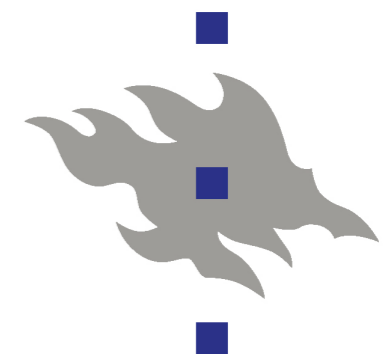
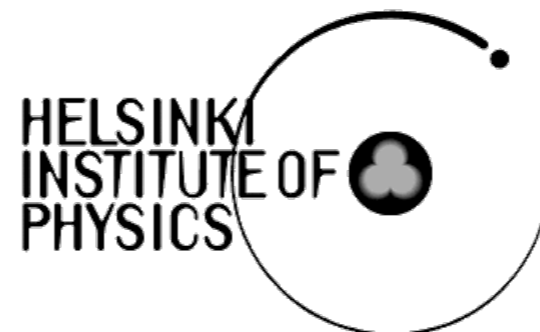


# Small $x$ gluon PDF from LHCb exclusive $J/\psi$ data

Chris A. Flett

In collaboration with Alan Martin  
Misha Ryskin  
Thomas Teubner



# Introduction

- Inclusive processes do not well constrain small  $x$ /Regge limit domain of PDFs
- Exclusive processes offer sensitive probe of this domain but as of yet not included in global analyses PDF determination - why?

1. Off forward kinematics imply susceptibility to  $GPD$  over conventional PDFs
2. Reliability and stability of theoretical predictions

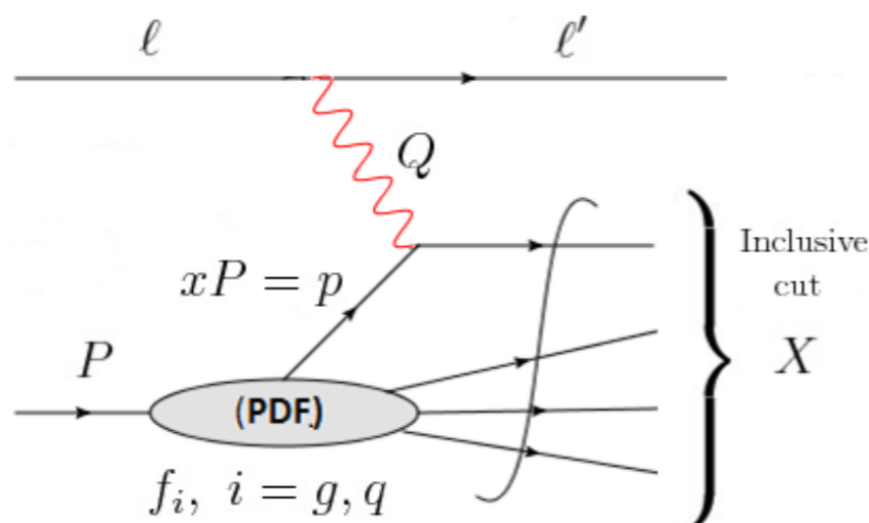
- As higher CM energies are realised at LHC, pushed towards small  $x$  domain,  $W \sim 1/x$

$$\left. \frac{d\sigma}{dt}(\gamma^* p) \right|_{t=0} = \frac{\Gamma_{ee}^{J/\psi} M_{J/\psi}^3 \pi^3}{48\alpha_{em}} \left[ \frac{\alpha_s(\bar{Q}^2)}{\bar{Q}^4} R_g x g(x, \bar{Q}^2) \right]^2 \left( 1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

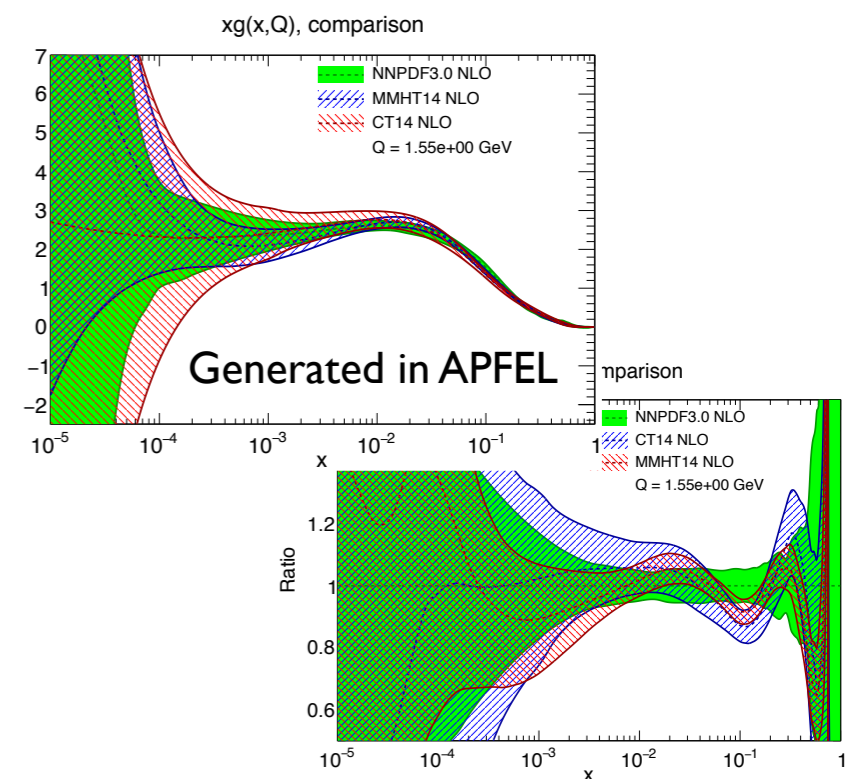
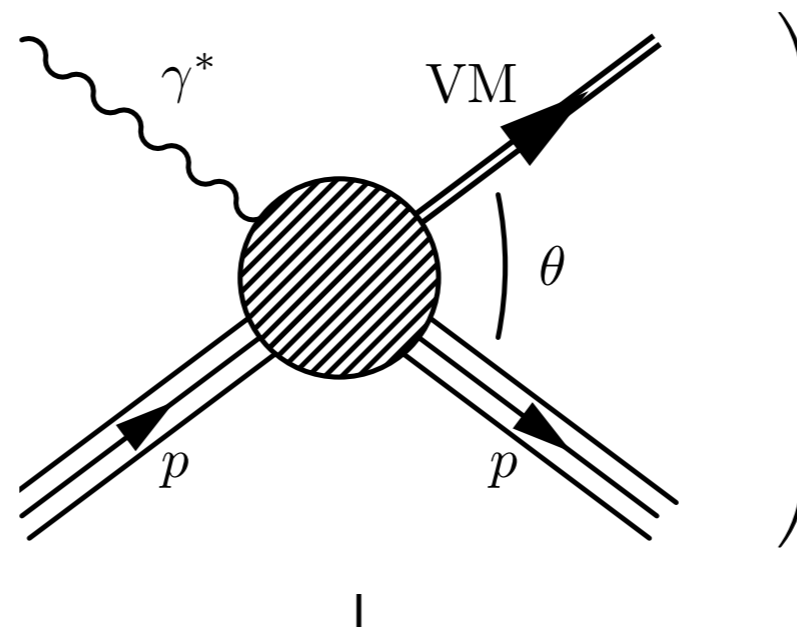
**Inclusive** - included in global parton analyses

**Exclusive** - can we use the data?

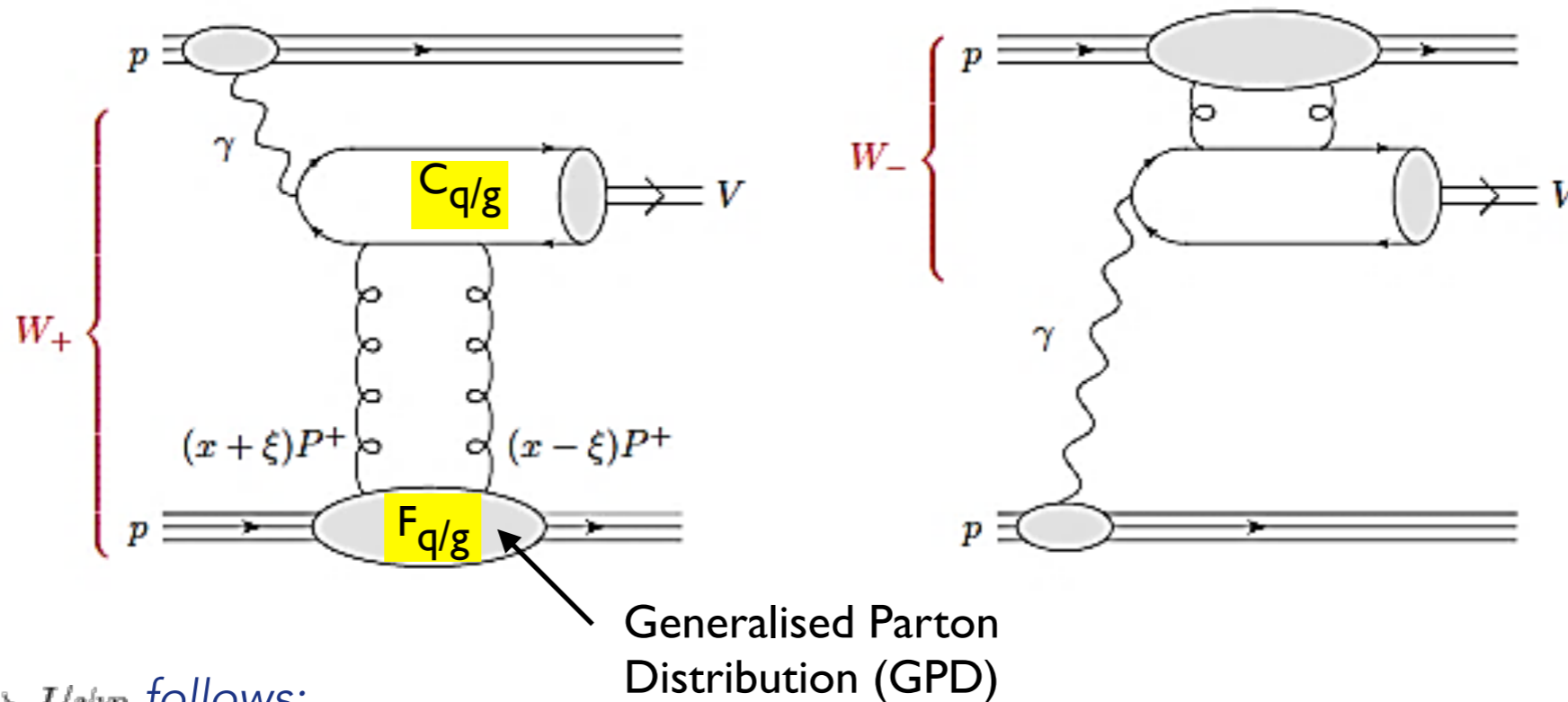
Ryskin 1993



e.g. DIS



# General Set up and assumptions



Setup for  $\gamma p \rightarrow J/\psi p$  follows:

