The $(g-2)_\mu$ in the Standard Model: review of the calculation of hadronic contributions

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ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

CHIPP Plenary meeting 2021
Spiez, June 10-11, 2021

Outline

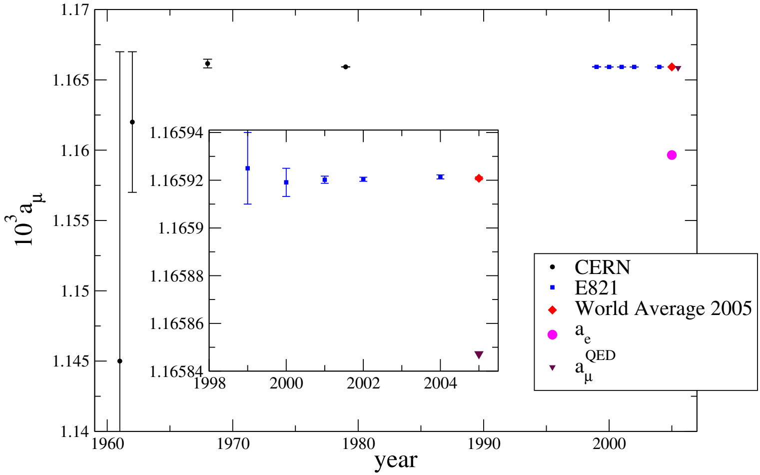
Introduction: $(g-2)_\mu$ in the Standard Model

Hadronic Vacuum Polarization contribution to $(g-2)_\mu$

Hadronic light-by-light contribution to $(g-2)_\mu$

Conclusions and Outlook

History of a_μ measurements



Present status of $(g-2)_\mu$, experiment vs SM

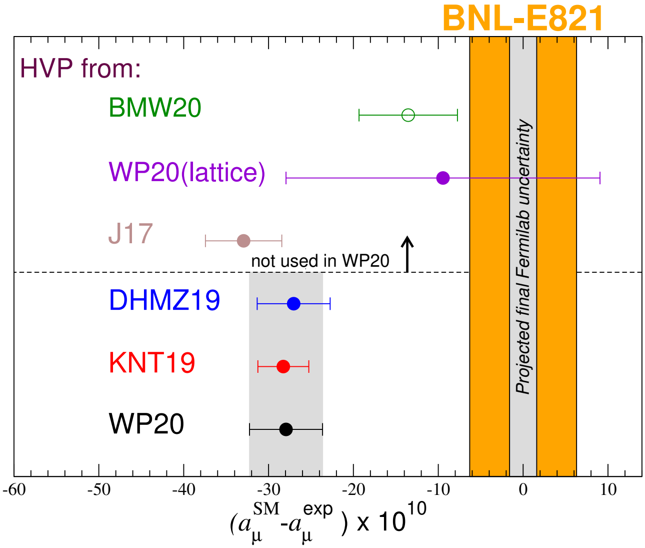
$$a_\mu(BNL) = 116\,592\,089(63) \times 10^{-11}$$

$$a_\mu(FNAL) = 116\,592\,040(54) \times 10^{-11}$$

$$a_\mu(Exp) = 116\,592\,061(41) \times 10^{-11}$$

Present status of $(g-2)_\mu$, experiment vs SM

Before the Fermilab result



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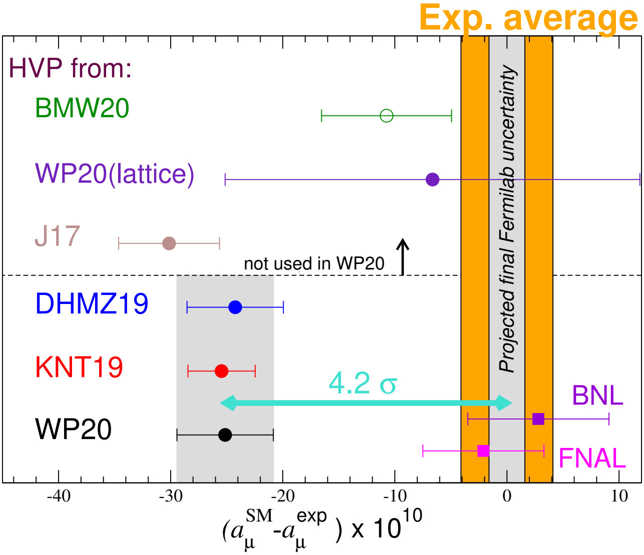

