New Results for Reggeons using FRG and Wilson Regularization and ε – expansion

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Outline

- Motivation
- N-Pomerons fields
- FRG approximation and interaction of Pomerons
- Numerical results for 2 Pomerons Interaction
- Summary and Outlook

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JHEP 05 (2024) 032
Two Pomerons: arXiv to appear

Motivation

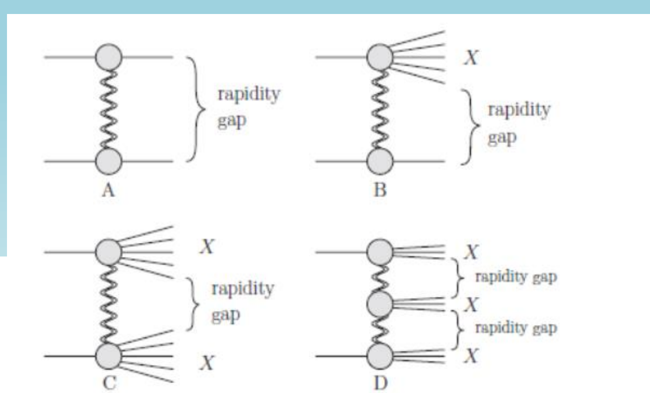
- High-Energy Scattering processes are performed using QCD and in particular the parton distributions.

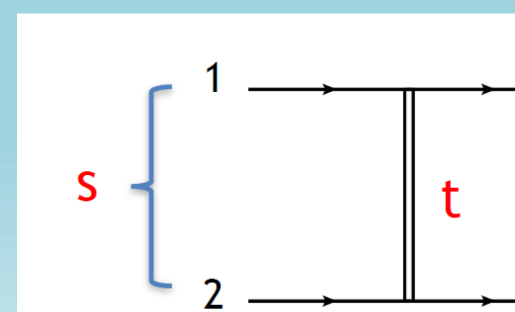
- For Diffractive process, in the Regge Limit $(s > t)$, t-channel exchange dominate and then becomes an effective 2+1dimensional: transversal space and rapidity (Lipatov effective action/CGC/Dipole approximantion)

- At very small transverse distances: pQCD and BFKL Pomeron (1958)

- At very large transverse distances before QCD era, there was the Reggeon Field Theory description introduced by Gribov

- The Pomeron is usually related to gluonic Exchange: state of two gluon and C=1

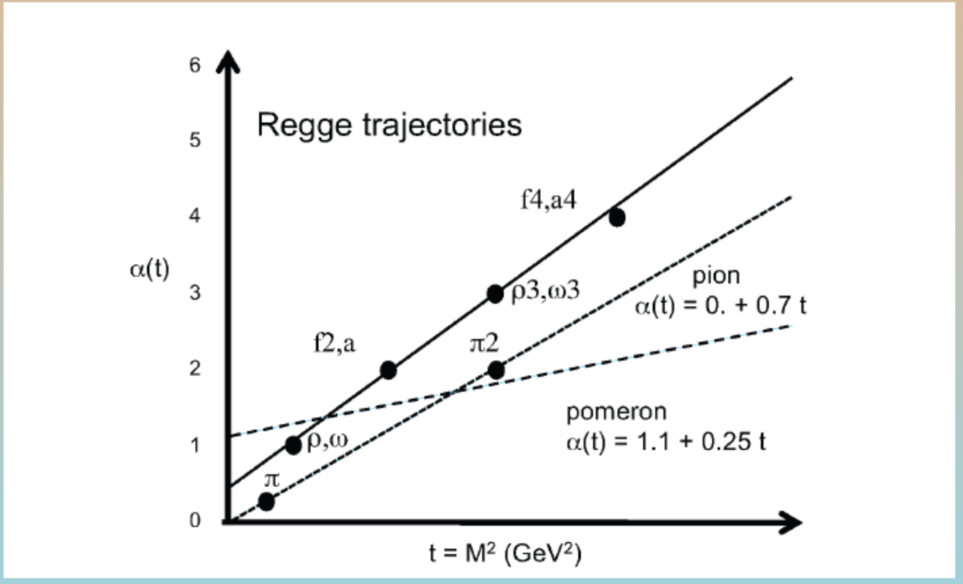




ZEUS Collaboration 1995
Result from HERA: evidence for the Pomeron

Reggeon Field Theory before QCD

- V. N. Gribov introduce in the 60´s: RFT
- Scattering amplitude at high energies for hadrons is according Regge Theory: the exchange of "quasi particles" characterized by its Regge trajectories : $\alpha_i\,(t)$

