Cosmological phase transitions and lattice Monte-Carlo

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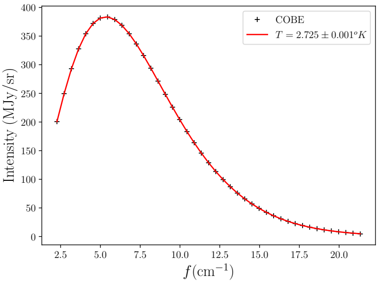
Nordic Lattice Meeting
University of Helsinki
23 August 2022

Hot Big Bang



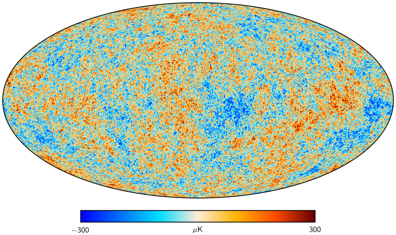
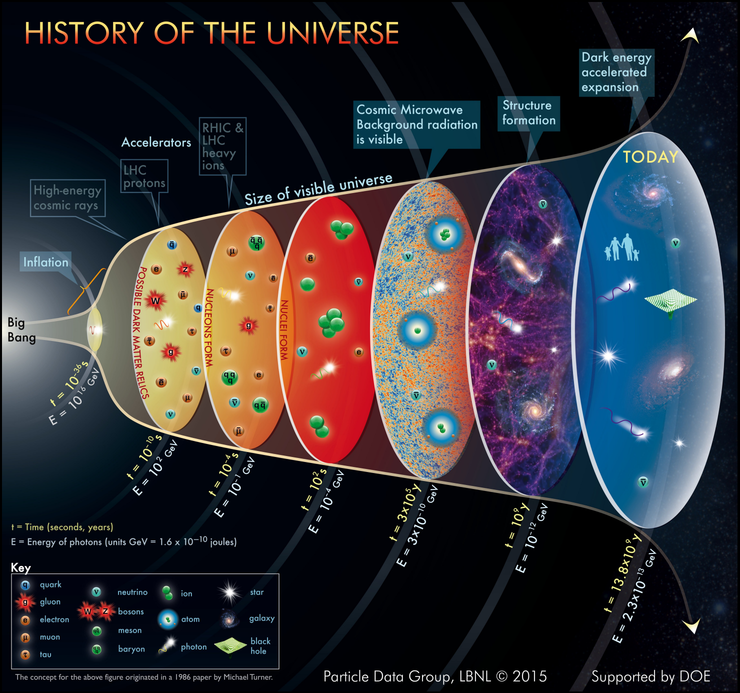


Figure: Spectrum of cosmic microwave background versus a blackbody spectrum (COBE), and temperature anisotropies (Planck).

- Matter was very close to thermal in the early universe.
- Lots of interesting thermal physics.



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

Why cosmological phase transitions?

Why not?

Are there really no phase transitions in particle cosmology?
From $T \sim$ meV, all the way up to inflation (10^{lots} GeV)?

Observable remnants $\Rightarrow$ new probe of particle physics

Such as $(n_B - n_{\bar{B}})/s$, topological defects, magnetic fields, gravitational waves, ... $\Rightarrow$ new probe of particle physics

Cosmological first-order phase transitions

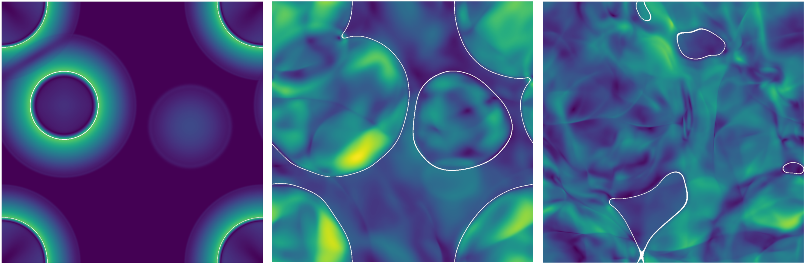


Figure: Cutting et al. arXiv:1906.00480.

- Bubbles nucleate, expand and collide
- This creates long-lived fluid flows
- And creates gravitational waves

Gravitational waves

- Gravitational waves directly observed by LIGO/Virgo →
- Future experiments will significantly improve sensitivity ↓

