Polylogarithms, homology of linear groups, and Steinberg modules

Report of Contributions

https://indico.global/e/9643

Contribution ID: **1** Type: **not specified**

Polylogarithms, K-theory, and motives I

Monday 9 June 2025 09:00 (50 minutes)

In this mini-course I will try and paint a general landscape where polylogarithms interact with algebraic K-theory via the formalism of motives. The goal will be to explain the idea behind Goncharov's program, which aims at computing the rational K-theory of fields via explicit complexes involving formal versions of multiple polylogarithms. The emphasis will be put on concrete computations with « low-dimensional » objects such as Li₂ and K₃.

Presenter: DUPONT, Clement

Contribution ID: **2** Type: **not specified**

Polylogarithms, K-theory, and motives II

Monday 9 June 2025 11:30 (50 minutes)

In this mini-course I will try and paint a general landscape where polylogarithms interact with algebraic K-theory via the formalism of motives. The goal will be to explain the idea behind Goncharov's program, which aims at computing the rational K-theory of fields via explicit complexes involving formal versions of multiple polylogarithms. The emphasis will be put on concrete computations with « low-dimensional » objects such as Li₂ and K₃.

Presenter: DUPONT, Clement

Contribution ID: **3** Type: **not specified**

Polylogarithms, K-theory, and motives III

Tuesday 10 June 2025 10:00 (50 minutes)

In this mini-course I will try and paint a general landscape where polylogarithms interact with algebraic K-theory via the formalism of motives. The goal will be to explain the idea behind Goncharov's program, which aims at computing the rational K-theory of fields via explicit complexes involving formal versions of multiple polylogarithms. The emphasis will be put on concrete computations with « low-dimensional » objects such as Li₂ and K₃.

Presenter: DUPONT, Clement

Contribution ID: **4** Type: **not specified**

Algebraic structures of Steinberg modules 1: Ring structure

Wednesday 11 June 2025 10:00 (50 minutes)

I will begin by reviewing the definition of Steinberg modules and apartments. By concatenating apartments, one can endow the direct sum of all Steinberg modules with the structure of an associative ring. I will also describe a space-level construction of this product due to Galatius—Kupers—Randal-Williams. This ring is not graded-commutative. However, it is commutative in an equivariant sense. I will introduce the necessary formalism and draw connections with the theory of representation stability. Using this framework, I will describe a version of the Church—Farb—Putman vanishing conjecture for congruence subgroups of SL_n(Z) proposed by myself, Napgal, and Patzt.

Presenter: MILLER, Jeremy

Contribution ID: **5** Type: **not specified**

Algebraic structures of Steinberg modules 2: Indecomposables

Thursday 12 June 2025 09:00 (50 minutes)

I will begin by describing a conjecture of Rognes regarding high connectivity of a certain simplicial complex called the common basis complex. This complex appears when studying rank filtrations of algebraic K-theory. I will describe the relationship between this simplicial complex and Steinberg modules. In particular, the homology of the common basis complex computes the derived commutative indecomposables of the equivariant ring built out of the Steinberg modules. Using this observation, we will see that Rognes'connectivity conjecture is intimately related to the Church—Farb—Putman vanishing conjecture. Following joint work with Patzt and Willson, I will sketch a proof of this connectivity conjecture in the case of fields and describe the Koszul dual of the ring built out of Steinberg modules. These results are equivariant versions of results of Galatius—Kupers—Randal-Williams. Time permitting, I will describe similar results for other families of groups such as symplectic groups (joint with Scalamandre and Sroka) and Aut(F_n) (joint with Brück and Piterman).

Presenter: MILLER, Jeremy

Contribution ID: **6** Type: **not specified**

Algebraic structures of Steinberg modules 3: Hopf algebras

Friday 13 June 2025 10:00 (50 minutes)

I will describe how Steinberg modules not only form a ring but in fact form a bi-algebra in a "duoidal"sense. This endows the homology of general linear groups with Steinberg module coefficients with the structure of a Hopf algebra (work joint with Ash and Patzt, independently work of Brown—Chan—Galatius—Payne). I will describe applications to the unstable cohomology of SL_n(Z) and GL_n(Z) due to Brown—Chan—Galatius—Payne and Brown—Hu—Panzer. These results resolve a question of Lee. This duoidal bi-algebra structure also has implications for the cohomology of congruence subgroups, such as work of Ash on injectivity of multiplication maps.