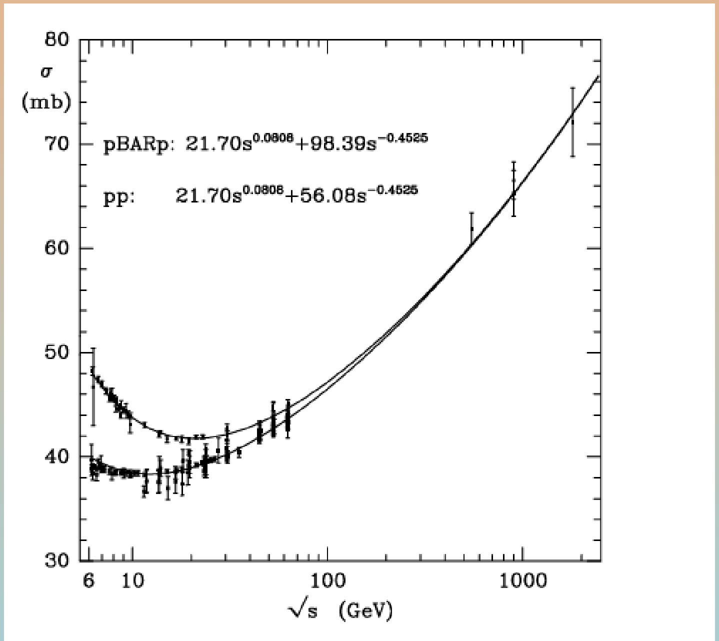


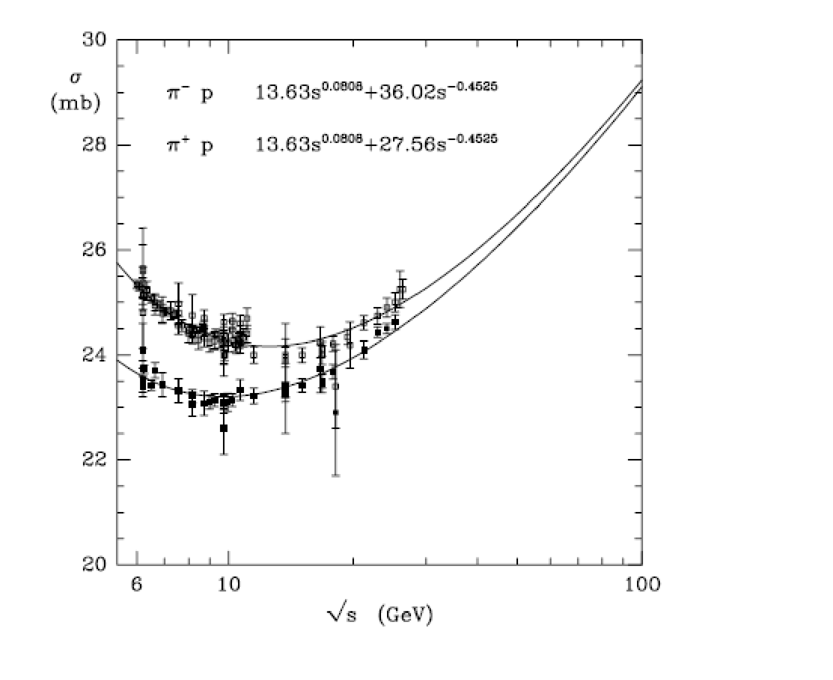
- the total Cross section, is given by: $\sigma_T = A_i s^{\alpha_i(0)-1}$
- Leading Pole: is Called Pomeron with vacuum quantum numbers

$$\alpha(t) = \alpha_0 + \alpha' t = 1 + (\alpha_0 - 1) + \alpha' t$$

$\mu = \alpha_0 - 1$ is the Pomeron intercept and α' is the Pomeron slope

A. Donnachie and Landshoff : Phys. Lett. B 727 (2013) 500 arXiv 1309.1292





- cross section grows with energy

  Unexpected behavior: Soft Pomeron exchange

$$\alpha_P(t) = 1.08 + 0.25\,(\mathrm{GeV}^{-2})t$$

Hard Pomeron pQCD
BFKL Kernel

Balinsky, Fadin, Kuraev, Lipatov (1977)

- For Hard processes short Transversal distances
  we consider pQCD → Hard Pomeron dominate scattering → Diffractive Scattering

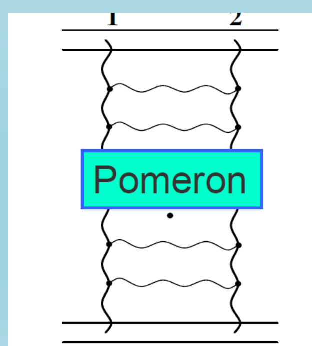
- The BFKL Pomeron which has been studied up to NLO in perturbation theory is a composite states of Reggeized gluons.
  Resumation of the Ladder at leading log approximation (multi Regge Kinemmatics MRK) Lipatov, Bartels

- **The intercept of the Pomeron is related with the eigenvalues of the BFKL Kernel**

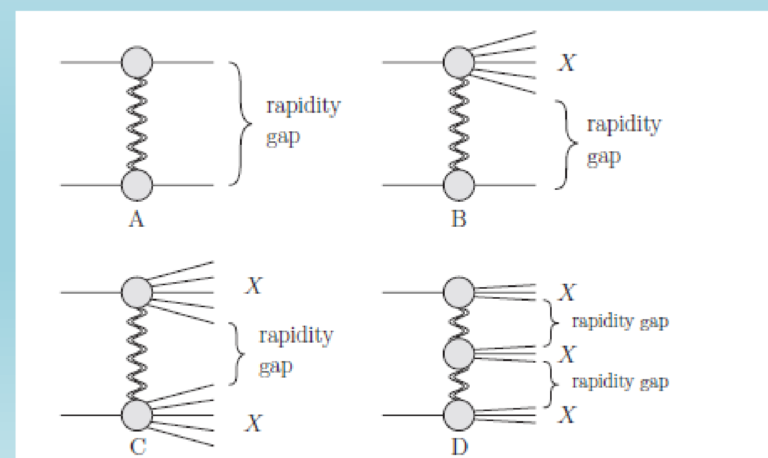
$\omega_0(\gamma) = \alpha_s N_c/\pi\, [2\,\Psi(1) - \Psi(\gamma) - \Psi(1-\gamma)]$ $\Psi(\gamma) = \frac{d}{d\gamma} Ln\, \Gamma(\gamma)$ Digamma function

$$\alpha_P(0) \approx 1 + \frac{\alpha_s N_c}{\pi} 4\ \ln 2 \approx 1 + 0.5295 \quad Hard\ BFKL/QCD$$

$$p + p \to p + X$$



$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$



QCD Description and BFKL Kernel

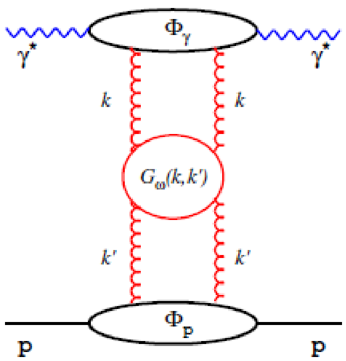
Balitsky, Fadin, Kuraev and Lipatov 1977

*A(s,***k***,***k´***)* is the amplitude for the scattering of a gluon with transverse momentum **k** off another gluon with transverse momentum **k´** at center of mass energy $\sqrt{s}$ and it is found to obey an evolution equation

$$\frac{\partial}{\partial Y} A(\mathbf{k}, \mathbf{k}', Y) = \frac{\alpha_s N_c}{\pi} \int \frac{d^2 q}{2\pi\,(k-q)^2} \left[A(q, k, Y') - \frac{k^2}{2\, q^2} A(k, k', Y) \right]$$

The kernel is obtained by summing all graphs which contribute an effective "gluon ladder".
Using a Mellin Transformation the BFKL evolution equation can be solved in terms of the $\phi_\omega(k)$ eigenfunctions of the Kernel

$$\omega\, \phi_\omega(k) = \bar{\alpha} \int \frac{d^2 k'}{2\pi} \widetilde{K}(k, k') \phi_\omega(k')$$



