# ~~QCD theory~~
# Formation of quark-gluon plasma in QCD

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Particle physics day, Helsinki, October 2017

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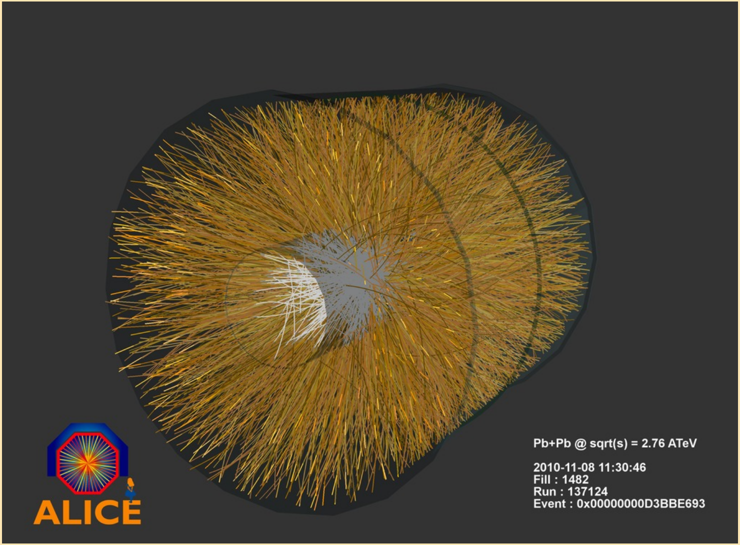
# Outline

- Heavy ion collision: big picture
- Initial state: small-$x$ gluons
- Production of particles in weak coupling: gluon saturation
- 2 ways of understanding glue
  - Counting particles
  - Measuring gluon field
- For practical phenomenology: add geometry

# A heavy ion event at the LHC

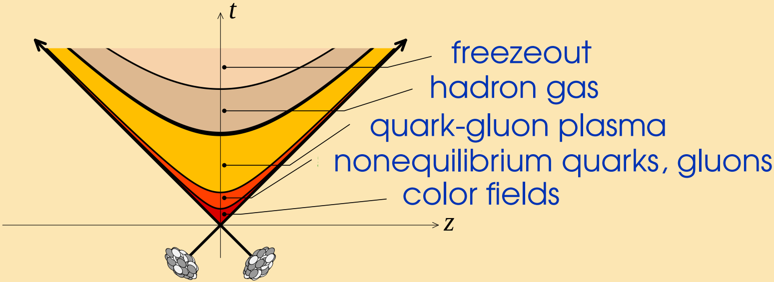



How does one understand what happened here?

# Heavy ion collision in spacetime

The purpose in heavy ion collisions: to create QCD **matter**,
i.e. system that is large and lives long
compared to the microscopic scale

$$t \gg \frac{1}{T} \qquad L \gg \frac{1}{T} \qquad T > 200\text{MeV}$$



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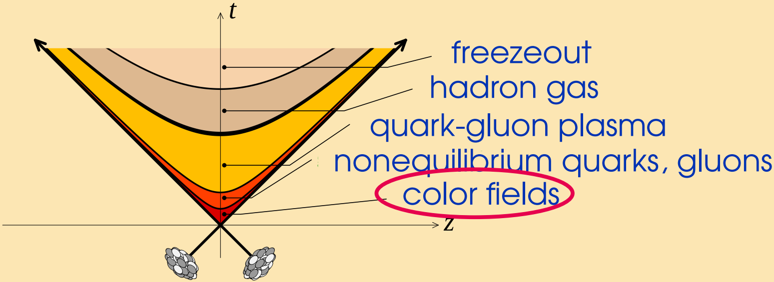


Concentrate here on the **earliest stage**

# Cascade of gluons

## Electric charge

- At rest: Coulomb electric field
- Moving at high velocity:
  Coulomb field is cloud of photons
  ("equivalent photon approximation")

$$\frac{dN}{d\omega} \sim \omega^{-1} \quad (\text{when } \omega \to 0)$$

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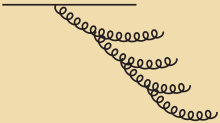
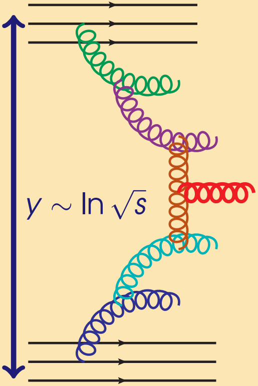
## Color charge

