

The gravity track of Higgs inflation

(work with Vera-Maria Enckell, Kari Enqvist,
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Also starring



- Eemeli Tomberg today here at 16.45:
Primordial black holes from Higgs inflation?
- Lumi-Pyry Wahlman on Wednesday in A315 at 14.15: **Can we Probe Gravitational Degrees of Freedom through Inflation?**



Using what you have



$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2 + \xi h^2}{2} R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

$$V(h) = \frac{\lambda}{4} (h^2 - v^2)^2 \simeq \frac{\lambda}{4} h^4$$

- Non-minimal coupling $\xi h^2 R$ is the only new dimension 4 operator for the combined Einstein-Hilbert + SM action.
- The coupling ξ is generated by renormalisation, even if it is classically zero.
- Non-minimal coupling enables Higgs inflation, which uses the only known scalar field that may be elementary.
(Bezrukov and Shaposhnikov: 0710.3755)



The many faces of Einstein gravity



$$S = \int d^4x \sqrt{-g} \left[\frac{1 + \xi h^2}{2} g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- Usually the gravitational degrees of freedom are taken to be the metric and its derivatives.
- In the Palatini formulation, the metric and the connection are independent degrees of freedom.
- In the Einstein-Hilbert case, the metric and the Palatini formulation are equivalent.
- With a non-minimally coupled scalar field, they are different physical theories. (Bauer and Demir: 0803.2664)



To the Einstein frame



$$S = \int d^4x \sqrt{-g} \left[\frac{1 + \xi h^2}{2} g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

- The Einstein frame is reached with the conformal transformation
 $g_{\alpha\beta} \rightarrow (1 + \xi h^2)^{-1} g_{\alpha\beta}$
- In the Palatini case, the conformal transformation does not affect the Ricci tensor.
- To recover canonical kinetic term, define new field χ :

- metric: $\frac{d\chi}{dh} = \sqrt{\frac{1 + \xi h^2 + 6\xi^2 h^2}{(1 + \xi h^2)^2}} \simeq \frac{\sqrt{6}}{h} \Rightarrow h \propto e^{\chi/\sqrt{6}}$

- Palatini: $\frac{d\chi}{dh} = \sqrt{\frac{1 + \xi h^2}{(1 + \xi h^2)^2}} \simeq \frac{1}{\sqrt{\xi} h} \Rightarrow h \propto e^{\sqrt{\xi} \chi}$

- Polynomial potential is transformed into exponential potential.



Higgs potential in Metric vs Palatini



$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} g^{\alpha\beta} \partial_\alpha \chi \partial_\beta \chi - U(\chi) \right]$$

- We get a different Einstein frame potential depending on the gravitational degrees of freedom:

- metric: $U(\chi) \equiv \frac{V[h(\chi)]}{[1 + \xi h(\chi)^2]^2} \simeq \frac{\lambda}{4\xi^2} (1 - 2e^{-\frac{2}{\sqrt{6}}\chi})$

- Palatini: $U(\chi) \simeq \frac{\lambda}{4\xi^2} (1 - 8e^{-2\sqrt{\xi}\chi})$

- The potential is exponentially flat.



Predictions of Higgs inflation on the plateau



- On the exponentially flat plateau, we get:

- metric: $n_s = 1 - \frac{2}{N}$, $r = \frac{12}{N^2}$, $\frac{U}{\epsilon} = \frac{\lambda}{3\xi^2} N^2$

