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Complex potential at T>0 from fine lattices

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+ HotQCD COLLABORATION

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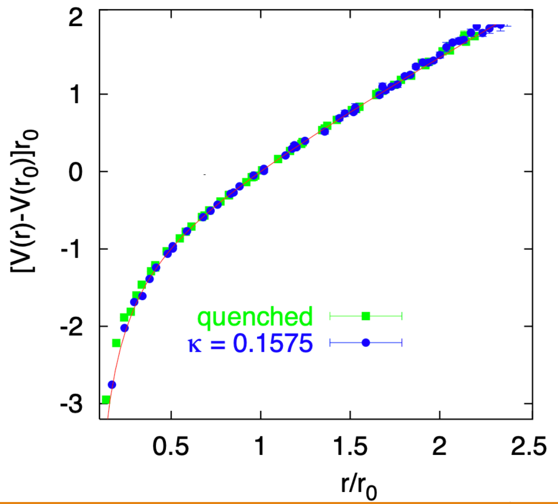
Introduction

Bound states of static quark and anti-quark pair: Probe for existence of Quark Gluon plasma in Heavy ion collisions (Alexander Rothkopf,Heavy quarkonium in extreme conditions,Physics Reports,Volume 858,2020,).

- Time evolution in Real-Time suffers from sign problem. (QFT suffers from sign problem; see Daniel's talk tomorrow)
- If separation of scales is present: Use EFTs (NRQCD and pNRQCD): describe physics in form of potential?

At T=0 Schrodinger like potential picture has been observed (G. Bali Phys.Rept. 343 (2001) 1-136).

$$\mathrm{i}\ \partial_t W_\square(t,r) = \Phi(t,r) W_\square(t,r)$$
$$V(r) = \lim_{t\to\infty} \Phi(t,r)$$



Introduction

- It remains to be seen if the potential picture even holds for T>0, if it does how is it modified? ; Affirmative indications in Quenched systems (A.Rothkopf and Y Burnier, *Phys.Rev.D* 95 (2017) 5, 054511).
- Spectral function is a link between real and imaginary time: (A Rothkopf ,T Hatsuda, S Sasaki Phys.Rev.Lett. 108 (2012) 162001).

$$\mathrm{W}_{\Box}(r,t) = \int d\omega e^{-i\omega t} \rho_{\Box}(r,\omega) \quad \Longleftrightarrow \quad W_{\Box}(r,\tau) = \int d\omega e^{-\omega\tau} \rho_{\Box}(r,\omega)$$

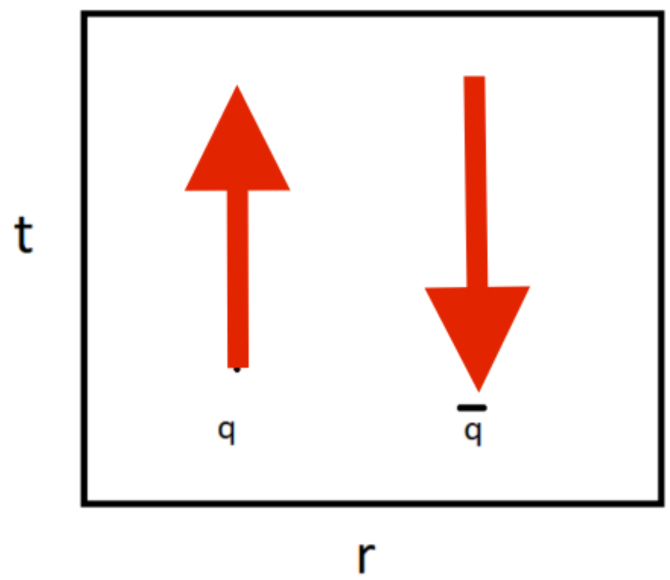
- Computation of Spectral Function : Ill-posed Inverse problem
- Potential linked to the dominant peak position and width of Spectral Function (Yannis Burnier and Alexander Rothkopf 1208.1899).
- In HTL regime there exists a complex potential with screened real part (M.Laine et. al JHEP 03 (2007), 054).

Lattice setup

- (2+1)-flavour QCD - configurations generated by HotQCD and TUMQCD collaborations.
- Using highly improved staggered quark (HISQ) action.