White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	$-98.3(7)$
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice , *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 061(41)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	252(59)

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White Paper (2020): $(g-2)_\mu$, experiment vs SM

White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier
Simon Eidelman
Aida El-Khadra (co-chair)
Martin Hoferichter
Christoph Lehner (co-chair)
Tsutomu Mibe (J-PARC E34 experiment)
(*Andreas Nyffeler* until summer 2020)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

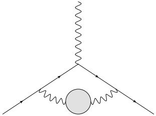
White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

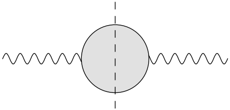
Muon $g-2$ Theory Initiative
Workshops:

- First plenary meeting, Q-Center (Fermilab), 3-6 June 2017
- HVP WG workshop, KEK (Japan), 12-14 February 2018
- HLbL WG workshop, U. of Connecticut, 12-14 March 2018
- Second plenary meeting, Mainz, 18-22 June 2018
- Third plenary meeting, Seattle, 9-13 September 2019
- Lattice HVP workshop, virtual, 16-20 November 2020
- Fourth plenary meeting, KEK (virtual), 28 June-02 July 2021

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$



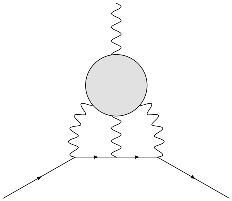


  - unitarity and analyticity $\Rightarrow$ dispersive approach
  - $\Rightarrow$ direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$
  - e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
  - alternative approach: lattice, becoming competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)

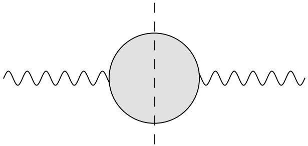


- earlier: "*it cannot be expressed in terms of measurable quantities*"
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- lattice QCD is becoming competitive

(Mainz, RBC/UKQCD)

HVP contribution: Master Formula

Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\Pi(q^2) \propto \sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)

$$a_\mu^{\text{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	-0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

2π: comparison with the dispersive approach

The 2π channel can itself be described dispersively $\Rightarrow$ more constrained theoretically Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18)

Energy range	ACD18	CHS18	DHMZ19	KNT19
$\leq 0.6\,\mathrm{GeV}$		110.1(9)	110.4(4)(5)	108.7(9)
$\leq 0.7\,\mathrm{GeV}$		214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq 0.8\,\mathrm{GeV}$		413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq 0.9\,\mathrm{GeV}$		479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq 1.0\,\mathrm{GeV}$		495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
$[0.6, 0.7]\,\mathrm{GeV}$		104.7(7)	104.2(5)(5)	104.4(5)
$[0.7, 0.8]\,\mathrm{GeV}$		198.3(9)	199.8(0.9)(1.2)	198.9(7)
$[0.8, 0.9]\,\mathrm{GeV}$		66.6(4)	67.5(4)(6)	66.6(3)
$[0.9, 1.0]\,\mathrm{GeV}$		15.3(1)	15.5(1)(2)	15.3(1)
$\leq 0.63\,\mathrm{GeV}$	132.9(8)	132.8(1.1)	132.9(5)(6)	131.2(1.0)
$[0.6, 0.9]\,\mathrm{GeV}$		369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$\left[\sqrt{0.1}, \sqrt{0.95}\,\right]\,\mathrm{GeV}$		490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

WP(20)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$\begin{aligned} a_\mu^{\text{HVP, LO}} &= 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10} \\ &= 693.1(4.0) \times 10^{-10} \end{aligned}$$

WP(20)

The BMW result

Borsanyi et al. Nature 2021

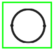
State-of-the-art lattice calculation of $a_\mu^{\text{HVP, LO}}$ based on

- current-current correlator, summed over all distances, integrated in time with appropriate kernel function
- using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin-breaking effects

