The effect of the large-scale structure on cosmological parameter extraction

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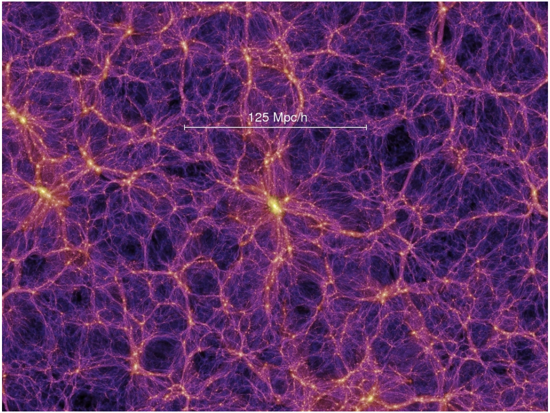
An inhomogeneous universe

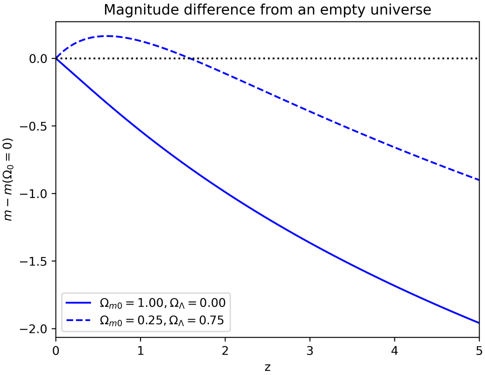
- Cosmological principle: On large enough length scales the universe is homogeneous and isotropic.
- This breaks down at the about 100 Mpc.
- How does this affect the metric of the spacetime?
- How does this affect observable cosmological parameters?



Source: Millenium Simulation, (Springel et al. 2005)

Extracting cosmological parameters

- Most observations are received via electromagnetic radiation.
- An important example: The luminosity distance -redshift relation.
- Inhomogeneities can be treated using various approximations...
- ...but we are entering the era of precision cosmology.



What now?

Questions:

- How do inhomogeneities at different scales and geometries affect the observed luminosity distances and redshifts?
- How do common approximation methods compare to a full relativistic calculation?

How hard can it be?

1. Solve the metric from the Einstein equations.
2. Calculate the observed redshifts and luminosity distances by tracing light rays in this background metric.
3. Comparison with observations and other methods, constraints on the inhomogeneities.

Solving the Einstein equations

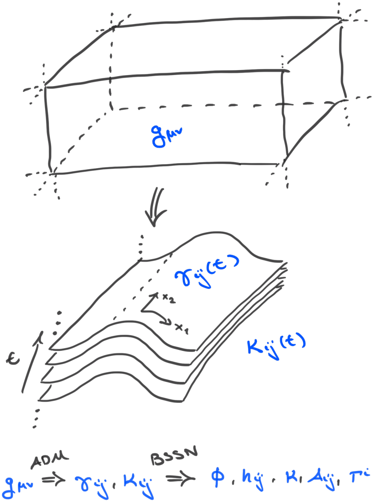
- The Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

  in their usual form cannot be solved on a computer.

- The equations must be expressed as an initial value problem. The most common approach is the ADM-formalism.

- Stability issues force various transformations of equations known as the BSSN-formalism.



Solving the Einstein equations

$$ds^2 = -dt^2 + e^{4\phi}\bar{\gamma}_{ij}dx^i dx^j$$

Synchronous gauge

$$\partial_t \phi = -\frac{1}{6}K$$

$$\partial_t K = \bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}K^2 + 4\pi\rho$$

$$\partial_t \bar{\gamma}_{ij} = -2\bar{A}_{ij}$$

$$\partial_t \bar{A}_{ij} = e^{-4\phi}R_{ij}^{TF} + K\bar{A}_{ij} - 2\bar{A}_{ik}\bar{A}^k{}_j$$

