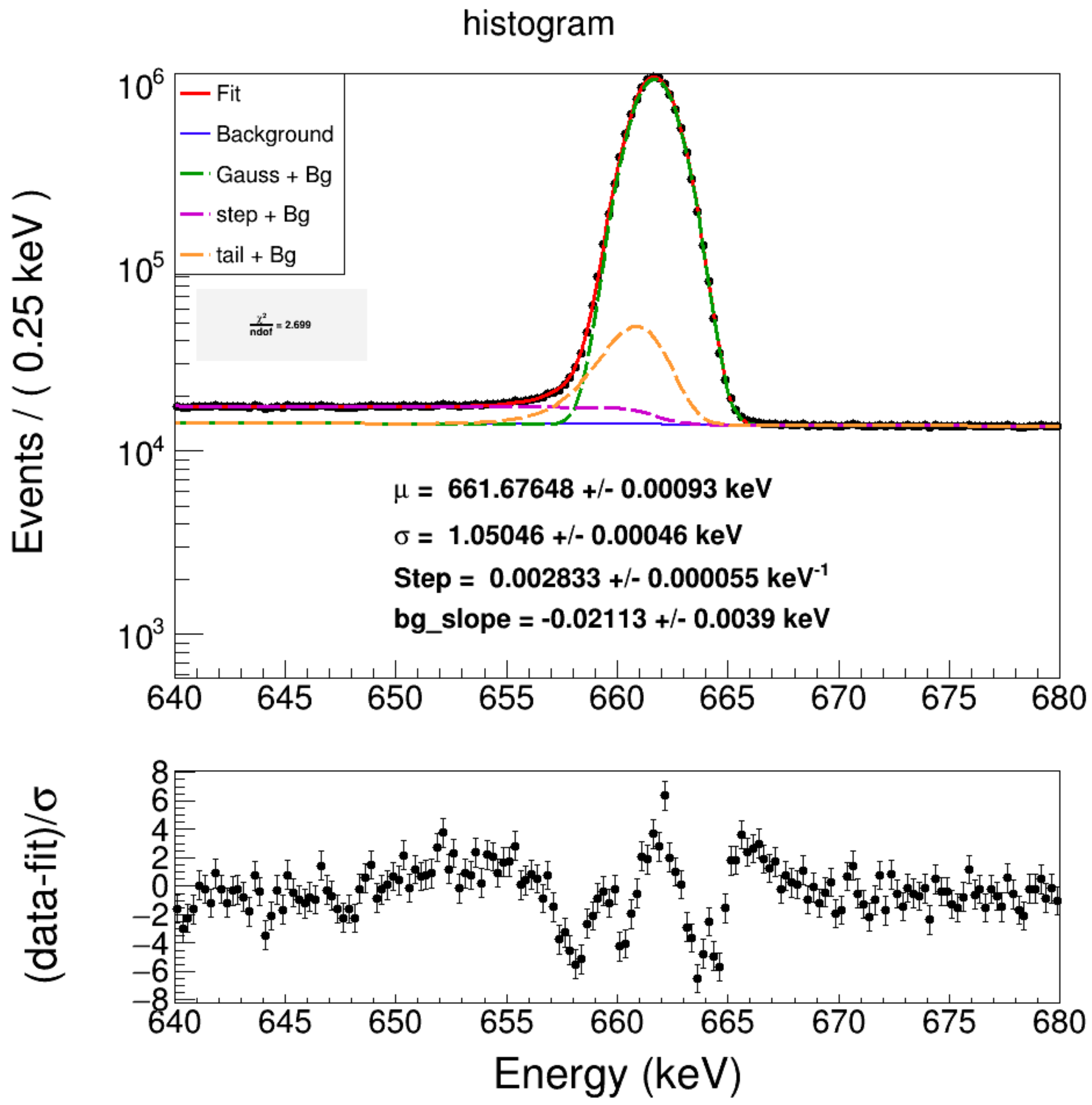
- Model
  - Gaussian
  - Low energy tail
  - Step function
  - Linear background

$$\frac{f_G}{\sqrt{2\pi}\sigma} \times \exp\left[-\frac{1}{2}\left(\frac{E-\mu}{\sigma}\right)^2\right] +$$