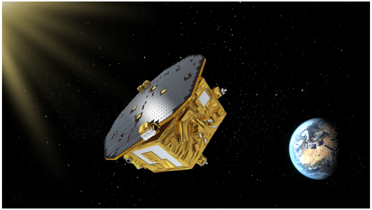


Figure: LISA Pathfinder

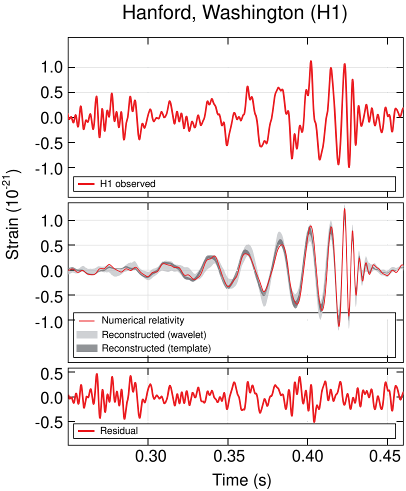
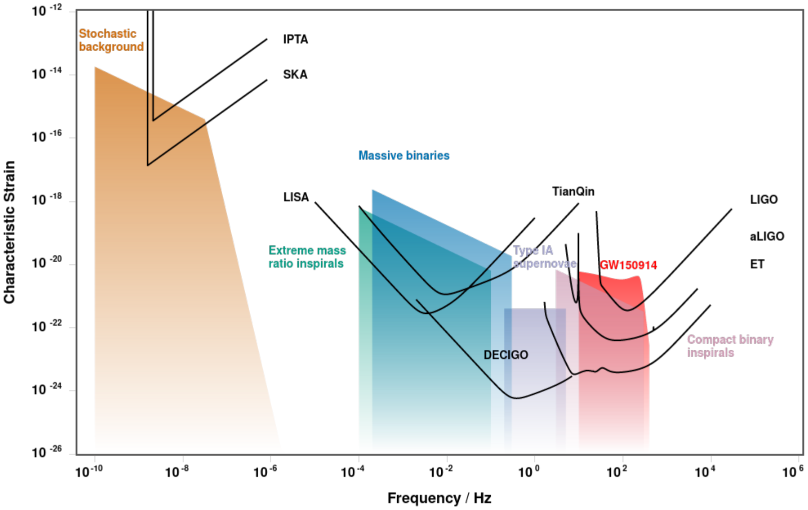


Figure: GW150914 1602.03837

The gravitational wave spectrum



gwplotter.com

The gravitational wave spectrum

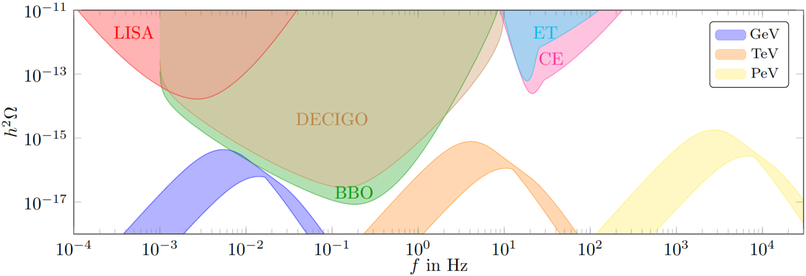


Figure: $SU(N)$ confinement transitions, Huang et al. arXiv:2012.11614

Gravitational waves from phase transitions: the pipeline

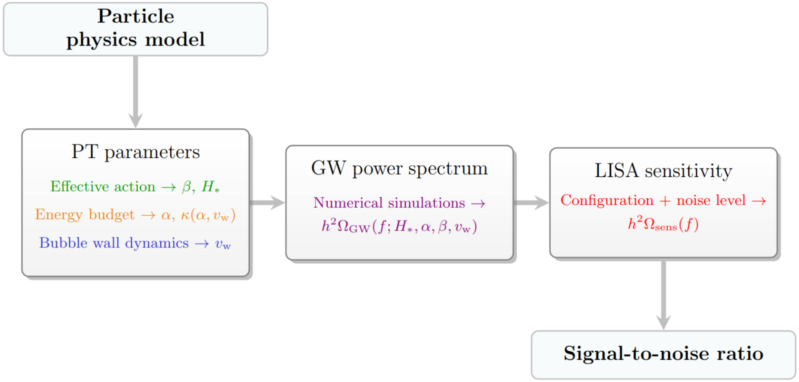
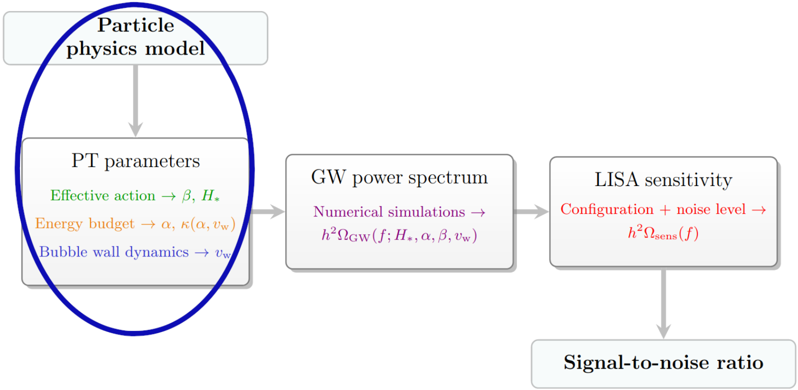


Figure: The Light Interferometer Space Antenna (LISA) pipeline $\mathscr{L} \to \mathrm{SNR}(f)$, Caprini et al. 1910.13125.