Presenter: MILLER, Jeremy

Contribution ID: **7** Type: **not specified**

Multiple polylogarithms and the Steinberg module I

Wednesday 11 June 2025 09:00 (50 minutes)

In this minicourse I will explain a surprising connection between multiple polylogarithms and the Steinberg module. In the first lecture I will outline Goncharov's programme relating multiple polylogarithms and algebraic K-theory of a field, describe the Hopf algebra structure for multiple polylogarithms and the basic properties of its different families of generators (multiple polylogarithms, iterated integrals, and correlators).

Presenter: RADCHENKO, Danylo

Contribution ID: **8** Type: **not specified**

Multiple polylogarithms and the Steinberg module II

Thursday 12 June 2025 10:00 (50 minutes)

In the second lecture, following a joint work with Charlton and Rudenko, I will define certain spaces of multiple polylogarithms on algebraic tori, and show that they are isomorphic to the tensor square of the Steinberg module of rationals. I will discuss some implications of this isomorphism both for polylogarithms and for the Steinberg module.

Presenter: RADCHENKO, Danylo

Contribution ID: **9** Type: **not specified**

Multiple polylogarithms and the Steinberg module III

Friday 13 June 2025 09:00 (50 minutes)

Finally, in the third lecture, following a work-in-progress by Kupers-Rudenko-Sierra, I will talk about a conjectural relation between the first homology group of GL with coefficients in (the space of indecomposables of) the tensor square of the Steinberg module and the conjectural motivic Lie coalgebra from Goncharov's programme.

Presenter: RADCHENKO, Danylo

Contribution ID: **10** Type: **not specified**

E_∞-algebras of general linear groups I

Monday 9 June 2025 10:00 (50 minutes)

For a field F, the collection of all group chains $C_*(GL_n(F); k)$ of general linear groups over F assemble into an E_∞-algebra **BGL**, which is equipped with an $\mathbb{N}$-grading by the rank n: the multiplicative structure is induced by block-sum of matrices. (Something similar can be done with F replaced by more general rings.) One may and should treat this object **BGL** as an algebraic object in its own right, albeit of a derived nature. In analysing partial presentations for this object, one is led to many things of interest to this workshop: algebraic K-theory of F, its rank filtration, coinvariants and higher homology of (split) Steinberg modules, and of the tensor-square of Steinberg modules, (pre-)Bloch groups, … Information about such partial presentations usually leads to new result of homological stability flavour for the groups $GL_n(F)$. For example, for an infinite field F one can quickly recover the theorem of Suslin—Nesterenko relating $H_n(GL_n(F), GL_{n-1}(F))$ to Milnor K-theory.

In these lectures I will first describe in conceptual terms the homotopical context in which to discuss such things, including the notion of indecomposables and its relation to the iterated bar construction. I will then describe concrete combinatorial models of the objects produced by the abstract machinery, namely the split Tits building and its higher analogues. Finally, I will describe some applications of this theory to homological stability and to the homology of Steinberg modules.

Presenter: RANDAL-WILLIAMS, Oscar

Contribution ID: **11** Type: **not specified**

E_∞-algebras of general linear groups II

Tuesday 10 June 2025 09:00 (50 minutes)

For a field F, the collection of all group chains $C_*(GL_n(F);k)$ of general linear groups over F assemble into an E_∞-algebra **BGL**, which is equipped with an $\mathbb{N}$-grading by the rank n: the multiplicative structure is induced by block-sum of matrices. (Something similar can be done with F replaced by more general rings.) One may and should treat this object **BGL** as an algebraic object in its own right, albeit of a derived nature. In analysing partial presentations for this object, one is led to many things of interest to this workshop: algebraic K-theory of F, its rank filtration, coinvariants and higher homology of (split) Steinberg modules, and of the tensor-square of Steinberg modules, (pre-)Bloch groups, … Information about such partial presentations usually leads to new result of homological stability flavour for the groups $GL_n(F)$. For example, for an infinite field F one can quickly recover the theorem of Suslin—Nesterenko relating $H_n(GL_n(F), GL_{n-1}(F))$ to Milnor K-theory.

In these lectures I will first describe in conceptual terms the homotopical context in which to discuss such things, including the notion of indecomposables and its relation to the iterated bar construction. I will then describe concrete combinatorial models of the objects produced by the abstract machinery, namely the split Tits building and its higher analogues. Finally, I will describe some applications of this theory to homological stability and to the homology of Steinberg modules.

