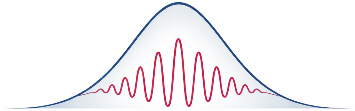


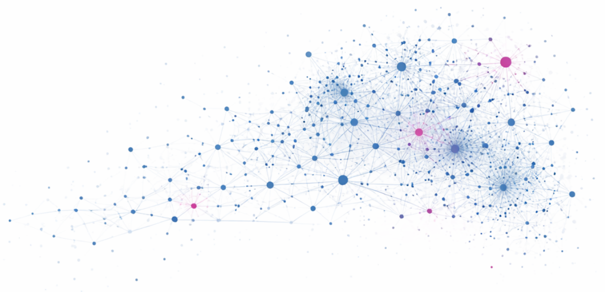
CLASSICAL APPROXIMATION OF QUANTUM MEAN-FIELD DYNAMICS

SRS Les Diablerets - August 31, 2026

Chiara Saffirio

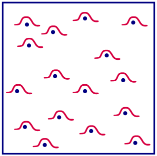
(University of Basel and University of Freiburg)

How can simple macroscopic laws emerge
from extremely complex microscopic dynamics?



Derivation of Effective Evolution Equations

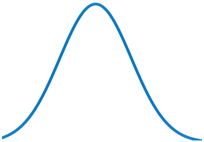
MICRO SCALE



Many-body

description

MACRO SCALE



PDE

Effective Equation

$\longrightarrow$

`scaling limit`

DERIVATION PROBLEM

(Hilbert's 6th problem)

Derivation of Effective Evolution Equations

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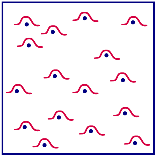

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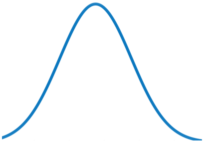

PDE

Effective Equation

$$\rightleftarrows$$

BOTTOM-UP AND TOP-DOWN APPROACHES

DERIVATION OF EFFECTIVE EVOLUTION EQUATIONS

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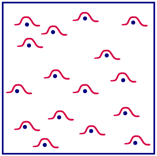

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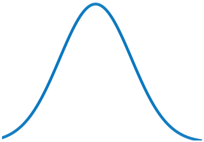

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Effective Equation

$$\longrightarrow$$
$$\longleftarrow$$

BOTTOM-UP AND TOP-DOWN APPROACHES

QUANTITATIVE AND CONSTRUCTIVE METHODS

Mean-Field Effective Equations: Examples

⋆ **Vlasov Equation**

$$\partial_t f + v \cdot \nabla_x f + E_f \cdot \nabla_v f = 0$$

$$\left.\begin{array}{l}\textit{astrophysics}\\ \textit{plasma physics}\\ \textit{semiconductor devices}\end{array}\right\} \textit{Coulomb}$$

MEAN-FIELD EFFECTIVE EQUATIONS: EXAMPLES

⋆ VLASOV EQUATION

$$\partial_t f + v \cdot \nabla_x f + E_f \cdot \nabla_v f = 0$$

astrophysics
plasma physics
semiconductor devices
} *Coulomb*

⋆ HARTREE–FOCK EQUATION

$$i\varepsilon\partial_t \rho = [H_\rho, \rho]$$

large atoms and molecules
quantum chemistry
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BEC-BCS cross-over
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⋆ **Schrödinger/Dirac–Maxwell**

$$\cdots$$

relativistic
QED } *screened Coulomb*

What makes effective theories work?

Many effective theories in physics are extremely successful, but rarely we understand *how* or *why* they work so well.

Mathematical physics tries to answer these questions

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Mathematical physics tries to answer these questions

- quantitative estimates:
  control on the accuracy and time of validity
- emergence of mechanisms:
  correlations and interactions across scales reveal how emergent phenomena appear
- modeling:
  from simplified structures to arbitrarily complex equations

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Many effective theories in physics are extremely successful,
but rarely we understand *how* or *why* they work so well.

