

# Algebra and Quantum Geometry of BPS Quivers

## Monday 20 January 2025

### Non-toric brane webs, Calabi-Yau 3-folds, and 5d SCFTs (09:30 - 10:30)

- **Presenter:** BOUSSEAU, Pierrick

One of the most remarkable predictions of string/M-theory is the existence of 5-dimensional superconformal field theories (5d SCFTs). There are two main approaches for constructing these 5d SCFTs, using either M-theory on canonical 3-fold singularities, or intersecting branes in Type IIB string theory. A natural question is to compare these two approaches. The answer is well-known for webs of 5-branes in Type IIB string theory, where the M-theory dual canonical 3-fold singularity is a toric Calabi–Yau 3-fold. In this talk, building on recent advances in mirror symmetry and enumerative geometry, I will provide an answer for the more general case of webs of 5-branes with 7-branes and explain how to construct the M-theory dual non-toric Calabi-Yau 3-fold. This is joint work with Valery Alexeev and Hulya Arguz.

### Mock modularity of Calabi-Yau threefolds (11:00 - 12:00)

- **Presenter:** ALEXANDROV, Sergey

I'll explain the modular properties, and their physical origin, of the generating functions of rank 0 DT invariants of Calabi-Yau threefolds counting D4-D2-D0 bound states in type II compactifications of string theory. These properties can be used to find generating functions up to a finite number of coefficients, the so-called polar terms, which in turn can be fixed using wall-crossing. I'll show how this program is realized for compact one-parameter Calabi-Yau threefolds and 2 units of D4-brane charge, leading to explicit mock modular forms encoding BPS indices. For higher charges, the polar terms remain so far inaccessible, but I'll present a general solution of modular anomaly equations in terms of indefinite theta series.

### Nekrasov's gauge origami and Oh-Thomas's virtual cycles (16:30 - 17:30)

- **Presenter:** KOOL, Martijn

Nekrasov's 4D ADHM quiver is a generalisation of the 2D and 3D ADHM quivers. I will give a mathematical definition of the corresponding "gauge origami" partition function in terms of certain torus localized Oh-Thomas virtual cycles. I will relate the orientation problem of gauge origami to the one of Hilbert schemes of points on affine 4-space, which was previously analysed in physics by Nekrasov-Piazzalunga ("Magnificent Four") and in algebraic geometry in a joint work with J. Rennemo. I will provide a conjectural sheaf-theoretic interpretation of gauge origami in terms of certain moduli of 2-dimensional framed sheaves on  $(\mathbb{P}^1)^4$ . Joint work with N. Arbesfeld and W. Lim.

### Towards relative Donaldson-Thomas theory in dimension 4 (18:00 - 19:00)

- **Presenter:** ZHAO, Gufang

The goal of this talk is to explore definitions of numerical invariants from moduli spaces originated from relative Donaldson-Thomas theory in dimension 4. The definition of invariants is formulated as functoriality of the cohomology or K-theory of these moduli spaces. Existence in examples will be given. Partially based on joint works with Cao, Toda, and Zhou.

# Tuesday 21 January 2025

## Riemann-Hilbert correspondence and wall-crossing structures (09:30 - 10:30)

- **Presenter:** SOIBELMAN, Yan

Generalized Riemann-Hilbert correspondence proposed by Maxim Kontsevich and myself in 2015 relates two areas of mathematics: deformation quantization and Floer theory. The aim of my talk is to revisit various aspects of the generalized RH-correspondence. In particular I am going to explain how wall-crossing structures relate local and global versions of the RH-correspondence.

## Universal Virasoro constraints for additive theories (11:00 - 12:00)

- **Presenter:** BOJKO, Arkadij

Recently, Virasoro constraints were formulated for sheaf-counting theories which was motivated by their original appearance on the Gromov-Witten side. The main difference lies in the ability to take direct sums of sheaves which allowed us (B.-Lim-Moreira) to connect these constraints to the geometric vertex algebra of Joyce. The upshot, that moduli spaces satisfying Virasoro constraints determine physical states of the vertex algebra, holds in larger generality than just for sheaves. In this talk, I will recall our original statement and explain how it can be extended to representations of quivers. This is already sufficient to illustrate how to formulate it for any additive theory. One can then use wall crossing expressed in terms of Joyce's vertex algebra, to prove Virasoro constraints in many cases.

## Scattering diagram for noncommutative resolutions (16:30 - 17:00)

- **Presenter:** DESCOMBES, Pierre

Scattering diagrams techniques allows to compute the BPS invariants of quiver with potential in terms of some initial data, which are expected to be simple in physically sensible examples (eg, they have been determined for class S theory). We are interested in quiver with potential giving noncommutative resolution of CY3 singularities: in this case, the space of stability conditions of the quiver is divided into chambers, corresponding to different (commutative) resolutions of the singularity, related by Mori transformations. Using techniques from the theory of Bridgeland stability condition, we prove that the initial data of the quiver are supported on the wall between these chambers, hence can be determined from the birational geometry of the resolutions. In particular, this gives a complete description of the initial data for toric quivers.

## BPS categories for quivers and Higgs bundles (17:30 - 18:30)

- **Presenter:** PADURARIU, Tudor

I will report on joint work with Yukinobu Toda about a refinements of BPS invariants using derived categories of coherent sheaves. We propose two conjectural dualities between BPS categories for Higgs bundles, which can be interpreted as a version of SYZ mirror symmetry and of the Dolbeault Langlands equivalence. I will also discuss some results about BPS categories for quivers, with a special focus on quivers with one vertex and multiple loops, which feature prominently when studying Higgs bundles.

# Wednesday 22 January 2025

## Topological correlators of massive $N = 2$ SQCD and 5d $N = 1$ SYM on $S^1$ (18:00 - 19:00)

- **Presenter:** MANSCHOT, Jan

TBA

# Thursday 23 January 2025

## Overview of DT invariants from Exponential Networks (09:30 - 10:30)

- **Presenter: ROMO, Mauricio**

I will give an overview on how to compute DT invariants of toric CYs, using exponential networks. I will present an proposal on the interpretation of these networks from the perspective of special Lagrangians on the mirror. As time allows, I will also present some new results involving DT invariants with the inclusion of noncompact divisors.

## TBA (11:00 - 12:00)

- **Presenter: EKHOLM, Tobias**

## Membranes and Maps (16:30 - 17:00)

- **Presenter: SCHUELER, Yannik**

In joint work with Andrea Brini, we proposed a definition of the refined topological string on a Calabi-Yau threefold  $X$  with a torus action in terms of the equivariant Gromov-Witten theory of the fivefold  $X \times \mathbb{C}^2$ . In my talk, I will outline the construction, mention some of its features, and especially discuss an anticipated equivariant generalisation of Gopakumar-Vafa invariants. This expectation based on a conjectural relation to the M2-brane index of the fivefold, which we expect to hold for arbitrary CY5 with a torus action. I will present evidence for for the conjecture in the context of local curves.

## TBA (17:30 - 18:30)

- **Presenter: GAIOTTO, Davide**

# Friday 24 January 2025

**TBA (09:30 - 10:30)**

- **Presenter: SHENDE, Vivek**

**TBA (11:00 - 12:00)**

- **Presenter: DOTSENKO, Vladimir**