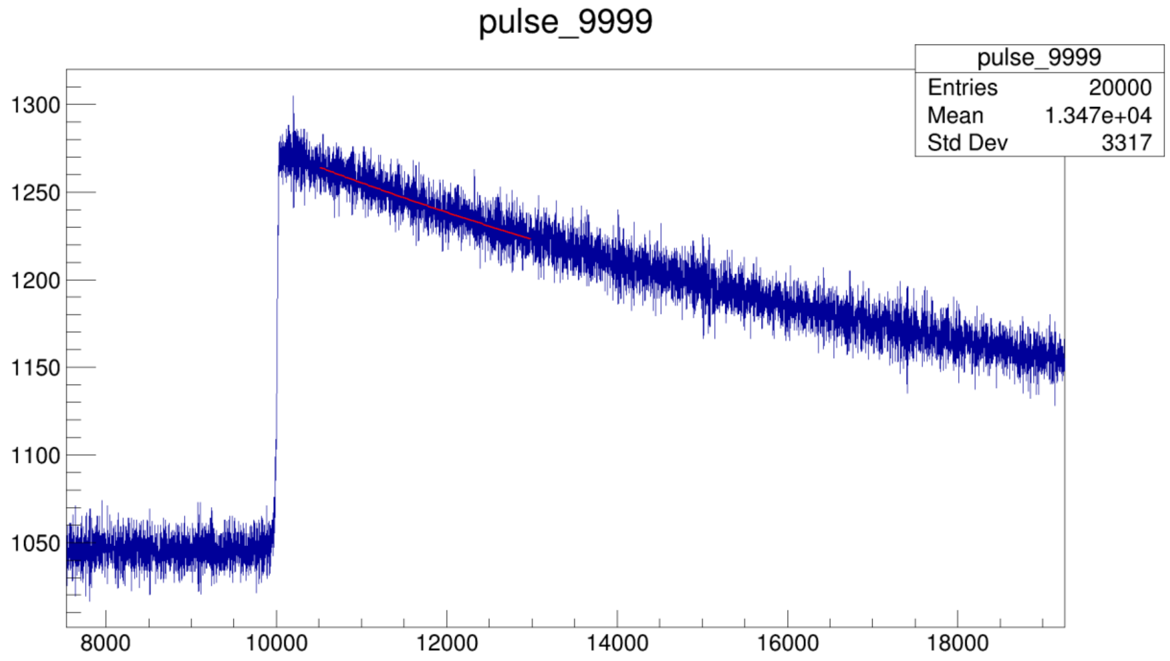
$$\frac{f_T}{2\beta}\exp\left[\frac{E-\mu}{\beta}+\frac{1}{2}\left(\frac{\sigma}{\beta}\right)^2\right]\operatorname{erfc}\left[\frac{E-\mu}{\sqrt{2}\sigma}\right] +$$