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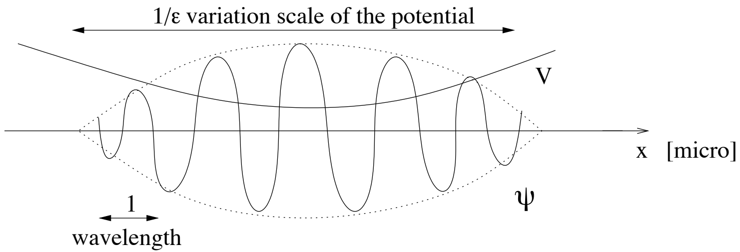
Many effective equations arise when micro and macro scales separate.

An important instance of this phenomenon appears in
the semiclassical regime,
where quantum dynamics develops oscillations at small scale.

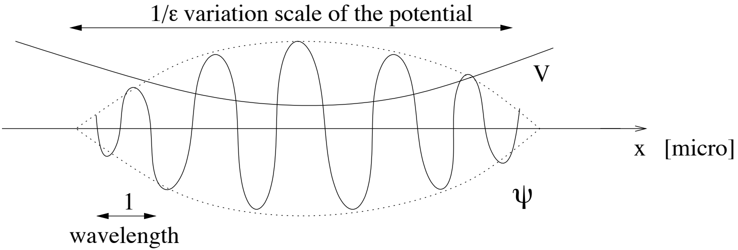
Focus: Semiclassical Regime



A quantum mechanical system is in the semiclassical regime whenever all the data live on a much bigger scale than the quantum wavelength

SCALE SEPARATION BETWEEN THE WAVELENGTH
AND ANY OTHER LENGTH SCALES

Focus: Semiclassical Regime



Mathematical difficulties:

- ⋆ multiscale problem (coexistence of scales)
- ⋆ oscillations $\leftrightarrow$ correlations and/or cancellations
- ⋆ singular limit $\leftrightarrow i\varepsilon\partial_t\psi = H\psi \Longrightarrow \partial_t\psi = \frac{1}{i\varepsilon}H\psi$

CLASSICAL MECHANICS

- the state of a particle is a point in the phase space, i.e. $(x, v) \in \mathbb{R}^d \times \mathbb{R}^d$
- observables are functions $f(x, v)$ on the phase space

Bridging the classical and quantum worlds

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- the state of a particle is an element of a Hilbert space $\mathfrak{h}$
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BRIDGING THE CLASSICAL AND QUANTUM WORLDS

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The Weyl quantization provides a canonical way

$$f(x, v) \quad \longleftrightarrow \quad \boldsymbol{\rho}_f := \mathrm{Op}_\varepsilon^W(f)$$

A simple example:

$$f(x, v) = v^2 + x^2 \quad \longleftrightarrow \quad \boldsymbol{\rho}_f = -\frac{\varepsilon^2}{2}\Delta + x^2$$

The Dynamical Problem I

Theorem 1 (Egorov '69)

Let h be a smooth classical Hamiltonian, $H := \boldsymbol{\rho}_h$ its Weyl quantization and $U(t) := e^{-itH/\varepsilon}$ be the corresponding quantum evolution.
If f is a smooth classical observable and $\boldsymbol{\rho}_f$ its Weyl quantization, then

$$U(t)^* \boldsymbol{\rho}_f U(t) = \boldsymbol{\rho}_{f \circ \Phi_t} + O(\varepsilon)$$

for fixed t, where Φ_t is the Hamiltonian flow generated by h.

Interpretation

- ⋆ Classical observables evolve by transport along the Hamiltonian flow

$$f \mapsto f \circ \Phi_t$$

- ⋆ Quantum observables evolve by conjugation

$$\boldsymbol{\rho}_f \mapsto U(t)^* \boldsymbol{\rho}_f U(t)$$

- ⋆ Egorov's theorem $\Rightarrow$ these two evolutions agree up to an error of order ε.

The Dynamical Problem II

The proof exploits the correspondence principle of quantum mechanics:

$$\frac{i}{\varepsilon}\,[\,\boldsymbol{\rho}_f\,,\,\boldsymbol{\rho}_g\,]\;\simeq\;\{\,f\,,\,g\,\}$$

where $\{\,f\,,\,g\,\}$ denote the Poisson brackets between f and g.