The BMW result

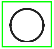
Borsanyi et al. Nature 2021

Isospin-symmetric



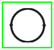
Connected light

$633.7(2.1)_{stat}(4.2)_{syst}$



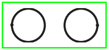
Connected strange

$53.393(89)_{stat}(68)_{syst}$



Connected charm

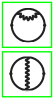
$14.6(0)_{stat}(1)_{syst}$



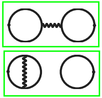
Disconnected

$-13.36(1.18)_{stat}(1.36)_{syst}$

QED isospin breaking: valence

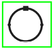


Connected $-1.23(40)_{stat}(31)_{syst}$



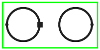
Disconnected $-0.55(15)_{stat}(10)_{syst}$

Strong–isospin breaking



Connected

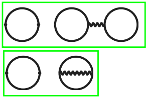
$6.60(63)_{stat}(53)_{syst}$



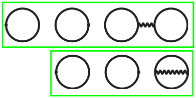
Disconnected

$-4.67(54)_{stat}(69)_{syst}$

QED isospin breaking: sea



Connected $0.37(21)_{stat}(24)_{syst}$



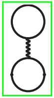
Disconnected $-0.040(33)_{stat}(21)_{syst}$

Other

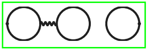
Bottom; higher-order; perturbative

$0.11(4)_{tot}$

QED isospin breaking: mixed



Connected $-0.0093(86)_{stat}(95)_{syst}$



Disconnected $0.011(24)_{stat}(14)_{syst}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{tot}$

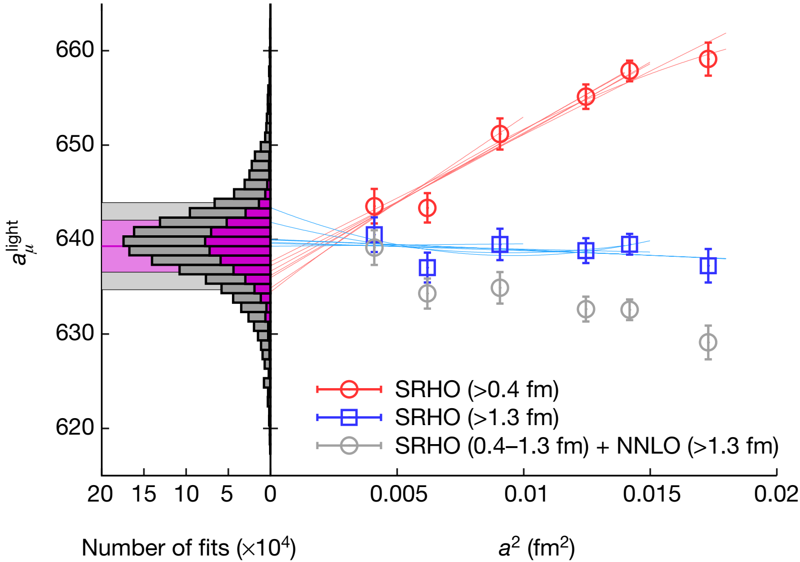
Isospin-breaking

$0.0(0.1)_{tot}$

$$a_\mu^{\text{LO-HVP}}\,(\times 10^{10}) = 707.5(2.3)_{stat}(5.0)_{syst}(5.5)_{tot}$$