$\widetilde{K}(k, k')$ **is the BFKL kernel**

Scattering process can be described using BFKL Green Function : G(t , t´, Y)

A(x, Q^2) = $\int dt\, dt'\, \Phi_\gamma(Q^2, t)$ G(t , t´, Y) $\Phi_P(t')$ where Y = ln(1/x) Rapidity

$\Phi_\gamma(Q^2$, t) describe the coupling of the gluon (perturbatively calculable) with transverse momentum k to a photon of virtuality Q^2 and Φ_P(t) describes the coupling of a gluon of transverse momentum k´ to the target proton

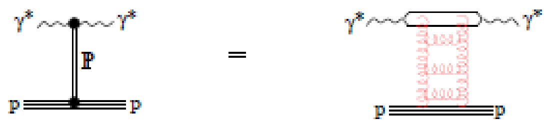


FIG. 4: The virtual photon interacts via its hadronic fluctuations which are $\bar{q}q$ dipoles and more complicated Fock states. The Pomeron exchange is illustrated as a perturbative ladder.

Soft Pomeron vs Hard Pomeron

$\alpha_P(0) \approx 1 + 1 + 0.5295 \quad Hard\ BFKL/QCD$

For Hard processes UV region
short Transversal distances
Large Momenta, large but finite energies

$\alpha_P(0) \approx 1.08 \quad soft$

For soft processes
largest Transversal distances
small Momenta, large but finite energies

$\alpha_{P,k}(0)$

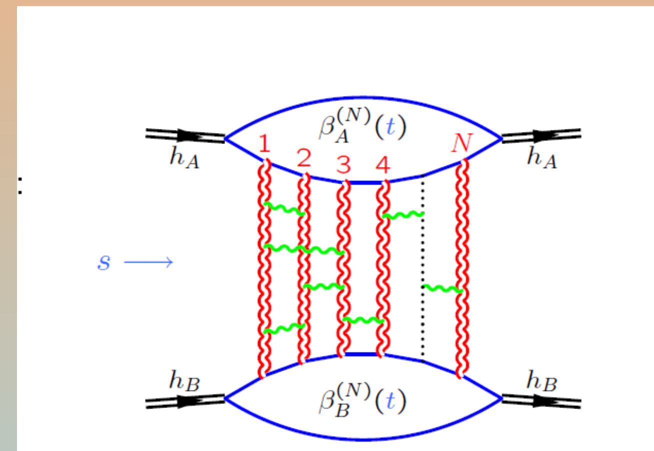
*we can connect Hard –Soft Pomeron regions of different sizes and different sorts of Pomeron using Functional Renormalization Group**

How we can study another states with 3, 4 gluons ?

1973 Lukaszuk and Nicolescu proposal the Odderon as the odd partner of the Pomeron (C =-1) 3 gluon

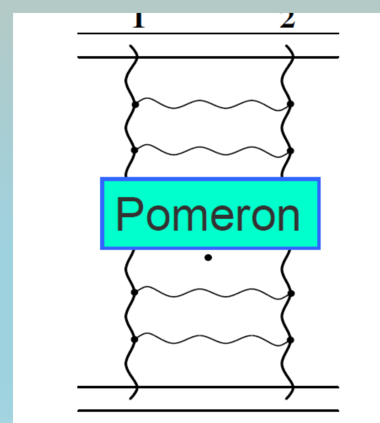
1980 J. Bartels; J. Kwiecinski and M. Praszalowicz

Multi-Reggeons equation BKP

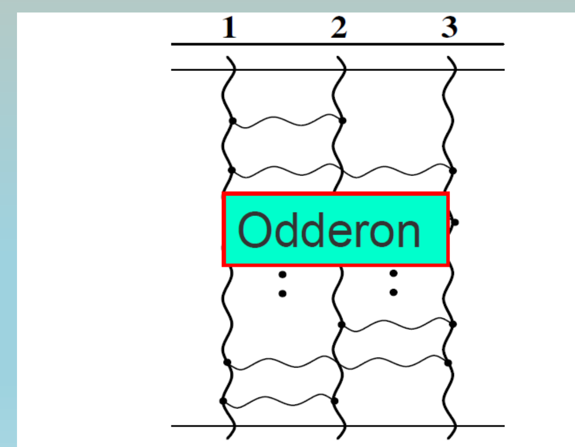


What we can study:

- Solutions
- Intercept and the Slope



$$\frac{\partial f}{\partial Y} = K_{12} \otimes f$$



$$\frac{\partial O}{\partial Y} = K_{12} \otimes O + K_{23} \otimes O + K_{31} \otimes O$$

Odderon is a crucial test of QCD

- pQCD the Odderon was studied and is bound state of three reggeized gluons

1980 J. Bartels; J. Kwiecinski and M. Praszalowicz
Ewerz: Odderon in QCD hep-ph 0306137

Solutions for the BKP equation:

Hard Odderon

- **Janik - Wosiek 1999** with an intercept $\alpha_O = 0.96$
- **Bartels, Lipatov, Vacca (BLV 2000)** with an intercept exactly equal to one

- Lattice and Spectroscopy several calculations, all indicating a low intercept.

However, the way in which this intercept is identified in lattice calculations is not conclusive.

H. B. Meyer and M. J. Teper, Phys. Lett. B605 (2005) 344
H. B. Meyer, PhD thesis at Oxford, hep-lat/0508002

- t-dependence of elastic cross section shows difference between pp and $p\bar{p}$ (evidence for existence of Odderon)

***Phys. Lett.B* 778 (2018) 414-418**

Did TOTEM experiment discover the Odderon?

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Abstract

The present study shows that the new TOTEM datum $\rho^{pp} = 0.098 \pm 0.01$ can be considered as the first experimental discovery of the Odderon, namely in its maximal form.

Keywords: Froissaron, Maximal Odderon, total cross sections, the phase of the forward amplitude.