Ivanov, Schäfer, Szymanowski, Krasnikov, 04

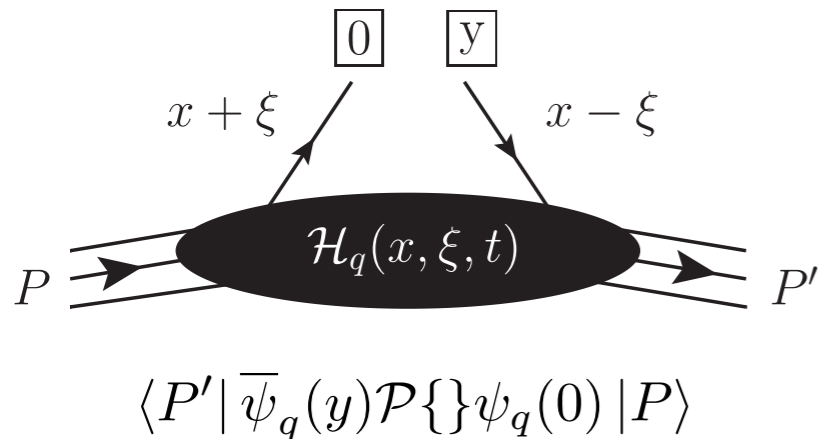
- Assume a factorisation  $F_{q/g} \otimes C_{q/g} \otimes \phi_{Q\bar{Q}}^V$
- Leading zeroth order term in rel. velocity (NRQCD)
- Colour singlet exchange between hard and soft sectors

$$A \propto \int_{-1}^1 dx \left[ C_g(x, \xi) F_g(x, \xi) + \sum_{q=u,d,s} C_q(x, \xi) F_q(x, \xi) \right]$$

# GPDs and the Shuvaev transform

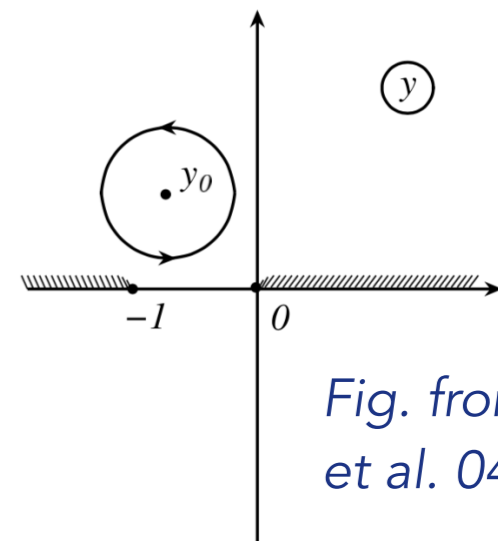
GPDs generalise PDFs: outgoing/incoming partons carry different momentum fractions

*Müller 94; Radyushkin 97; Ji 97*



**Shuvaev:** Relates GPDs to PDFs at small  $x$  under physically motivated assumptions c.f analyticity

*Shuvaev 99 Martin et al. 09*



*Fig. from Ivanov et al. 04*

Idea: Conformal moments of GPDs = Mellin moments of PDFs

*(up to corrections of order  $\xi^2$ )*

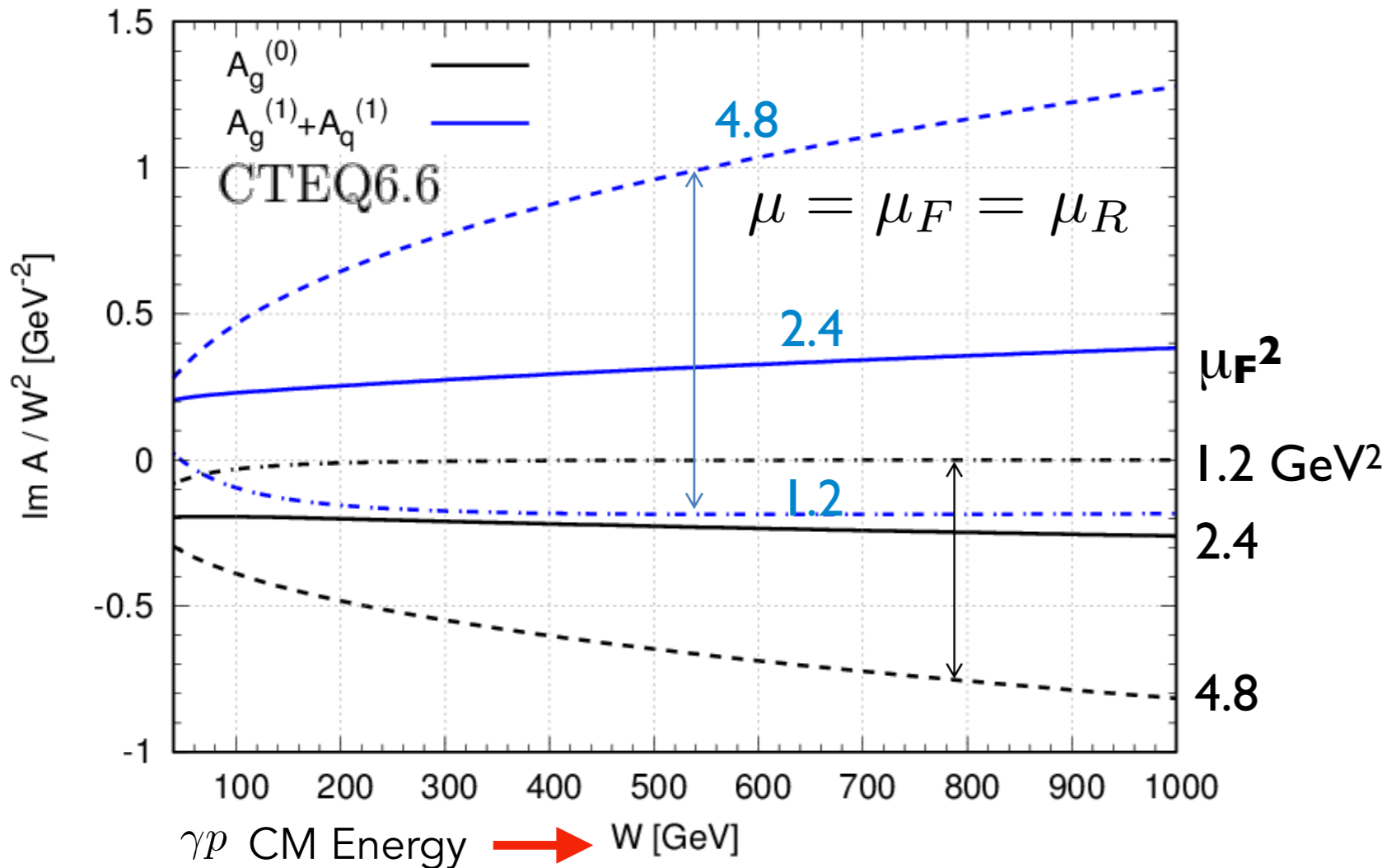
- Construct GPD grids in multidimensional parameter space  $x, \xi/x, qsq$  with forward PDFs from LHAPDF
- Costly computationally due to slowly converging double integral transform
- Regge theory considerations  $\Rightarrow$  Shuvaev transform valid in space like (DGLAP) region only. In time like (ERBL) region imaginary part of coefficient is zero

# Stability of prediction I

## NLO in $\overline{\text{MS}}$ scheme

D. Ivanov, B.Pire, L.Szymanowski, J.Wagner, hep-ph/0401131  
 S.P.Jones, PhD thesis, Liverpool (2014)

- A. Bad perturbative convergence  $|\text{NLO}_{\text{correctn.}}| > |\text{LO}|$  and
- B. Strong dependence on scale  $\mu_F$  opp. sign



**Disclaimer:** Plots generated using existing global partons. Here, CTEQ6.6

Can do better...

# Stability of prediction II

## 'Scale Fixing'

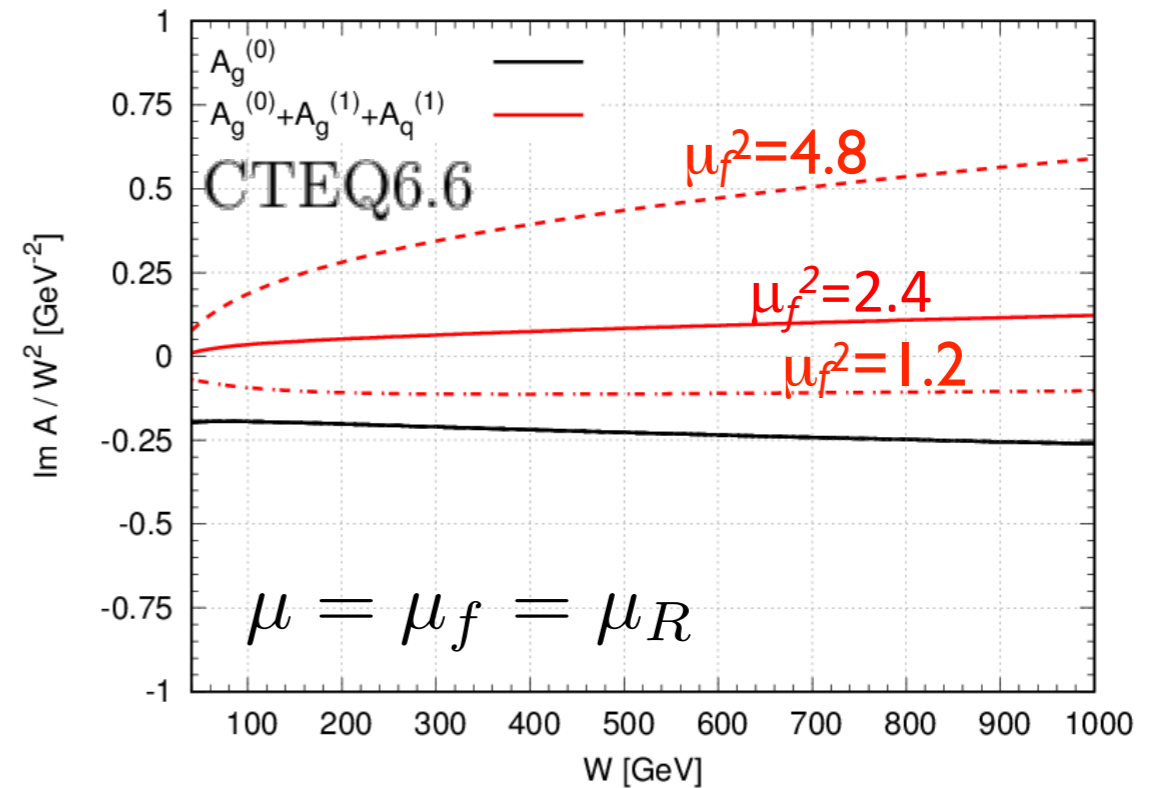
'Optimal' factorisation scale  $\mu_F = m$   
 eliminates large logs at NLO