Gravitational waves from phase transitions: the pipeline



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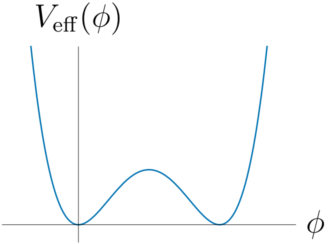
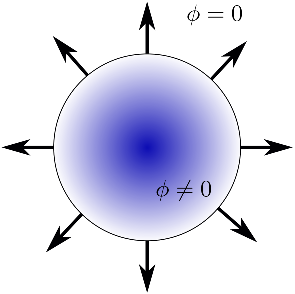
Phase transition parameters





Equilibrium (hom.)

- order of transition
- T_c, critical temperature
- $\Delta\theta_c$, latent heat
- c_s^2, sound speed

Near-equilibrium

- Γ, bubble nucleation rate $\Rightarrow$ T_*, $\Delta\theta_*$, α_*, β/H_*

Nonequilibrium

- v_w, bubble wall speed

Overview

1. Motivation
2. Theoretical uncertainties
3. Scale hierarchies in phase transitions
4. Lattice Monte-Carlo for phase transitions
5. Conclusions

Theoretical uncertainties

Standard approach to computing parameters

One-loop resummed approximation is based on

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + \underbrace{\frac{1}{2}\int_{P\in\mathbb{R}^4} \log(P^2 + V''_{\text{tree}})}_{\text{Coleman-Weinberg}}$$

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$$\underbrace{-T\int_{p\in\mathbb{R}^3} \log\left[1 + n_{\text{B}}\left(\frac{\sqrt{p^2 + V''_{\text{tree}}}}{T}\right)\right]}_{\text{thermal}}$$

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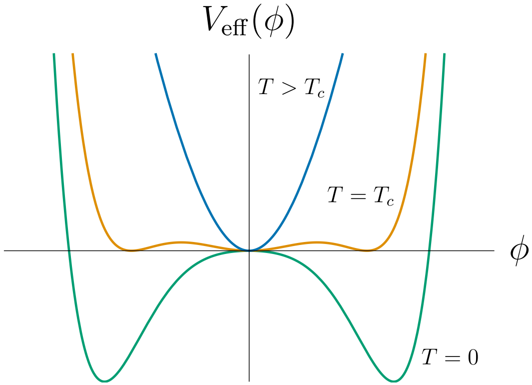
$$\underbrace{-\frac{T}{12\pi}\left((V''_{\text{tree}} + \Pi_T)^{3/2} - (V''_{\text{tree}})^{3/2}\right)}_{\text{daisy}}.$$

- Solve $\Re V_{\text{eff}}{}'(\phi, T) = 0 \Rightarrow$ phases
- Solve $-\partial_r^2\phi - 2\partial_r\phi + \Re V_{\text{eff}}{}'(\phi, T) = 0 \Rightarrow$ critical bubble

Phase transitions



For there to be a phase transition, thermal/quantum fluctuations should modify the potential at leading order,

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}.$$

Perturbative failures

- Perturbation theory predicts a first-order electroweak phase transition in the Standard Model for all m_{H}.
- Transition is actually a crossover for $m_{\mathrm{H}} \approx 125$ GeV.

Kajantie et al. '96

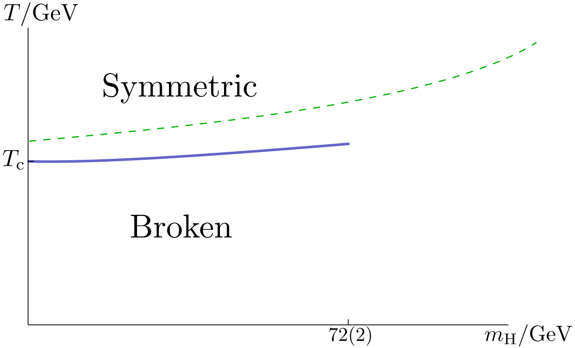
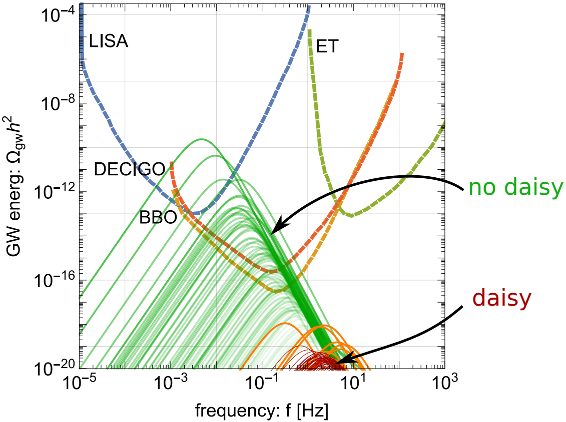


Figure: Phase diagram of electroweak sector.