21 Jan 2018

1. Introduction

Very recently, the TOTEM experiment released the following values at $\sqrt{s} = 13$ TeV of pp total cross section σ^{pp} and ρ^{pp} parameter [1]

The Odderon is defined as a singularity in the complex j-plane, located at $j = 1$ when $t = 0$ and which contributes to the odd-under-crossing amplitude F_{-}. It was first introduced in 1973 on the theoretical basis of

T. Csörgö, R. Pasechnik and A. Ster ***Eur. Phys. J. C* 79 (2019) 1, 62**

M. Broilo E.G.S. Luna M.J. Menon ***arXiv:1803.06560***

Csorgo et al. EPJC 81 (2021) 2

- There are evidence for the non-perturbative Odderon

BFKL regularized equation and N-Pomeron

In the presence of an infrared cutoff and with running α_s the piece of the ω −cut between $\omega = \omega_0$ and zero is replaced by an infinite sequence of discrete poles, which accumulate at zero

L. N. Lipatov, Sov. Phys. JETP 63 (1986) 904

We need IR regulator in the propagator in order study the ladder diagram in the NLO.

BFKL with IR regulator it not new:

- **Lipatov 1986** — **IR Regulator**
- **Braun and Vacca 1999** — **bootstrap conditions**
- **Levin, Lipatov and Siddikov 2014** — **mass regulator**
- **Kowalski, Lipatov, Ross 2014** — **boundary conditions**
- **Bartels, Contreras and Vacca 2019** — **Wilsonian / ERG regulator**

- We introduce the following momentum regulator for the propagator for the Gluon

$$\frac{1}{q^2} \to \frac{1}{[q^2+R_k(q^2)]}$$

$$R_k(q^2) = (k^2 - q^2)\,\theta(k^2 - q^2) \quad \text{Wilsonian Regulator}$$

Trajectory Gluons

$$\omega_g(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2k \frac{1}{k^2(q-k)^2}$$

$$\omega_{gk}(q^2) = -q^2 \frac{\bar{\alpha}_s}{4\pi} \int d^2l \frac{1}{[l^2 + R_k(l^2)]\,[(q-l)^2 + R_k((q-l)^2]}$$

- BFKL Kernel

$$K_{\rm BFKL}(q', q-q'; q'', q-q'') = \frac{\bar{\alpha}_s}{2\pi} \left(-q^2 \frac{(q'-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2} + \frac{q''^2(q-q')^2 + q'^2(q-q'')^2}{(q'-q'')^2 + R_k(q'-q'')^2)} \right)$$

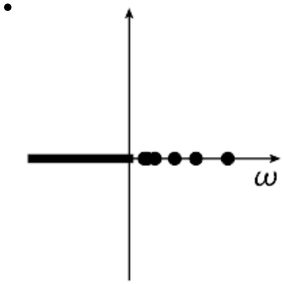
What is important this result

$\bar{\alpha}_s(\mathrm{q}^2) = \alpha_s(\mathrm{q}^2)\frac{N_c}{\pi}$

BFKL kernel in the color singlet state of the t-channel

- ✓ **the BFKL spectral descomposition kernel with the Wilsonian IR regulated and with running coupling constant: is discrete.**

$$\omega\, \phi_\omega(k) = \int \frac{d^2k'}{2\pi} \overline{\alpha}(k,k')\widetilde{K}(k,k')\phi_\omega(k')$$



- Eigenvalues intercept
- Eigenfunctions: bound states n-Pomeron fields

Slope

$$\alpha_s(\mathrm{q}^2) = \frac{3.41}{\beta_0\left[\ln(\mathrm{q}^2 + R_0^2) + \ln\frac{m_h^2}{\Lambda_{QCD}^2}\right]}$$

- ✓ $\alpha\,(q^2) = \omega(q^2) \sim\ \omega^0 + q^2\alpha'$ $\quad \alpha' = \frac{d\omega}{dq^2}$ *when* $q^2 \rightarrow$

Wave Function Pomeron

- The support of the Wave Function, is defined in the UV region

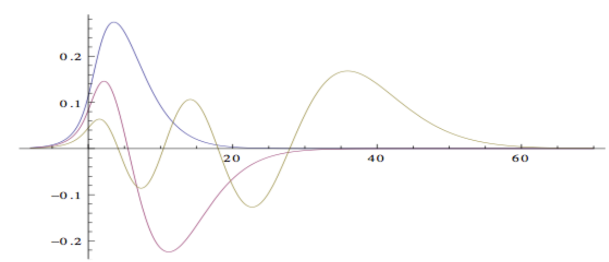


Figure 22: three leading wavefunctions (No 1,2,5) as a function of $\ln q^2$.

J. Bartels, C. Contreras G. P Vacca JHEP 1901(2019) 004

Numerical Analyses for Odderon

- Using Numerical analysis
- Massive infrared regulator
- Running coupling constant

Eigenvalues are consistent with the BLV solution:

$$\omega_{\text{Odderon}} = 0.00003 \text{ for } \alpha_s = 0.2$$

Slops are relative small and different from Pomeron

WF: all the leading eigenstate are in UV region

*M. Braun and G. P. Vacca similar result using bootstrap (private communication)

Bartels, Contreras and Vacca *JHEP* 04 (2020) 183

How we can test this N - Pomeron

- Ellis, Kowalski and Ross: Using the N-BFKL Pomerons started to fit the small-x and low-Q^2 HERA data F2/DIS

J. Ellis, H. Kowalski and D. A. Ross, Phys. Lett. B 668 (2008) 51 Kowalski, Lipatov, Ross and Watt: arXiv: 1005.0355

- All these approaches only use the contribution of the 3 discreet BFKL Pomeron

- No attempt has been made to introduce the triple Pomeron vertex and to study its QCD effect.

- In the field theory based upon reggeized gluons these vertex functions would lead to an extremely hard problem to solve

- To study the interactions of these Regge poles and in order to be able to move from larger to small distances it will be convenient to consider the local approximation and to make use of the well-known formalism of RFT

- BFKL Pomeron into a reggeon field theory which includes corrections to the BFKL Pomeron based upon interaction vertices, in particular the triple Pomeron vertex

The FRG for Reggeons Field Theory

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

Let us define here the theory which we want to investigate. We shall consider a set of fields and their conjugate ψ_i , $\psi_i^\dagger$, for $i = 1..N$, whose evolution and interactions are defined (Gribov action)

$$\Gamma_k = \int d^D x d\tau [Z_i(\frac{1}{2}\psi_i^\dagger \overleftrightarrow{\partial}_\tau \psi_i - \alpha_i' \psi_i^\dagger \nabla^2 \psi_i) + m_{i,l}(\psi_i \psi_l^\dagger + \psi_i^\dagger \psi_l) - \mu_i \psi_i^\dagger \psi_i - V_k[\psi_i^\dagger, \psi_i; \lambda_j]]$$

$$V(\psi, \psi^\dagger) = \sum_{jkl} \frac{i}{2} \alpha' \lambda_{j,kl} \left(\psi_j^\dagger \psi_k \psi_l + \psi_j \psi_k^\dagger \psi_l^\dagger\right)$$