S.P.Jones, A.D.Martin, M.G.Ryskin, T.Teubner, 1507.06942

Resummation of  $(\alpha_s \ln(1/\xi) \ln(\mu_F/m)^n)$

terms into LO PDF, leaving remnant  
 NLO coefficient  
 and residual,  $\mu_f$ , scale dependence

Fix:  $\mu_F^2 = 2.4 \text{ GeV}^2$

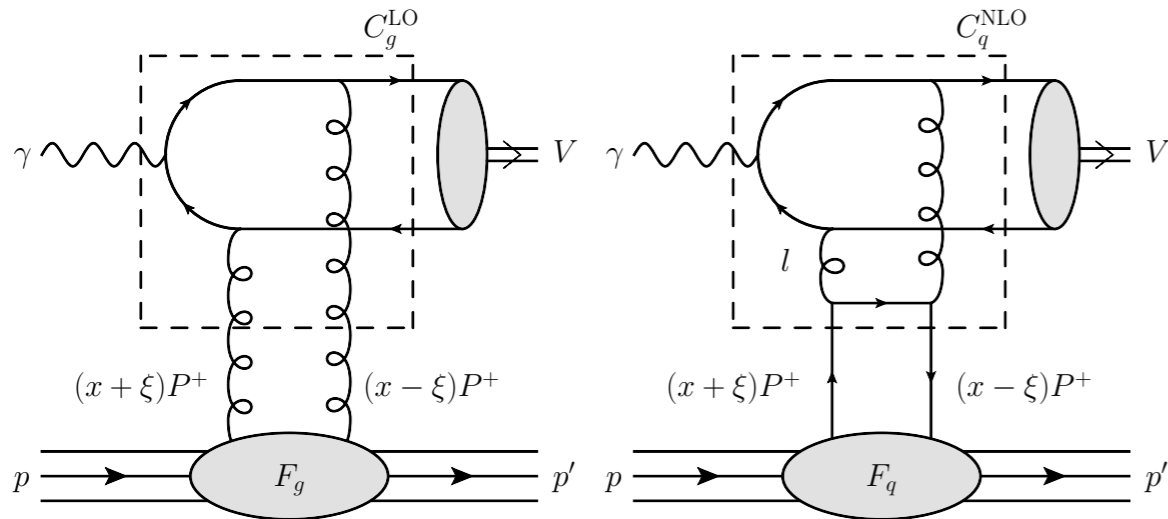


$$A(\mu_f) = C^{\text{LO}} \times \text{GPD}(\mu_F) + C^{\text{NLO}}(\mu_F) \times \text{GPD}(\mu_f)$$

Look for another sizeable correction that can reduce variations further  
 -> implementation of a 'Q0' cut

# Stability of prediction III

'Q0' cut S.P.Jones, A.D.Martin, M.G.Ryskin, T.Teubner, 1610.02272



Subtract DGLAP contribution

NLO ( $|\ell^2| < Q_0^2$ )

from known NLO MSbar coefficient function to avoid a double count with input GPD at  $Q_0$ .

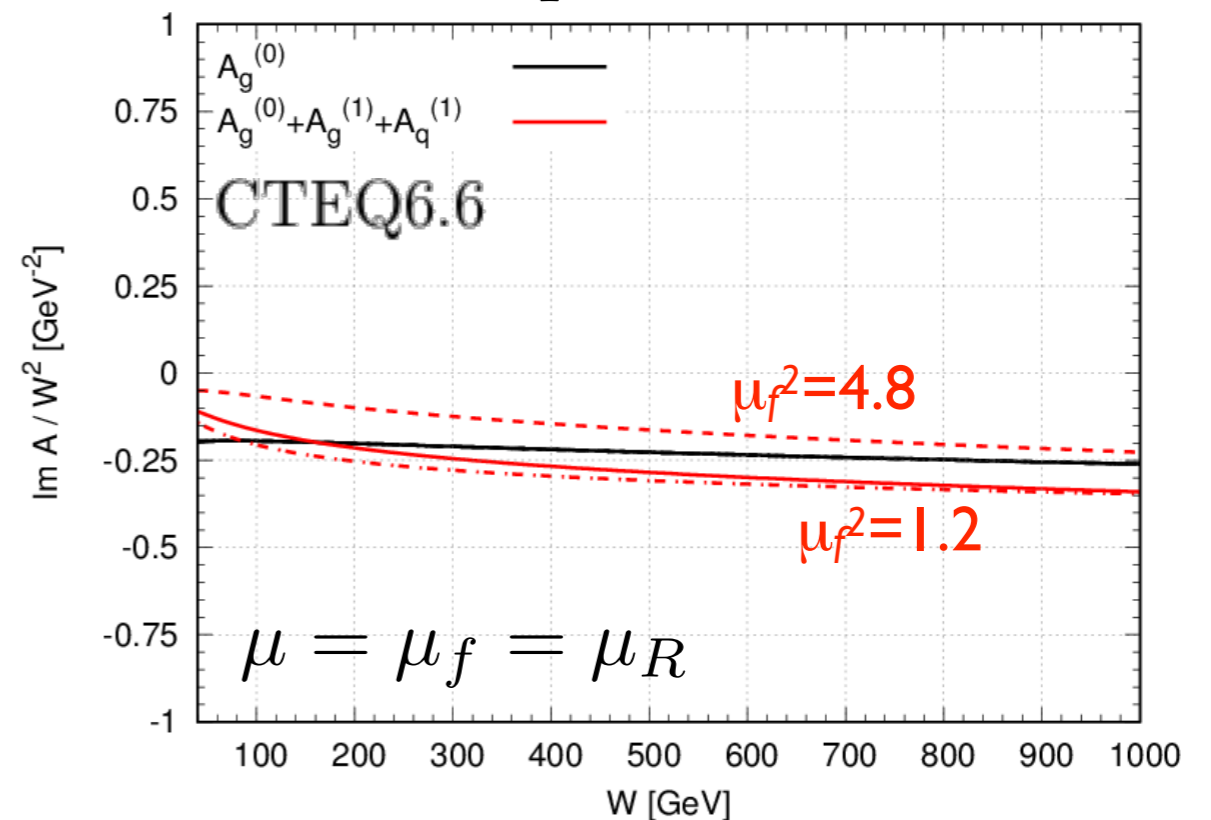
Typically power suppressed, but sizeable here

$$\mathcal{O}(Q_0^2/m_c^2)$$



How do these predictions compare with the data at HERA and LHCb?

Fix:  $\mu_F^2 = 2.4 \text{ GeV}^2$

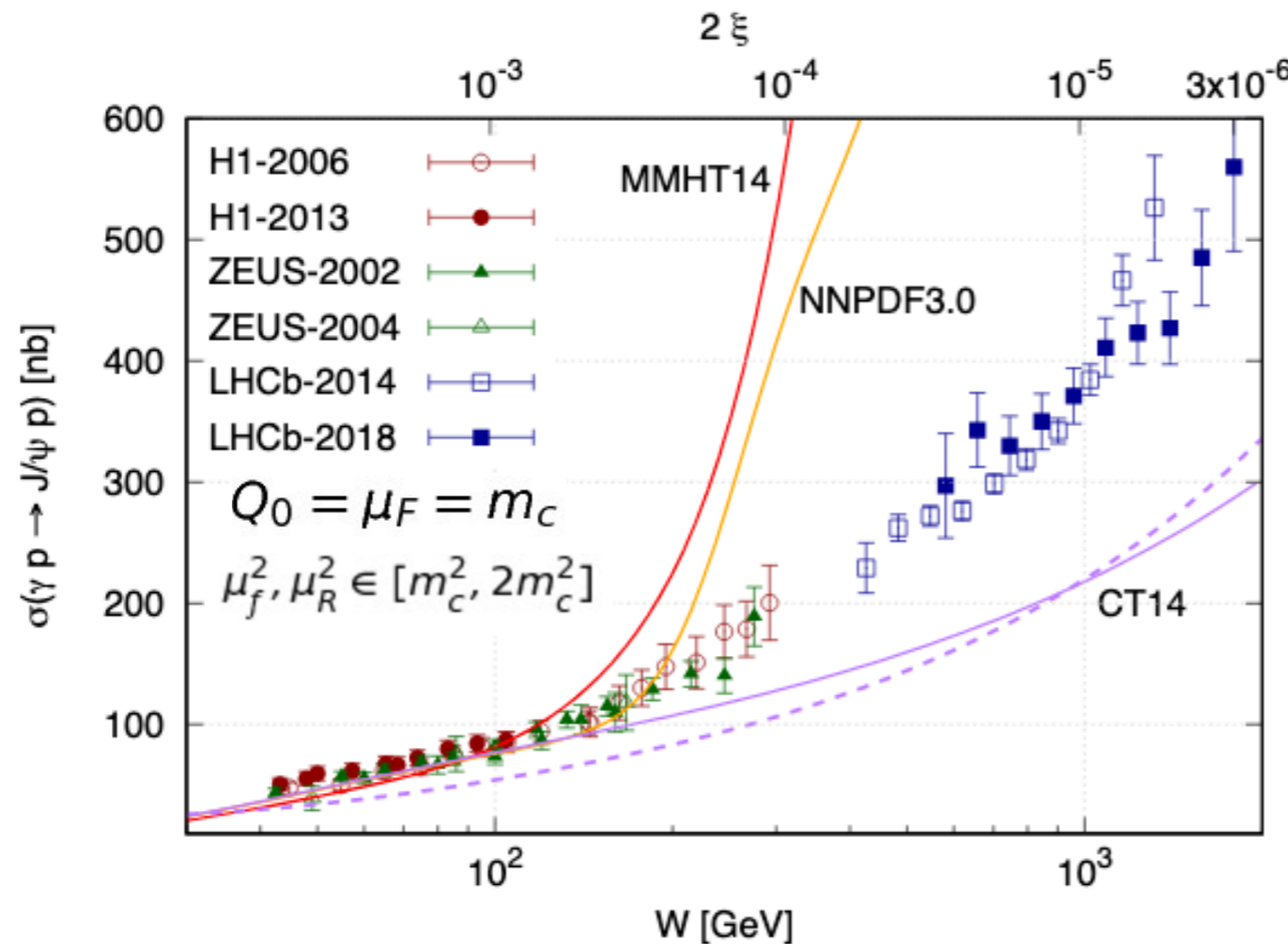




# Towards the bigger picture

Plot demonstrates good scale stability of our NLO predictions in LHCb regime

Predictions at optimal scale (solid) agree better with HERA data



CAF, S.P.Jones, A.D.Martin,  
M.G.Ryskin, T.Teubner,  
1907.06471 & 1908.08398

Diversity between predictions based on current global PDFs in unconstrained phase space -> important message