- Charge has cloud of gluons
- But now: gluons are source of
  new gluons: cascade

$$\frac{dN}{d\omega} \sim \omega^{-1-\mathcal{O}(\alpha_s)}$$

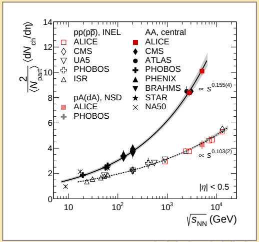


# Initial state is small $x$

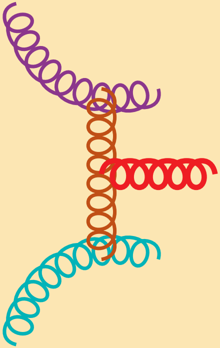



- Gluons ending in central rapidity region: multiple splittings from valence quarks
- Emission probability $\alpha_s\, dx/x$
  $\implies$ rapidity plateau for $\Delta y \ll 1/\alpha_s$
- Many gluons, in fact

$$N \sim \sum_n \frac{1}{n!}(\alpha_s \ln\sqrt{s})^n \sim \sqrt{s}^{\alpha_s}$$



# Particle production



- Perturbative gluon production

$$\frac{dN_g}{dy\, d^2\mathbf{p}\, d^2\mathbf{x}} \sim \frac{1}{{p_T}^4} \quad \text{(dimensionally)}$$

- Total number of gluons

$$\int d^2\mathbf{p} \frac{dN_g}{dy\, d^2\mathbf{p}\, d^2\mathbf{x}} \sim \int \frac{dp_T}{{p_T}^3}$$

- Transverse energy (gauge invariant)

$$\int d^2\mathbf{p} \frac{dN_g}{dy\, d^2\mathbf{p}\, d^2\mathbf{x}} \sim \int \frac{dp_T}{{p_T}^2}$$

Something needs to happen to regulate this; either at

- Confinement scale: strings, meson/baryon dof's
- Weak coupling scale: saturation $\Longrightarrow$ at high $\sqrt{s}$

# 2 ways of doing weak coupling QCD at small $x$

- Parton picture, Infinite Momentum Frame (IMF)
  - Measure numbers of partons: **parton distributions**
  - Calculate their interactions as pQCD scattering

Talk by Paakkinen

+ Correct physical picture at large $x$, large $p_T$
- But including saturation is hard

- Target rest frame picture
  - Small-$x$ gluons = classical color field
  - Quantify as eikonal scattering amplitude of probe

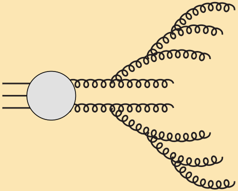
Talk by Hänninen

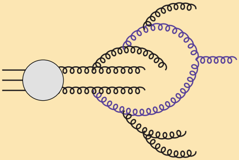
+ Gluon saturation "automatic": consequence unitarity!
- Only high energy: connecting to large-$x$, $p_T$ tricky.

# Gluon saturation in IMF: nonlinear interactions

Saturation when phase space density of gluons large

- Number of gluons $xG(x, Q^2)$
- Size of one gluon $\sim 1/Q^2$
- Transverse space available $\pi R_p^2$
- Coupling $\alpha_s$





**Estimate in terms of gluon numbers**

Nonlinearities important when

$$\pi R_p^2 \sim \alpha_s xG(x, Q^2)\frac{1}{Q^2}$$

Solve for $Q^2 = Q_s^2$; "saturation scale"

(LHC kinematics: $Q_s \approx 1 .. 2$ GeV)

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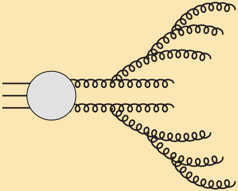
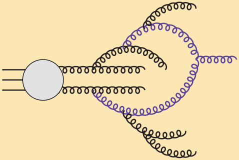


**Estimate in terms of gluon numbers**

Nonlinearities important when

$$\pi R_p^2 = \kappa \frac{dN_g}{dy} \frac{1}{Q^2}$$

Solve for $Q^2 = Q_s^2$; "saturation scale"



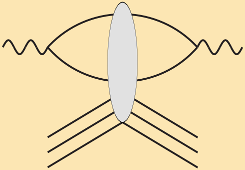
(LHC kinematics: $Q_s \approx 1 .. 2$ GeV)

