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E_∞-algebras of general linear groups III

Tuesday 10 June 2025 11:30 (50 minutes)

For a field F, the collection of all group chains $C_*(GL_n(F);k)$ of general linear groups over F assemble into an E_∞-algebra **BGL**, which is equipped with an $\mathbb{N}$-grading by the rank n: the multiplicative structure is induced by block-sum of matrices. (Something similar can be done with F replaced by more general rings.) One may and should treat this object **BGL** as an algebraic object in its own right, albeit of a derived nature. In analysing partial presentations for this object, one is led to many things of interest to this workshop: algebraic K-theory of F, its rank filtration, coinvariants and higher homology of (split) Steinberg modules, and of the tensor-square of Steinberg modules, (pre-)Bloch groups, ... Information about such partial presentations usually leads to new result of homological stability flavour for the groups $GL_n(F)$. For example, for an infinite field F one can quickly recover the theorem of Suslin—Nesterenko relating $H_n(GL_n(F), GL_{n-1}(F))$ to Milnor K-theory.

In these lectures I will first describe in conceptual terms the homotopical context in which to discuss such things, including the notion of indecomposables and its relation to the iterated bar construction. I will then describe concrete combinatorial models of the objects produced by the abstract machinery, namely the split Tits building and its higher analogues. Finally, I will describe some applications of this theory to homological stability and to the homology of Steinberg modules.

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