

Speaker: Alex Scott

Title: Induced subgraphs and nearly linear independent sets

Abstract: The quantitative version of Ramsey's theorem, due to Erdos and Szekeres, tells us that every graph on n vertices without a clique of size t contains an independent set of size at least n^{1-c_t} . But what subgraphs do we need to exclude to ensure that a graph on n vertices has an independent set of size $n^{1-o(1)}$? Clearly, we must exclude a clique, and considering random graphs shows that we must also exclude a forest. We show that this is enough. We also discuss a fractional version of the Gyarfás-Sumner Conjecture, and extensions to the multicolour setting.

This is joint work with Tung Nguyen and Paul Seymour.

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Speaker: Eoin Long

Title: Distinct degrees and homogeneous sets

Abstract: Given an n -vertex graph G , let $\text{hom}(G)$ denote the size of a largest homogeneous set in G and let $f(G)$ denote the maximal number of distinct degrees appearing in an induced subgraph of G . The relationship between these parameters has been well studied by several researchers over the last 40 years, beginning with Erdős, Faudree and Sós in the Ramsey regime when $\text{hom}(G) = O(\log n)$.

In this talk I hope to discuss a recent theorem which provides the complete extremal relationship between these parameters (asymptotically), showing that any n -vertex graph G satisfies

$$\max (f(G) \cdot \text{hom}(G), f(G)^{3/2} \cdot \text{hom}(G)^{1/2}) \geq n^{1-o(1)}.$$

This relationship is tight (up to the $n^{-o(1)}$ term) for all possible values of $\text{hom}(G)$, from order $\log n$ to n , as demonstrated by appropriately generated Erdős-Rényi random graphs.

Joint work with Laurențiu Ploscaru.

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Speaker: Peter Allen:

Title: Counting relative to random sets

Abstract: Conlon and Gowers in 2016 described a general approach to proving sparse random analogues of extremal results in combinatorics, such as bounding the minimum and maximum number of triangles in any subgraph of $G(n,p)$ with a given number of edges. The general part of this approach is a functional-analytic statement which, given a sparse setting, constructs a dense model. However, there is a condition which must be shown to hold with high probability to apply the dense model theorem. In Conlon and Gowers' work, there is a technical difficulty with the probabilistic part which leads to a rather involved proof, which applies only in a restricted setting (for example, they can handle triangles but not triangles with a pendant edge), and with quite poor bounds on 'high probability'.

We revisit Conlon and Gowers' approach, and show how to avoid their technical problem, giving a simpler proof of their counting result which applies in a general setting and with optimal probability

bounds. As a corollary, we prove the 'Counting KLR' theorem of Conlon, Gowers, Samotij and Schacht, but for general hypergraphs and with optimal probability bounds.

This is joint work with Julia Boettcher, Joanna Lada and Domenico Mergoni.

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Speaker: Oliver Janzer

Title: Tight general bounds for the extremal numbers of 0-1 matrices

Abstract: A zero-one matrix M is said to contain another zero-one matrix A if we can delete some rows and columns of M and replace some 1-entries with 0-entries such that the resulting matrix is A . The extremal number of A , denoted $\text{ex}(n,A)$, is the maximum number of 1-entries that an $n \times n$ zero-one matrix can have without containing A . The systematic study of this function for various patterns A goes back to the work of Furedi and Hajnal from 1992, and the field has many connections to other areas of mathematics and theoretical computer science. The problem has been particularly extensively studied for so-called acyclic matrices, but very little is known about the general case (that is, the case where A is not necessarily acyclic). We prove the first asymptotically tight general result by showing that if A has at most t 1-entries in every row, then $\text{ex}(n,A) \leq n^{2-1/t+o(1)}$. This verifies a conjecture of Methuku and Tomon.

Our result also provides the first tight general bound for the extremal number of vertex-ordered graphs with interval chromatic number 2 , generalizing a celebrated result of Furedi, and Alon, Krivelevich and Sudakov about the (unordered) extremal number of bipartite graphs with maximum degree t in one of the vertex classes.

Joint work with Barnabas Janzer, Van Magnan and Abhishek Methuku.

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Speaker: Michael Krivelevich

Title: Minimum degree conditions for graph rigidity

Abstract: Graph rigidity is one of the most classical subjects in graph theory, studying geometric properties of graphs. Formally, a graph $G=(V,E)$ is d -rigid if a generic embedding of its vertex set V into \mathbb{R}^d is rigid, namely, every continuous motion of its vertices preserving the lengths of the edges of G necessarily preserves all pairwise distances between the vertices of G .

I will present sufficient minimum degree conditions for d -rigidity. Specifically, we will see that for $d = O(n/\log^2 n)$, if an n -vertex graph G has minimum degree $\delta(G) \geq n/2+d-1$, then it is d -rigid, and this is optimal up to a constant factor of 2 in the coefficient of d .