"EKRT" model: turn this argument quantitative by

- Counting final state gluons
- Fitting constant $\kappa$ to data

# Target rest frame

Example: DIS process $\gamma^* + A \to X$



- $\gamma^*$ consists of partons, to first approximation a $q\bar{q}$ dipole
- Partons traverse target gluon field

**Eikonal (high energy) limit**

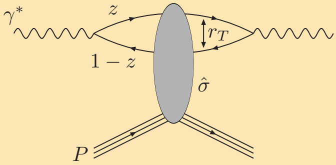
Interaction: Wilson line in target **classical color field**

$$V = \mathbb{P}\exp\left\{-ig\int \mathrm{d}x^+ A_a^- t^a\right\}$$

Scattering amplitude for dipole probe:

$$\mathcal{N}(\mathbf{x}-\mathbf{y}) = 1 - \frac{1}{N_c}\left\langle \mathrm{Tr}\, V^\dagger(\mathbf{x})V(\mathbf{y})\right\rangle \quad \leq \quad 1$$

# Gluon saturation in dipole picture: unitarity




- Scattering amplitude must satisfy unitarity limit $\mathcal{N} \leq 1$ (probability $< 1$)
- Two gluon exchange:

$$\mathcal{N}(r) \sim \alpha_s \frac{r^2 xG(x, Q^2 \sim 1/r^2)}{\pi R_p^2}$$

2-gluon exchange wrong when $Q^2 \sim \frac{1}{r^2} \lesssim Q_s^2 \sim \frac{\alpha_s xG(x,Q_s^2)}{\pi R_p^2}$

**Gluon saturation in target rest frame**

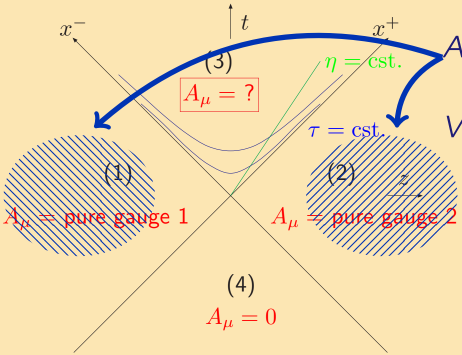
- Degree of freedom is scattering amplitude (not number of partons)
- Saturation appears as unitarity constraint
  $\Longrightarrow$ Built into formalism; does not look dynamical.

# Gluon fields in AA collision

Now two colliding nuclei $\Longrightarrow$ two color currents

$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x})\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x})\delta(x^+)$$

**C**lassical **Y**ang-**M**ills




2 pure gauges

$$A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x})\partial_i V^\dagger_{(1,2)}(\mathbf{x})$$

$V_{(1,2)}(\mathbf{x})$: same Wilson line

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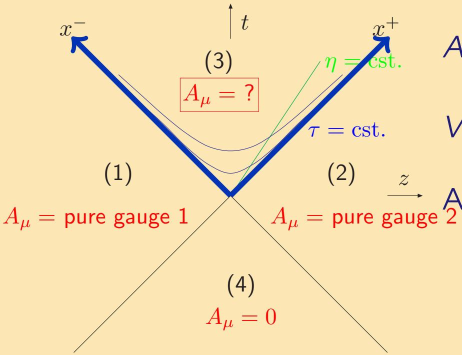


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$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

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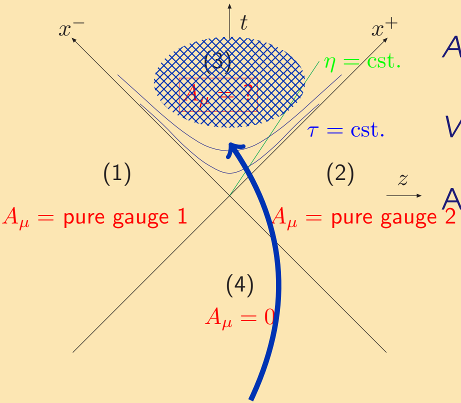


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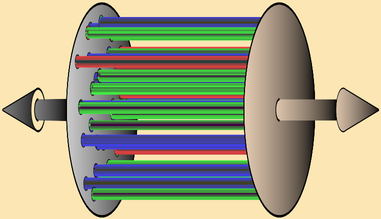
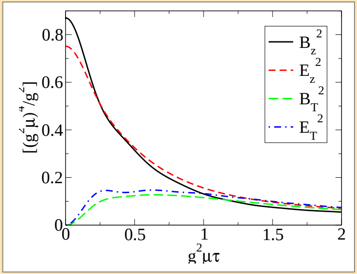
Solve numerically equations of motion for $\boldsymbol{\tau > 0}$

