Hard probes in the Glasma

Simulating jets and heavy quarks in the early stages of heavy-ion collisions using colored particle-in-cell methods

by Dana Avramescu

University of Jyväskylä, Center of Excellence in Quark Matter

in collaboration with V. Băran, V. Greco, A. Ipp, T. Lappi, H. Mäntysaari, D. I. Müller, M. Ruggieri

Particle Physics Day, Helsinki, 24 November 2022

General outline

1. Introduction

   Initial stage of heavy-ion collisions • Features of the Glasma

2. Motivation

   Effect of Glasma on hard probes • Context in the literature

3. Methodology

   Glasma numerical implementation • Colored particle-in-cell solver

4. Results

   Momentum broadening • Heavy quarks probing the Glasma

5. Conclusion

   Summary of results

Initial stage for heavy-ion collisions

- Heavy-ion collision $\leftrightarrow$ multi-stage process with each stage $\mapsto$ effective theory
- Initial stage using Color Glass Condensate $\mapsto$ EFT for high energy QCD

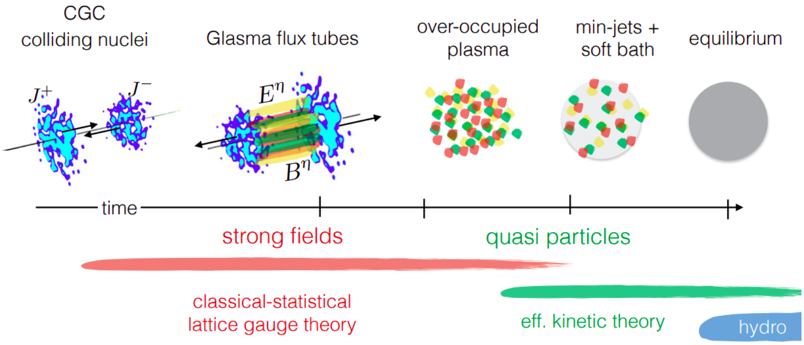

Figure from S. Schlichting talk @ Initial Stages 2016 [Equilibration in weak coupling approaches]

Classical colored fields

- High energy nucleus $\rightarrow$ many gluons $\Leftrightarrow$ high occupation numbers for gluon fields $\Rightarrow$ classical colored fields

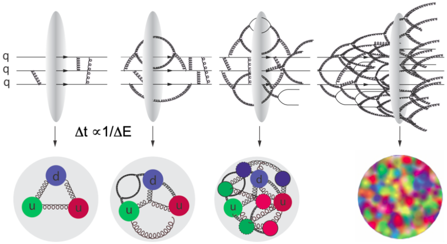

Figure from F. Salazar's talk @ INT program
[Probing QCD at High Energy and Density with Jets]

- Classical Yang-Mills field equations

covariant derivative

field strength tensor

gluons gauge field

$$\left(\mathcal{D}_\mu \; F^{\mu\nu}\right)\left[A^\mu\right] = J^\nu$$

color current of nucleus

- MV model for color charges $\mapsto J^\mu$
- Color electromagnetic fields $\equiv$ Glasma

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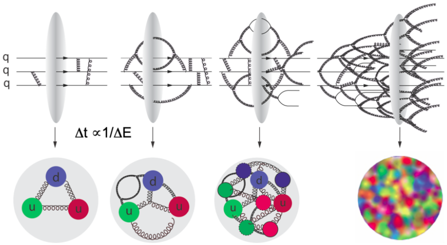

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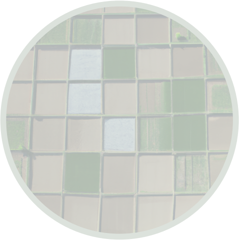
Any field

Colored field

Lattice field



Fields images generated using DALL·E 2 OpenAI

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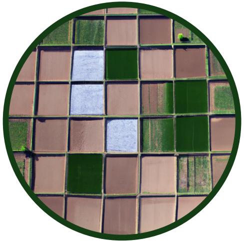