As a byproduct of our arguments, we also show that for $d = O(n/\log^2 n)$, every n -vertex graph G of minimum degree d has a partition into $d+1$ non-empty parts, with an edge of G between every pair of parts. (This is to say that the so-called pseudoachromatic number of G is at least $d+1$.)

This number of parts is optimal.

The talk is based on joint work with Alan Lew and Peleg Michaeli.

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Speaker: Shoham Letzter
Title: Pancyclicity of highly connected graphs

Abstract: A well known result due to Erdős and Chvatál (1972) asserts that if the vertex connectivity of a graph is at least as large as its independence number, then the graph has a Hamilton cycle. We extend this by showing that if the graph has sufficiently many vertices and its vertex connectivity is strictly larger than the independence number, then the graph is pancyclic, meaning that it has a cycle of length l for every l between 3 and the order of the graph. This proves a conjecture of Jackson and Ordaz (1990) for large graphs, and improves upon a recent result of Draganić, Munhá-Correia and Sudakov.

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Speaker: Yuval Wigderson
Title: Graph decompositions, Ramsey theory, and random graphs

Abstract: A basic result of probabilistic combinatorics, originally due to Erdős and Rényi, is the determination of the threshold at which the random graph $G_{n,p}$ contains a triangle with high probability. But one can also ask more refined versions of this question, where we ask not just for one triangle but for many triangles which interact in complicated ways. For example, what is the threshold at which we can no longer partition $G_{n,p}$ into two triangle-free subgraphs?

In this talk, I will discuss the proof of the Kohayakawa–Kreuter conjecture, which gives a general answer to all such questions. Rather surprisingly, a key step of the proof is a purely deterministic graph decomposition statement, closely related to classical results such as Nash-Williams' tree decomposition theorem, whose proof uses techniques from combinatorial optimization and structural graph theory.

Based on joint works with Micha Christoph, Eden Kuperwasser, Anders Martinsson, Wojciech Samotij, and Raphael Steiner.

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Speaker: Gal Kronenberg
Title:

Abstract:

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Speaker: Julian Sahasrabudhe
Title: Diagonal multicolour Ramsey numbers

Abstract: In this talk I will discuss some of the ideas behind the recent improvement to the upper bounds on the diagonal multi-colour Ramsey numbers. In particular, a connection with a geometric perspective on the problem.

This is based on joint work with Balister, Bollobás, Campos, Griffiths, Hurley, Morris and Tiba.

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Speaker: Lior Gishboliner

Title: Edge-modification problems for hypergraphs

Abstract: The edge-modification problem for an r -uniform hypergraph H is the algorithmic problem of determining, for an input r -uniform hypergraph G , the smallest number of edges that needs to be deleted in order to turn G into an H -free hypergraph. Alon, Shapira and Sudakov proved that for a (2-uniform) non-bipartite graph H , this problem is NP-hard, and even NP-hard to approximate up to error $n^{2-\epsilon}$ for any fixed $\epsilon > 0$. Addressing a question of Ailon and Alon, we generalize this result to r -uniform hypergraphs.

This is a joint work with Yevgeny Levanzov and Asaf Shapira.

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Speaker: Matthew Kwan

Title: Entangled states are typically incomparable

Abstract: For two independent random vectors x, y on the n -simplex, what is the probability that x and y are comparable in the majorisation order? For different distributions of x and y , this question has been (independently) studied in probabilistic combinatorics and in quantum information theory. In particular, with an appropriate distribution for x and y , this is equivalent to studying the probability that for two randomly entangled quantum states ψ and ϕ , it is possible to transform ϕ into ψ via local operations and classical communication. With Vishesh Jain and Marcus Michelen, we proved a conjecture of Nielsen, and some related predictions of Cunden, Facchi, Florio and Gramegna, in this direction.

In this talk I plan to introduce the subject, outline our proof of Nielsen's conjecture, and mention some open problems.

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Speaker: Liana Yepremyan

Title: Counterexample to Babai's lonely colour conjecture

Abstract: Motivated by colouring minimal Cayley graphs, in 1978, Babai conjectured that no-lonely-colour graphs have bounded chromatic number. We disprove this in a strong sense by constructing graphs of arbitrarily large girth and chromatic number that have a proper edge-colouring in which each cycle contains no colour exactly once.

Joint work with James Davies and Meike Hatzel.

