Tachyonic production of dark relics: classical lattice vs quantum 2PI in Hartree truncation

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Lattice vs 2PI Hartree

- Quantum fields out of equilibrium $\rightarrow$ Quantum transport theory.
- Out of equilibrium physics is important during inflation, early universe phase transitions, supernovae and so on...
- The evolution can be solved using 2PI Hartree methods, but how good are the solutions?

The setup: Oscillating curvature at the end of inflation can feed particle production of a non-minimally coupled dark scalar.

This presentation is based on our work in Kainulainen et al. (2024).

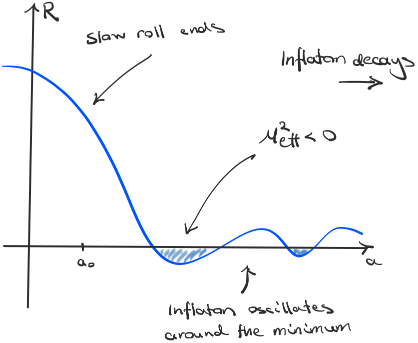
Gravitational particle production

Spectator field action

$$\mathcal{S}_\chi = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla^\mu\chi)(\nabla_\mu\chi) - \frac{1}{2}m_\chi^2\chi^2 + \frac{\xi}{2}R\chi^2 - \frac{\lambda}{4}\chi^4\right] .$$

- Inflaton ϕ with $V(\phi) = \frac{1}{2}m_\phi^2\phi^2$.
- The spectator field can form a viable component of dark matter[1].

Oscillating curvature at the end of inflation sources χ.



[1] See Markkanen and Nurmi (2017) and Fairbarn et al. (2019).

2PI Hartree in a rush

1. Evolution equations from 2PI + Truncation

2. Moment expansion + Wigner space

3. Numerical solutions + Physical quantities

$$\frac{\delta \Gamma_{2PI}[\chi, \Delta\chi]}{\delta \chi(x)} = 0$$

$$\frac{\delta \Gamma_{2PI}[\chi, \Delta\chi]}{\delta \Delta\chi(x,y)} = 0$$

$$\Downarrow$$

$$(\partial_\eta^2 + m_{\rm eff}^2)\,\sigma_\chi = 2\lambda \sigma_\chi^3$$

$$\left(\frac{1}{4}\partial_\eta^2 + |\vec{k}|^2 + m_{\rm eff}^2\right)\rho_{0k} = \rho_{2k}$$

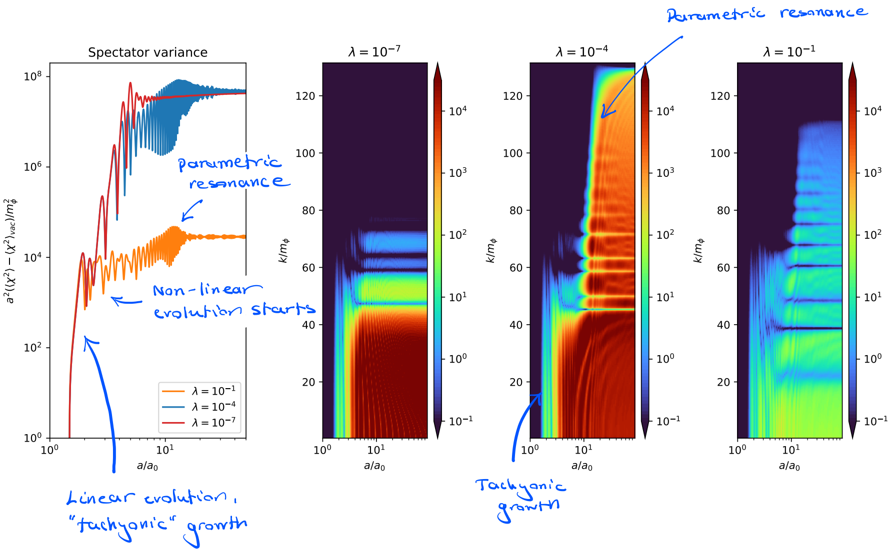
$$\partial_\eta \rho_{2k} - \frac{1}{2}\partial_\eta (m_{\rm eff}^2)\,\rho_{0k} = 0$$

$$\Downarrow$$

$n_k, \bar{n}_k, \langle T^{\mu\nu} \rangle$ etc...