## Repeat Disclaimer:

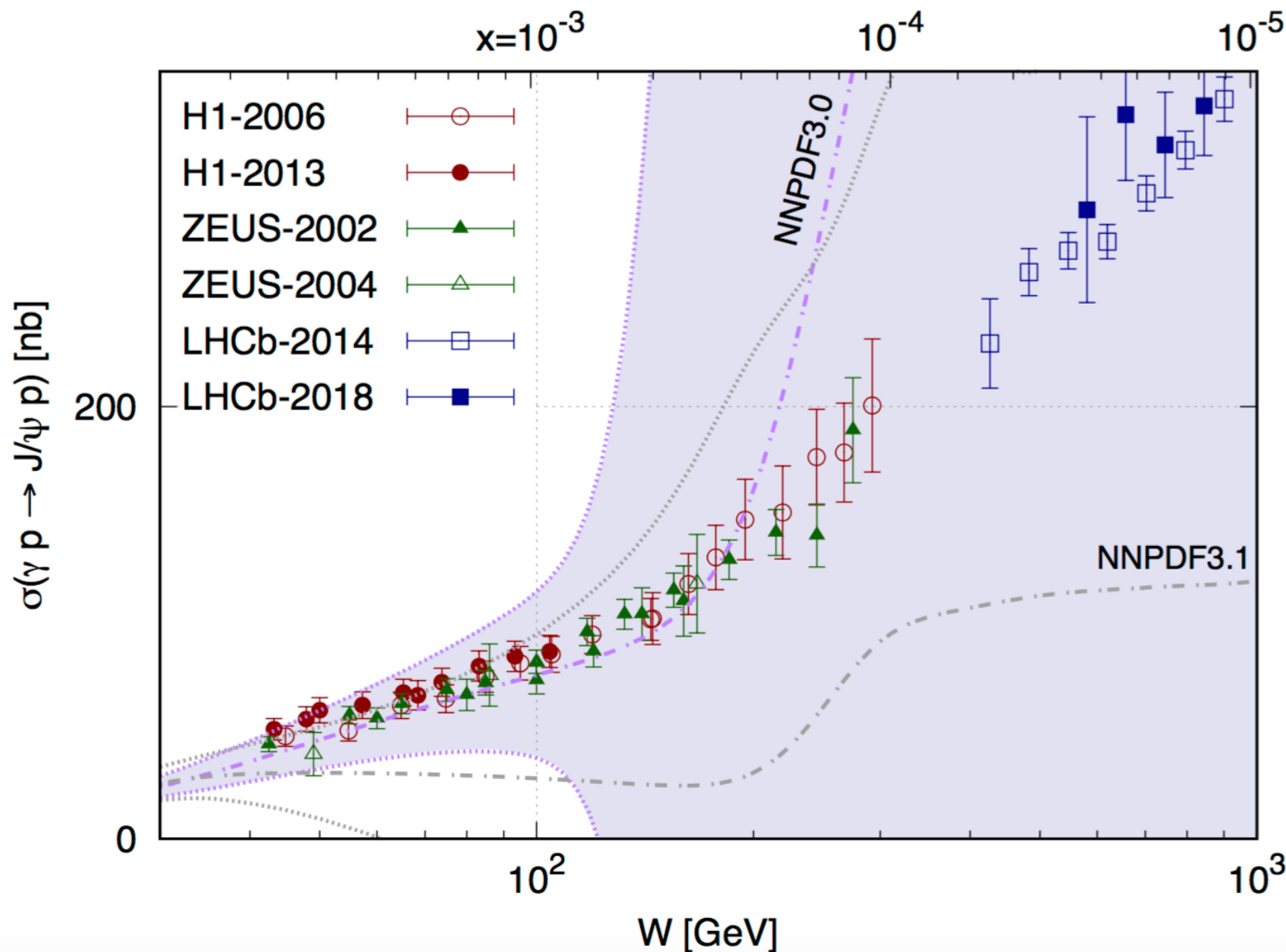
Convoluting with existing global partons. Here, MMHT14, NNPDF3.0 & CT14

$$\frac{\text{Re}\mathcal{M}}{\text{Im}\mathcal{M}} \sim \frac{\pi}{2} \lambda = \frac{\pi}{2} \frac{\partial \ln \text{Im}\mathcal{M}/W^2}{\partial \ln W^2} \quad \text{with} \quad \mathcal{M} \sim x^{-\lambda}$$



Error budgets: errors due to parameter variations in global fits  $\gg$  experimental uncertainty and scale variations in the theoretical result

..... exclusive data now in a position to readily improve global analyses



*Exclusive* LHCb data will constrain small  $x$  growth whilst *exclusive* HERA data will improve determination of partons in regime with data constraints already from diffractive DIS HERA data

# Extraction of low x gluon PDF via exclusive J/psi

Left

**Approach 1:** Fit a low x gluon PDF ansatz to the data

Right

**Approach 2:** Bayesian reweight current global PDF analyses

	$\lambda$	$n$	$\chi^2_{\min}$	$\chi^2_{\min}/\text{d.o.f}$
NNPDF3.0	0.136	0.966	44.51	1.04
MMHT14	0.136	1.082	47.00	1.09
CT14	0.132	0.946	48.25	1.12

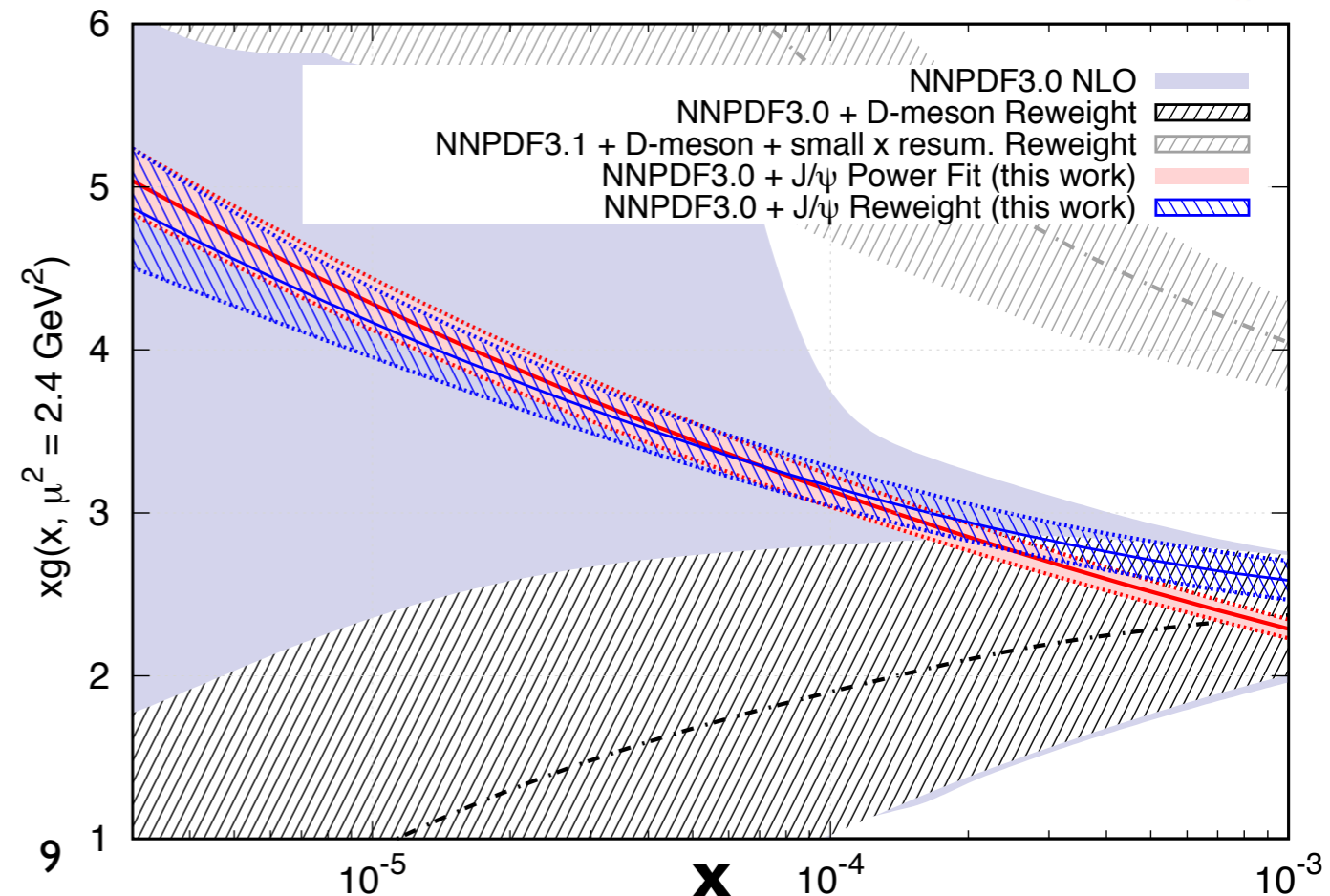
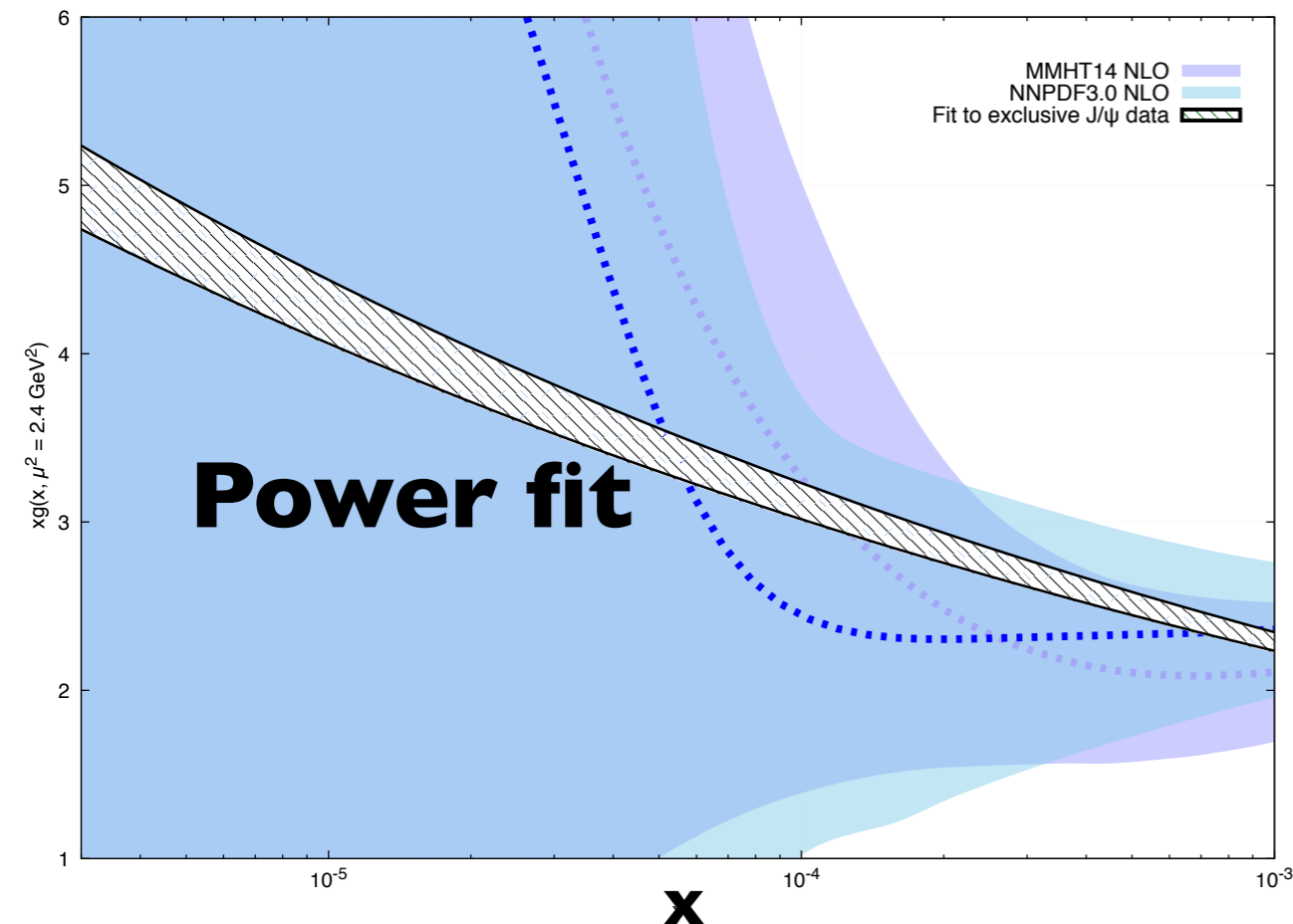
$$xg^{\text{new}}(x, \mu_0^2) = nN_0 (1-x) x^{-\lambda}$$