Fields images generated using DALL·E 2 OpenAI

...but a very particular color field

Monet field

Van Gogh field

Klimt field

Fields paintings generated using DALL·E 2 OpenAI

Features of the Glasma fields

- Strongly coupled $\Rightarrow$ non-linear regime, out-of-equilibrium classical colored fields
- Gluon saturation built in: the saturation momentum Q_s $\rightarrow$ the only physical parameter, here $Q_s = 2\,\mathrm{GeV}$
- Fields become dilute after $\delta\tau \simeq Q_s^{-1}$
- Fields arrange themselves in correlation domains of transverse size $\delta x_T \simeq Q_s^{-1}$
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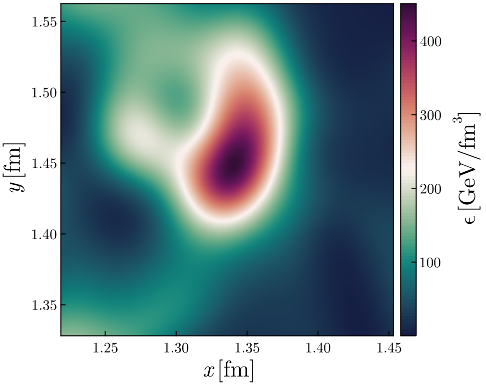

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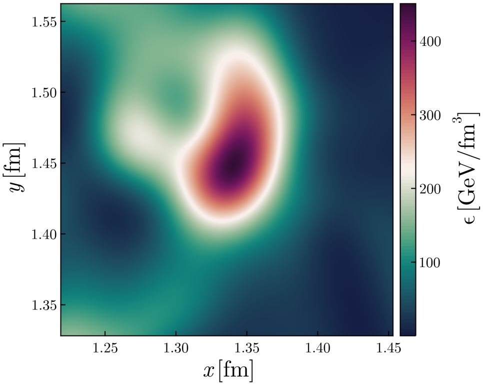

Motivation

Glasma fields

hard probes

- *Question:* What is the effect of initial stage on the early-produced partons ?
- *Prerequisite:* Glasma fields numerically solved using real-time lattice gauge theory
- *Task:* Develop a colored particle-in-cell solver for particles in Glasma background fields
  Inspired by the colored particle-in-cell method for solving the equations of motion of particles interacting with Yang-Mills fields
- *Goal:* Systematic study the impact of the Glasma stage on heavy quarks and jets
  Quantifiable by evaluating momentum broadening δp^2 and transport coefficients $\hat{q}$ for jets and κ for heavy quarks in the Glasma

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Glasma fields ↓ initial stage; hard probes ↓ early-produced partons

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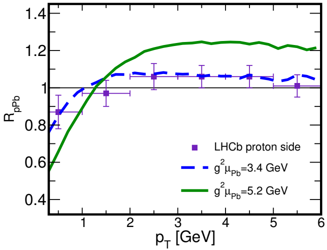
Literature breadcrumbs

- 2018 — M. Ruggieri *et al.* [arXiv:1805.09617]
  Heavy quarks probing the Glasma in p-Pb collisions
- 2020 — Jet momentum broadening in the pre-equilibrium Glasma
- 2020 — Heavy quark diffusion in an overoccupied gluon plasma
- 2022 — Transport of hard probes through glasma
- 2022 — Momentum broadening of heavy quarks and jets in the Glasma

- Heavy quarks in Glasma
- Diffusion, momentum broadening, nuclear modification factor



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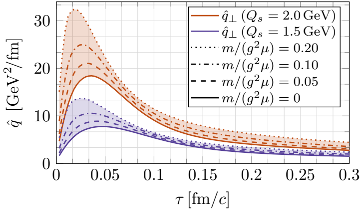
- Glasma solved on the lattice
- Momentum broadening and $\hat{q}$ for light-like jets



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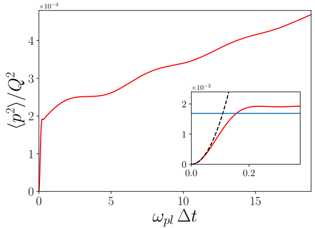
2022 Momentum broadening of heavy quarks and jets in the Glasma

- Yang-Mills fields on the lattice
- Momentum broadening and κ for static heavy quarks



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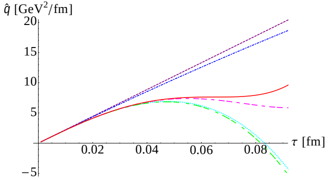
2022 — M. Carrington *et al.* [arXiv:2202.00357]
Transport of hard probes through glasma

2022 — Momentum broadening of heavy quarks and jets in the Glasma

- Glasma in τ-expansion, hard probes transport using Fokker-Planck
- Jet quenching $\hat{q}$ in Glasma