Theoretical uncertainties

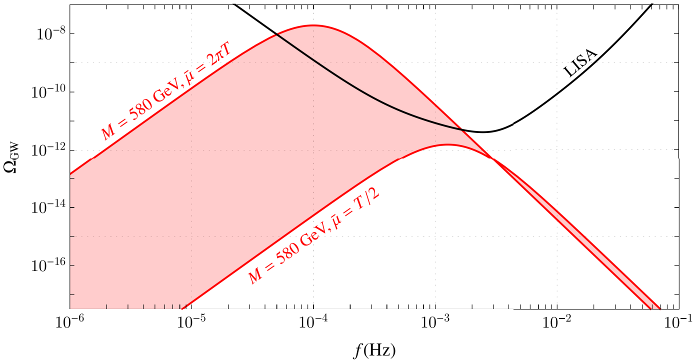


GW signals in two different 1-loop approximations for

$$\mathscr{L} = \mathscr{L}_{SM} + \frac{a_2}{2}(\Phi^\dagger\Phi)\sigma^2 + \frac{1}{2}(\partial\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{b_4}{4}\sigma^4$$

Carena, Liu & Wang 1911.10206

Theoretical uncertainties

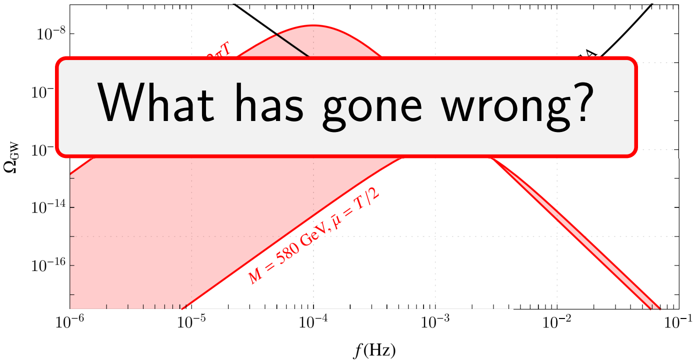


Renormalisation scale dependence of GW spectrum at one physical parameter point for

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{M^2}(\Phi^\dagger\Phi)^3.$$

Croon, OG, Schicho, Tenkanen & White 2009.10080

Theoretical uncertainties



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What has gone wrong?

Possible sources of theoretical uncertainties:

- nonperturbativity? Linde '80
- inconsistencies? E. Weinberg & Wu '87, E. Weinberg '92
- higher order perturbative corrections? Arnold & Espinosa '92
- gauge dependence or infrared divergences? Laine '94
- renormalisation scale dependence? Farakos et al. '94
- ...

Scale hierarchies in phase transitions

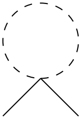
A hierarchy problem

Let's assume there is some very massive particle χ, $M_\chi \gg m_H$, coupled to the Standard Model Higgs Φ like

$$\mathscr{L} = \mathscr{L}_{\rm SM} + g^2 \Phi^\dagger \Phi \chi^\dagger \chi + \mathscr{L}_\chi .$$

If we integrate out χ, we find that the Higgs mass parameter gets a correction of the form

$$(\Delta m_H^2)\Phi^\dagger\Phi \; =$$



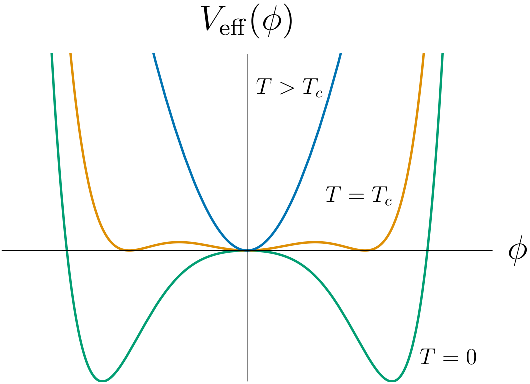
,

$$\sim g^2 M_\chi^2 \Phi^\dagger \Phi \; .$$

Relevant operators in the IR get large contributions from the UV,

$$\frac{\Delta m_H^2}{m_H^2} \sim g^2 \left(\frac{M_\chi}{m_H} \right)^2 .$$

Phase transitions



For there to be a phase transition, thermal/quantum fluctuations should modify the potential at leading order,

$$V_{\text{eff}} = V_{\text{tree}} + \Delta V_{\text{fluct}}.$$

Hierarchies in phase transitions

So, for there to be a phase transition, we need

$$\frac{\Delta V_{\text{fluct}}}{V_{\text{tree}}} \sim g^2 N \left(\frac{\Lambda_{\text{fluct}}}{\Lambda_{\text{tree}}} \right)^{\sigma} \overset{!}{\gtrsim} 1,$$

where $\sigma > 0$ for relevant operators.

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$\Rightarrow$ either:

(i) $g^2 N \gtrsim 1$, i.e. strong coupling

(ii) $\Lambda_{\text{fluct}} \gg \Lambda_{\text{tree}}$, i.e. scale hierarchy

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Perturbative phase transitions require scale hierarchies!