See Kainulainen and Koskivaara (2021) and Kainulainen et al. (2023) for details

2PI Hartree: Particle production



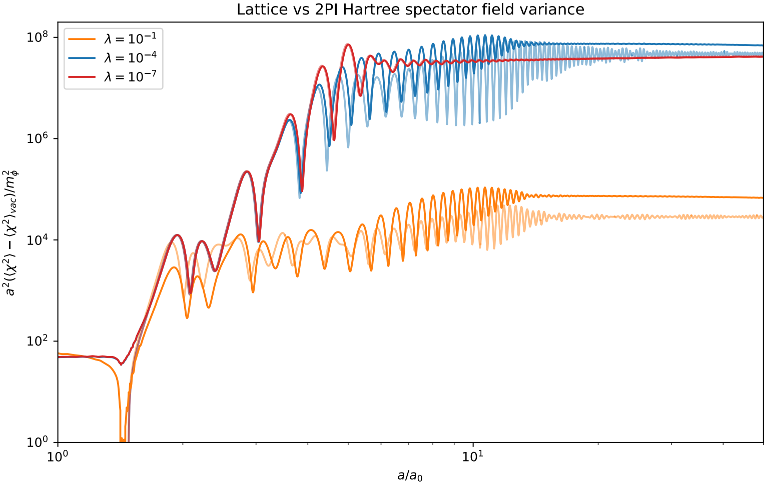
Lattice vs 2PI Hartree

How good is the Hartree approximation?

- A point of comparison from classical lattice simulations.
- The spectator scalar was evolved using a modified version of the CosmoLattice[1] code.
  - Leapfrog and 4th order velocity verlet evolvers.
  - Initialize a bit before the end of slow roll in BD-vacuum.
  - Running times: Lattice $\sim$ 1 hour, 2PI Hartree $\sim$ 2 minutes.

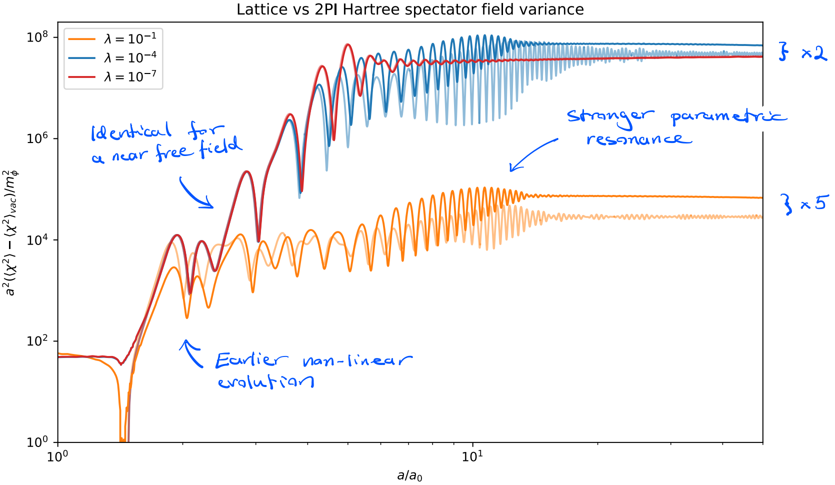
[1] `https://cosmolattice.net/`, arXiv:2006.15122, arXiv:2102.01031.