$N_\sigma^3 \times N_\tau$ lattices. $N_\tau = 10, 12, 16$ and $N_\sigma/N_\tau = 4$

- Calculate Wilson Line correlator in Coulomb Gauge.
- Fix box approach; temp range - 140MeV to 2GeV.
- Pion mass 160MeV, Kaon mass physical.

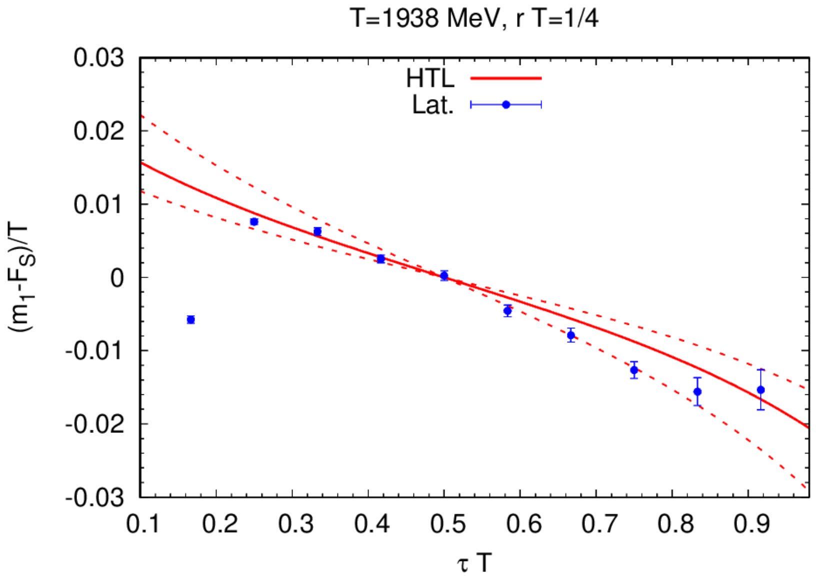


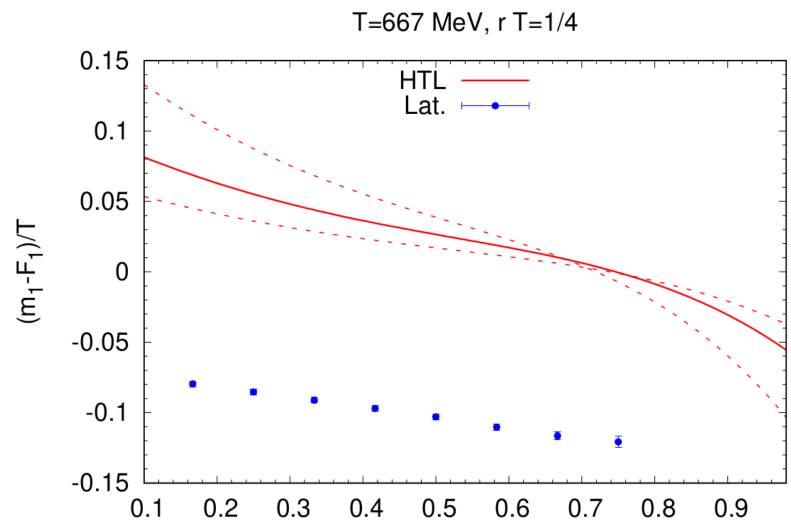
HTL comparison

Define effective mass of the correlation function:

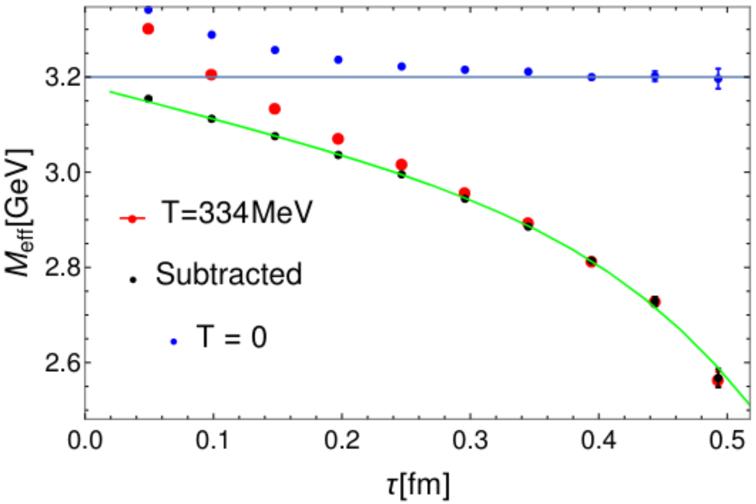
$$m_{eff}(r,\tau,T) = -\partial_\tau \ln W(r,\tau,T)$$

- Subtracting UV part using T=0 correlator results in linear behavior at small tau.
- Plot shows effective mass subtracted from Free Energies.
- HTL does not quantitatively fit the data except for some specific T and separation distance.