Infrared strong coupling

Due to the high occupancy of infrared bosons, the effective expansion parameter $\alpha_{\rm eff}$ grows

$$\alpha_{\rm eff} \sim g^2 \frac{1}{1 - e^{E/T}} \approx g^2 \frac{T}{E}$$

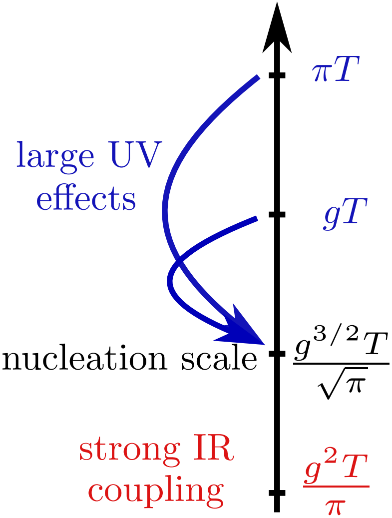
lighter modes are more strongly coupled:

$$\begin{aligned}
\text{hard}: &\quad E \sim \pi T \Rightarrow \alpha_{\rm eff} \sim g^2 \sim 0.03, \\
\text{soft}: &\quad E \sim gT \Rightarrow \alpha_{\rm eff} \sim g \sim 0.18, \\
\text{supersoft}: &\quad E \sim g^{3/2} T \Rightarrow \alpha_{\rm eff} \sim g^{1/2} \sim 0.42, \\
\text{ultrasoft}: &\quad E \sim g^2 T \Rightarrow \alpha_{\rm eff} \sim g^0 \sim 1.
\end{aligned}$$

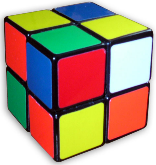
UV and IR problems

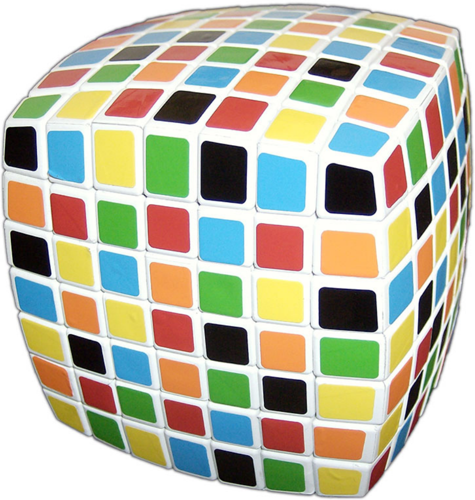
There are two main difficulties

- large UV effects break loop expansion $\leftarrow$ EFT
- IR becomes strongly coupled $\leftarrow$ lattice



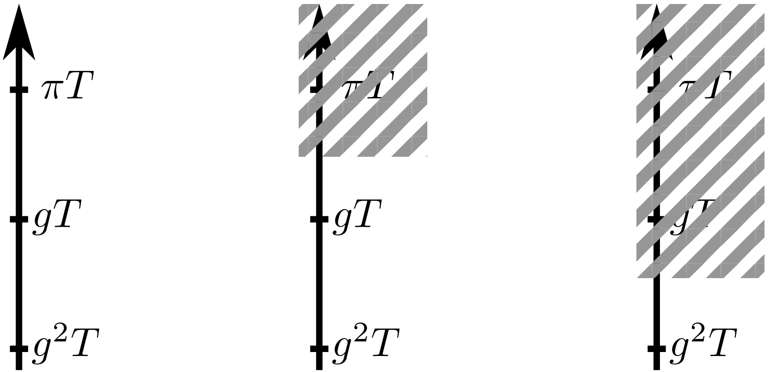
UV easy IR hard





Lattice Monte-Carlo for phase transitions

High temperature effective field theory



$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} \not{D} \psi$$

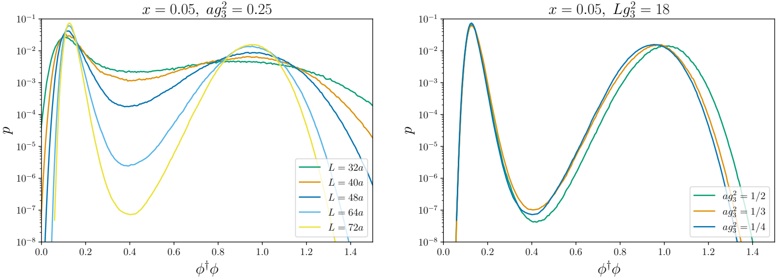
$$\mathscr{L}_{\text{eff}} = \frac{1}{4} F_{ij} F_{ij} + (D_i A_0)^2 + m_D^2 A_0^2 + \lambda A_0^4$$

$$\mathscr{L}_{\text{eff}} = \frac{1}{4} F_{ij} F_{ij}$$

Farakos et al. '94, Braaten & Nieto '95, Kajantie et al. '95

Equilibrium thermodynamics on the lattice

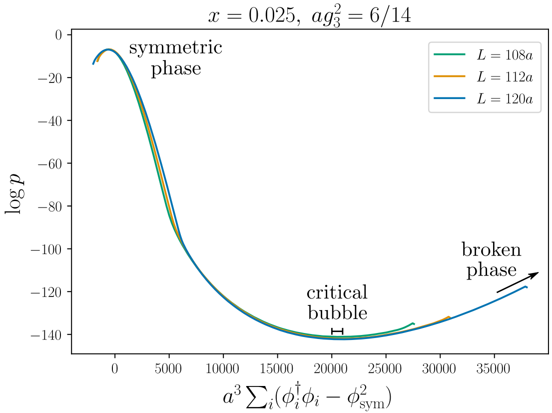
- Monte-Carlo simulations of 3d EFT sample the thermal distribution of field configurations, $p \propto e^{-H[\phi]/T}$.