The interaction N=2 discrete fields is described by the cubic potential V_k:

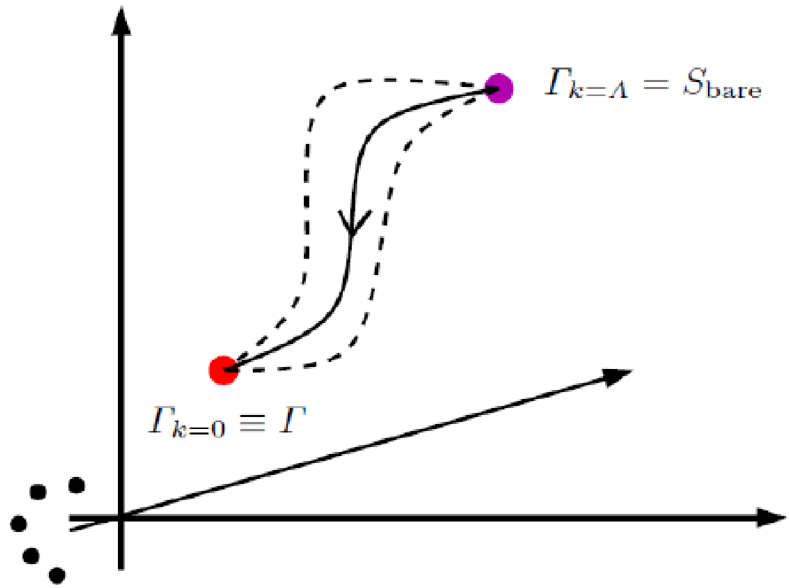
$$(\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6) = (\lambda_{1,11},\ \lambda_{1,22},\ \lambda_{2,11},\ \lambda_{2,22},\ \lambda_{1,21} = \lambda_{1,12},\ \lambda_{2,21} = \lambda_{2,12})$$

$$\begin{aligned} V_k[\psi_1, \psi_1^\dagger, \psi_2, \psi_2^\dagger] \ = \ & i\lambda_1 \psi_1^\dagger(\psi_1^\dagger + \psi_1)\psi_1 + i\lambda_2 \psi_2^\dagger(\psi_2^\dagger + \psi_2)\psi_2 + i\lambda_3 \psi_1^\dagger(\psi_2^\dagger + \psi_2)\psi_1 + \\ & i\lambda_4 \psi_2^\dagger(\psi_1^\dagger + \psi_1)\psi_2 + i\lambda_5(\psi_2^{\dagger 2}\psi_1 + \psi_2^2 \psi_1^\dagger) + i\lambda_6(\psi_1^{\dagger 2}\psi_2 i + \psi_1^2 \psi_2^\dagger), \end{aligned}$$

$$\begin{aligned} \Gamma_k[\psi_1, \psi_1^\dagger, \psi_2, \psi_2^\dagger] = & \int d^D x d\tau [Z_1(\frac{1}{2}\psi_1^\dagger \overleftrightarrow{\partial}_\tau \psi_1 - \alpha_1' \psi_1^\dagger \nabla^2 \psi_1) + Z_2(\frac{1}{2}\psi_2^\dagger \overleftrightarrow{\partial}_\tau \psi_2 - \alpha_2' \psi_2^\dagger \nabla^2 \psi_2) \\ & + m(\psi_1^\dagger \psi_2 + \psi_2^\dagger \psi_1) - \mu_1\ \psi_1^\dagger \psi_1 - \mu_2 \psi_2^\dagger \psi_2 - V_k[\psi_1, \psi_1^\dagger, \psi_2, \psi_2^\dagger]\ ]. \end{aligned}$$

Wetterich Equation 93

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$



Action $\Gamma_k(\psi_i, \psi_i^\dagger,\ \lambda_i)$

Regulator $R_k(q)$

Initial Condition $\Gamma_{k=\Lambda}(\phi, g_i)$

Interaction V = $\sum \lambda_{i;jk}\,(\psi_i^\dagger \psi_j \psi_k$ +cc)

$$R_1 = Z_1 \alpha_1'(k^2 - q^2)\Theta(k^2 - q^2)$$

$$R_2 = Z_2 \alpha_2'(k^2 - q^2)\Theta(k^2 - q^2) = r Z_2 \alpha_1'(k^2 - q^2)\Theta(k^2 - q^2),$$

$$r = \frac{\alpha_2'}{\alpha_1'}$$

Calculation

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} Tr\left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} \partial_t R_k\right]$$

$$\Gamma_k^{(2)} = \Gamma_{k,0}^{(2)} - V_k \quad ; \qquad G_{k,0} = \frac{1}{\Gamma_{k,0}^{(2)} + R_k},$$

$$(\Gamma_{k,0}^{(2)} + R_k - V_k)^{-1} = -G_{k,0}(1 + V_k G_{k,0} + V_k G_{k,0} V_k G_{k,0} + \dots).$$

$$\Gamma_{k,0}^{(2)} = k^D \begin{pmatrix} 0 & (-i\omega + \alpha_1' q^2 - k^2\alpha_1'\tilde{\mu}_1) & 0 & k^2\alpha_1'\tilde{m} \\ (i\omega + \alpha_1' q^2 - k^2\alpha_1'\tilde{\mu}_1) & 0 & k^2\alpha_1'\tilde{m} & 0 \\ 0 & k^2\alpha_1'\tilde{m} & 0 & (-i\omega + \alpha_2' q^2 - k^2\alpha_1'\tilde{\mu}_2) \\ k^2\alpha_1'\tilde{m} & 0 & (i\omega + \alpha_2' q^2 - k^2\alpha_1'\tilde{\mu}_2) & 0 \end{pmatrix}.$$

$$Tr[\ \dots\ ] \equiv \int\int \frac{dq^D}{(2\pi)^D}\frac{d\omega}{(2\pi)} Tr[\ \dots\ ].$$

$\Gamma_k(\phi, \lambda_i, \mu_i) = \sum_i g_i(k) O_i(\phi) \qquad \partial_t \Gamma_k \rightarrow \sum_i \partial_t g_i(k) * O_i(\phi) \rightarrow$
$\beta_i(k)$ asociadas al $\{O_i\}$ operator basis.