Lattice vs 2PI Hartree: Particle production



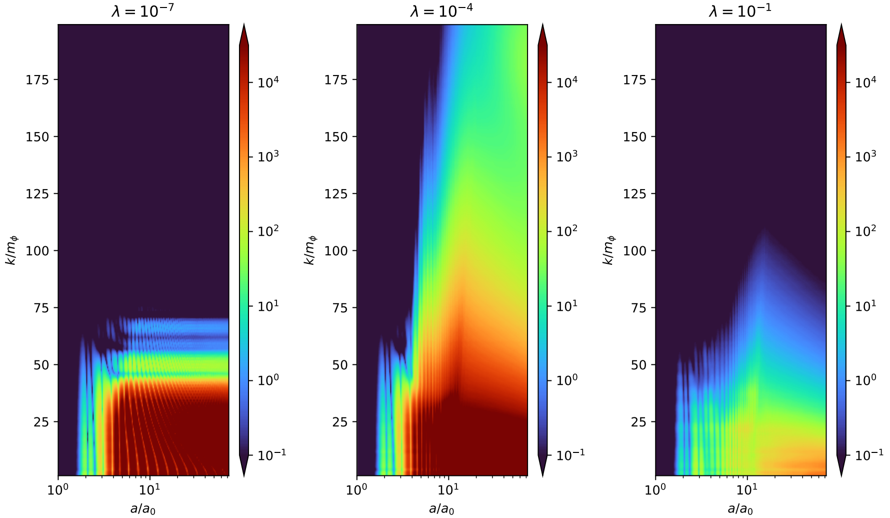
Solid = Lattice, Transparent = 2PI Hartree

Lattice vs 2PI Hartree: Particle production

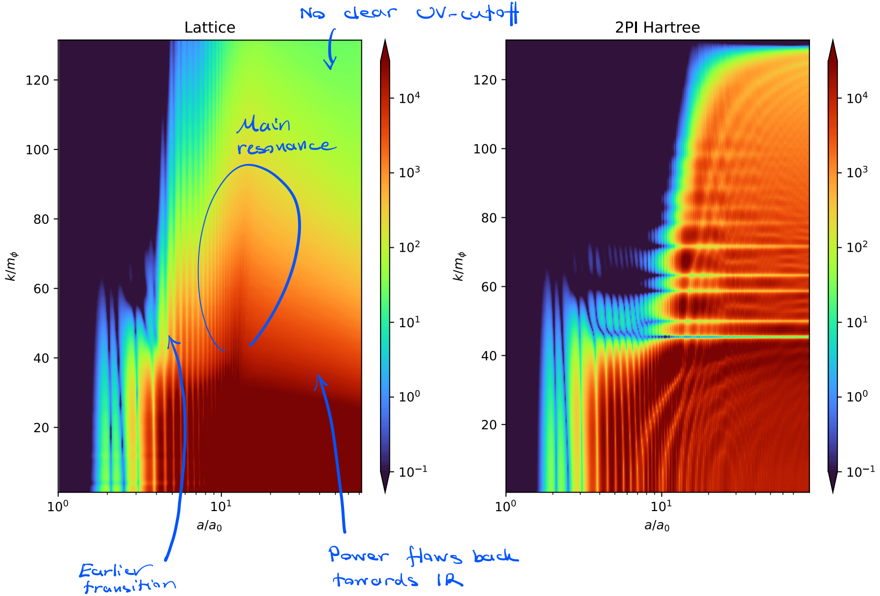


Solid = Lattice, Transparent = 2PI Hartree

Lattice: Spectra



Lattice vs 2PI Hartree spectra, $\lambda = 10^{-4}$



The next steps

- Qualitative agreement between 2PI Hartree and lattice.
- The parametric resonance is present and stronger on the lattice → Classical effect.
- The main difference is a direct coupling between different modes.

We need to go beyond Hartree for collision terms.