Simple example:

$$\frac{i}{\varepsilon}\,[\,x_j\,,\,i\varepsilon\partial_{x_k}\,]\;=\;\{\,x_j\,,\,v_k\,\}=\delta_{j,k}$$

Moreover:

$$\left[\,\frac{x}{i\varepsilon}\,,\,\boldsymbol{\rho}_f\,\right]\simeq\{x,f\}=\nabla_v f \qquad \left[\,\nabla\,,\,\boldsymbol{\rho}_f\,\right]\simeq\{-v,f\}=\nabla_x f$$

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How close are quantum and classical dynamics?

For which initial states?

On which time scales?

A Dynamical Approach

Pseudo-Differential Calculus + Fourier Integral Operators + PDEs

[C.S.‘19-‘20, Lafleche, C.S. ‘23-‘24, Chong, Lafleche, C.S. ‘23-‘24-‘25]

- Quantized problem and topology
- Reduction to a stability problem in quantized form + error term
  $\longrightarrow$ Unitary transformation (inspired by Bogoliubov trasformation)
- Stability of solutions of the limiting PDE in a chosen topology
  $\longrightarrow$ Correspondence principle of Quantum Mechanics
- Control on the error term (PDO and/or FIO)

THE HARTREE EQUATION

$$i\,\varepsilon\,\partial_t\,\boldsymbol{\rho} = \left[-\frac{\varepsilon}{2}^2\Delta + V * \mathrm{diag}(\boldsymbol{\rho})\,,\,\boldsymbol{\rho}\right]$$

THE VLASOV EQUATION

$$\partial_t f + v\cdot\nabla_x f + (\nabla V * \varrho_f)\cdot\nabla_v f = 0$$

[Narhofer, Sewell ‘81; Spohn ‘81; Lions, Paul ‘93; Figalli, Ligabò, Paul ‘14, . . .]

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WEYL QUANTIZATION

$$\boldsymbol{\rho}_f(x,y) = (2\pi\varepsilon)^{-3}\int f\left(\frac{x+y}{2}, v\right)\, e^{iv\cdot\frac{(x-y)}{\varepsilon}}\, dv$$

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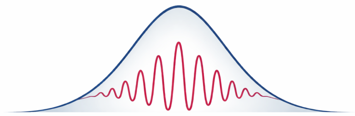

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WEYL QUANTIZED VLASOV EQUATION

$$i\,\varepsilon\,\partial_t\,\boldsymbol{\rho}_f = \left[-\frac{\varepsilon}{2}^{2}\Delta\,,\ \boldsymbol{\rho}_f\right] + A_{\boldsymbol{\rho}_f}$$

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A SIMPLE EXAMPLE

- Quantized problem and topology

$$\|\boldsymbol{\rho} - \boldsymbol{\rho}_f\|_{\mathcal{L}^1} \qquad \text{with} \qquad \|\boldsymbol{\rho}\|_{\mathcal{L}^p} = \varepsilon^{3/p}\mathrm{tr}\left(|\boldsymbol{\rho}|^p\right)^{1/p}$$

- Quantized problem and topology
- Reduction to a uniqueness problem in quantized form + error term

$\longrightarrow$ Unitary transformation (inspired by Bogoliubov trasformation)

$$\boxed{i\,\varepsilon\,\partial_t\,U(t) = H_{\boldsymbol{\rho}}\,U(t) \qquad i\,\varepsilon\,\partial_t\left\{U^*(t)\,(\boldsymbol{\rho}-\boldsymbol{\rho}_f)\,U(t)\right\} = \ldots}$$

Duhamel + trace norm

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- Uniqueness of solutions of the limiting PDE (Vlasov) in corresp. topology

$$\|f_1 - f_2\|_{L^1_{x,v}} \leq \|f_1^{\text{in}} - f_2^{\text{in}}\|_{L^1_{x,v}}\, e^{\Lambda(t)}$$

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$$\boxed{\|\boldsymbol{\rho}-\boldsymbol{\rho}_f\|_{\mathcal{L}^1} \leq \left(\|\boldsymbol{\rho}^{\mathrm{in}} - \boldsymbol{\rho}_{f^{\mathrm{in}}}\|_{\mathcal{L}^1} + \text{error term}\right) e^{\Lambda(t)}}$$

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- Control on the error term (PDO and/or FIO)

THEOREM [Laflèche, C.S. '23; Chong, Laflèche, C.S. '23-'24]