Beta Funtions: $\beta_i(k) = \partial_t g_i(k)$

Fixed Points Conditions $\partial_t \Gamma_k^* \cong 0 \quad y \quad t = \ln(\frac{k}{\Lambda})$

Flow equation 10 parameter

Beta Funtions: $g_i(k) = (\mu_i, \lambda_{1:11}, \lambda_{2:22}, \lambda_{1:12}, \lambda_{1:22}, \lambda_{2:21}, \lambda_{2:11})$

Dimensionaless variable

$$\tilde{\psi} = \sqrt{Z_1} k^{-D/2} \psi \ , \ \tilde{\chi} = \sqrt{Z_2} k^{-D/2} \chi \ , \ \tilde{V} = \frac{V}{\alpha'_1 k^{D+2}}$$

$$\tilde{m} = \frac{m}{\sqrt{Z_1 Z_2 \alpha'_1 k^2}} \ , \ \tilde{\mu}_1 = \frac{\mu_1}{Z_1 \alpha'_1 k^2} \ , \ \tilde{\mu}_2 = \frac{\mu_2}{Z_2 \alpha'_1 k^2}$$

$$\tilde{\lambda}_1 = \frac{\lambda_1 k^{D/2}}{Z_1^{3/2} \alpha'_1 k^2} \ , \ \tilde{\lambda}_2 = \frac{\lambda_2 k^{D/2}}{Z_2^{3/2} \alpha'_1 k^2} \ , \ \tilde{\lambda}_{3,6} = \frac{\lambda_{3,6} k^{D/2}}{Z_1 \sqrt{Z_2} \alpha'_1 k^2} \ , \ \tilde{\lambda}_{4,5} = \frac{\lambda_{4,5} k^{D/2}}{Z_2 \sqrt{Z_1} \alpha'_1 k^2}$$

$$\eta_i = -\frac{1}{Z_i} \partial_t Z_i \qquad \xi_i = -\frac{1}{\alpha'_i} \partial_t \alpha'_i$$

$$r = \alpha'_2 / \alpha'_1 \qquad \dot{r} = r(\xi_1 - \xi_2)$$

Betas Functions and stability matrix

$$\begin{aligned}
\dot{\mu}_1 &= (-2+\xi_1+\eta_1)\mu_1 + \textstyle\sum_{i,j} \lambda_i\lambda_j f_{1,ij} \\
\dot{\mu}_2 &= (-2+\xi_1+\eta_2)\mu_2 + \textstyle\sum_{i,j} \lambda_i\lambda_j f_{2,ij} \\
\dot{m} &= (-2+\xi_1+\tfrac{1}{2}(\eta_1+\eta_2))m + \textstyle\sum_{i,j} \lambda_i\lambda_j f_{m,ij} \\
\dot{\lambda}_1 &= (-2+\tfrac{D}{2}+\xi_1+\tfrac{3}{2}\eta_1)\lambda_1 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{1,ijk} \\
\dot{\lambda}_2 &= (-2+\tfrac{D}{2}+\xi_1+\tfrac{3}{2}\eta_2)\lambda_2 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{2,ijk} \\
\dot{\lambda}_3 &= (-2+\tfrac{D}{2}+\xi_1+\eta_1+\tfrac{1}{2}\eta_2)\lambda_3 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{3,ijk} \\
\dot{\lambda}_4 &= (-2+\tfrac{D}{2}+\xi_1+\eta_2+\tfrac{1}{2}\eta_1)\lambda_4 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{4,ijk} \\
\dot{\lambda}_5 &= (-2+\tfrac{D}{2}+\xi_1+\eta_2+\tfrac{1}{2}\eta_1)\lambda_5 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{5,ijk} \\
\dot{\lambda}_6 &= (-2+\tfrac{D}{2}+\xi_1+\eta_1+\tfrac{1}{2}\eta_2)\lambda_6 + \textstyle\sum_{i,j,k} \lambda_i\lambda_j\lambda_k f_{6,ijk} \\
\dot{r} &= r(\xi_2-\xi_1).
\end{aligned}$$

$$M_{ij} = \frac{\partial \beta_i}{\partial \lambda_j}\bigg|_{\lambda_{FP}}$$

Algorithm for Solving Nonlinear Equation Systems
Li, G. and Zeng, Z. 2008
Goulianas, et als. 2016

***Cancino and Contreras, Universe* 10 (2024) 3, 103**

Numerical Solution

- Two decoupled Pomerons

μ_1	μ_2	m	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	r
0.1333	0.1333	0.	1.1676	1.1676	0.	0.	0.	0.	1.
0.1053	0.1053	0.	1.0001	1.0001	0.	0.	0.	0.	1.

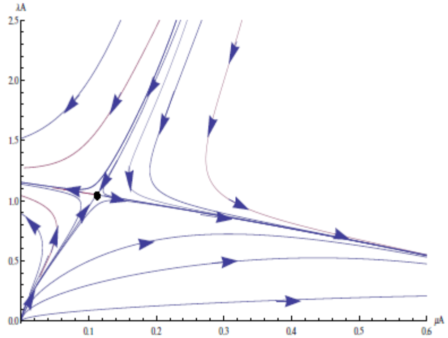
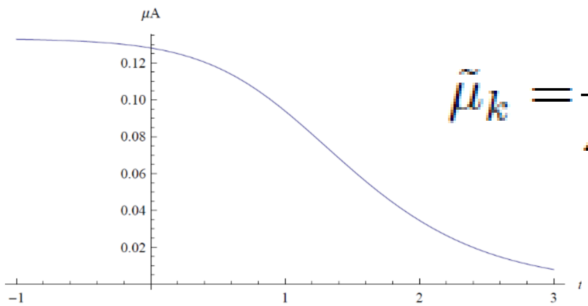


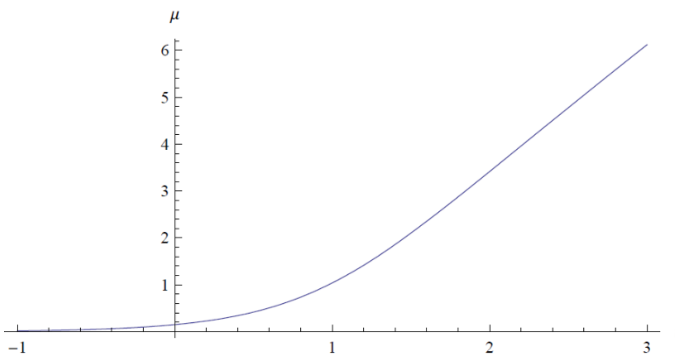
Fig.2: Some trajectories of the flow equation.