$$n_s = 0.96, r = 5 \times 10^{-3}, \xi = 4 \times 10^4 \sqrt{\lambda}$$

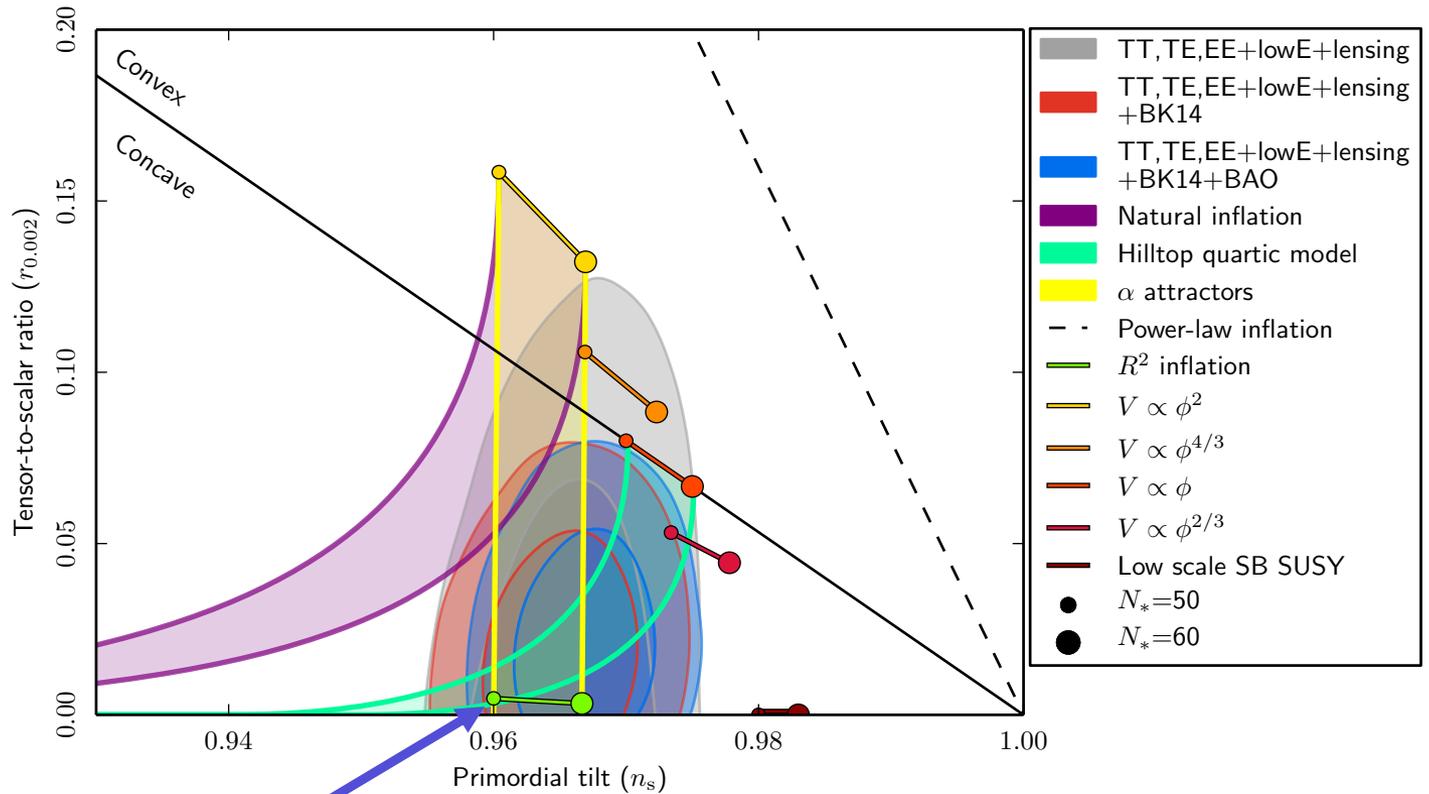
- Palatini: $n_s = 1 - \frac{2}{N}$, $r = \frac{2}{\xi N^2}$, $\frac{U}{\epsilon} = \frac{2\lambda}{\xi} N^2$

$$n_s = 0.96, r = \frac{8 \times 10^{-4}}{\xi} = \frac{8 \times 10^{-14}}{\lambda}, \xi = 10^{10} \lambda$$

- Reheating is fixed: $N \approx 50$. (Figueroa et al: 1504.04600, Rubio and Tomberg: 1902.10148)



The data likes Higgs inflation





Metric vs Palatini: R^2 term



$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (1 + \xi h^2) g^{\alpha\beta} R_{\alpha\beta} + \alpha R^2 - \frac{1}{2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - V(h) \right]$$