Spectral Function Model Fits



- Lattice data sensitive only to peak position (Ω) and effective width (Γ).
- Parametrize Correlator as: $C_{sub}(\tau, T) \quad \sim \quad \exp(-\Omega\tau + \frac{1}{2}\Gamma^2\tau^2 + O(\tau^3))$

$$\rho_r(\omega, T) \quad = \quad A(T)\exp\left(-\frac{[\omega - \Omega(T)]^2}{2\Gamma^2(T)}\right) + A^{\text{cut}}(T)\,\delta\left(\omega - \omega^{\text{cut}}(T)\right)$$

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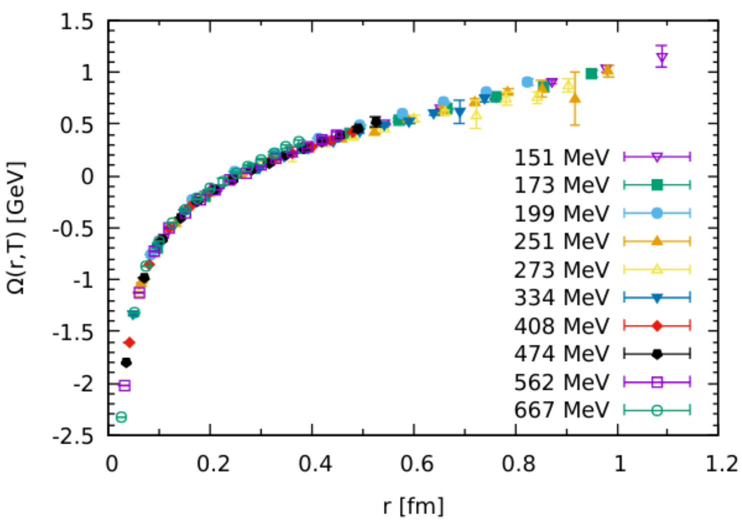
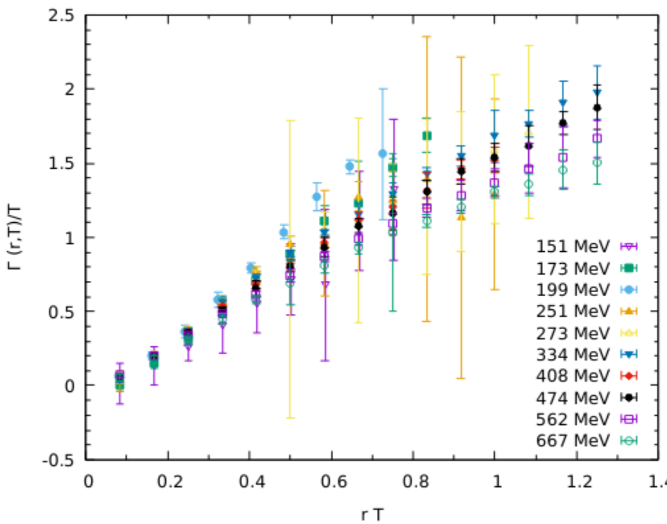
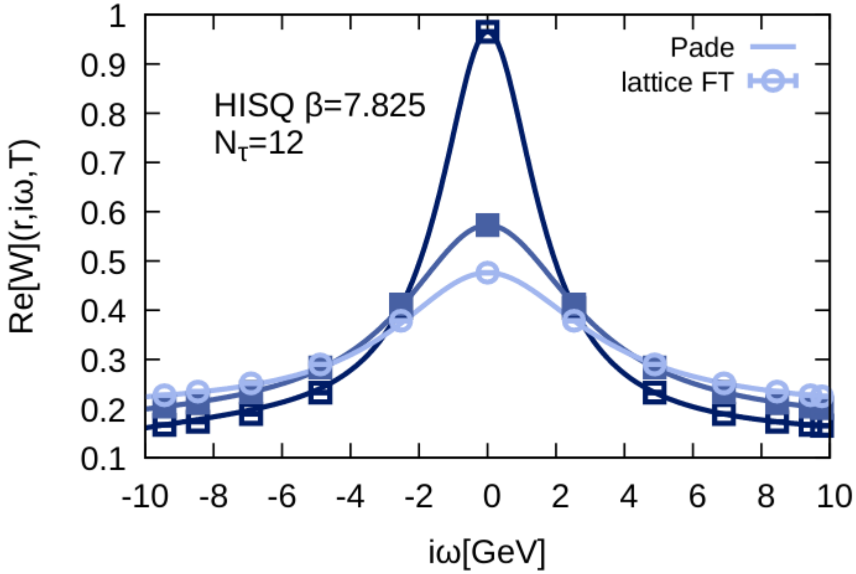