$$\partial_t \bar{\Gamma}^i = 2\bar{\Gamma}^i_{jk}\bar{A}^{kj} - \frac{4}{3}\bar{\gamma}^{ij}\partial_j K + 12\bar{A}^{ij}\partial_j\phi$$

$$\dot{\rho} = K\rho.$$

Evolution equations

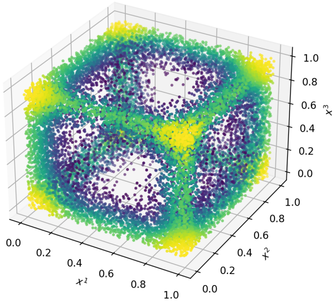
$$\bar{\gamma}^{ij}\bar{D}_i\bar{D}_j e^{\phi} - \frac{e^{\phi}}{8}\bar{R} + \frac{e^{5\phi}}{8}\bar{A}_{ij}\bar{A}^{ij} - \frac{e^{5\phi}}{12}K^2 = -2\pi e^{5\phi}\rho.$$

$$\bar{D}_j\bar{A}^{ij} = -6\bar{A}^{ij}\partial_j\phi + \frac{2}{3}\bar{\gamma}^{ij}\partial_j K.$$

Constraint equations

Initial conditions

- Obtained by solving the linearized constraint equations at $z = 100$.
- Sufficient resolution can be achieved by demanding cubic lattice symmetries.
- For now, we assume energy content is just cold dark matter.



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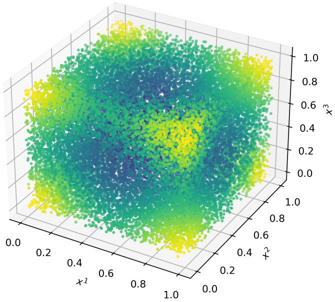


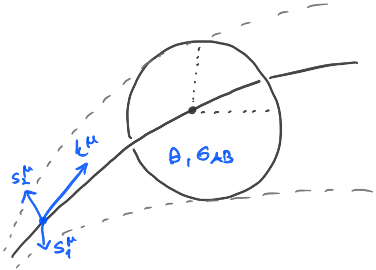
Solving the trajectories of light rays

- The light trajectories and redshift are solved from the geodesic equation.
- Luminosity distance and apparent magnitude:

$$D_L := \sqrt{\frac{L}{4\pi F}}$$

$$m = M + 5\log_{10}\frac{D_L}{\mathrm{Mpc}} + 25$$

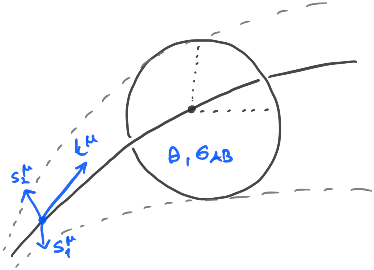
- Luminosity distances can be calculated by considering congruences of nearby geodesics.



Solving the trajectories of light rays

$$\frac{dx^i}{dt} = (k^0)^{-1} k^i$$

$$k^\mu \nabla_\mu k^\nu = 0$$



$$D_L = (1+z)^2 D_A$$

$$\frac{d}{du} \ln D_A = \frac{1}{2}\theta$$

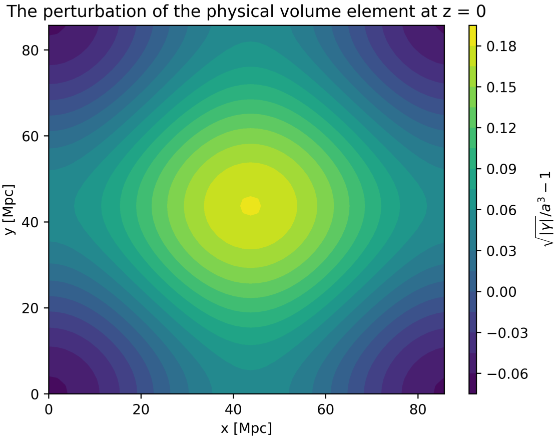
$$\frac{d\theta}{du} = -\frac{1}{2}\theta^2 - 2\sigma^2 - R_{\mu\nu} k^\mu k^\nu$$