$$\lambda = 0.136 \pm 0.006$$

$$n = 0.966 \pm 0.025$$

CAF, A.D. Martin, M.G. Ryskin, T. Teubner, 2006. 13857

$$N_{\text{eff}} \ll N_{\text{rep}}$$

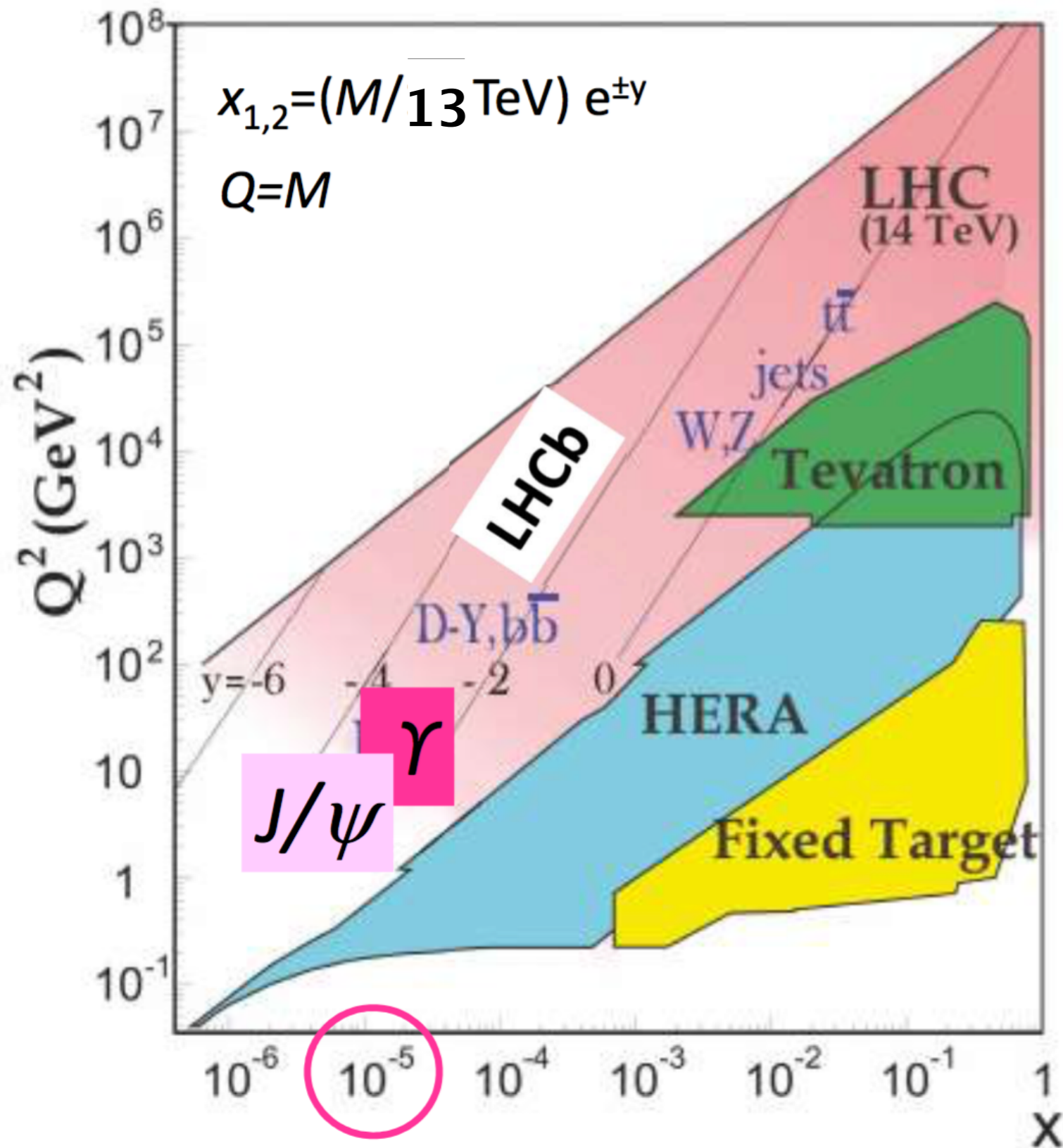


# Summary

- Conventional  $\overline{\text{MS}}$  NLO coll. fact. result unreliable and unstable
- Systematic taming via 'Q0' cut and resummation of large logarithmic contributions collectively reduce wild scale variations
- Predictions at cross section level exhibit good scale stability and central values in agreement of data within 1 sigma error bands
- MMHT14' and NNPDF3.0 largely overshooting data in LHCb regime
- Impossible to describe growth of J/psi cross section with energy, observed by the LHCb, using gluons obtained from fit to open charm (decreasing with decreasing x). Tension observed between extracted gluons from exclusive and inclusive sector through J/psi and D channels resp.
- Inconsistencies in the D sector from the experimental side? (see backup slides)
- Upshot: In a position to finally use exclusive J/psi data (easier to collect and theory result now improved) in a global fitter framework

**Thank you**

# Kinematic coverage

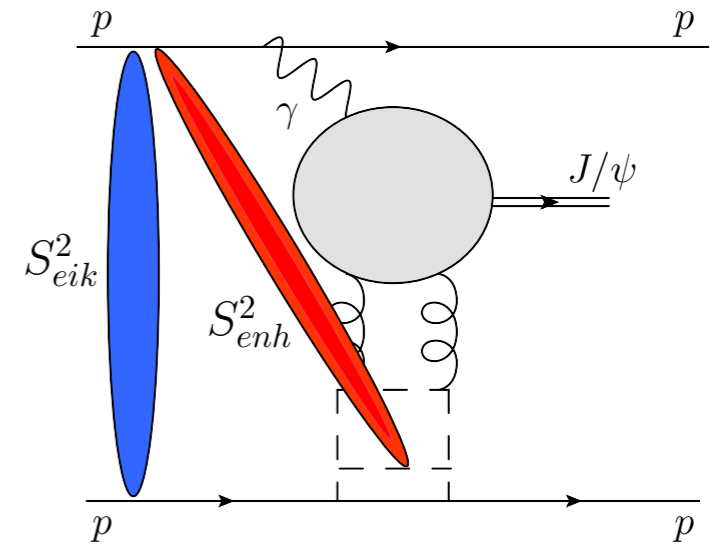
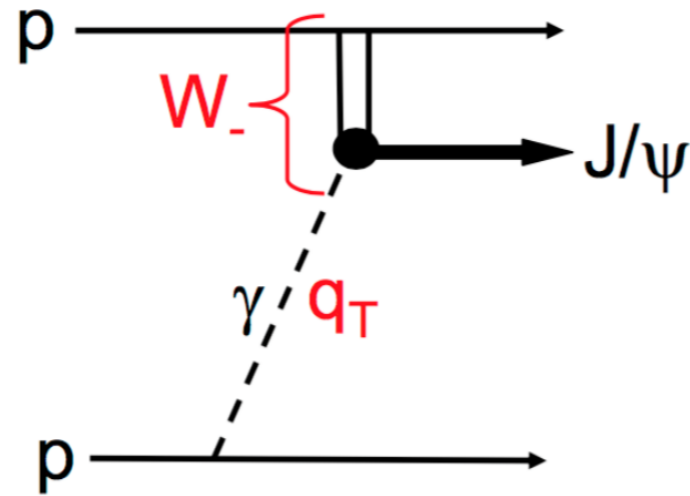
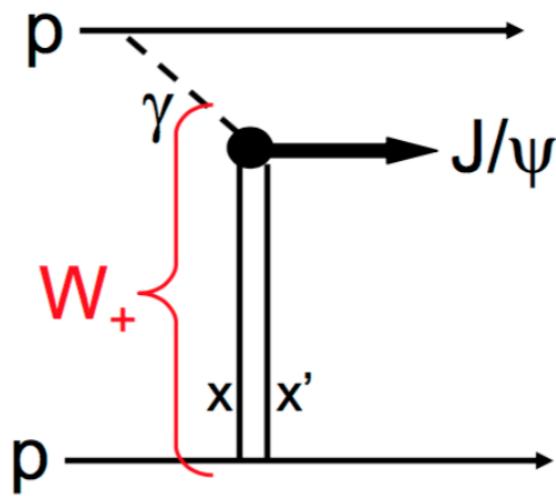


LHCb with  $2 < y < 4.5$   
can probe gluon  
down to  $x \sim 10^{-5}$

exclusive  $J/\psi, Y$   
[ $Q=M_V/2$  (scale)]

Why are these  
LHCb data not used  
in global PDF fits ??