Evolution equations:

$$\frac{1}{4}\partial_\eta^2 \rho_0 + \omega_u^2 \rho_0 - \rho_2 = -\langle C_A \rangle$$

$$\partial_\eta \rho_1 = \langle C_H \rangle$$

$$\partial_\eta \rho_2 - \frac{1}{2}\partial_\eta(\mu^2)\rho_0 = \langle k_0 C_H \rangle$$

Collision integral:

$$\langle C \rangle = \frac{1}{2}\left(\Pi^> \otimes \Delta^< - \Pi^< \otimes \Delta^>\right)$$

$\Pi =$ 

For our setup:

$\langle C \rangle \sim 2$ momentum integrals to leading order

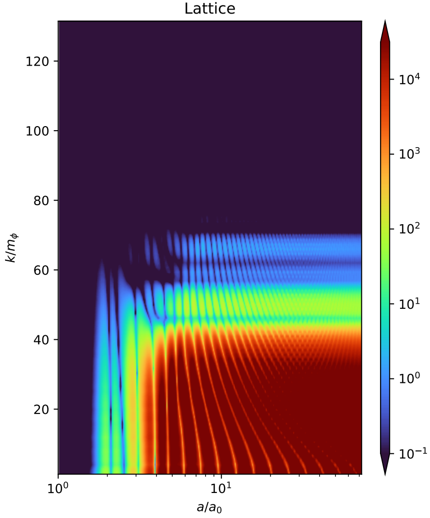
Summary

- Resonance effects cannot be ignored with gravitational particle production.
- 2PI Hartree and the lattice agree on the amount of particle production up to $\mathcal{O}(1)$.
- Hartree is not sufficient to predict the momentum distribution.

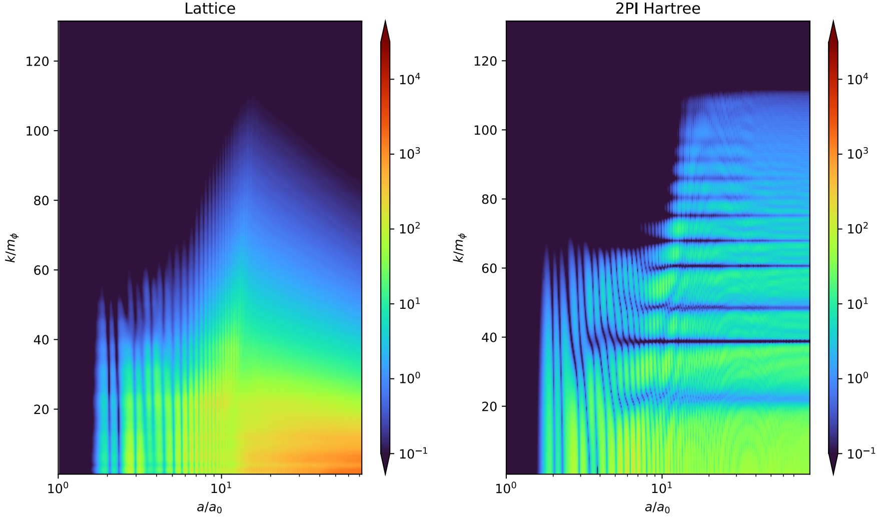
- What to expect from 2PI with collision terms?
  - Correct power spectrum + thermalization
  - Coupling to other fields, fermions (?) etc...
  - Quantify off-shell corrections
  - Hopefully still as fast as lattice

Backup-slides

Lattice vs 2PI Hartree spectra, $\lambda = 10^{-7}$



Lattice vs 2PI Hartree spectra, $\lambda = 10^{-1}$



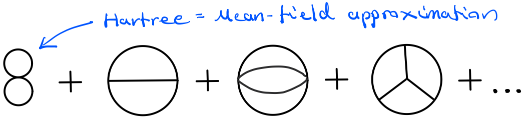
2PI Hartree calculation

- 2PI-effective action: (Δ_0 = classical propagator).