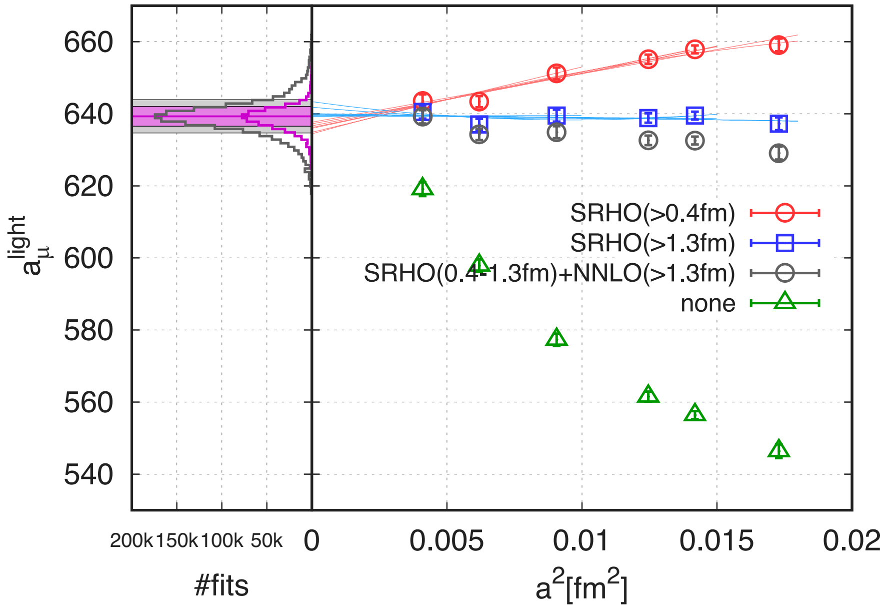
The BMW result

Borsanyi et al. Nature 2021



The BMW result

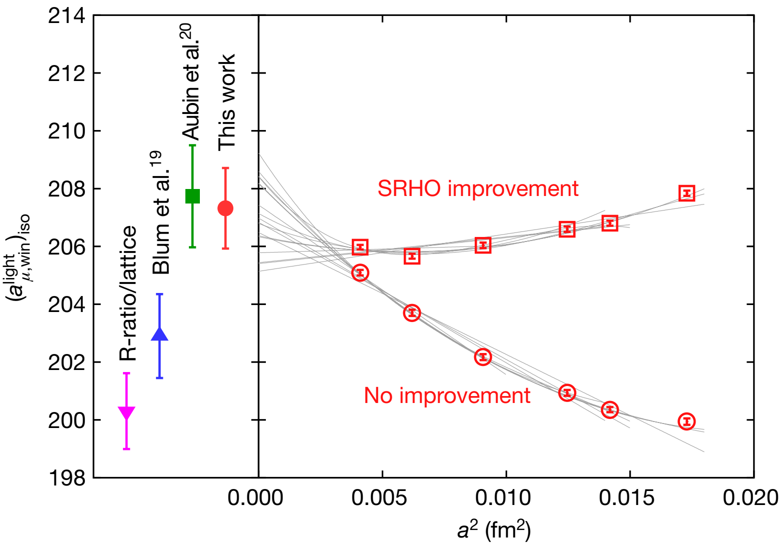
Borsanyi et al. Nature 2021



The BMW result

Borsanyi et al. Nature 2021

Article



Consequences of the BMW result

A shift in the value of $a_\mu^{\text{HVP, LO}}$ would have consequences:

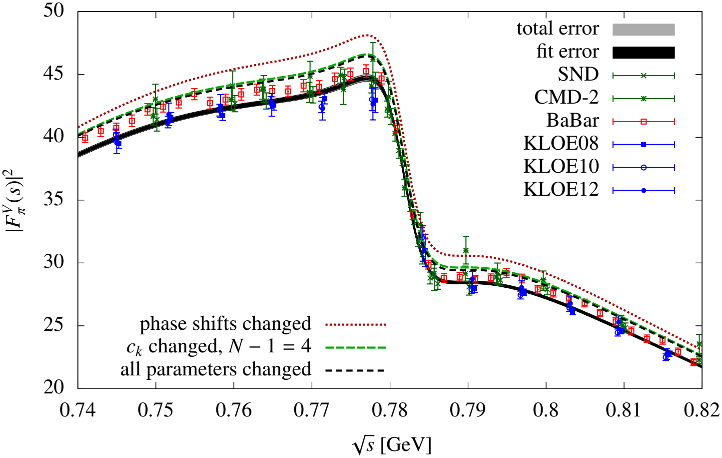
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad \Delta\sigma(e^+e^- \to \text{hadrons})$
- $\Delta\alpha_{\text{had}}(M_Z^2)$ is determined by an integral of the same $\sigma(e^+e^- \to \text{hadrons})$ (more weight at high energy)
- changing $a_\mu^{\text{HVP, LO}}$ necessarily implies a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$: size depends on the energy range of $\Delta\sigma(e^+e^- \to \text{hadrons})$
- a shift in $\Delta\alpha_{\text{had}}(M_Z^2)$ has an impact on the EW-fit
- to save the EW-fit $\Delta\sigma(e^+e^- \to \text{hadrons})$ must occur below $\sim$ 1 (max 2) GeV

  Crivellin, Hoferichter, Manzari, Montull (20)/Keshavarzi, Marciano, Passera, Sirlin (20)/Malaescu, Schott (20)
- or the need for BSM physics would be moved elsewhere

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?