$$\frac{d}{du}\sigma_{AB} = -\theta\sigma_{AB} - \left[R_{\alpha\mu\beta\nu} s_A^\alpha s_B^\beta k^\mu k^\nu - \frac{1}{2}\delta_{AB} R_{\mu\nu} k^\mu k^\nu \right]$$

$$k^\mu \nabla_\mu s_A^\nu = 0$$

An example calculation

- Sinusoidal configuration with the length scale ≈ 100 Mpc
- End density contrast ≈ 10 %, almost linear evolution.
- Work in progress, take any numbers with a grain of salt.



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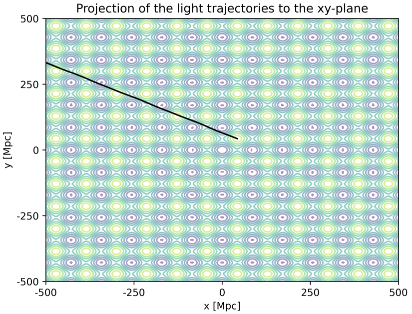

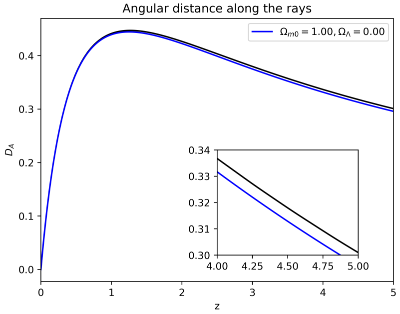

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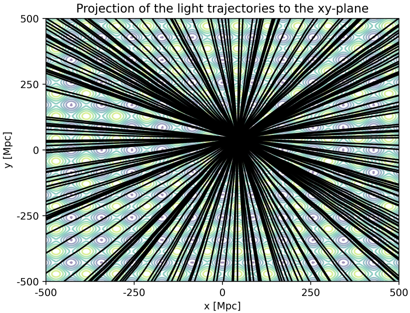

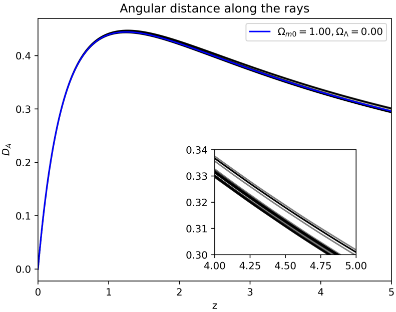

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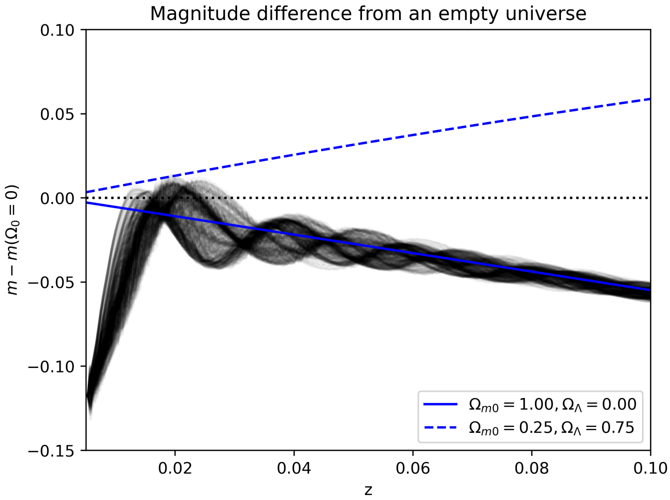


What is to come?

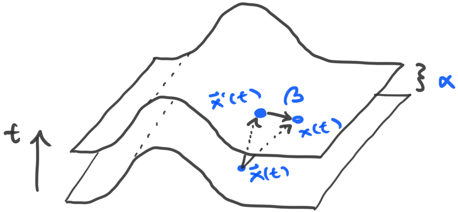
- Literature so far has studied mostly very symmetric models, such as LTB-models. Do the results differ for less symmetric spacetimes?
- The effects of at the scale of 100 Mpc appear small. What about larger structures?
- How much dark energy changes the picture? Initial estimates can be obtained using ΛLTB-models.
- What are the effects on the CMB-dipole?