$$\Gamma_{2\mathrm{PI}}[\sigma, \Delta] = S[\sigma] - \frac{i}{2}\mathrm{Tr}_{\mathcal{C}}[\ln(\Delta)] + \frac{i}{2}\mathrm{Tr}_{\mathcal{C}}[\Delta_0^{-1}\Delta] + \Gamma_2[\sigma, \Delta],$$

where

$$\Gamma_2[\sigma, \Delta] =$$



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- Equations of motion:

$$\frac{\delta \Gamma_{2\mathrm{PI}}}{\delta \sigma} = 0, \qquad \frac{\delta \Gamma_{2\mathrm{PI}}}{\delta \Delta} = 0,$$

which result in (non-renormalized equations)

$$\left[\Box_x + \tilde{m}^2 + \lambda\sigma^2 + 3\lambda\Delta_{xx}\right]\sigma = 0, \qquad \tilde{m}^2 = a^2\left(m^2 - \xi\left(1 - \frac{1}{6}\right)R\right)$$

$$\left[\Box_x + \tilde{m}^2 + \lambda\sigma^2 + 3\lambda\Delta_{xx}\right] i\Delta_{xy}^{ab} = \delta^{ab}\delta^{(4)}(x-y).$$

2PI Hartree-calculation

- Wigner transform and write the moment equations:

$$\left(\partial_\eta^2 + M_{\text{eff}}^2\right)\sigma_\chi = 2\lambda\sigma_\chi^3,\ \sigma_\chi \equiv a\chi.$$

$$\left(\frac{1}{4}\partial_\eta^2 + |\mathbf{k}|^2 + M_{\text{eff}}^2\right)\rho_{0\mathbf{k}} = \rho_{2\mathbf{k}},\ \rho_{n\mathbf{k}} \equiv \int \frac{dk_0}{2\pi} k_0^n \Delta_{\mathbf{k}}^{<}(k_0,\eta),$$

$$\partial_\eta X_{\mathbf{k}} = 0,\ X_{\mathbf{k}} = 2\rho_{0\mathbf{k}}\rho_{2\mathbf{k}} - \left(|\mathbf{k}|^2 + M_{\text{eff}}^2\right)\rho_{0\mathbf{k}}^2 - \frac{1}{4}(\partial_\eta \rho_{0\mathbf{k}})^2.$$

- The effective mass:

mean-field contribution

$$M_{\text{eff}}^2 = a^2 m_\chi^2 - a^2\left(\xi - \frac{1}{6}\right)R + 3\lambda a^2\langle\chi\rangle^2 + 3\lambda a^2\left(\langle\chi^2\rangle - \langle\chi^2\rangle_{\text{vac}}\right)$$

$$+ \frac{3\lambda}{16\pi^2}\left[M_{\text{eff}}^2 \ln\left(\frac{M_{\text{eff}}^2}{a^2 m_\chi^2}\right) - M_{\text{eff}}^2 + a^2 m_\chi^2\right].$$

Collision terms

$$\frac{\delta\Gamma[\sigma,\Delta]}{\delta\sigma}=0$$

Mean field equation

$$(\Delta_0^{-1}-\Pi_H)\otimes\Delta_H+\Gamma\otimes\mathcal{A}=0$$

$$(\Delta_0^{-1}-\Pi_H)\otimes\mathcal{A}+\Gamma\otimes\Delta_H=0$$

} Pole equations

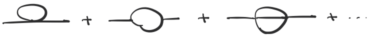
Collision integral

$$(\Delta_0^{-1}-\Pi_H)\otimes\Delta^{<}-\Pi^{<}\otimes\Delta_H=\frac{1}{2}\left(\Pi^{>}\otimes\Delta^{<}-\Pi^{<}\otimes\Delta^{>}\right)$$

↑ Kadanoff-Baym equation

Self-energy:

$$\Pi=\frac{\delta\Gamma_{int}}{\delta\Delta}=$$



An example contribution to the collision integral:

$$\otimes\Delta\sim\int_w\Delta(u,w)^3\Delta(w,v)=\dots=\text{integral involving moments}$$

