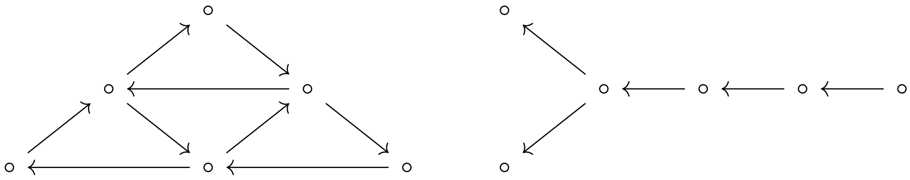
CLUSTER ALGEBRAS

Exercise 1 - Mutations of quivers

(1) Show that all orientations of a tree (with no frozen vertices) are mutation equivalent to each other.

(2) Is the mutation equivalence class of a tree T equal to all the orientations of T ?

(3) Show that the two following quivers are mutation equivalent.



Exercise 2 - Examples of cluster algebras

(1) For the following quivers, prove that the associated cluster algebra is of finite type, describe all the seeds, the cluster variables and the exchange graph.

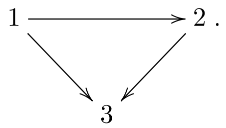
$$1 \qquad\qquad 1 \longrightarrow \boxed{2} \qquad\qquad \boxed{2} \longleftarrow 1 \longrightarrow \boxed{3} \qquad\qquad \boxed{2} \longrightarrow 1 \longrightarrow \boxed{3}$$

(2) Same question for

$$1 \longrightarrow 2 \longrightarrow \boxed{3}.$$

(3) Check that the Kronecker quiver $1 \rightrightarrows 2$ is mutation finite but has an infinite number of cluster variables. One could specialize the initial variables at $1, 1$ and find a relation with Fibonacci numbers.

(4) Let $Q =$



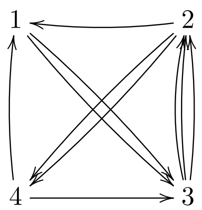
(a) Show that there are infinitely many cluster variables for the corresponding cluster algebra.

(b) Let x_1, x_2, x_3 denote the initial cluster variables. Do the cluster variables x_3 and $\frac{x_1 + x_1^2 x_2 + x_3}{x_2 x_3}$ belong to a common cluster ?

Exercise 3 - Laurent phenomenon

Consider the following quiver

$Q =$



(1) What quiver do you obtain after mutating at the vertex 1.

(2) Find a sequence of mutations which generates a sequence of cluster variables satisfying the recursive relation $z_{n+2} z_{n-2} = z_{n+1} z_{n-1} + z_n^2$.

(3) The somos-4 sequence is the sequence satisfying this recurrence and the initial conditions $z_0 = z_1 = z_2 = z_3 = 1$. Prove that all the terms of this sequence are integers.