Thank you

Backup slides

ADM-formalism

$$
\begin{aligned}
\partial_t \gamma_{ij} &= -2\alpha K_{ij} + D_i \beta_j + D_j \beta_i, \\
\partial_t K_{ij} &= -D_i D_j \alpha + \alpha (R_{ij} - 2K_{ik} K^k{}_j + K K_{ij}) - 8\pi\alpha (S_{ij} - \frac{1}{2}\gamma_{ij}(S - \tilde{\rho})) \\
&\quad + \beta^k D_k K_{ij} + K_{ik} D_j \beta^k + K_{jk} D_i \beta^k, \\
R + K^2 - K_{ij} K^{ij} &= 16\pi\tilde{\rho}, \\
D_j (K^{ij} - \gamma^{ij} K) &= 8\pi S^i.
\end{aligned}
$$



BSSN-formalism

- Discretizations of the ADM-equations are numerically unstable due to mixed second derivatives in the Ricci tensor.
- The solution is to cancel the problem terms by adding zero in the form of a suitable multiple of the constraint equations. This involves introducing an auxiliary field $\bar{\Gamma}^i = \bar{\gamma}_{jk}\bar{\Gamma}^i_{jk}$.
- In addtion, stability is improved by conformally transforming the metric.

$$\gamma_{ij} \to e^{4\phi}\bar{\gamma}_{ij}$$

$$K_{ij} \to e^{4\phi}\left(\bar{A}_{ij} + \frac{1}{3}\bar{\gamma}_{ij}K\right)$$

BSSN-formalism

$$\partial_t \phi = -\frac{1}{6}\alpha K + \frac{1}{6}\partial_i \beta^i + \beta^k \partial_k \phi$$

$$\partial_t K = -D^2 \alpha + \alpha(\bar{A}_{ij}\bar{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\tilde{\rho} + S) + \beta^i \partial_i K$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \bar{A}_{ij} - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k - 4\bar{\gamma}_{ij}\beta^k \partial_k \phi + e^{-4\phi}(D_i \beta_j + D_j \beta_i)$$

$$\partial_t \bar{A}_{ij} = e^{-4\phi}\left((-D_i D_j \alpha)^{TF} + \alpha(R_{ij}^{TF} - 8\pi S_{ij}^{TF})\right) + \alpha(K\bar{A}_{ij} - 2\bar{A}_{ik}\bar{A}^k{}_j) + \beta^k \partial_k \bar{A}_{ij} + \bar{A}_{ik}\partial_j \beta^k + \bar{A}_{jk}\partial_i \beta^k - \frac{2}{3}\bar{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -\bar{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\bar{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\bar{A}^{ij}\partial_j \phi\right) + \beta^j \partial_j \bar{\Gamma}^i - \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{ki}\partial_k \partial_j \beta^j + \bar{\gamma}^{kj}\partial_k \partial_j \beta^i$$

Linearized initial conditions

- Choose $K =$ constant and $\bar{A}_{ij} = 0$ at the initial time. This satisfies the momentum constraint. Also choose $h_{ij} = 0$ at the initial timeslice.
- The Hamiltonian constraint in the synchronous gauge:

$$\nabla^2 \psi = \left(\frac{1}{12} K^2 - 2\pi\rho \right) \psi^5,$$

  where $\psi = e^{\phi}$.
- Expanding linearly around the flat FLRW-background results in

$$\nabla^2 \psi = -\frac{3}{4} \dot{a}^2 \delta$$

- Consider a sinusoidal density perturbation $\delta \propto \sin(2\pi x/L)$. The resulting metric perturbation is $\delta\psi \propto L^2 \sin(2\pi x/L)$.