- Below $1-2$ GeV only one significant channel: $\pi^+\pi^-$
- Strongly constrained by analyticity and unitarity ($F_\pi^V(s)$)
- $F_\pi^V(s)$ parametrization which satisfies these
  $\Rightarrow$ small number of parameters GC, Hoferichter, Stoffer (18)
- $\Delta a_\mu^{\text{HVP, LO}} \quad \Leftrightarrow \quad$ shifts in these parameters
  analysis of the corresponding scenarios GC, Hoferichter, Stoffer (21)

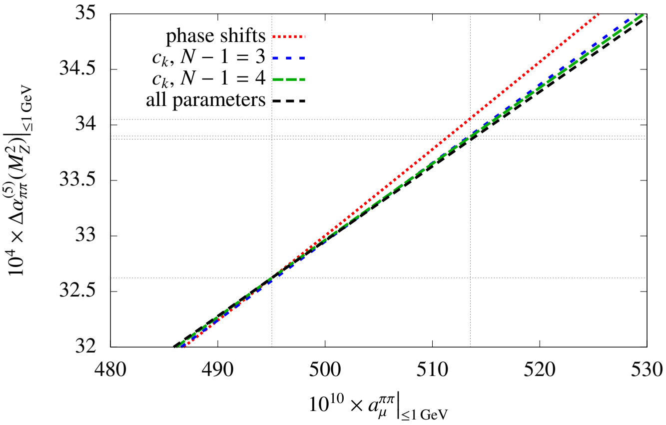
Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



GC, Hoferichter, Stoffer (21)

Tension [BMW20 vs e^+e^- data] stronger for KLOE than for BABAR

Changes in $\sigma(e^+e^- \to \text{hadrons})$ below 1 GeV?



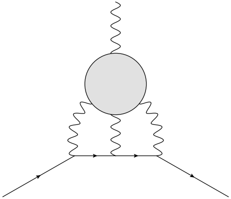
GC, Hoferichter, Stoffer (21)

$$10^4 \Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = \begin{cases} 272.2(4.1) & \text{EW fit} \\ 276.1(1.1) & \sigma_{\text{had}}(s) \end{cases}$$

Calculating the HLbL contribution

Calculating the HLbL contribution is complicated

- 4-point function of em currents in QCD



- a data-driven approach like for HVP seemed hopeless
  "*it cannot be expressed in terms of measurable quantities*"
  but has been recently developed and used

  GC, Hoferichter, Procura, Stoffer=CHPS (14,15,17), Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- lattice QCD is an alternative and is making fast progress

  RBC/UKQCD (20), Mainz (19,20)

HLbL contribution: Master Formula

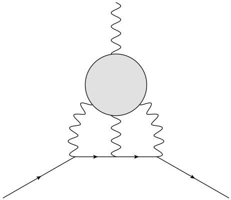
$$a_\mu^{\text{HLbL}} = \frac{2\alpha^3}{48\pi^2}\int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\sum_{i=1}^{12} T_i(Q_1,Q_2,\tau)\bar{\Pi}_i(Q_1,Q_2,\tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta:

$$Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$$

The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$. CHPS (15)

- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment: their imaginary parts are related to measurable subprocesses



Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	−1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

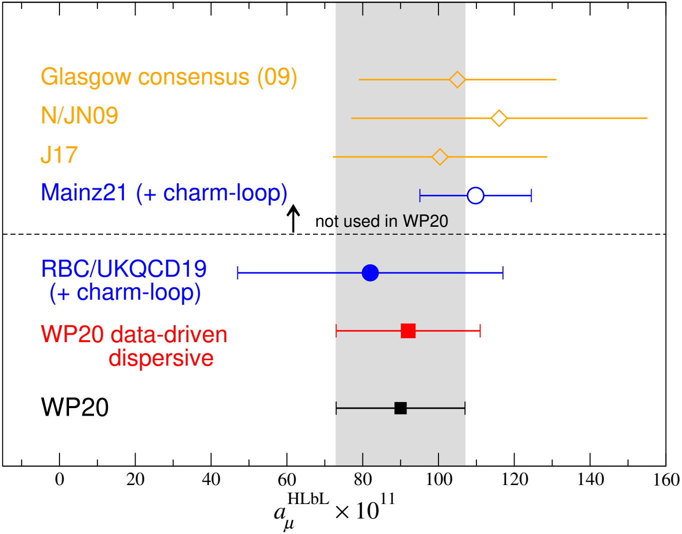
- significant reduction of uncertainties in the first three rows: low-energy region well constrained by a dispersive approach