- Efficient update algorithms known. Kajantie et al. '95
- Superrenormalisability $\Rightarrow$ exact lattice-continuum relations.

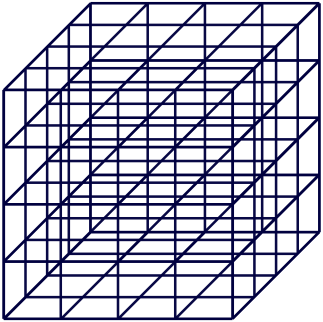
Laine '95

Bubble nucleation on the lattice

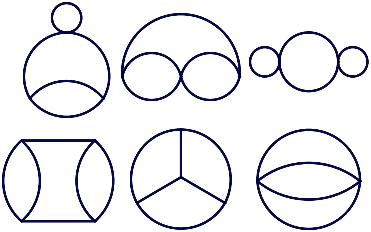


$$\Gamma = \underbrace{\left\langle \frac{\delta_{\text{tunnel}}}{2N_{\text{crossings}}} \left| \frac{\Delta(\phi^\dagger\phi)}{\Delta t} \right|_{\varphi_C^2} \right\rangle}_{\text{dynamical}} \underbrace{\frac{P\left(|\phi^\dagger\phi - \varphi_C^2| < \epsilon/2\right)}{\epsilon V P\left(\phi^\dagger\phi < \varphi_C^2\right)}}_{\text{statistical}}.$$

Lattice vs perturbation theory



vs

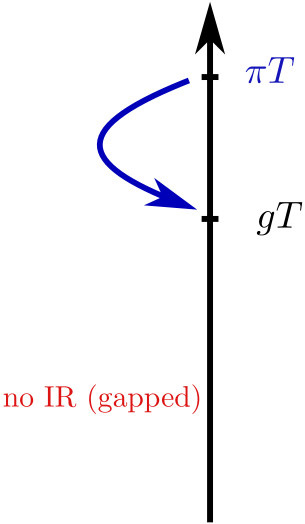


Real scalar model

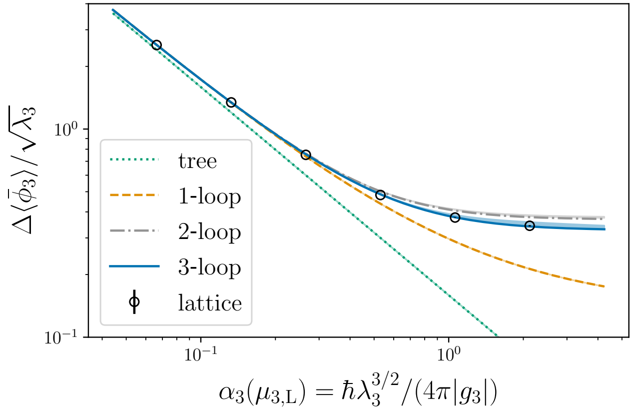
A simple model,

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{3!}\phi^2 + \frac{\lambda}{4!}\phi^4 + J_1\phi + J_2\phi^2$$

with only two scales.



Results: real scalar model

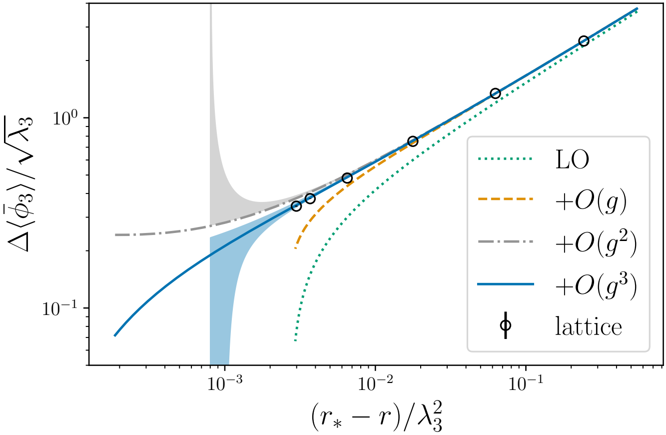


Perturbation theory converges towards the lattice for the jump in the scalar condensate in

$$\mathscr{L} = \frac{1}{2}(\partial\phi)^2 + \frac{m_3^2}{2}\phi^2 + \frac{g_3}{3!}\phi^2 + \frac{\lambda_3}{4!}\phi^4$$