Pade' Interpolation

- Transform the Euclidean correlator into Matsubara frequency space.
- Implement Pade approximation in the form of continued fraction according to Schlessinger prescription (L. Schlessinger, Phys. Rev. 167, 1411 (1968)).
- This is interpolation of data and not fitting. Does not require minimization.
- Obtain pole structure from rational function: Directly related to the peak position (Ω) and width (Γ).



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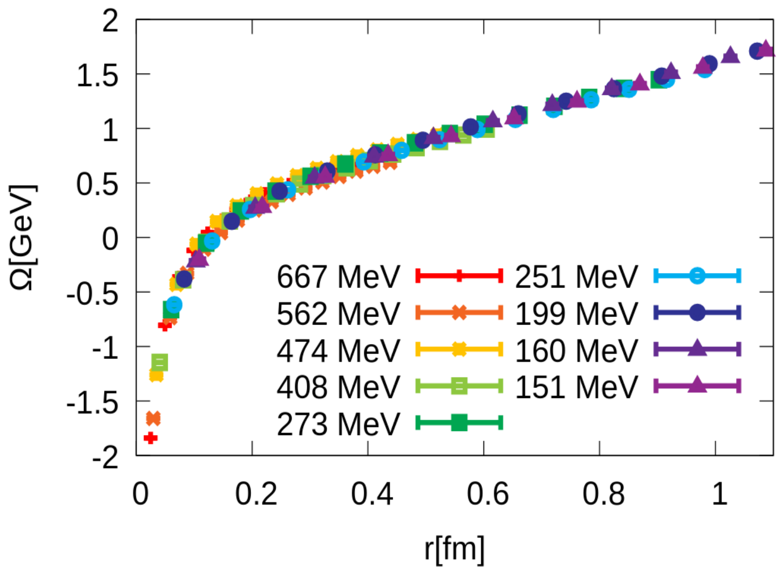
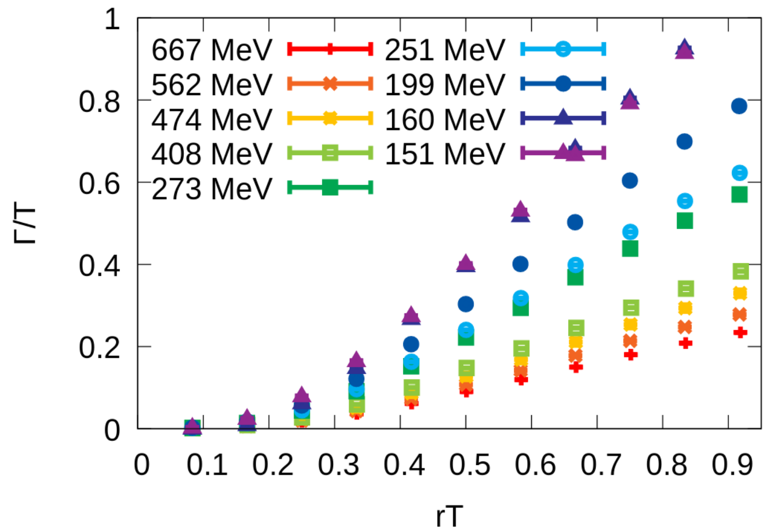