Presenter: RANDAL-WILLIAMS, Oscar

Contribution ID: **12** Type: **not specified**

E_∞-algebras of general linear groups III

Tuesday 10 June 2025 11:30 (50 minutes)

For a field F, the collection of all group chains $C_*(GL_n(F);k)$ of general linear groups over F assemble into an E_∞-algebra **BGL**, which is equipped with an $\mathbb{N}$-grading by the rank n: the multiplicative structure is induced by block-sum of matrices. (Something similar can be done with F replaced by more general rings.) One may and should treat this object **BGL** as an algebraic object in its own right, albeit of a derived nature. In analysing partial presentations for this object, one is led to many things of interest to this workshop: algebraic K-theory of F, its rank filtration, coinvariants and higher homology of (split) Steinberg modules, and of the tensor-square of Steinberg modules, (pre-)Bloch groups, … Information about such partial presentations usually leads to new result of homological stability flavour for the groups $GL_n(F)$. For example, for an infinite field F one can quickly recover the theorem of Suslin—Nesterenko relating $H_n(GL_n(F), GL_{n-1}(F))$ to Milnor K-theory.

In these lectures I will first describe in conceptual terms the homotopical context in which to discuss such things, including the notion of indecomposables and its relation to the iterated bar construction. I will then describe concrete combinatorial models of the objects produced by the abstract machinery, namely the split Tits building and its higher analogues. Finally, I will describe some applications of this theory to homological stability and to the homology of Steinberg modules.

Presenter: RANDAL-WILLIAMS, Oscar

Contribution ID: **13** Type: **not specified**

Cusp forms and the depth filtration

Wednesday 11 June 2025 11:30 (50 minutes)

The goal of this talk is to explain the mechanism by which cusp forms of certain congruence subgroups of SL₂(Z) impose relations in the depth filtration of the motivic fundamental group of the category of mixed Tate motives unramified over the ring Z[mu_N,1/N]. The known depth 2 relations in the case N=1, first observed by Ihara and Takao, were proved by Francis Brown and Hain–Matsumoto. (Both proofs use a period computation due to Brown). One should be able to similarly establish depth 2 relations when N is a prime number > 5. The first steps towards this goal were taken by Eric Hopper in arXiv:2208.01153 using the elliptic KZB connection for the universal family of elliptic curves with a cyclic subgroup of order N removed, which was written down by Calaque and Gonzalez. I will explain his work and how it isolates the period computations that control the relations.

Presenter: HAIN, Richard

Contribution ID: **14** Type: **not specified**

Cocycles for GL_n and graph complexes from differential forms

Monday 9 June 2025 18:05 (50 minutes)

We study invariant differential forms on the space of positive definite matrices. Integration of these forms gives rise to functionals that satisfy relations due to Stokes' theorem. In certain degrees, these relations allow us to interpret the integrals as cocycles for the general linear group or graph complexes. This construction explains many new cohomology classes. Perhaps surprisingly, for even n, the same differential form behaves very differently in the two contexts—so much so that the cohomology classes we construct for the odd graph complex are not pullbacks from our classes for GL_{2n}.

This is joint work (arXiv:2406.12734) with Francis Brown and Simone Hu.

Presenter: PANZER, Erik

Contribution ID: **15** Type: **not specified**

Fun with Tits buildings

Friday 13 June 2025 11:30 (50 minutes)

I'm going to talk about a couple of cool facts about affine and linear Tits buildings for the real numbers. One is that you can get a Solomon-Tits theorem if you take any collection of hyperplanes and their intersections, rather than taking all subspaces. The other is that if you suspend the Tits building twice, then the apartments become cubes, and this leads to a beautiful geometric picture of the products and coproducts in linear Tits buildings.

As an application, we make new computations of scissors congruence groups for polytopes with restrictions on them, such as having vertices in a given number field, or only rectilinear boxes. And we describe the Hopf algebra structure on the coinvariants of the Steinberg module in a way that lifts more easily to spectra. This is joint work with Kupers, Lemann, Miller, and Sroka, and with Klang, Kuijper, Mehrle, and Wittich.

Presenter: MALKIEWICH, Cary

Contribution ID: **16** Type: **not specified**

The Steinberg module and the Church–Farb–Putman conjecture

Monday 9 June 2025 17:00 (50 minutes)

In this talk, I will introduce the Steinberg module, and describe its relationship to a conjecture of Church–Farb–Putman on the high-degree rational cohomology of SL_n(Z). The talk will feature work joint with Benjamin Brück, Jeremy Miller, Peter Patzt, and Robin Sroka.