- We reproduce the values of the critical exponents universality class of Percolation
- The convergence is under control with the increasing the local truncation

1980 Cardy y Sugar found that the RFT is in the same Universality class of "Percolation"



$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$



Percolation and Monte Carlo Simulation:
The critical Exponent ν = 0.73 with is related with our ν = - 1/(most negative eigenvalue)"
(L. Canet, B. Delamotte, N. Wschebor,…)

truncation	3	4	5	6	7	8	9	10	11	12
exponent ν	0.53	0.59	0.59	0.78	0.76	0.72	0.72	0.74	0.74	0.73
mass $\tilde{\mu}$	0.111	0.274	0.386	0.429	0.341	0.388	0.388	0.400	0.399	0.397

***Bartels, Contreras and Vacca; JHEP* 03 (2016) 201**

- # Pomeron-Odderon Interaction

$\mu_1 \to \mu_1$	$\mu_2 \to \mu_2$	$\lambda \to \lambda_1$	$\lambda_2 \to \lambda_4$	$\lambda_3 \to \lambda_5$	$r \to r$
0.1111	0.1108	1.0503	1.4467	0.	0.9218

$$\begin{aligned} V_3 &= -\mu_P \psi^\dagger \psi + i\lambda \psi^\dagger (\psi + \psi^\dagger)\psi - \\ &\quad -\mu_O \chi^\dagger \chi + i\lambda_2 \chi^\dagger (\psi + \psi^\dagger)\chi + \lambda_3 (\psi^\dagger \chi^2 + \chi^{\dagger 2} \psi). \end{aligned}$$

$$\begin{aligned} V_4 &= \lambda_{41} (\psi\psi^\dagger)^2 + \lambda_{42} \psi\psi^\dagger (\psi^2 + \psi^{\dagger 2}) + \lambda_{43} (\chi\chi^\dagger)^2 + i\lambda_{44} \chi\chi^\dagger (\chi^2 + \chi^{\dagger 2}) \\ &\quad + i\lambda_{45} \psi\psi^\dagger (\chi^2 + \chi^{\dagger 2}) + \lambda_{46} \psi\psi^\dagger \chi\chi^\dagger + \lambda_{47} \chi\chi^\dagger (\psi^2 + \psi^{\dagger 2}). \end{aligned}$$

$$\begin{aligned} V_5 &= i\Big(\lambda_{51} (\psi\psi^\dagger)^2 (\psi + \psi^\dagger) + \lambda_{52}\, \psi\psi^\dagger (\psi^3 + \psi^{\dagger 3}) + \lambda_{53}\, \chi\chi^\dagger (\psi^3 + \psi^{\dagger 3}) + \lambda_{54}\, \psi\psi^\dagger \chi\chi^\dagger (\psi + \psi^\dagger)\Big) \\ &\quad + \lambda_{55} (\chi^2 \psi^{\dagger 3} + \chi^{\dagger 2} \psi^3) + \lambda_{56} (\chi^2 \psi^{\dagger 2} \psi + \chi^{\dagger 2} \psi^\dagger \psi^2) + \lambda_{57} (\chi^2 \psi^\dagger \psi^2 + \chi^{\dagger 2} \psi^{\dagger 2} \psi) \\ &\quad + i\Big(\lambda_{58} (\chi^4 \psi^\dagger + \chi^{\dagger 4} \psi) + \lambda_{59} (\chi\chi^\dagger)^2 (\psi + \psi^\dagger)\Big) \\ &\quad + \lambda_{510}\, \chi\chi^\dagger (\chi^2 \psi + \chi^{\dagger 2} \psi^\dagger) + \lambda_{511}\, \chi\chi^\dagger (\chi^2 \psi^\dagger + \chi^{\dagger 2} \psi). \qquad (4) \end{aligned}$$

***Bartels, Contreras and Vacca; Phys. Rev. D* 95 (2017) 1, 014013**

Flow Equations:

$$
\begin{aligned}
\dot{\mu}_P &= (-2+\eta_P+\zeta_P)\mu_P + 2A_P\frac{\lambda^2}{(1-\mu_P)^2} - 2A_O r\frac{\lambda_3^2}{(r-\mu_O)^2} \\
\dot{\mu}_O &= (-2+\eta_O+\zeta_P)\mu_O + 2(A_P+A_O r)\frac{\lambda_2^2}{(1+r-\mu_P-\mu_O)^2} \\
\dot{\lambda} &= (-2+D/2+\zeta_P+\frac{3}{2}\eta_P)\lambda + 8A_P\frac{\lambda^3}{(1-\mu_P)^3} - 4A_O r\frac{\lambda_2\lambda_3^2}{(r-\mu_O)^3} \\
\dot{\lambda}_2 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_2 \\
&+\frac{2\lambda\lambda_2^2(6A_P+5A_O r)+4\lambda_2^3(A_P+A_O r)-4\lambda_2\lambda_3^2(A_P+2A_O r)}{(1+r-\mu_P-\mu_O)^3} \\
&+\frac{2A_P\lambda\lambda_2^2(r-\mu_O)^2}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^3} - \frac{4A_O r\lambda_2\lambda_3^2(1-\mu_P)^2}{(1-\mu_O)^2(1+r-\mu_P-\mu_O)^3} \\
&+\frac{2\lambda\lambda_2^2(3A_P+A_O r)(r-\mu_O)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^3} - \frac{4\lambda_2\lambda_3^2(A_P+3A_O r)(1-\mu_P)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^3} \\
\dot{\lambda}_3 &= (-2+D/2+\zeta_P+\frac{1}{2}\eta_P+\eta_O)\lambda_3 \\
&+\frac{2\lambda_2^2\lambda_3(A_P+2A_O r)}{(r-\mu_O)(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3(2A_P+A_O r)}{(1-\mu_P)(1+r-\mu_P-\mu_O)^2} \\
&+\frac{2\lambda_2^2\lambda_3 A_O r(1-\mu_P)}{(r-\mu_O)^2(1+r-\mu_P-\mu_O)^2} + \frac{4\lambda\lambda_2\lambda_3 A_P(r-\mu_O)}{(1-\mu_P)^2(1+r-\mu_P-\mu_O)^2}.
\end{aligned}
$$