Let $\boldsymbol{\rho}$ be a solution to the Hartree equation and $\boldsymbol{\rho}_f$ be the Weyl quantization of a solution f to the Vlasov equation with initial data $\boldsymbol{\rho}^{\mathrm{in}}$ and f^{in}, respectively. Under assumptions on f^{in}, there exists $\Lambda(\cdot) \in C(\mathbb{R};\mathbb{R}_+)$ such that

$$\|\boldsymbol{\rho} - \boldsymbol{\rho}_f\|_{\mathcal{L}^p} \leq \left(\|\boldsymbol{\rho}^{\mathrm{in}} - \boldsymbol{\rho}_{f^{\mathrm{in}}}\|_{\mathcal{L}^p} + \varepsilon\right) e^{\Lambda(t)}$$

Precise statements

Theorem [Laflèche, C.S. '23; Chong, Laflèche, C.S. '23-'24]

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$$W_{2,\varepsilon}(\boldsymbol{\rho}, f) \leq \left(W_{2,\varepsilon}(\boldsymbol{\rho}^{\text{in}}, f^{\text{in}}) + \varepsilon\right) e^{\Lambda(t)}$$

$W_{2,\varepsilon}$ quantum Wasserstein distance [Golse, Mouhot, Paul '15]

CLASSICAL 2-WASSERSTEIN DISTANCE

$$W_2(f_1, f_2) = \left\{ \inf_{\gamma \in \Gamma(f_1, f_2)} \int |z_1 - z_2|^2 d\gamma(z_1, z_2) \right\}^{\frac{1}{2}}$$

QUANTUM WASSERSTEIN

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QUANTUM WASSERSTEIN PSEUDO-DISTANCE [Golse, Mouhot, Paul '15]

$$W_{2,\hbar}(f, \boldsymbol{\rho}) = \left\{ \inf_{\boldsymbol{\gamma} \in \Gamma(f, \boldsymbol{\rho})} \int \hbar^3 \mathrm{tr}\,(\mathbf{c}_\hbar(z) \boldsymbol{\rho}) dz \right\}^{\frac{1}{2}}$$

where $\mathbf{c}_\hbar$ is the operator acting on functions φ as

$$\mathbf{c}_\hbar(z)\varphi(y) = \left(|x - y|^2 + |v - (-i\hbar\nabla_y)|^2 \right) \varphi(y)$$

with $z = (x, v) \in \mathbb{R}^{2d}$, and $\boldsymbol{\gamma}$ operator valued function such that

$$\hbar^3 \mathrm{tr}\,(\boldsymbol{\gamma}(z)) = f(z) \qquad \int_{\mathbb{R}^6} \boldsymbol{\gamma}(z) dz = \boldsymbol{\rho}$$

What the method achieves

- constructive method for dynamical problems
  - → time-dependent unitary transformation reminiscent of Bogoliubov transformation in many-body quantum mechanics
- control at the level of observables
  - → low (*pure*) and high (*mixed*) regularity states
  - → general initial data
- optimal error bounds
  - → ε
- time dependence: problem- and data-dependent
  - → dispersive properties
  - → mixing properties (see quantum Landau damping [Smith '25])
- applications to other problems/areas

- MANY-BODY QUANTUM SYSTEMS
  - $\longrightarrow$ Fermi gas - Coulomb systems [Chong, Laflèche, C.S. '24]

Applications and Perspectives

- Many-body quantum systems
  - → Fermi gas - Coulomb systems [Chong, Laflèche, C.S. '24]
  - → Derivation of QED models [Leopold '26; Leopold, C.S. '26]
  - → Fractional Quantum Hall effect [Rougerie et al. '24, '25]
  - → Dirac equation [Golse, Leopold, Mauser, Moeller, C.S. '26]

Applications and Perspectives

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- **Weyl Law**
  - → [Benedikter et al. ‘25]
  - → [Cardenas ‘25]
  - → [Chong, Lafleche, C.S. ‘26]

- **Quantum Landau Damping**
  - → [You ‘24; Smith ‘25; Nguyen, You ‘26]

Open Problems and Outlook

- More on the Dirac equation (decoherence)
- Mathematical theory of superconductivity (BCS–BEC cross-over)
- Collisional models
- General relativity models (Einstein–Vlasov)

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Thanks!