Get energy density, pressure $\Longrightarrow$ plug into hydrodynamics

# Result: initial gluon fields






- Initial condition is longitudinal $E$ and $B$ field,
- Depend on transverse coordinate with correlation length $1/Q_s \implies$ gluon correlations

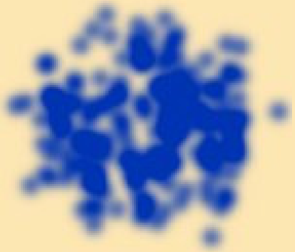
**What was the use of classical approximation?**

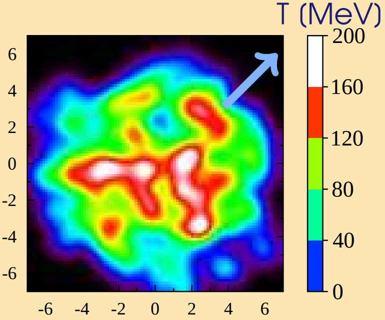
- Enables nonlinear treatment, which makes result finite
- No cutoff imposed by hand

# Fluctuating geometry

A heavy ion is not smooth:

- Nucleon positions fluctuate from event to event
- Initial plasma is spatially very anisotropic






- Interactions/collectivity

\+ Temperature/pressure gradients

⟹ Anisotropic force, acceleration → momentum anisotropy in final state

# Add nucleon positions and shape

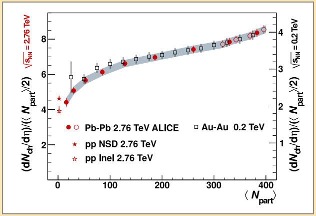
Confinement scale physics

Model by Monte Carlo Glauber:

- Draw positions of nucleons
- Nucleon ⊥ profile ∼ $R_p$
- Jargon of the field:

  Participant nucleons

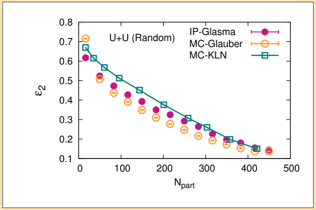
  Binary collisions (NN collisions)




**Understanding collision requires**

- Consistently combine
  1. **QCD** particle production
  2. Geometry ∼ MC Glauber
- Hydrodynamical evolution
- Successfully compute multitude of soft observables

next talk Kim



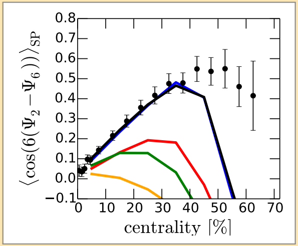
# Conclusions: from QCD to heavy ion event

- Formation of QCD plasma:
  QCD in nontrivial regime:
  - High gluon phase space density: nonperturbative
  - Dynamic phenomena:
    no imaginary time lattice!
  - But possible in weak coupling
    (or very strong coupling talk Jokela )
- Contact with experiments:
  - Combine with confinement scale nuclear geometry
  - Spacetime evolution via hydrodynamics: multiple soft observables

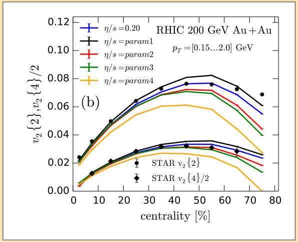
    $\Longrightarrow$ Properties of QCD matter

Hydro calculations from Eskola et al $\Longrightarrow$

Colors: different viscosity



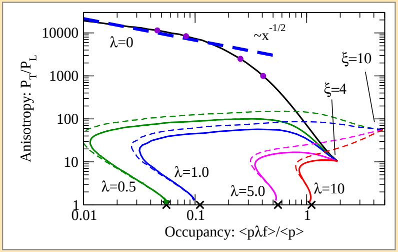

Event plane correlators




Momentum harmonics

# Thermalization at weak coupling

- Initial state classical, $f(k) \sim \frac{1}{\alpha_s} \gg 1$
- Expansion, scattering off $\perp$ plane $\Longrightarrow$ occupy more phase space $\Longrightarrow f(k)$ decreases
- Kinetic theory for $f(k) \ll \frac{1}{\alpha_s}$
- Major problem is **isotropization**
- Close to isotropy: 2nd order hydro




Kurkela, Zhu `arXiv:1506.06647`

Kinetic theory simulation in expanding system, initial anisotropy to a final equilibrium.

LHC estimate $\tau \sim 1$fm