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid, Kubis, Leupold, Schneider (18)

- 1 – 2 GeV and asymptotic region (short distance constraints) have been improved, but still work in progress (see WP(20))

  Melnikov, Vainshtein (04), (.......), Bijnens, Hermansson-Truedsson, Laub, Rodríguez-Sánchez (20,21)

Situation for HLbL



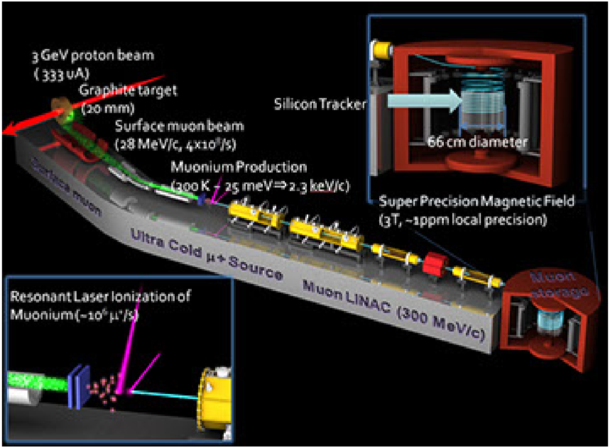
Conclusions

- The WP provides the current status of the SM evaluation of $(g-2)_\mu$: 4.2σ discrepancy with experiment (w/ FNAL)
- Evaluation of the HVP contribution based on the dispersive approach: 0.6% error $\Rightarrow$ dominates the theory uncertainty
- Recent lattice calculation [BMW(20)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  If confirmed $\Rightarrow$ discrepancy with experiment $\searrow$ below 2σ
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(20), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ potential 7σ discrepancy
- Improvements on the theory side:
  - HVP data-driven:
    Other e^+e^- experiments are available or forthcoming:
    SND, BESIII, CMD3, BaBar $\Rightarrow$ Further error reductions
  - HVP lattice:
    BMW result must be confirmed (or refuted) by others.
    Difference to data-driven evaluation must be understood
  - HLbL data-driven: goal of $\sim$ 10% uncertainty within reach
  - HLbL lattice: RBC/UKQCD $\Rightarrow$ similar precision as Mainz.
    Good agreement with data-driven evaluation.

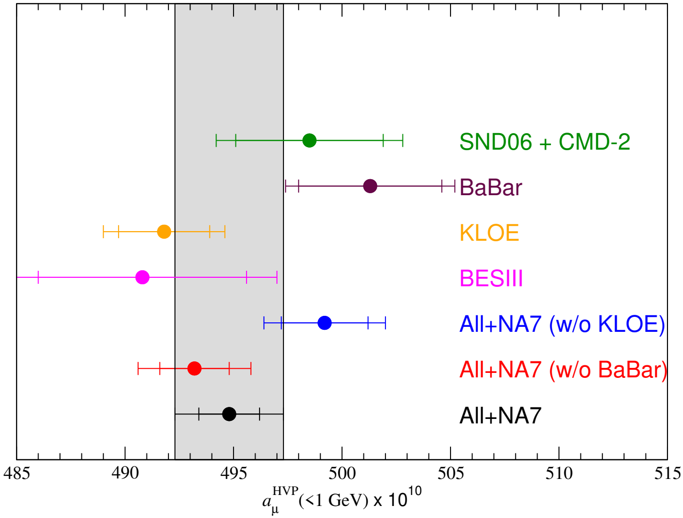
Future: Muon $g-2$/EDM experiment @ J-PARC