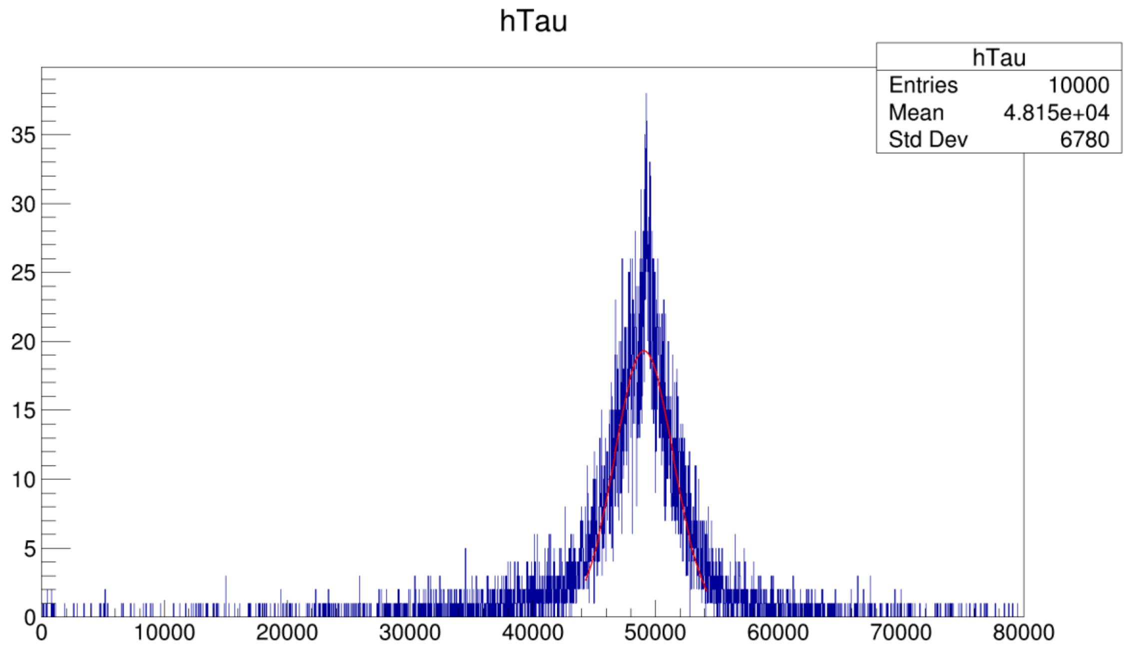
$$\frac{S}{2}\times \operatorname{erfc}\left[\frac{E-\mu}{\sqrt{2}\sigma}\right] +$$