# General Set up and assumptions



LHCb data

$$\frac{d\sigma(pp)}{dy} = S^2(W_+) \left( k_+ \frac{dn}{dk_+} \right) \sigma_+(\gamma p) + S^2(W_-) \left( k_- \frac{dn}{dk_-} \right) \sigma_-(\gamma p)$$

survival probability factors

LHCb 'data'

photon flux

HERA gives  $W_-$

$$W_{\pm}^2 = M_{J/\psi} \sqrt{s} e^{\pm|y|} \Rightarrow x_{\pm} = \begin{cases} 10^{-5} \\ 0.02 \end{cases} \text{ at } y = 4, \sqrt{s} = 13 \text{ TeV}$$

# Shuvaev Transform

**Full Transform:**

$$\mathcal{H}_q(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \operatorname{Im} \int_0^1 \frac{ds}{y(s) \sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left( \frac{q(x')}{|x'|} \right),$$
$$\mathcal{H}_g(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \operatorname{Im} \int_0^1 \frac{ds(x + \xi(1 - 2s))}{y(s) \sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left( \frac{g(x')}{|x'|} \right),$$
$$y(s) = \frac{4s(1 - s)}{x + \xi(1 - 2s)}.$$

[ Shuvaev et. al 1999 ]



# Shuvaev Transform cont.

The conformal moments  $H_i^N$  of the GPDs are given by

$$H_i^N \equiv \int_{-1}^1 dx R_{N,i}(x_1, x_2) H_i(x, \xi), \quad i = q, g, \quad \text{Ohrndorf, 82}$$

The conformal moments are polynomials in even powers of  $\xi$ ,

$$H_i^N = \sum_{k=0}^{\lfloor (N+1)/2 \rfloor} c_{k,i}^N \xi^{2k} = c_{0,i}^N + c_{1,i}^N \xi^2 + c_{2,i}^N \xi^4 + \dots, \quad , c_{0,i}^N = f_i^N$$

Leading term is Mellin moment of PDF

- Provided inverse exists then can relate GPDs to PDFs with suppression of order  $x$  (i.e. good low  $x$  approx )



# Shuvaev Transform cont.

Widely debated, certain conditions needing upheld, e.g lack of singularities in  
Re  $N > 1$  plane e.g Diehl, Kugler, 08

Regge theory considerations => condition met Martin, Nockles, Ryskin, Teubner, 09

- Can check in physically motivated ansatz, e.g MSTW2008 global partons input parametrisation

$$xg(x, Q_0^2) = A_g x^{\delta_g} (1-x)^{\eta_g} (1 + \epsilon_g \sqrt{x} + \gamma_g x) + A_{g'} x^{\delta_{g'}} (1-x)^{\eta_{g'}}.$$

Martin,  
Stirling, Thorne,  
Watt, 09

Expand about  $x \sim 0$

$$xg(x, Q_0^2) = A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}} + \dots,$$

Mellin transform:

$$\begin{aligned} xg^N(Q_0^2) &= \int_0^1 dx x^{N-1} (A_g x^{\delta_g} + A_{g'} x^{\delta_{g'}}) + \dots \\ &= \frac{A_g}{N + \delta_g} + \frac{A_{g'}}{N + \delta_{g'}} + \dots, \end{aligned}$$

Fits to data (including 1sig. errors) suggest  $\delta_g > -1$  and  $\delta_{g'} > -1$

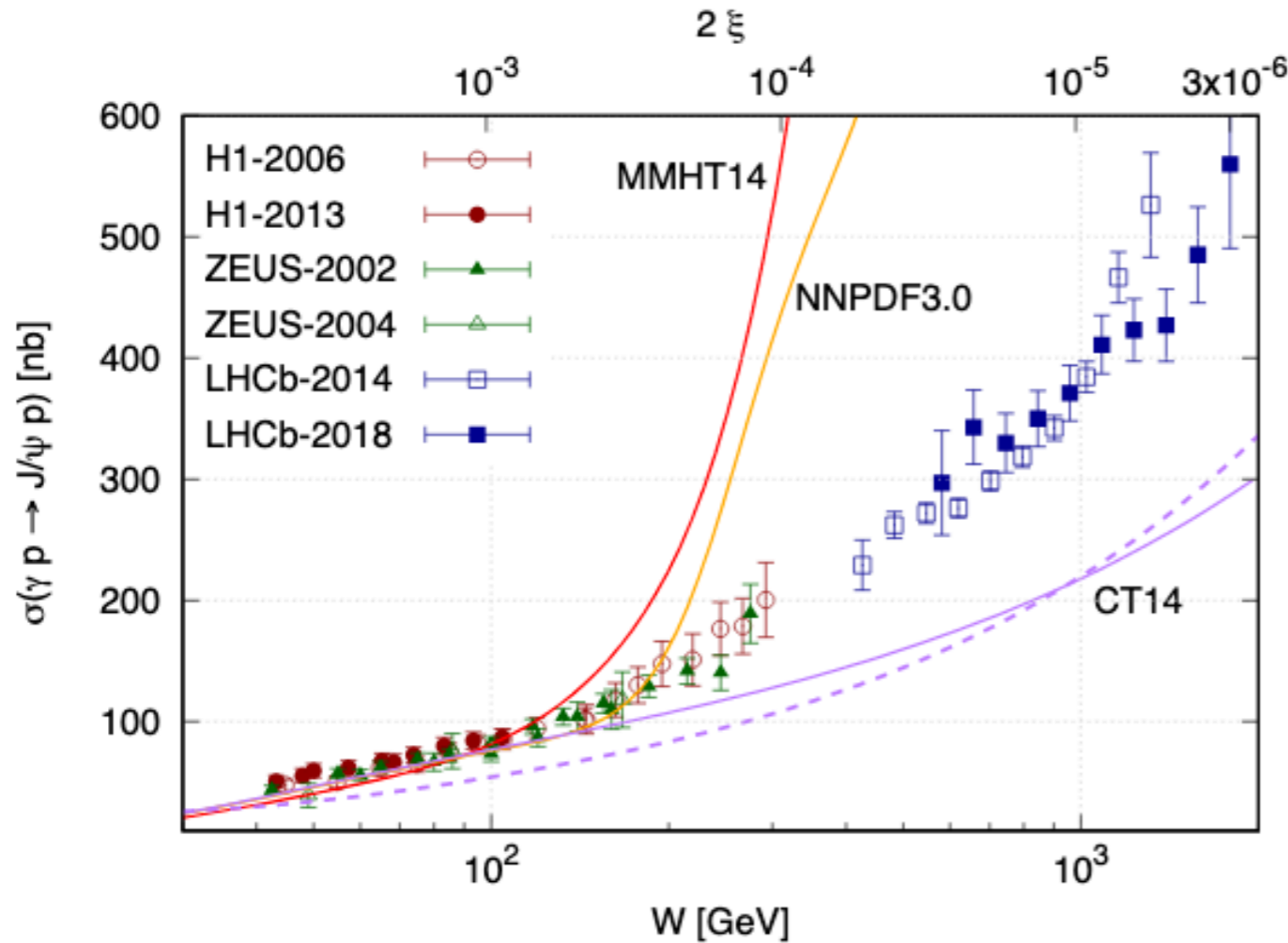
- **Shuvaev transform describes HVM and GDVCS data well**

*Kumericki, Muller, 10*

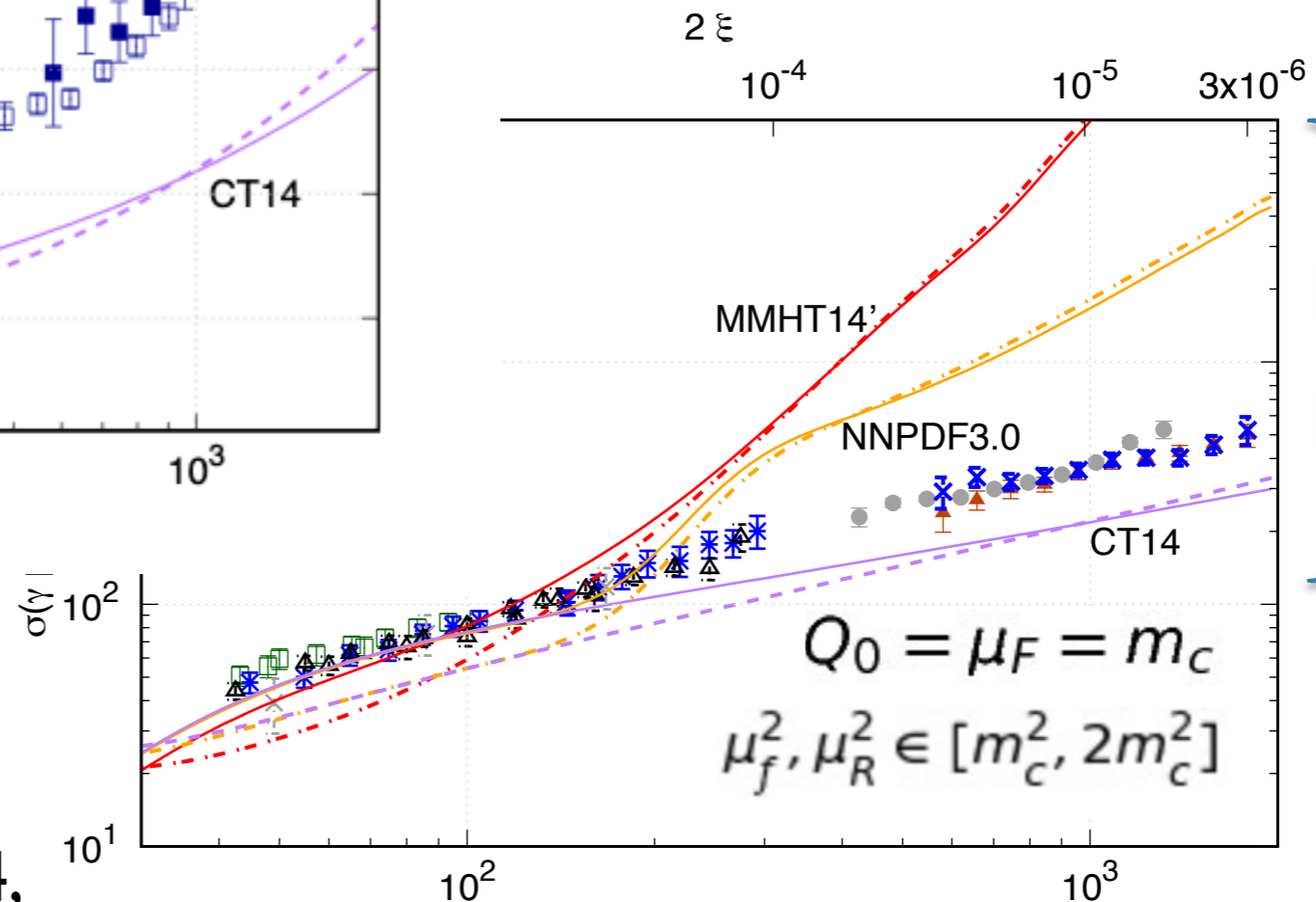
# Cross section stability

Plots demonstrates good scale stability of our NLO predictions in LHCb regime

Predictions at optimal scale (solid) agree better with HERA data



CAF, S.P.Jones, A.D.Martin,  
M.G.Ryskin, T.Teubner,  
1907.06471 & 1908.08398



Diversity  
in  
prediction  
->  
important  
message

**Repeat Disclaimer:**  
Convoluting with existing  
global partons. Here, MMHT14,  
NNPDF3.0 & CT14

$$\frac{\text{Re}\mathcal{M}}{\text{Im}\mathcal{M}} \sim \frac{\pi}{2} \lambda = \frac{\pi}{2} \frac{\partial \ln \text{Im}\mathcal{M}/W^2}{\partial \ln W^2} \quad \text{with } \mathcal{M} \sim x^{-\lambda}$$