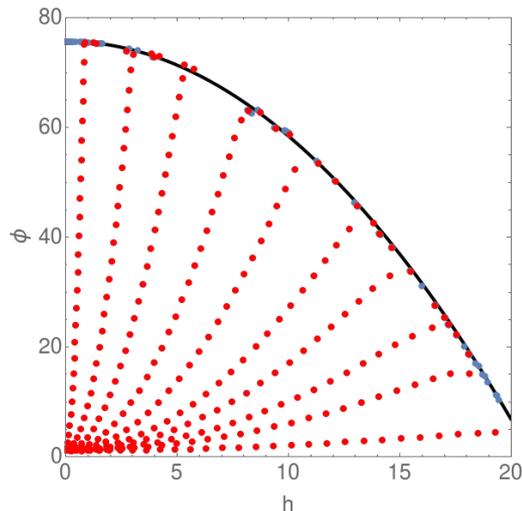
- Loop corrections generate an R^2 term in the action.
- Its effect is completely different in the metric and in the Palatini formulation.



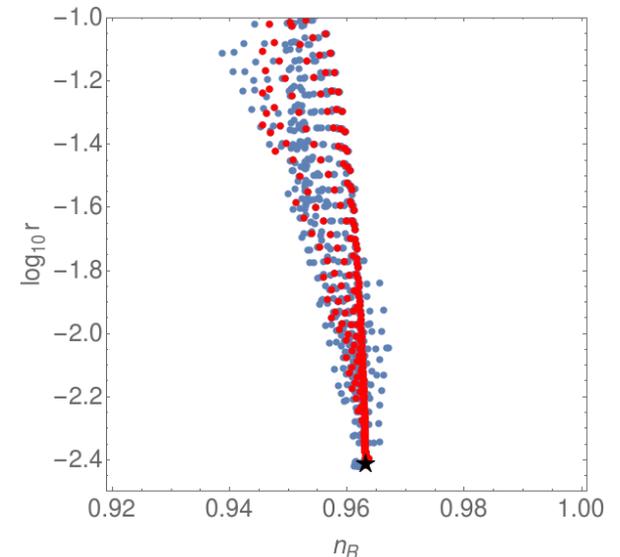
R^2 à la metric: two-field model



- In the metric case, the R^2 term adds a scalar degree of freedom, so we get a two-field model. (Wang and Wang: 1701.06636, Ema: 1701.07665, Zhang, Huang and Sasaki: 1712.09896, He, Starobinsky and Yokoyama: 1804.00409, Gundhi and Steinwachs: 1810.10546, Enckell, Enqvist, SR, Wahlman: 1812.08754)
- The R^2 term destabilises Higgs inflation. (Enckell, Enqvist, SR, Wahlman: 1812.08754)



Trajectories in field space.



Results for observables.



R^2 à la Palatini: saving your favourite model



$$U(\chi) = \frac{V[h(\chi)]}{[1 + \xi h(\chi)^2]^2}$$

- In the Palatini case, the new scalar field does not get a kinetic term and can be integrated out. (Enckell, Enqvist, SR, Wahlman: 1810.05536)

- The action reduces to

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} \partial^\alpha \chi \partial_\alpha \chi + \frac{\alpha}{2} (1 + 8\alpha U) (\partial^\alpha \chi \partial_\alpha \chi)^2 - \frac{U}{1 + 8\alpha U} \right].$$

- The only effect is to change the tensor power spectrum, giving $r \rightarrow r/(1 + 8\alpha U)$.
- This can be used to rescue any scalar field model where r was excluded by the data.



Features unique to Palatini



- In the metric case, the metric and the Riemann tensor are the only geometrical tensors.
- In the Palatini case, we also have the non-metricity tensor $\nabla_{\gamma} g_{\alpha\beta}$.
- Now there are many more scalars, with no correspondence in the metric case.