Presenter: WILSON, Jennifer

Contribution ID: **17** Type: **not specified**

The common basis complex and the partial decomposition poset

Tuesday 10 June 2025 18:00 (50 minutes)

With Piterman-Welker, we showed that Rognes' common basis complex is homotopy equivalent to a certain poset of partial decompositions. I will give an idea of our mostly combinatorial argument and also mention other contexts to which it applies, such as symplectic groups, automorphisms of free groups and matroids.

Presenter: BRÜCK, Benjamin

Contribution ID: **18** Type: **not specified**

Goncharov's programme, and symmetries of weight 6 multiple polylogarithms

Thursday 12 June 2025 11:30 (50 minutes)

In his programme to investigate Zagier's Polylogarithm Conjecture on the values of $\zeta_F(m)$, Goncharov gave a conjectural criterion – the Depth Conjecture – to determine the depth (number of variables) of a linear combination of multiple polylogarithms using the motivic coproduct. I will give an overview of this conjecture and its implications; in particular this Conjecture explains why all multiple polylogarithms of weight 2, and weight 3 can be expressed via Li_2 and Li_3 respectively.

In weight 4, this Conjecture predicts that $Li_{3,1}(x, y)$ (rather, some small modification thereof) should satisfy the dilogarithm 5-term relation independently in each variable, modulo depth 1 terms Li_4. This was established by Gangl, via an explicit 122-term reduction to Li_4's, and conceptually understood by Goncharov and Rudenko in their proof of Zagier's Conjecture for $zeta_F(4)$.

In weight 6, Matveiakin and Rudenko were likewise able to show that $Li_{4,1,1}(x, y, z)$ (or small modification thereof) satisfies the dilogarithm 5-term in each variable, but only modulo depth 2 and *modulo the 6-fold symmetries* $\lambda \mapsto \lambda^{-1}, 1-\lambda$. Goncharov's Depth Conjecture predicts that $Li_{4,1,1}$ should also satisfy these symmetries modulo depth 2, but unlike in weight 4, these symmetries proved much harder to establish. I will show how to explicitly derive these symmetries from the degenerations of the Matveiakin-Rudenko quadrangular polylogarithms to boundary components of $\overline{\mathfrak{M}}_{0,9}$. This establishes Matveiakin-Rudenko's 5-term reduction unconditionally, and hence Goncharov's Depth Conjecture in weight 6, depth 3.

Presenter: CHARLTON, Steven

Contribution ID: **19** Type: **not specified**

Goncharov's conjecture and higher Chow group

Thursday 12 June 2025 17:00 (50 minutes)

I will talk about my recent result (arXiv:2404.06271) which states that for any n, the cohomology of polylogarithmic complex in degree (n-1) and weight n is isomorphic to the appropriate graded piece of algebraic K-theory. This gives a new case of Goncharov's conjecture stating that graded pieces of algebraic K-theory should be isomorphic to the cohomology of the polylogarithmic complex.

Algebraic K-theory can be computed as Bloch's higher Chow group. Elements in the polylogarithmic complex correspond to some explicit algebraic cycles. I will define some explicit map from higher Chow groups to the cohomology of the polylogarithmic complex. The definition of this map was motivated by the analytical properties of so-called Chow dilogarithm which was studied by A. Goncharov.

Presenter: BOLBOCHAN, Vasily

Contribution ID: **20** Type: **not specified**

Tropicalizations of locally symmetric varieties

Tuesday 10 June 2025 17:00 (50 minutes)

I will discuss recent work constructing tropicalizations of locally symmetric varieties. Beyond being of interest just in tropical geometry, I will discuss how such tropicalizations have applications to the cohomology of moduli spaces as well as to the cohomology of arithmetic groups.

Presenter: BRUCE, Juliette

Contribution ID: **21** Type: **not specified**

The Rognes rank spectral sequence and Goncharov program via E_∞-homology.

Thursday 12 June 2025 18:00 (50 minutes)

In this talk I will explain ongoing work with Kupers and Rudenko on a new approach to understanding the Rognes rank spectral sequence and the Goncharov program using the E_∞-homology of the E_∞-algebra associated to symmetric monoidal category of vector spaces over a field. One of the new ideas is to compute the Koszul dual Lie cobracket on the indecompodables of this algebra and use it to understand the d^1 differential of the Rognes rank spectral sequence. I will also mention some applications of these tools to weight 3 polylogarithms and algebraic K theory, and some open conjectures.

Presenter: SIERRA, Ismael