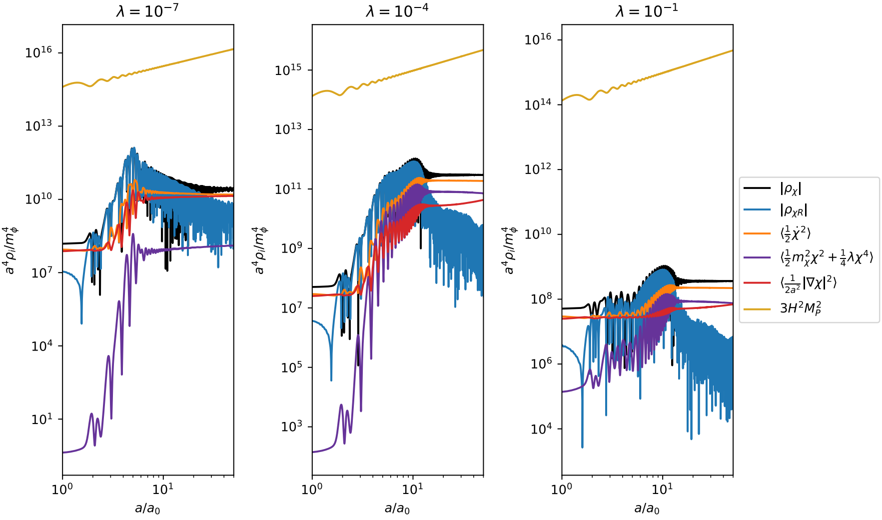
Memory integral. Needs a suitable approximation.

A fermionic case is discussed in Kainulainen and Parkkinen (2024) [Preprint].

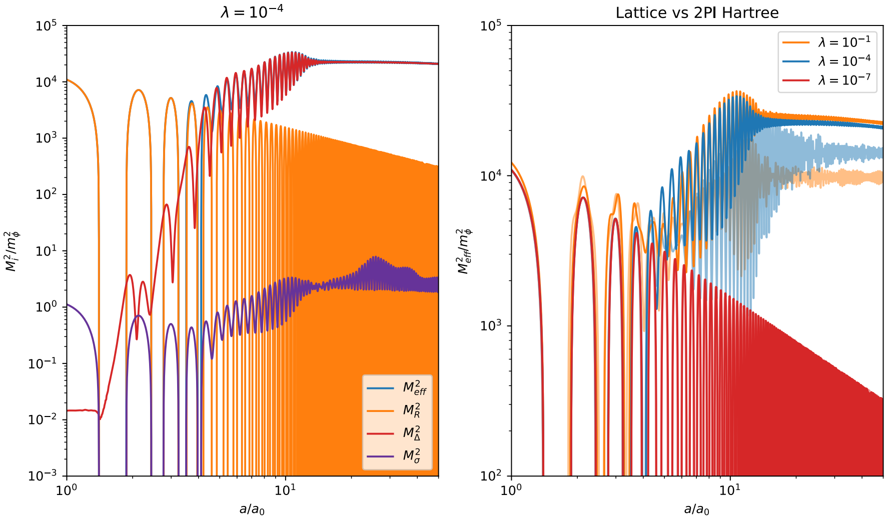
Some lattice details

- Main modifications to CosmoLattice:
  - Inflaton sector was calculated separately and a was input to CosmoLattice.
  - Added the extra R-term to the CosmoLattice evolution kernel.
  - Modified the energy calculation to include the non-minimal coupling.
- Evolution was calculated using leapfrog method. A higher order verlet integrator was used for comparison for select runs.
- We used an integration box with the size H_0^{-1}, with the grid size $N = 512$. Larger grids were used for testing.
- Initial state was the continuum Bunch-Davies vacuum corrected for the non-minimal coupling.
- The evolution is reliable up to $a/a_0 \approx 30$. Beyond this the solution develops resolution-dependent effects.

Energy components on the lattice



Lattice vs 2PI Hartree: Effective masses



Right panel: Solid = Lattice, Transparent = 2PI Hartree