Kinetic terms for the metric



- We have new kinetic terms, such as

$$h\nabla_{\alpha}h\nabla_{\beta}g^{\beta\alpha} \qquad g_{\alpha\beta}\nabla_{\gamma}g^{\gamma\alpha}\nabla_{\delta}g^{\delta\beta}$$

- Mixing of Higgs and metric kinetic terms sources non-metricity (SR: 1811.09514).
- Going to the Einstein frame and canonical field now has more possibilities:

$$\frac{d\chi}{dh} = \pm\sqrt{K(h)} = \pm\sqrt{\frac{\sum_{n=0}^6 c_n h^{2n}}{\sum_{n=0}^7 d_n h^{2n}}}$$

- By tuning the constants, we can generate an inflection point, α -attractor or the potentials

$$U \propto 1 - a\chi^2, U \propto 1 - a\chi^{-2/3}, U \propto \chi^2, U \propto \chi^{4/3}$$



Higgs as a window to gravity



- Higgs inflation is a conservative possibility using only known degrees of freedom.
- Different formulations of general relativity become inequivalent theories when the matter couples to the connection.
- Higgs opens a window to gravitational degrees of freedom.
- In the Palatini case, r is suppressed.
- In the metric case, R^2 term leads to a complicated two-field model. In the Palatini case, it is harmless.
- In the Palatini case, new gravitational terms can completely change the effective potential. (More to come: teleparallel, Ashtekar, ...)





R^2 à la metric: two-field model details



- In the metric case, the R^2 term adds a scalar degree of freedom, so we get a two-field model. (Wang and Wang: 1701.06636, Ema: 1701.07665, Zhang, Huang and Sasaki: 1712.09896, He, Starobinsky and Yokoyama: 1804.00409, Gundhi and Steinwachs: 1810.10546, Enckell, Enqvist, SR, Wahlman: 1812.08754)
- The action becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{\varphi + \xi h^2 + 6\xi^2 h^2}{2(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha h \partial_\beta h - \frac{3\xi h}{(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha h \partial_\beta \varphi - \frac{3}{4(\varphi + \xi h^2)^2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - \hat{V}(h, \varphi) \right]$$

$$\hat{V}(h, \varphi) = \frac{\lambda}{4} \frac{(h^2 - v^2)^2}{(\varphi + \xi h^2)^2} + \frac{1}{8\alpha} \frac{(\varphi - 1)^2}{(\varphi + \xi h^2)^2}$$



Kinetic terms for the metric



- Considering terms with only up to 2 derivatives (and demanding the equations of motion can be derived without adding boundary terms), the action is (SR: 1811.09514)

$$Q_{\alpha\beta\gamma} \equiv \nabla_{\alpha} g_{\beta\gamma}$$

$$Q_{\gamma} \equiv g^{\alpha\beta} Q_{\alpha\beta\gamma}$$

$$\hat{Q}_{\alpha} \equiv g^{\beta\gamma} Q_{\alpha\beta\gamma}$$

$$\begin{aligned}
 S &= \int d^4x \sqrt{-g} \left[\frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} K(h) g^{\alpha\beta} \nabla_{\alpha} h \nabla_{\beta} h - V(h) \right. \\
 &\quad + A_1(h) \nabla_{\alpha} h \nabla_{\beta} g^{\beta\alpha} + A_2(h) g^{\alpha\beta} g_{\gamma\delta} \nabla_{\alpha} h \nabla_{\beta} g^{\gamma\delta} + B_1(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_{\alpha} g^{\gamma\epsilon} \nabla_{\beta} g^{\delta\eta} \\
 &\quad + B_2(h) g_{\gamma\delta} \nabla_{\alpha} g^{\beta\gamma} \nabla_{\beta} g^{\alpha\delta} + B_3(h) g_{\alpha\beta} \nabla_{\gamma} g^{\gamma\alpha} \nabla_{\delta} g^{\delta\beta} \\
 &\quad \left. + B_4(h) g^{\alpha\beta} g_{\gamma\delta} g_{\epsilon\eta} \nabla_{\alpha} g^{\gamma\delta} \nabla_{\beta} g^{\epsilon\eta} + B_5(h) g_{\gamma\delta} \nabla_{\alpha} g^{\alpha\beta} \nabla_{\beta} g^{\gamma\delta} \right] \\
 &= \int d^4x \sqrt{-g} \left[\frac{1}{2} F(h) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - \frac{1}{2} K(h) g^{\alpha\beta} \nabla_{\alpha} h \nabla_{\beta} h - V(h) \right. \\
 &\quad - A_1(h) \nabla_{\alpha} h \hat{Q}^{\alpha} - A_2(h) \nabla_{\alpha} h Q^{\alpha} + B_1(h) Q_{\gamma\alpha\beta} Q^{\gamma\alpha\beta} + B_2(h) Q_{\gamma\alpha\beta} Q^{\beta\gamma\alpha} \\
 &\quad \left. + B_3(h) \hat{Q}_{\alpha} \hat{Q}^{\alpha} + B_4(h) Q_{\alpha} Q^{\alpha} + B_5(h) Q_{\alpha} \hat{Q}^{\alpha} \right].
 \end{aligned}$$