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Speaker: Nemanja Draganić

Title: Resolution of the Erdős-Faudree problem on Hamiltonian subsets of Dirac graphs

Abstract: In 1996, in his last written paper, Erdős asked a question he formulated together with Faudree: does there exist a positive epsilon such that any $(n+1)$ -regular graph on $2n$ vertices has at least $\epsilon \cdot 2^{2n}$ Hamiltonian subsets? We answer this question in the affirmative in a strong form by showing that any such graph has at least $(1/2 - o(1)) \cdot 2^{2n}$ Hamiltonian subsets. This is tight up to lower-order terms.

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Speaker: Istvan Tomon

Title: Discrepancy of graphs and hypergraphs

Abstract: The discrepancy of a graph (or a hypergraph) measures the maximum deviation of the sizes of its induced subgraphs from their expected size. This notion is closely related to many other extensively studied functions of graphs, such as maximum cut, minimum bisection, and spectral gap. I will talk about how to attack several problems in this area with the help of linear algebraic techniques. Based on joint works with Eero Räty and Benny Sudakov.

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Speaker: Richard Montgomery

Title: Transversal decompositions of random Latin squares

Abstract: Not every Latin square has a transversal, but, as Kwan has shown, almost every Latin square does. I will discuss a proof that, moreover, almost every Latin square can be decomposed into disjoint transversals.

Joint work with Candida Bowtell

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Speaker: Huy Tuan Pham

Title: Random Cayley graphs: Dense and sparse

Abstract: Given a finite group G , a random Cayley graph $G(p)$ is a Cayley graph of G whose symmetric generating set is chosen by including each set $\{g, g^{-1}\}$ independently with probability p .

In the dense regime where p is constant, I will discuss results on clique and independence numbers of random Cayley graphs of general groups, a proof of a conjecture of Alon and Orlitsky, as well as progress towards a conjecture of Alon on existence of Ramsey Cayley graphs in arbitrary groups. These questions are naturally connected with some fundamental problems in additive combinatorics. Surprisingly, our insights suggest that in many of these problems the group structure is superfluous and can be replaced by much more general combinatorial structures.

In the sparse regime, I will discuss some recent improvements on upper bounds to the independence number of $G(p)$. In the polynomially sparse regime, the improved estimates on the independence

number of $G(p)$ lead to improved bounds on a question of Ben Green on the largest subset of \mathbb{Z}_N which cannot be written as a sumset $A+A$.

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Speaker: Matthew Jenssen

Title: Lower tails for triangle counts in the critical window

Abstract: The classical lower-tail problem for triangles in random graphs asks the following: given $\eta \in [0,1]$, what is the probability that $G(n,p)$ contains at most η times the expected number of triangles? When $p = o(n^{-1/2})$ or $p = \omega(n^{-1/2})$ the asymptotics of the logarithm of this probability are known via Janson's inequality in the former case and regularity or container methods in the latter case.

We prove for the first time asymptotic formulas for the logarithm of the lower tail probability when $p = c n^{-1/2}$ for c constant. Our results apply for all c when $\eta \geq 1/2$ and for c small enough when $\eta < 1/2$. For the special case $\eta = 0$ of triangle-freeness, our results prove that a phase transition occurs as c varies (in the sense of a non-analyticity of the rate function), while for $\eta \geq 1/2$ we prove that no phase transition occurs.

Our method involves ingredients from algorithms and statistical physics including rapid mixing of Markov chains and the cluster expansion. We complement our asymptotic formulas with efficient algorithms to approximately sample from $G(n,p)$ conditioned on the lower tail event.

Joint work with Will Perkins, Aditya Potukuchi and Michael Simkin.

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Speaker: Oleg Pikhurko

Title: Constructions of Turán systems that are tight up to a multiplicative constant

Abstract: The Turán density $t(s,r)$ is the asymptotically smallest edge density of an r -graph for which every set of s vertices contains at least one edge. The question of estimating this function received a lot of attention since it was first raised by Turán in 1941. A trivial lower bound is $t(s,r) \geq 1/\binom{s}{r}$. In the early 1990s, de Caen conjectured that $t(r+1,r)$ grows faster than $O(1/r)$ and offered 500 Canadian dollars for resolving this question.

I will give an overview of this area and present a construction disproving this conjecture by showing more strongly that for every integer R there is C such that $t(r+R,r) \leq C/\binom{r+R}{r}$, that is, the trivial lower bound is tight for every fixed R up to a multiplicative constant $C=C(R)$.

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Speaker: Mehtaab Sawhney

Title: Hitting time mixing for the random transposition walk

Abstract: Consider shuffling a deck of n cards labeled 1 through n by at each time step picking one card uniformly at random with your left hand and one uniformly at random with your right hand and swapping them. We prove that at the first time one touches all the cards, the deck is well shuffled in total variation distance. The proof is inspired by classical proof in random graphs; especially that at the minimum degree 2 threshold the random graph contains a Hamiltonian cycle.

Joint w. Vishesh Jain