Spectral Function Extraction using Bayesian Method

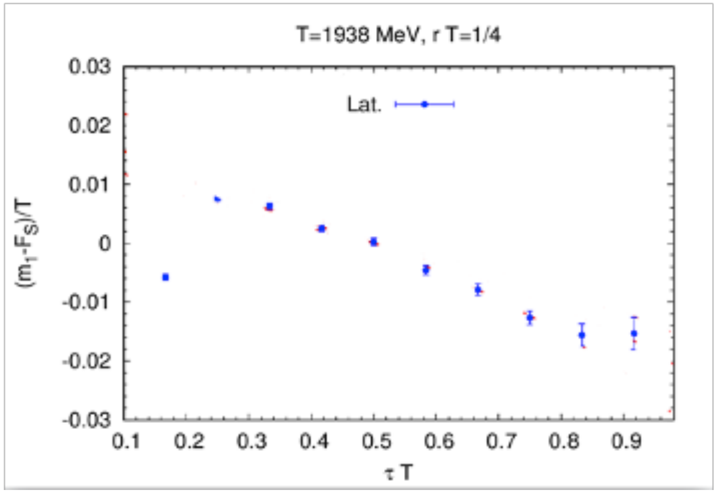
$$P[\rho|D,I] \propto P[D|\rho,I]P[\rho|I] = \exp[-L + \alpha S_{\rm BR}]$$

L is the usual quadratic distance used in chi-square fitting.

The prior probability $P(\rho|I) = \exp(\alpha S_{BR})$ acts as a regulator

$$\mathrm{S_{BR}} = \int d\omega \big(1 - \frac{\rho(\omega)}{m(\omega)} + \log\big[\frac{\rho(\omega)}{m(\omega)}\big]\big).$$

- Look for the most probably spectrum by locating the extremum of the posterior.
- Effective masses at high T show non-monotonicity at small tau; non-positive spectral function.---cannot use Bayesian Methods.



HTL inspired fits

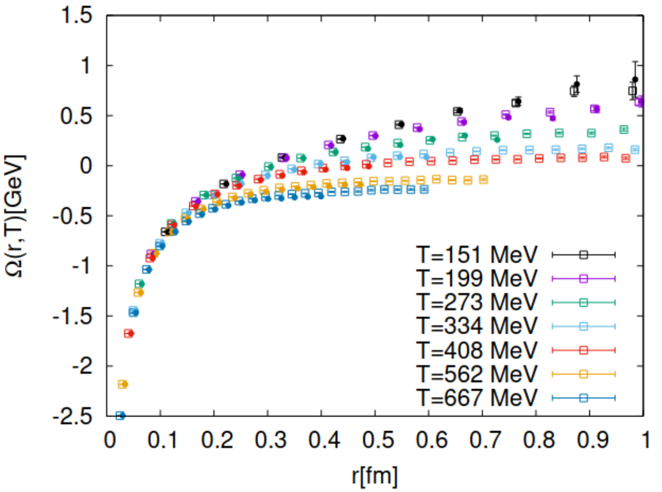
Peak position (Ω) and width (Γ) interpreted as the real and imaginary part of thermal static energy Es (D. Bala and S. Datta, Phys. Rev. D 101, 034507(2020)).

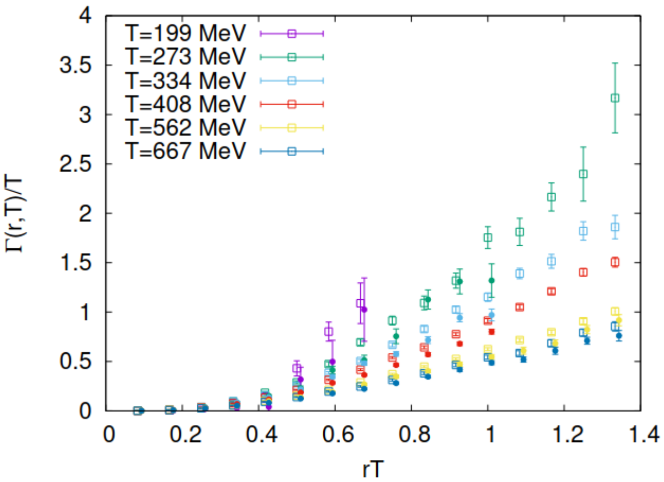
$$E_s(r,T) = \lim_{t\to\infty} i\frac{\partial \log W(r,t,T)}{\partial t} = \Omega(r,T) - i\Gamma(r,T).$$