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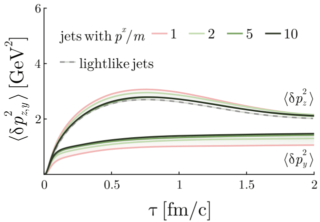
- 2020 — Heavy quark diffusion in an overoccupied gluon plasma
- 2022 — Transport of hard probes through glasma
- 2022 — D. Avramescu *et al.* [arXiv:2208.04781] Momentum broadening of heavy quarks and jets in the Glasma

- Glasma and full particle dynamics solved on the lattice
- Realistic heavy quarks and jets



Glasma on the lattice

- Boost-invariant equations of motion

$$\frac{1}{\tau}\mathcal{D}_i\partial_\tau A^i + \mathrm{i}g\tau A^\eta \partial_\tau A^\eta = 0$$
$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A^i - \mathrm{i}g\tau^2 A^\eta \mathcal{D}_i A^\eta - \mathcal{D}_j F_{ji} = 0$$
$$\frac{1}{\tau^2}\partial_\tau \tau^2 \partial_\tau A^\eta - \mathcal{D}_i(\mathcal{D}_i A^\eta) = 0$$

- Glasma initial conditions

$$A^i(\tau, \vec{x}_\perp)\Big|_{\tau=0} = A_1^i(\vec{x}_\perp) + A_2^i(\vec{x}_\perp)$$

$$A^\eta(\tau, \vec{x}_\perp)\Big|_{\tau=0} = \frac{\mathrm{i}g}{2}[A_1^i(\vec{x}_\perp), A_2^i(\vec{x}_\perp)]$$

- Wilson lines on the lattice $\leftrightarrow$ gauge links $\mathrm{W}_{x,\mu} = \exp\{igaA_\mu(x)\}$
- Wilson loops on lattice $\leftrightarrow$ plaquettes $\mathrm{W}_{x,\mu\nu} \equiv \mathrm{W}_{x,\mu}\mathrm{W}_{x+\mu,\nu}\mathrm{W}^\dagger_{x+\mu,\mu}\mathrm{W}^\dagger_{x,\nu}$

- Glasma lattice implementation with plaquettes only in the transverse plane

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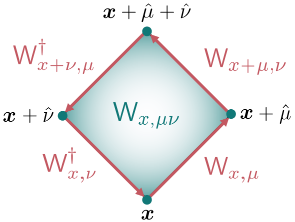

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Particle solver

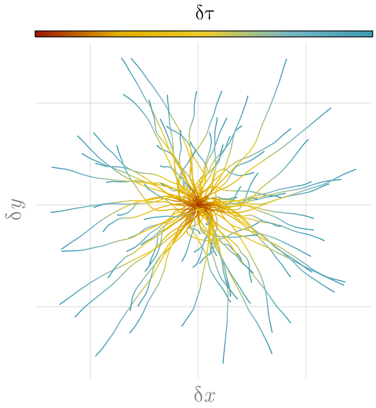
- Wong's equations $\leftrightarrow$ classical equations of motion for particles (x^μ, p^μ, Q) evolving in Yang-Mills fields A^μ

$$\underbrace{m}_{\text{mass}}\frac{\mathrm{d}\overbrace{x^\mu}^{\text{coordinate}}}{\mathrm{d}\underbrace{\tau}_{\text{proper time}}} = p^\mu, \qquad m\frac{\mathrm{D}\overbrace{p^\mu}^{\text{momentum}}}{\mathrm{d}\tau} = \underbrace{g}_{\text{coupling constant}}\frac{1}{T_R}\mathrm{Tr}\left\{\overbrace{Q}^{\text{color charge}} F^{\mu\nu}[A^\mu]\right\}p_\nu, \qquad \underbrace{m}_{\text{mass}}\frac{\mathrm{d}Q}{\mathrm{d}\tau} = -\mathrm{i}g[\overbrace{A_\mu}^{\text{gauge field}}, Q]\,p^\mu$$

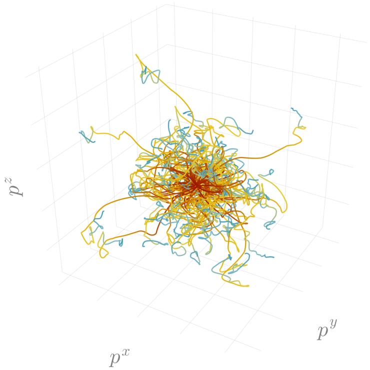
where $T_R = 1/2$ for quarks in the fundamental representation and $D/\mathrm{d}\tau$ is the covariant derivative in curvilinear coordinates