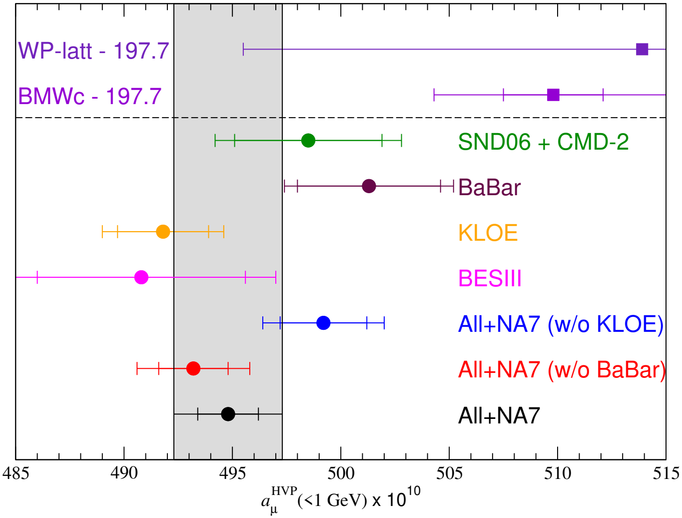
Credit: J-PARC

Backup Slides

BMW vs individual $\pi^+\pi^-$ experiments

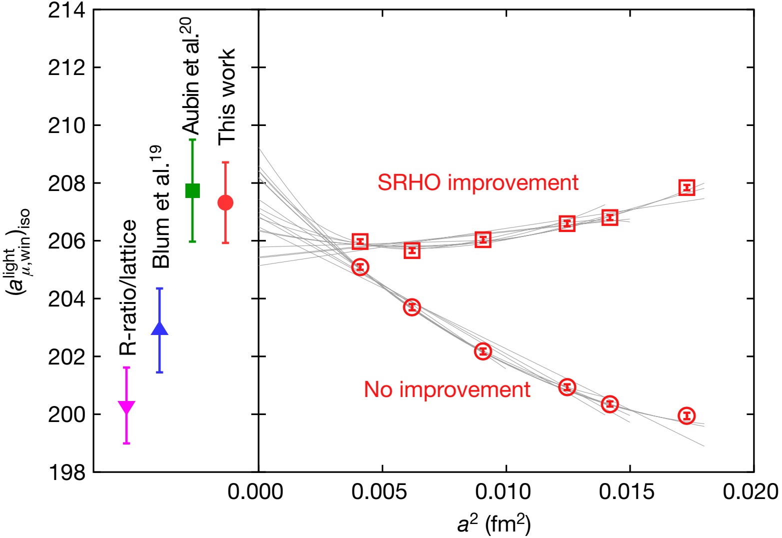


BMW vs individual $\pi^+\pi^-$ experiments



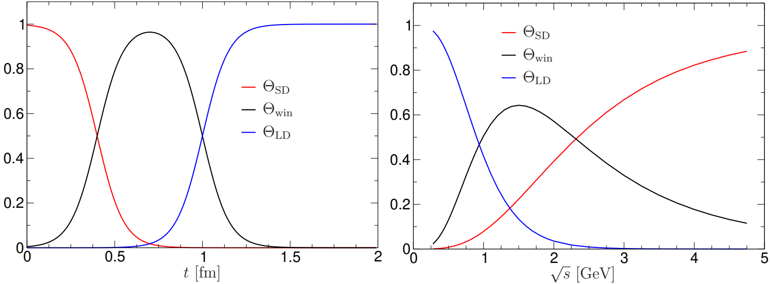
BMW vs individual $\pi^+\pi^-$ experiments