W (r, t, T) is the Fourier transform of the spectral function ρ_r (r, ω)

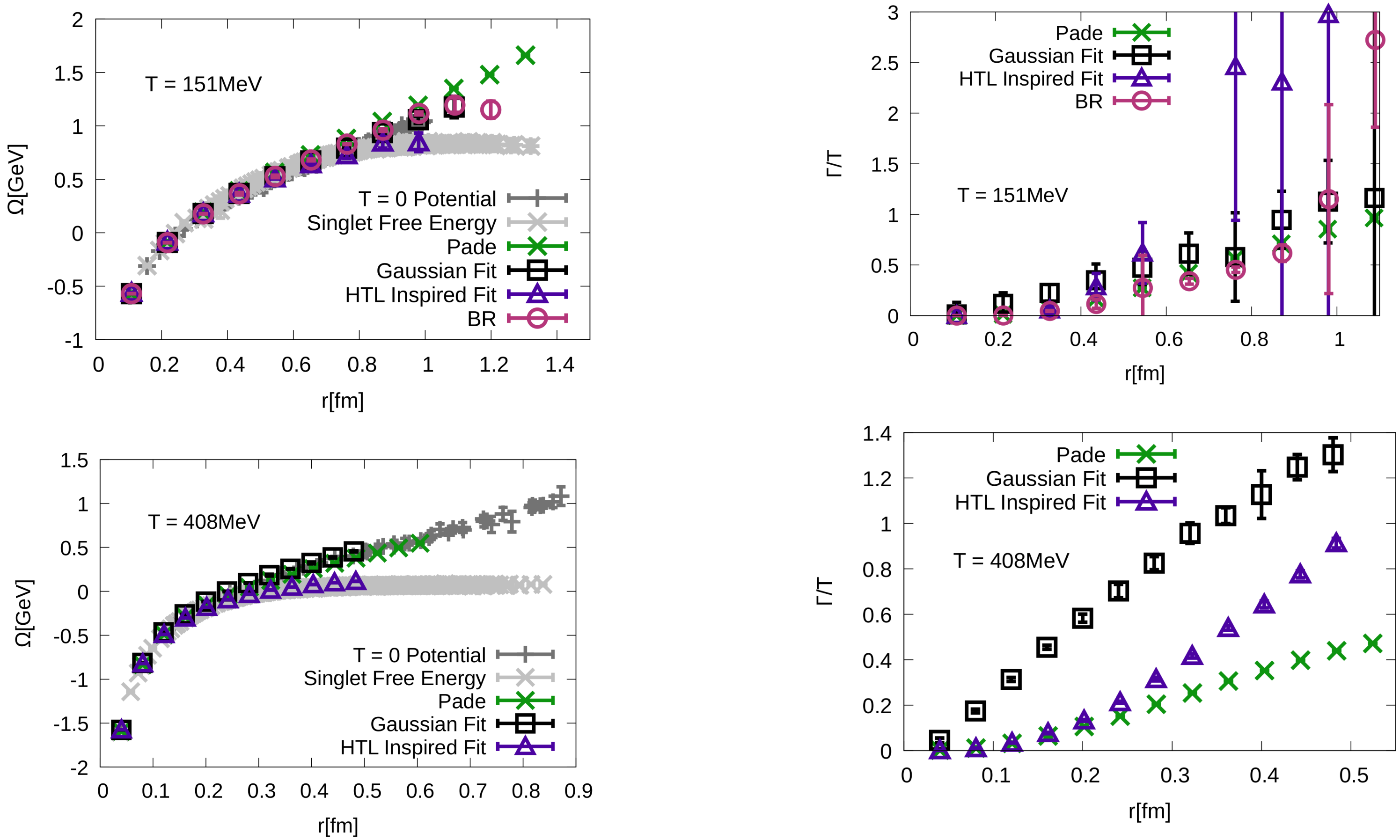
$$m_{eff}(r, n_\tau = \tau/a)a = \log\left(\frac{W(r,n_\tau,N_\tau)}{W(r,n_\tau+1,N_\tau)}\right)$$

$$= \Omega(r,T)\,a - \frac{\Gamma(r,T)aN_\tau}{\pi}\,\log\left[\frac{\sin(\pi n_\tau/N_\tau)}{\sin(\pi(n_\tau+1)/N_\tau)}\right]$$