OG 2101.05528

Results: approaching the second-order phase transition



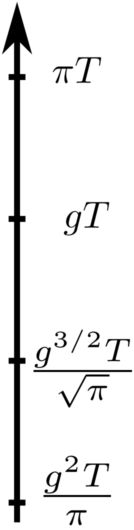
OG 2101.05528

SU(2) Higgs model

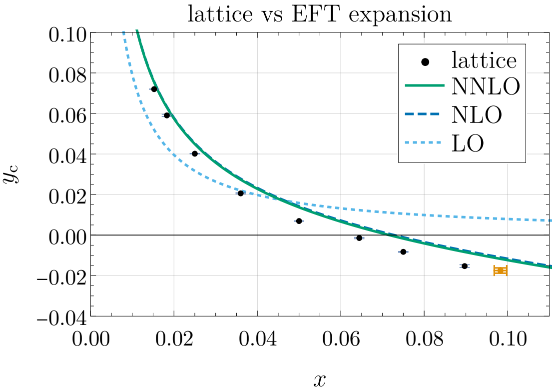
A more complicated model with all the scales

$$\mathscr{L} = \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \Phi)^\dagger D_\mu \Phi + m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

- large UV effects
- strongly coupled IR



Results: SU(2) Higgs model

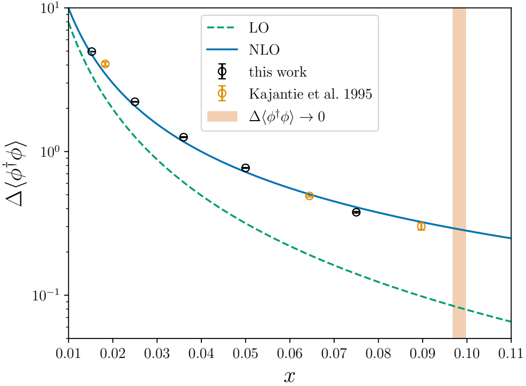


Kajantie et al. 95

OG, Güyer & Rummukainen 2205.07238

Ekstedt, OG & Löfgren 2205.07241

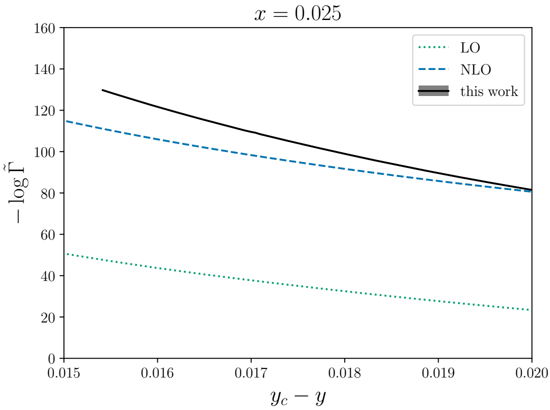
Results: SU(2) Higgs model



Kajantie et al. 95

OG, Güyer & Rummukainen 2205.07238

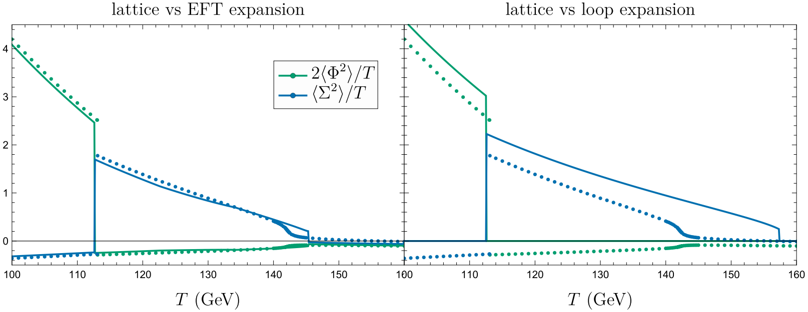
SU(2) Higgs model - bubble nucleation



Moore & Rummukainen '00

OG, Güyer & Rummukainen 2205.07238

Triplet extension of the Standard Model

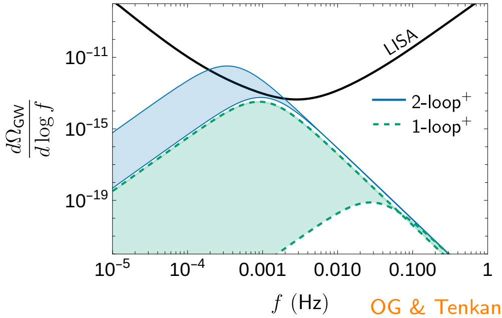


Perturbative EFT approach agrees reasonably well with lattice in triplet extension of Standard Model,

$$\mathscr{L} = \mathscr{L}_{\mathsf{SM}} + \frac{a_2}{2}\Phi^\dagger\Phi\Sigma^a\Sigma^a + \frac{1}{2}D_\mu\Sigma^a D_\mu\Sigma^a + \frac{m_\Sigma^2}{2}\Sigma^a\Sigma^a + \frac{b_4}{4}(\Sigma^a\Sigma^a)^2$$

OG & Tenkanen forthcoming

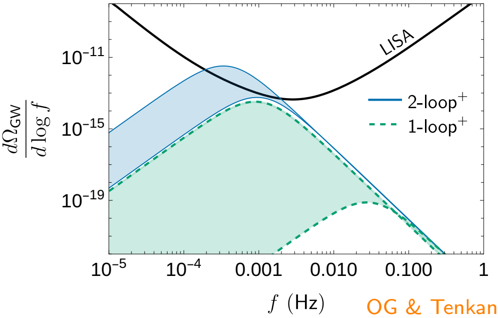
Conclusions



OG & Tenkanen 2104.04399

- Phase transitions may produce observable gravitational waves
- Large theoretical uncertainties in one-loop approximation
- EFT can solve problems from UV
- Lattice can solve problems from IR

Conclusions



OG & Tenkanen 2104.04399

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Thanks for listening!