Glasma spaghetti and noodles

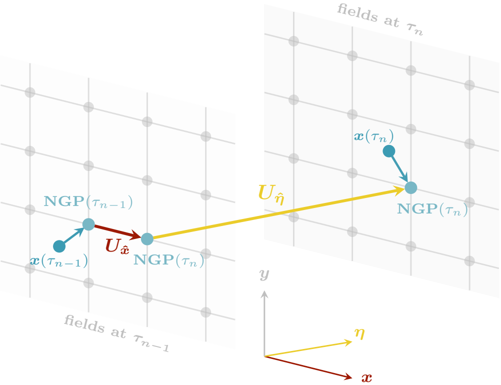
- Glasma spaghetti trajectories



- Glasma noodles momenta evolution



Color rotation on the lattice



- Lattice rotation of color charge inspired by the colored particle-in-cell method

$$Q(\tau_n) = \mathcal{U}(\tau_{n-1}, \tau_n)\, Q(\tau_{n-1})\, \mathcal{U}^\dagger(\tau_{n-1}, \tau_n)$$

with the Wilson line constructed as

Transverse gauge link

$$\mathcal{U}(\tau_{n-1}, \tau_n) = U_{x_{n-1}, \hat{x}} \cdot U_{x_{n-1}, \hat{\eta}}$$

Rapidity gauge link

- Symplectic solver which assures $Q \in \mathrm{SU(N)}$ and conservation of Casimir invariants

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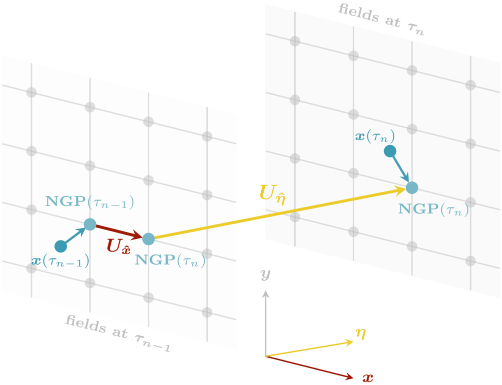

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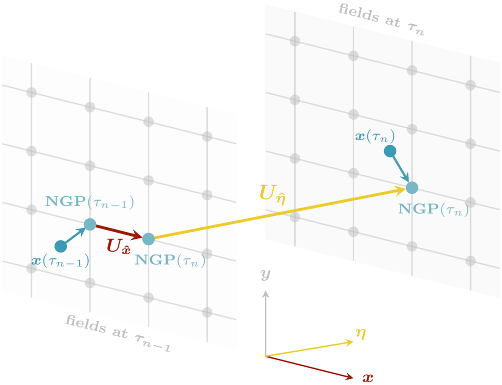

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Quantifying the effect of Glasma

- Momentum broadening $\leftrightarrow$ measure for the accumulated momentum of a probe in Glasma

$$\delta p_\mu^2(\tau) \equiv p_\mu^2(\tau) - p_\mu^2(\tau_{\text{form}})$$

- Derivative of momentum broadening $\leftrightarrow$ instantaneous transport coefficient

$$\kappa_{L,T}(\tau) \equiv \frac{\mathrm{d}}{\mathrm{d}\tau}\langle \delta p_{L,T}^2(\tau)\rangle$$

- Anisotropy transfer anisotropic Glasma $\mapsto$ hard probes

$$\text{heavy quark anisotropy} \equiv \frac{\langle \delta p_L^2\rangle}{\langle \delta p_T^2\rangle}$$

