Workshop on Resurgence, wall-crossing and geometry



Contribution ID: 10

Type: not specified

Integral Representations of Quantum Invariants of Three-Manifolds

Thursday 16 January 2025 09:30 (1 hour)

In this talk we focus on finite dimensional integral representations of topological invariants of three-manifolds which are motivated by resurgence.

Our first example will be the Witten-Reshetikhin-Turaev (WRT) quantum invariant of a three-manifold with a colored link. Our corresponding integral representation, which is joint with Andersen and Hindson, is motivated by Witten's works on analytic continuation of quantum Chern-Simons field theory. Our integral representation leads to a formal semi-classical expansion in terms of Chern-Simons invariants of flat connections in accordance with Witten's asymptotic expansion conjecture.

Additionally, I will present a new result, a proof of Witten's asymptotic expansion conjecture for the WRT invariants of a Seifert fibered integral homology three-sphere M, which is joint with Andersen, Han, Li, Sauzin and Sun. This proof is based on resurgence, quantum modularity and a previous result, which is joint with Andersen and inspired by work of Gukov, Marino and Putrov, namely the so-called radial limit theorem for M. This theorem concerns the BPS q-series invariant of M defined by Gukov, Pei, Putrov and Vafa, and the theorem asserts that as q tends to a root of unity, this series limits the WRT invariant of M at that root of unity.

Finally, based on joint work with Murakami, I will present a new integral formula for a certain average of the BPS q-series invariants of pairs (Y,b), where Y is a general negative definite plumbed homology three-sphere Y and b is a spin-c structure. This integral formula is used to establish, for this class of three-manifolds, the radial limit conjecture for non-semisimple quantum invariants due to Constantino, Gukov and Putrov. This asserts that the limit of this average of BPS q-series invariants is equal to the non-semisimple quantum invariant of Y defined by Costantino, Geer, Patureau and Mirand.

Presenter: MISTEGARD, William