$$C_0 + C_1 E$$

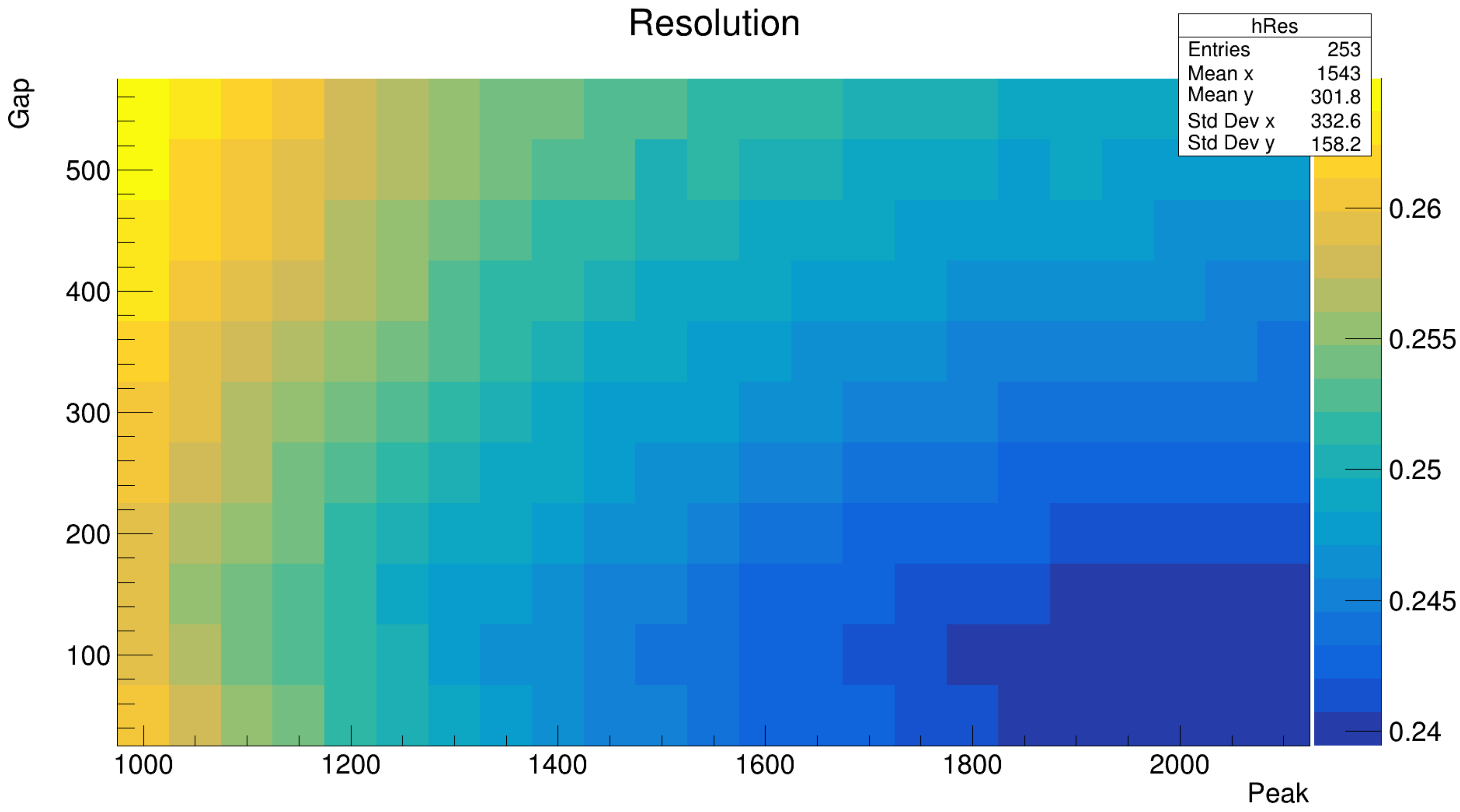


Online optimization – Trapezoid

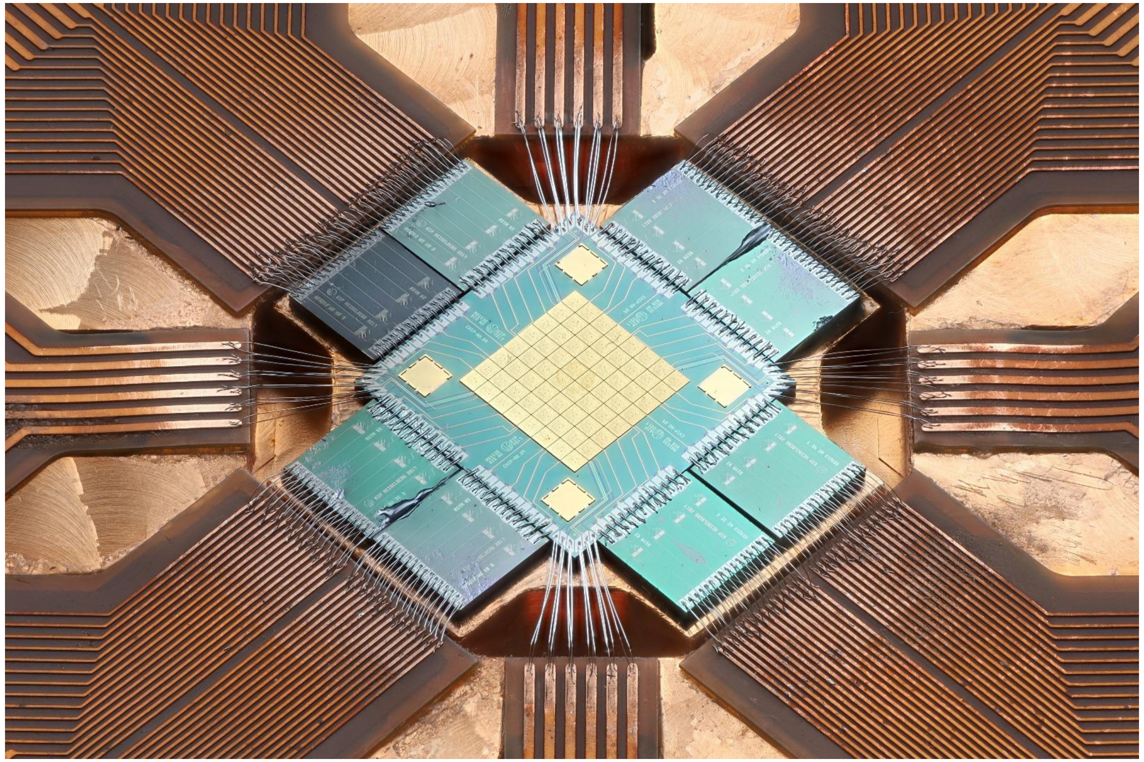


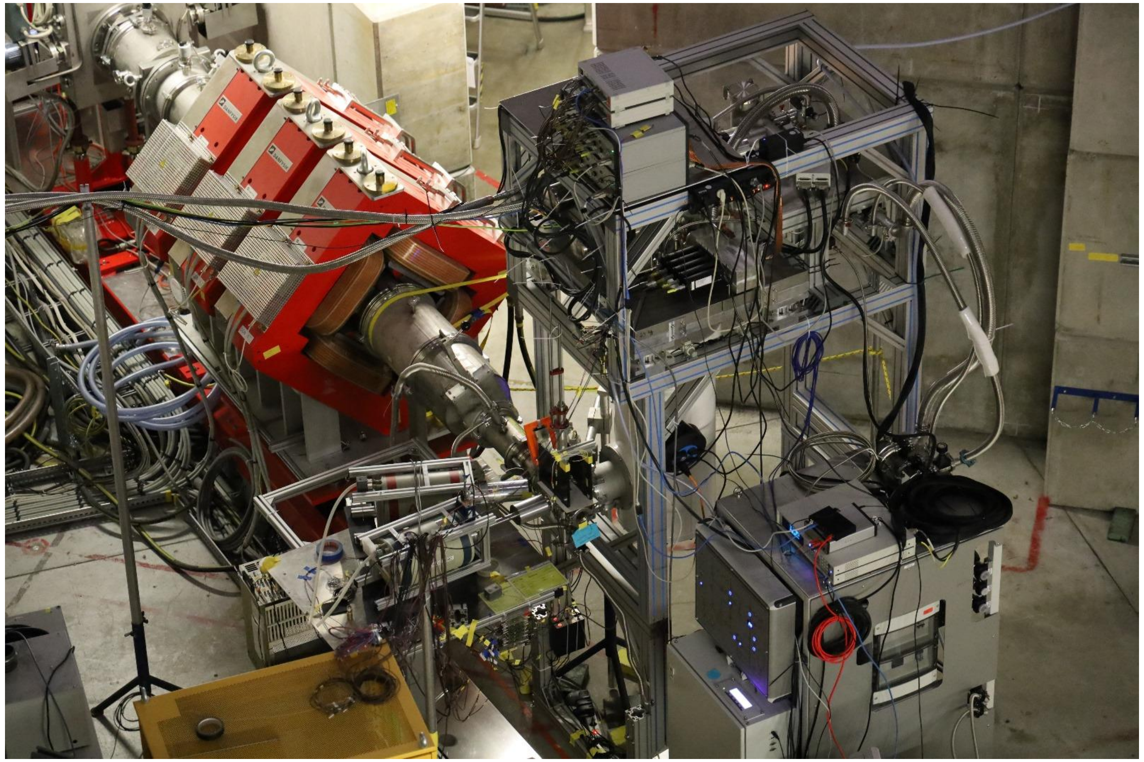


Online optimization – Trapezoid



MMC detector





Compare to other methods

- Limited to ~10^{-3}
- Electron scattering: A lot of disagreement → Conservative estimate 0.5-1% uncertainty on radii