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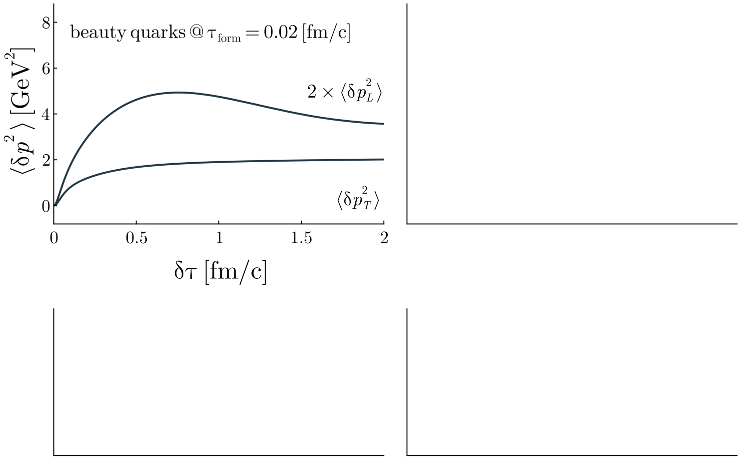
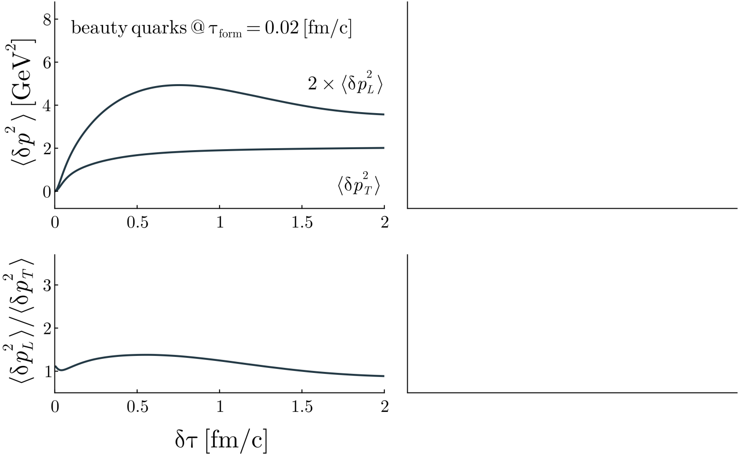
Heavy quark momentum broadening



- Longitudinal and transverse momentum broadening for beauty quarks with initial p_T
- Heavy quark anisotropy
- Dynamical quarks $\rightarrow$ finite mass, initial $p_T \in [0, 10]$ GeV
- Static quarks $\rightarrow$ infinitely massive $\Rightarrow \langle \delta p^2 \rangle \propto \langle EE \rangle$
- Deviations from static quark scenario, full dynamics matters
- Charm quarks are lighter than beauty but are formed later when the fields are dilute

Heavy quark momentum broadening



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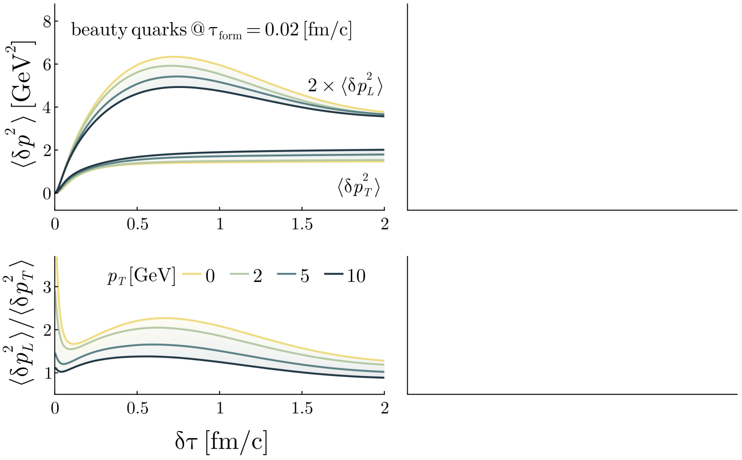

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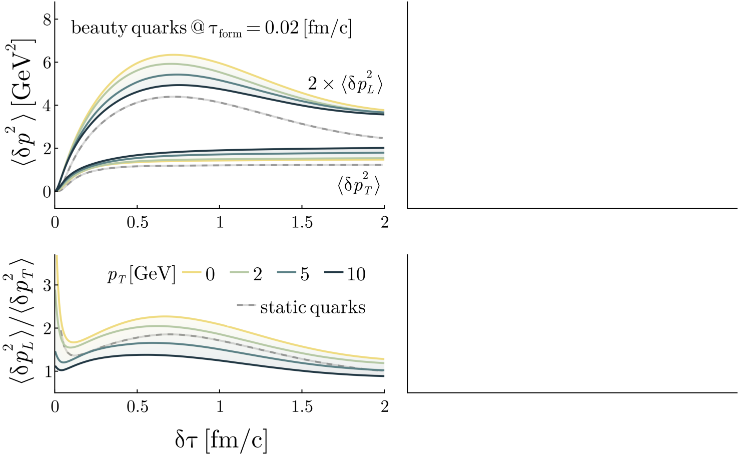