- Non-minimal coupling and kinetic mixing source non-metricity and torsion.



Coordinates in field space



$$U(\chi) = \frac{V[h(\chi)]}{[1 + \xi h(\chi)^2]^2}$$

- The action can be simplified by choosing suitable coordinates in field space.

$$h \rightarrow \chi(h)$$

$$g_{\alpha\beta} \rightarrow \Omega(h)^{-1} g_{\alpha\beta}$$

$$\Gamma_{\alpha\beta}^{\gamma} \rightarrow \Gamma_{\alpha\beta}^{\gamma} + \Sigma_{\alpha\beta}^{\gamma} = \Gamma_{\alpha\beta}^{\gamma} + g^{\gamma\delta} [\Sigma_1(h) g_{\alpha\beta} \partial_{\delta} h + 2\Sigma_2(h) g_{\delta(\alpha} \partial_{\beta)} h + 2\Sigma_3(h) g_{\delta[\alpha} \partial_{\beta]} h]$$

- All effects of the non-minimal connection are mapped onto the potential:

$$S = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} g^{\alpha\beta} \partial_{\alpha} \chi \partial_{\beta} \chi - U(\chi) \right] .$$

- The field transformation is $\frac{d\chi}{dh} = \pm \sqrt{K(h)}$, where $K(h)$ is a complicated function.



The Higgs case



- Let us consider the Higgs case with only dimension 4 operators.

$$U(\chi) = \frac{V[h(\chi)]}{[1 + \xi h(\chi)^2]^2}$$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} (f + \xi h^2) g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial\Gamma) - k g^{\alpha\beta} \partial_\alpha h \partial_\beta h - \frac{\lambda}{4} h^4 \right. \\ \left. - a_1 h \partial_\alpha h \hat{Q}^\alpha - a_2 h \partial_\alpha h Q^\alpha + (b_{10} + b_{11} h^2) Q_{\gamma\alpha\beta} Q^{\gamma\alpha\beta} + (b_{20} + b_{21} h^2) Q_{\gamma\alpha\beta} Q^{\beta\gamma\alpha} \right. \\ \left. + (b_{30} + b_{31} h^2) \hat{Q}_\alpha \hat{Q}^\alpha + (b_{40} + b_{41} h^2) Q_\alpha Q^\alpha + (b_{50} + b_{51} h^2) Q_\alpha \hat{Q}^\alpha \right]$$

$$Q_{\alpha\beta\gamma} \equiv \nabla_\alpha g_{\beta\gamma}$$

$$Q_\gamma \equiv g^{\alpha\beta} Q_{\alpha\beta\gamma}$$

$$\hat{Q}_\alpha \equiv g^{\beta\gamma} Q_{\alpha\beta\gamma}$$

- Here f , k , a_i , b_{i0} and b_{i1} are constants. We get

$$\frac{d\chi}{dh} = \pm \sqrt{K(h)} = \pm \sqrt{\frac{\sum_{n=0}^6 c_n h^{2n}}{\sum_{n=0}^7 d_n h^{2n}}}$$

- By tuning the constants, we can generate an inflection point, α -attractor or $U \propto 1 - a\chi^2$, $U \propto 1 - a\chi^{-2/3}$, $U \propto \chi^2$, $U \propto \chi^{4/3}$.