Comparison of Results



Quenched QCD

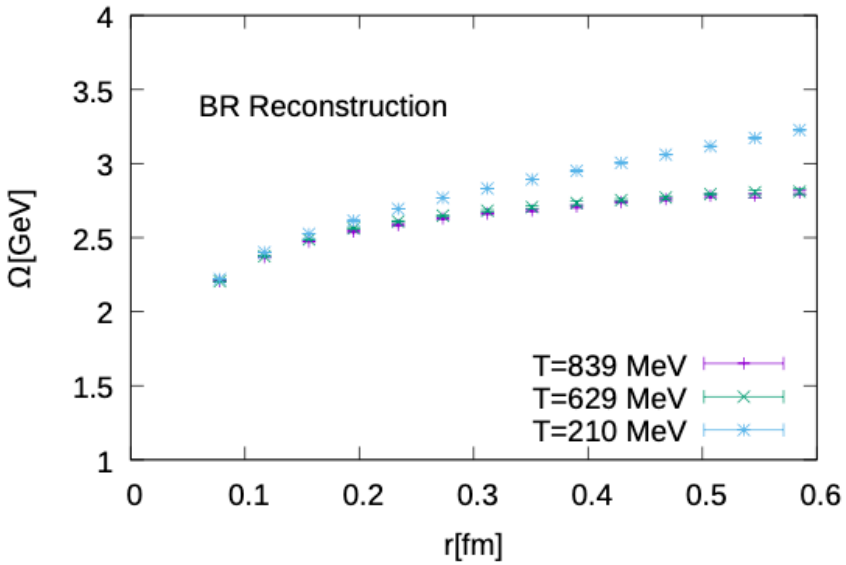
- We see that from the Gaussian fits and Pade' peak position (Ω) for HISQ Lattices is temperature independent:; Quite puzzling.
- Results obtained are very different from previous studies of Quenched Lattices (A Rothkopf, Y. Burnier 1607.04049):; Different methods used.
- Need further investigation:: Check new methods with Bayesian reconstruction.
- Check robustness of methods with new Quenched QCD lattices.