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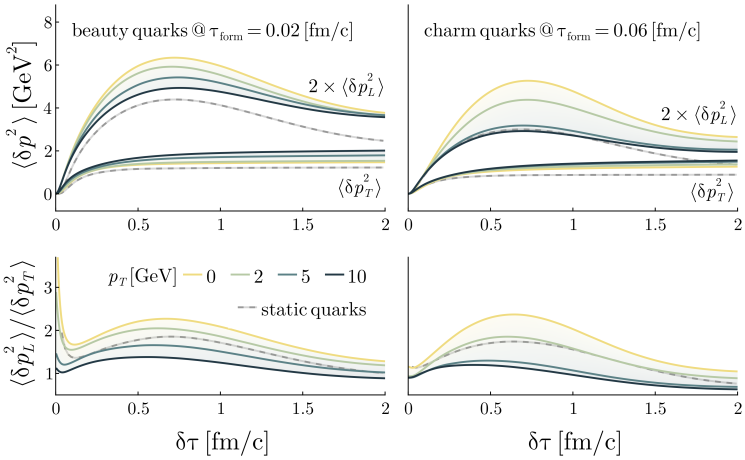

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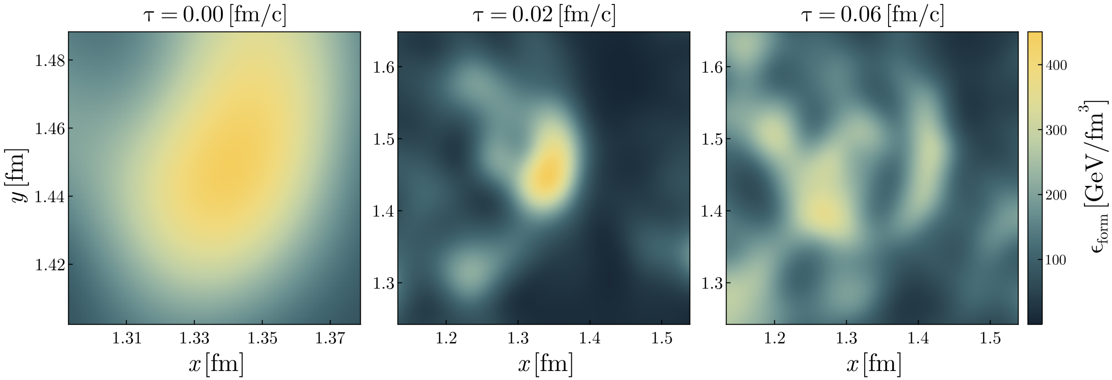
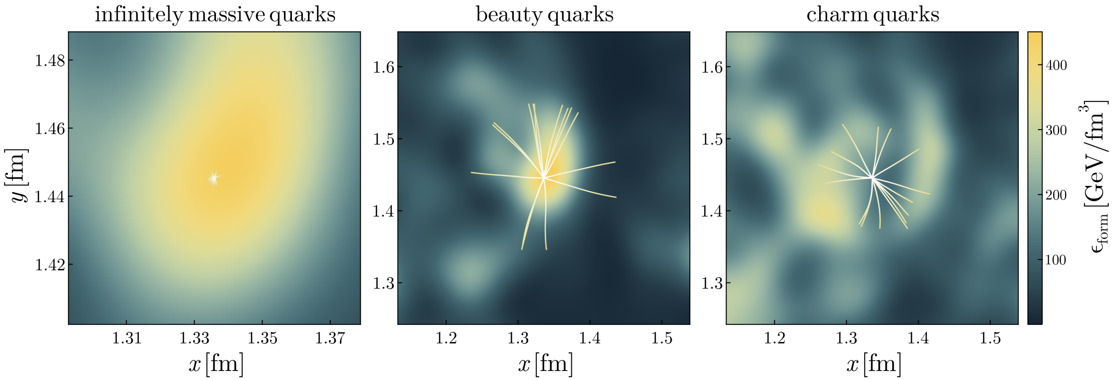
Heavy quarks in Glasma flux tubes



Proper time evolution of the energy density from a Glasma correlation domain

Heavy quarks in Glasma flux tubes



Trajectories of heavy quarks produced at the *center* of a Glasma flux tube

Conclusions

Summary

- Developed a numerical solver for probes in Glasma
- Used this solver to investigate *momentum broadening*, transport coefficients and *anisotropy* of *heavy quarks* and jets in Glasma
- Studied the effect of finite formation time, mass and initial transverse momentum

Future studies

- Investigate how *Glasma field correlators* affect the momentum broadenings of hard probes
- Compute other observables: *angular correlations of $Q\overline{Q}$ pairs*
- Include *energy loss* mechanisms of partons in Glasma *(backreaction, bremsstrahlung)*
- Extend the study to *3+1D Glasma*

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