Article

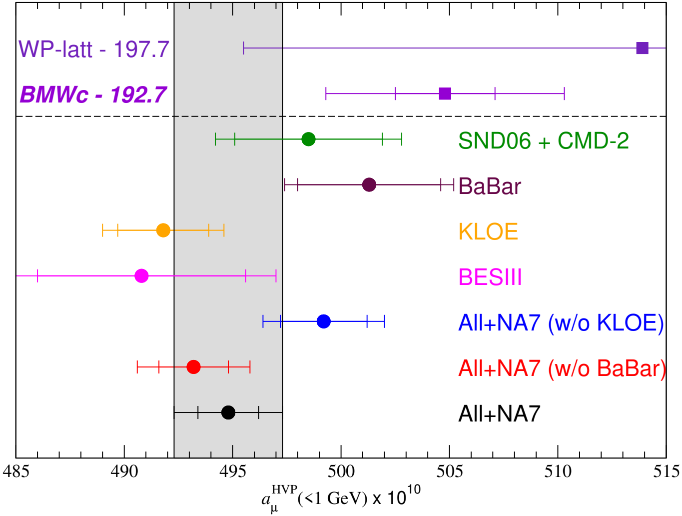


BMW vs individual $\pi^+\pi^-$ experiments

Weight functions for the window quantities



BMW vs individual $\pi^+\pi^-$ experiments



a_μ^{win} suggests that $\sim 5 \times 10^{-10}$ must come from above 1 GeV

GC, Hoferichter, Stoffer

Short-distance contraints

Two different kinematic configurations for large Q_i^2:

1. All momenta large

   Melnikov-Vainshtein (04), Bijnens et al (19)

$$\lim_{Q\to\infty} Q^4\,\bar{\Pi}_1(Q^2,Q^2,Q^2) = -\frac{4}{9\pi^2}.$$

2. $Q^2 \equiv Q_1^2 \sim Q_2^2 \gg Q_3^2$:

   Melnikov-Vainshtein (04)

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2\,\bar{\Pi}_1(Q^2,Q^2,Q_3^2) = -\frac{2}{3\pi^2}$$

   In fact, in the chiral (and large-N_c) limit

$$\lim_{Q\to\infty} Q^2\,\bar{\Pi}_1(Q^2,Q^2,Q_3^2) = -\frac{2}{3\pi^2 Q_3^2}$$

   the Q_3^2 dependence is known exactly

No individual contribution can satisfy these constraints

Approaches to satisfying the SDC

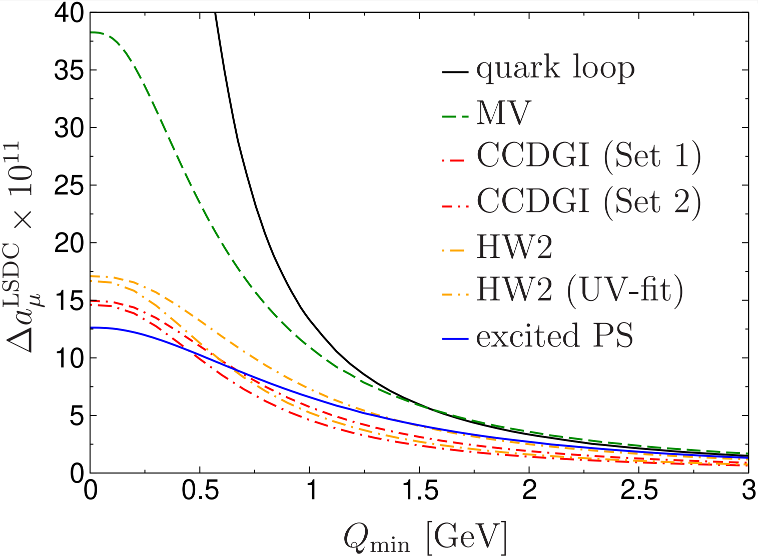
SDCs have been satisfied with three different approaches:

1. pion-pole modification — Melnikhov-Vainshtein (04)
2. Regge model of pseudoscalars — GC, Hagelstein, Hoferichter, Laub, Stoffer (19)
3. Holographic QCD: axial resonances — Leutgeb-Rebhan, Cappiello et al. (20)

	MV model	HW2/CCDGI (set 2)	Regge model
π^0/a_1	17	$(4-5)$	2.7(1.3)
η/f_1	10	$(4-5)$	3.4(1.3)
η'/f_1^*	12	$(6-8)$	6.5(2.0)
Total	38	$(9.6-13)$	12.6(4.1)

- SDCs satisfied with pseudoscalars or axials lead to similar effects on a_μ
- a fourth approach based on interpolants confirms the estimate of HW2/CCDGI and Regge model — Lüdtke, Procura (20)

Approaches to satisfying the SDC



WP(20)