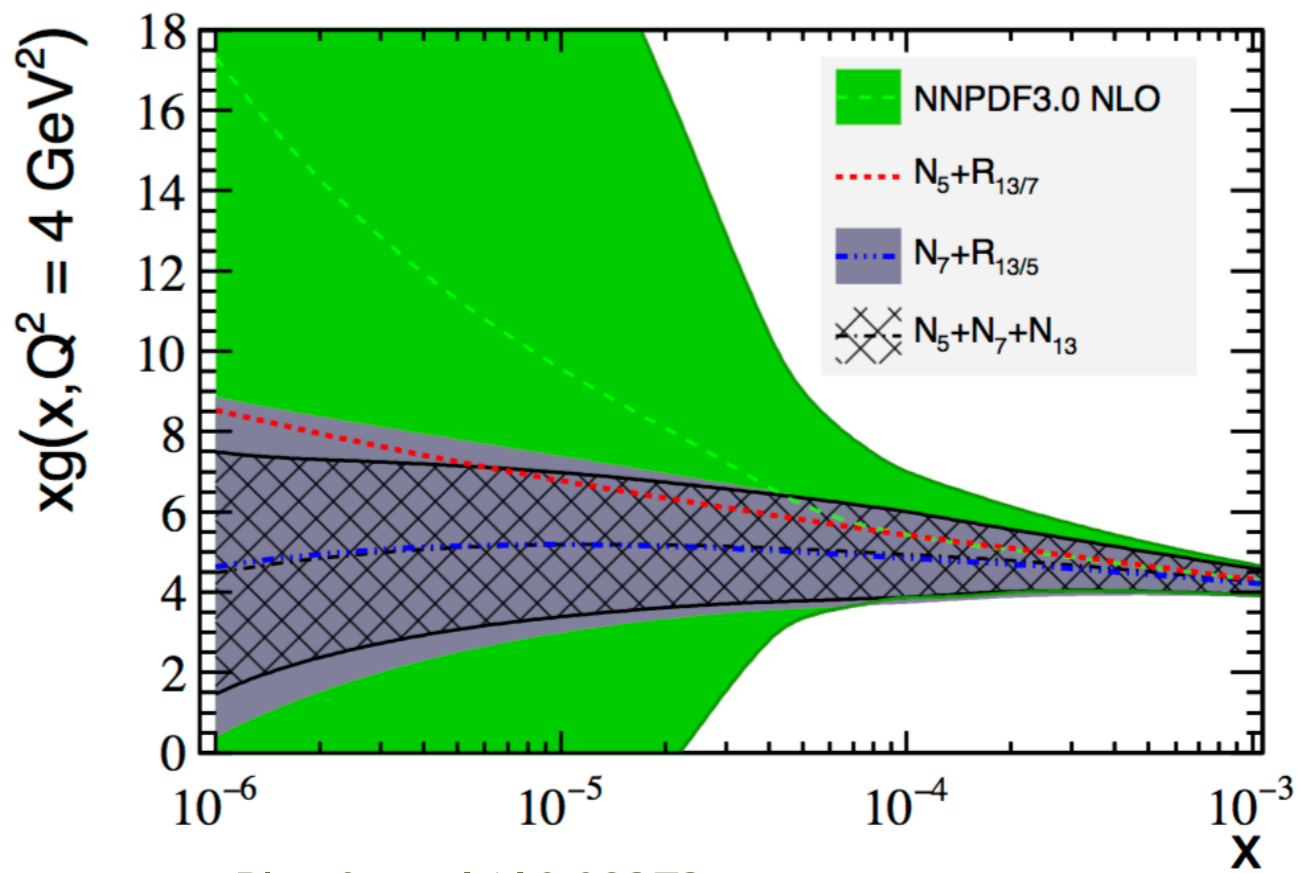
# Constraints from inclusive D meson production data

**Idea:** Construct ratios of observables in  $y$  and  $p_T$  bins to combat various uncertainties

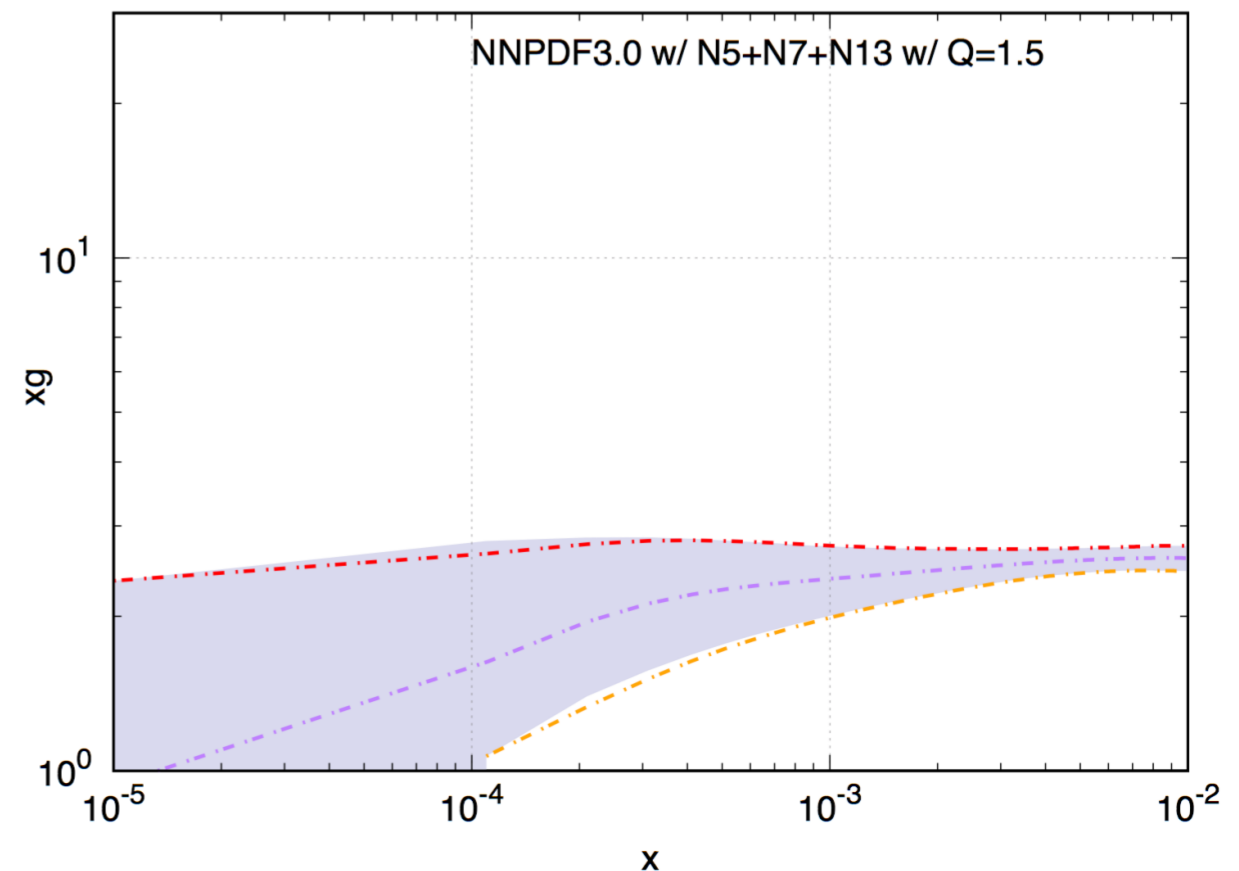
$$N_X^{ij} = \frac{d^2\sigma(X \text{ TeV})}{dy_i^D d(p_T^D)_j} \bigg/ \frac{d^2\sigma(X \text{ TeV})}{dy_{\text{ref}}^D d(p_T^D)_j}$$

$$R_{13/X}^{ij} = \frac{d^2\sigma(13 \text{ TeV})}{dy_i^D d(p_T^D)_j} \bigg/ \frac{d^2\sigma(X \text{ TeV})}{dy_i^D d(p_T^D)_j}$$

→ find decreasing gluon at the lowest  $x$  they may probe



Plot from 1610.09373



# Tension with the J/psi data

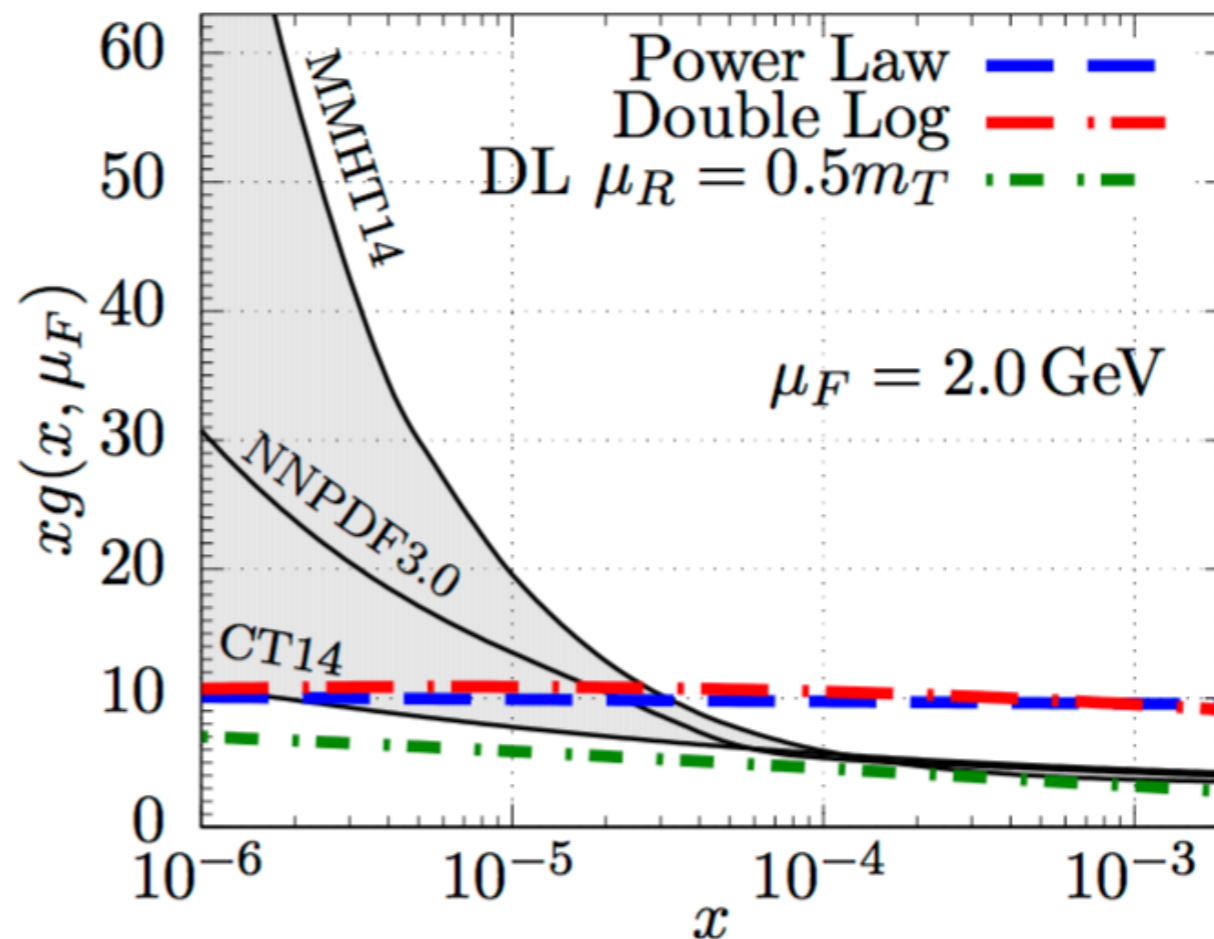
We need a much harder gluon at low  $x$  to describe the exclusive J/psi LHCb data.