Backup slides

High temperature effective field theory

Equilibrium thermodynamics

- Can be formulated in $\mathbb{R}^3 \times S^1$.

- Fields are expanded into Fourier (Matsubara) modes:

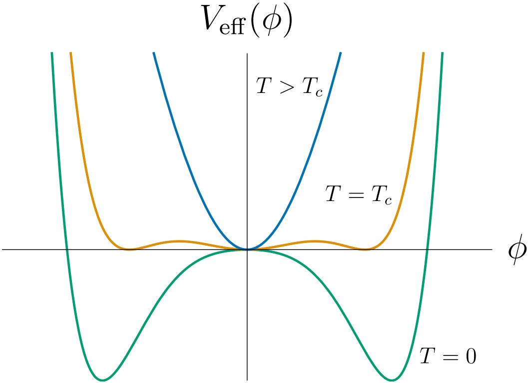
$$\Phi(x, \tau) = \sum_{n \text{ even}} \phi_n(x) e^{i(n\pi T)\tau} \leftarrow \text{boson}$$

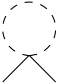
$$\Psi(x, \tau) = \sum_{n \text{ odd}} \psi_n(x) e^{i(n\pi T)\tau} \leftarrow \text{fermion}$$

- Masses of Matsubara modes are

$$m_n^2 = m^2 + (n\pi T)^2$$

Thermal mass hierarchies

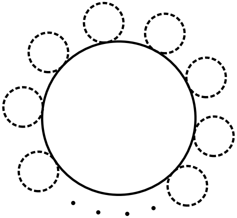


$m^2_{0,\text{eff}} = m^2 +$  $= m^2 + \# g^2 T^2$

- At $T \gg T_c$, thermal corrections dominate, so $m_{0,\text{eff}} \sim gT$ which is much less than πT.
- Near $T = T_c$, cancellations typically give $m_{0,\text{eff}} \ll gT$.

Resumming UV problems

$$\mathscr{L}_0 = \underbrace{\frac{1}{2}(\nabla\phi_0)^2 + \frac{1}{2}m^2\phi_0^2}_{\mathscr{L}_{\rm free}} + \underbrace{\frac{1}{4!}g^2\phi_0^4}_{\mathscr{L}_{\rm int}}.$$



Resummation by changing split between $\mathscr{L}_{\rm free}$ and $\mathscr{L}_{\rm int}$,

$$\mathscr{L}_{\rm free} \to \mathscr{L}_{\rm free} + \frac{1}{2}(m_{0,\rm eff}^2 - m^2)\phi_0^2,$$

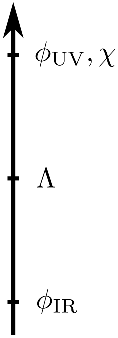
$$\mathscr{L}_{\rm int} \to \mathscr{L}_{\rm int} + \frac{1}{2}(m^2 - m_{0,\rm eff}^2)\phi_0^2.$$

Top-down EFT

- Split degrees of freedom $\{\phi, \chi\}$ based on energy $\rightarrow$

- Integrate out the UV modes:

$$\int \mathcal{D}\phi \int \mathcal{D}\chi \; e^{-S[\phi,\chi]} = \int \mathcal{D}\phi_{\mathsf{IR}} \left(\int \mathcal{D}\phi_{\mathsf{UV}} \mathcal{D}\chi \; e^{-S[\phi,\chi]} \right)$$
$$= \int \mathcal{D}\phi_{\mathsf{IR}} \; e^{-S_{\mathsf{eff}}[\phi_{\mathsf{IR}}]}$$



- Careful power-counting cancels dependence on Λ.

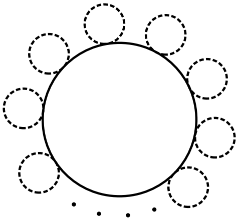
Burgess '21, Hirvonen '22

Resummations with EFT

By first integrating out the UV modes

$$S_{\text{eff}}[\phi_{\text{IR}}] = S_\phi[\phi_{\text{IR}}] - \log \int \mathcal{D}\phi_{\text{UV}}\mathcal{D}\chi \; e^{-S_\chi[\chi]-S_{\chi\phi}[\phi,\chi]},$$
$$\approx S_\phi[\phi_{\text{IR}}] + \int_x \frac{1}{2}(m_{\text{eff}}^2 - m^2)\phi_{\text{IR}}^2,$$

the daisy resummations arise naturally.



So do all other resummations necessary to resolve UV problems (i.e. large contributions to IR quantities from UV physics).