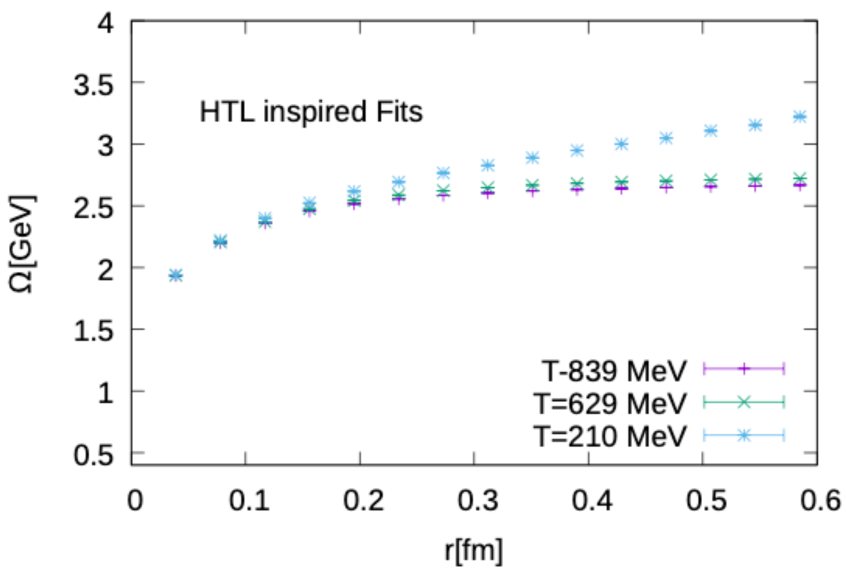
Lattice setup

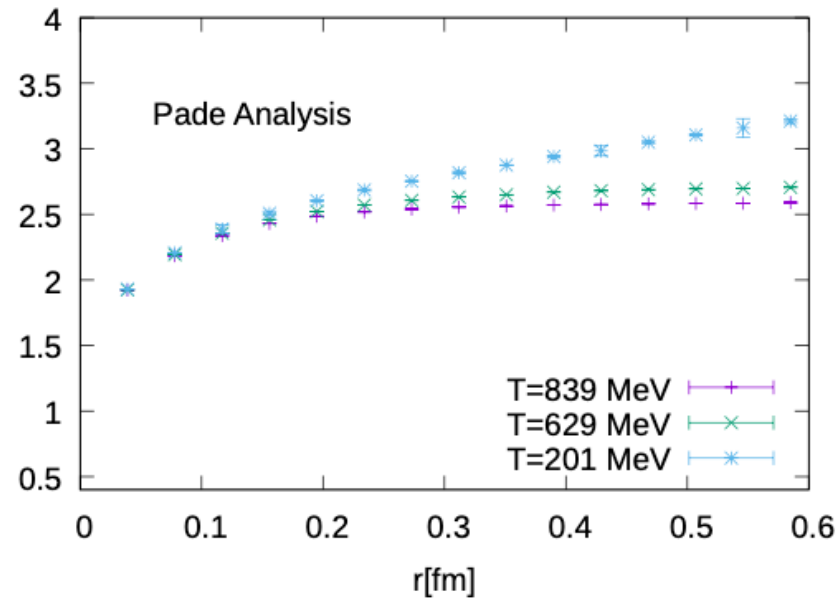
- Using anisotropic Wilson action configurations generated using OpenQCD code from FASTSUM collaboration.
- $N_s^3 \times N_\tau$ lattices. N_τ = 24,32,48,96,192 and N_s = 64.
- Use renormalized anisotropy : a_t / a_s = 4 (xi=3.5)
- Calculate Wilson Line correlator in Coulomb Gauge (simulatedQCD code).
- Fix box approach; temp range 105-839 MeV .

Preliminary Results

Tc= 270 MeV

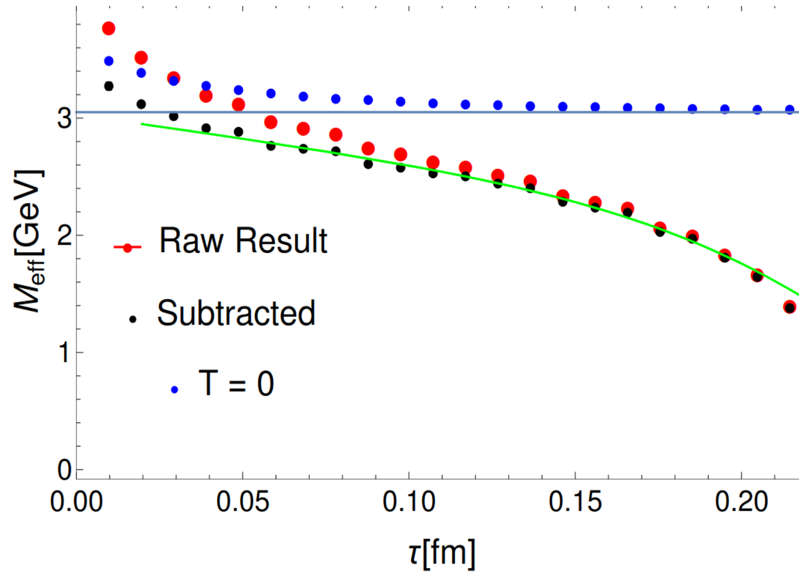






Effective masses and correlator

- M_{eff} at r/a = 12 for N_τ = 96 (blue), N_τ = 24 (T=839MeV) .
- Continuum extracted from N_τ = 96 and removed from N_τ = 24 (black).
- Lines shows exponential and gaussian plus delta function fits.
- Small τ behaviour shows T dependence.
- Cannot do Gaussian fits.



Summary

- Spectral functions of Wilson line correlators encode the real and imaginary part of the complex potential between static quark-antiquark pairs.
- We show analysis of spectral structure with four different methods.
- We see that from the Gaussian fits and Pade' peak position (Ω) is temperature independent in HISQ lattices.
- Results obtained are very different from previous quenched QCD studies.
- Peak position of new quenched study shows screening above T_c
- Quenched study is still work in progress.
- Need to understand origin of differences between quenched and dynamical QCD.