## What's the reconciliation?

Indications of **inconsistencies** in the inclusive D experimental measurement

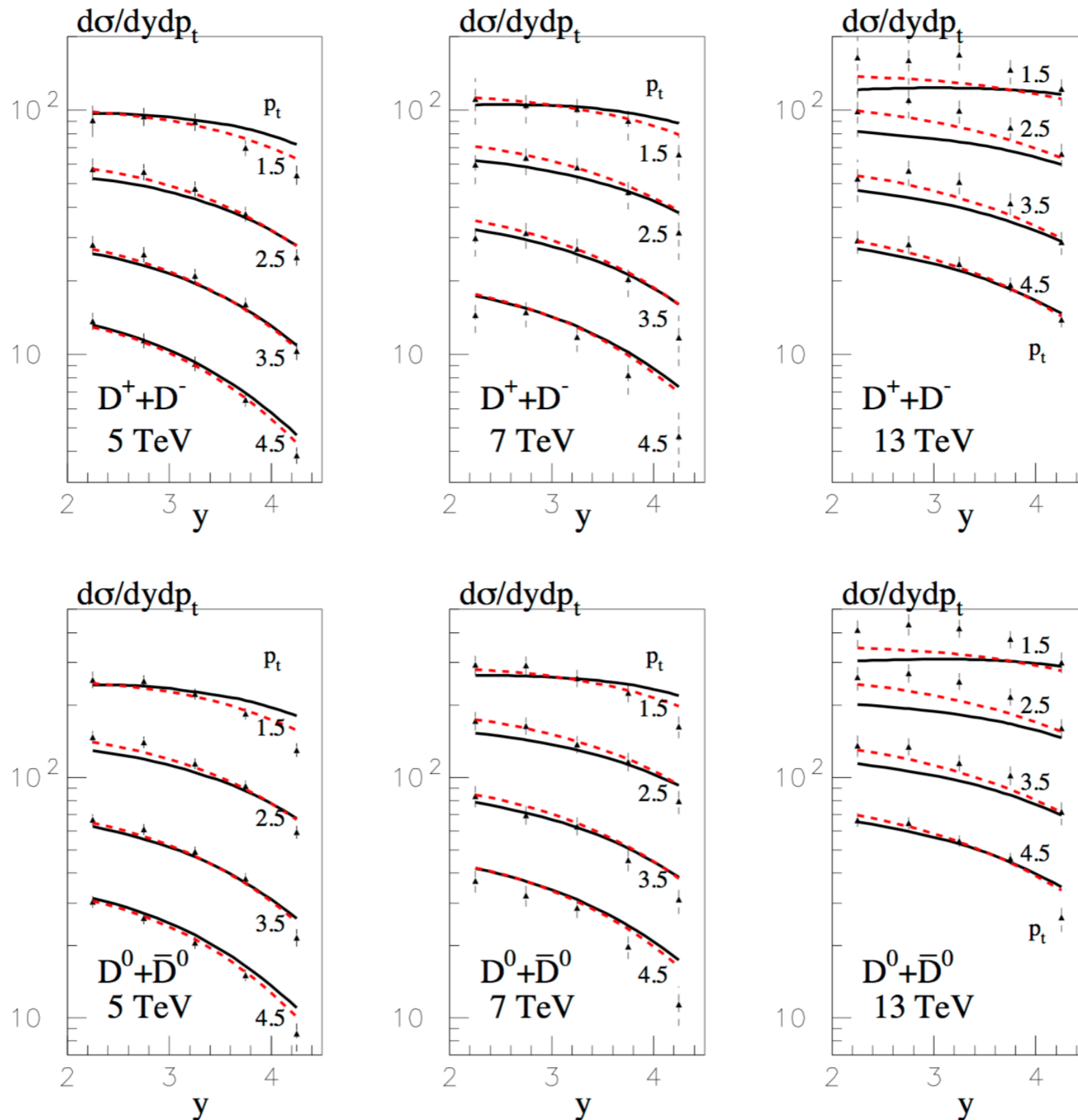
$$xg(x) = N \left( \frac{x}{x_0} \right)^{-\lambda}$$

$$xg(x, \mu^2) = N^{\text{DL}} \left( \frac{x}{x_0} \right)^{-a} \left( \frac{\mu^2}{Q_0^2} \right)^b \exp \left[ \sqrt{16(N_c/\beta_0) \ln(1/x) \ln(G)} \right]$$



Plot from 1712.06834

# Rapidity and energy dependence of open charm cross section



- Need *slower* increasing gluon with decreasing  $x$  to describe rapidity dependence
- Need *faster* increasing gluon with decreasing  $x$  to describe energy dependence

$$y \sim \ln(1/x) !!$$

dash

$Q_0=1$  GeV and  $\mu_F = \mu_R = 0.85m_T$

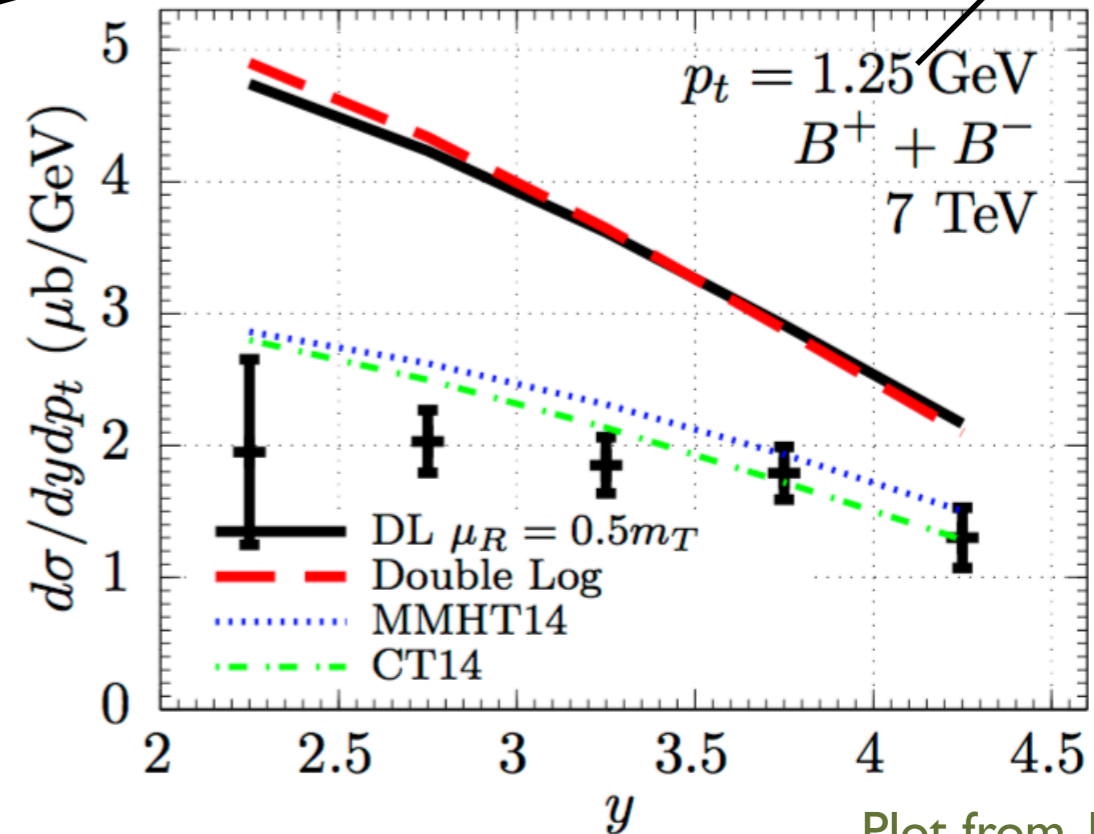
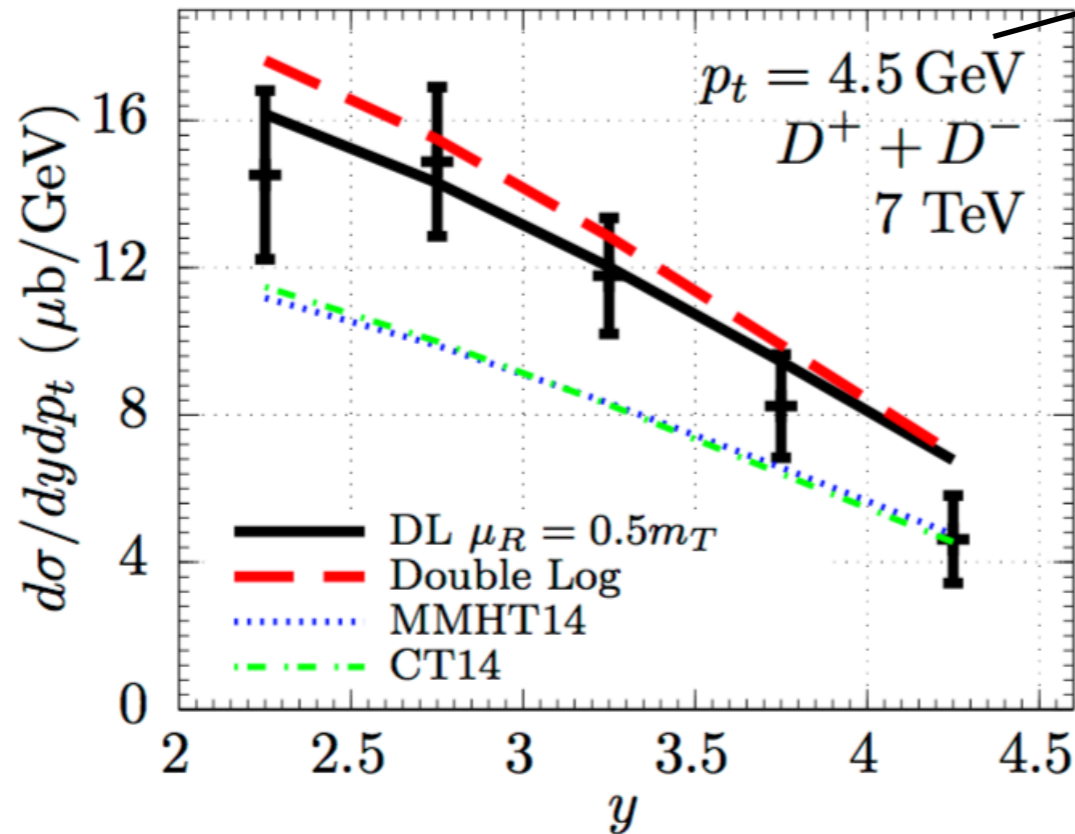
solid

$\mu_f = \mu_R = 0.5m_T$  and  $Q_0=0.5$  GeV



# Open beauty results

B sector has something to say...



$p_t$  chosen to sample gluon at same factorisation scale and  $x$

Gluon found through fit to D meson data fails to describe the B meson distribution

**Should we really trust the decreasing nature of the low scale, low  $x$  gluon obtained via fit to LHCb open charm data?**

Plot from 1712.06834