Fixed Points:

$$\mu_P = 0.274381, \; \mu_O = 0.26979$$

$\mu = \alpha_0 - 1$

$$\lambda = 1.34738, \; \lambda_2 = 1.79401, \; \lambda_3 = 0.$$

$$r = 0.88018$$

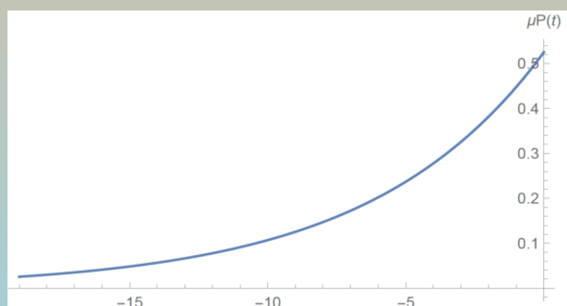
Anomalous dimensions :

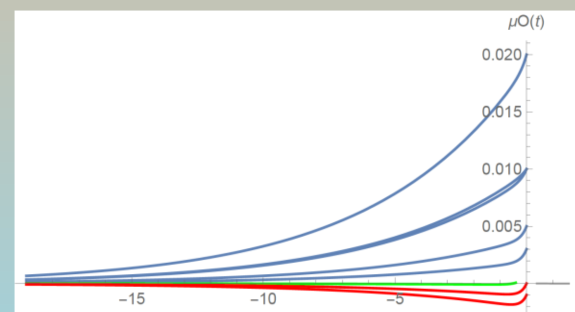
$\eta_P \simeq -0.33 \;, \eta_O \simeq -0.35$

$\xi_2 = \xi_1 = \zeta_P = \zeta_P = 0.16$

$$\alpha'_O = r\,\alpha'_P.$$

$\mu_i = \alpha_i - 1$





$$\tilde{\mu}_k = \frac{\mu_k}{Z_k \alpha'_k k^2}$$

The results are equivalent with the ε-expansion

arXiv 2001.0599 M. Braum and G. P. Vacca

Results II:

The convergence is under control with the increasing the local truncation

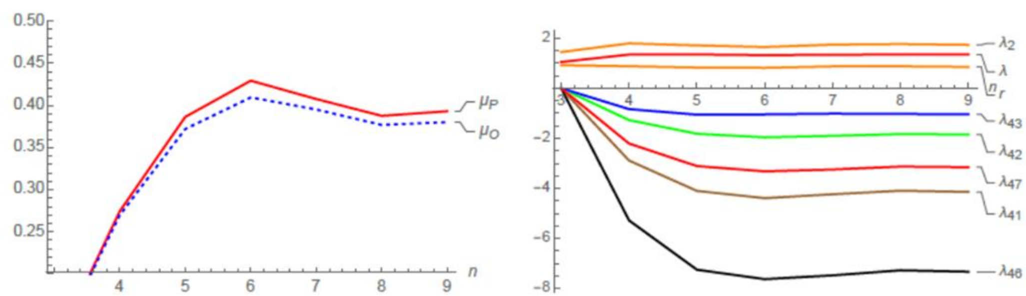
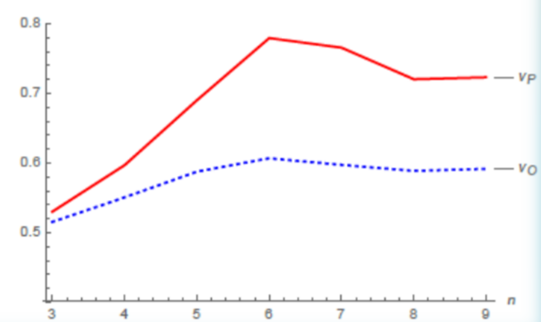


Figure 1: Values of the parameters of the fixed point solution of the LPA truncations for different orders n of the polynomial ($3 \leq n \leq 9$). The masses (which equal intercept minus one) μ_P (red curve) and μ_O (blue dotted curve) for the Pomeron and Odderon fields are in the left panel. The first non zero couplings $\lambda, \lambda_2, \lambda_{41}, \lambda_{42}, \lambda_{43}, \lambda_{46}, \lambda_{47}, r$ are reported on the right panel.



$$\nu_P \simeq 0.73, \ \nu_O \simeq 0.6.$$

Percolation

ν = - 1/(most negative eigenvalue)

- ## New Solution

Fixed Point IR

μ_1	μ_2	m	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	r
0.1384	0.1384	-0.0056	1.0118	1.0118	0.8150	0.8150	-0.1967	-0.1967	1.
0.1084	0.1084	-0.0042	0.8631	0.8631	0.6967	0.6967	-0.1664	-0.1664	1.

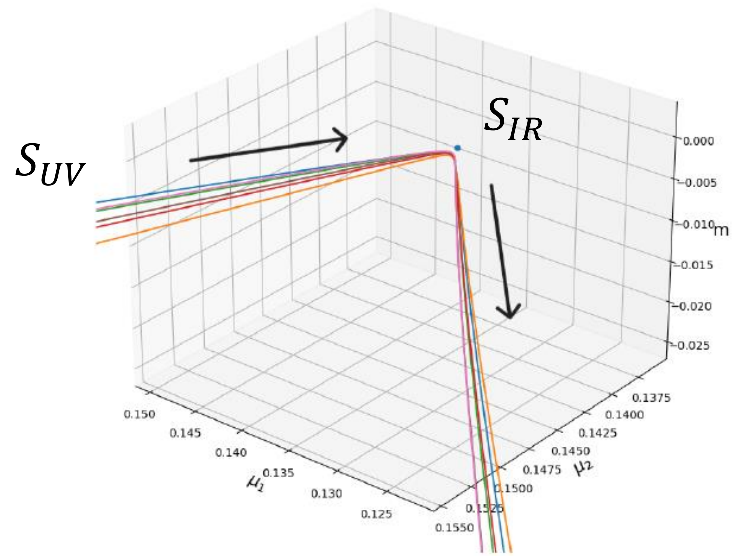
Critical Exponents θ_i of the stabilty matrix $g_{k,i} = g^*{}_i + c_a e^{\theta_i t}\, v_i^a$

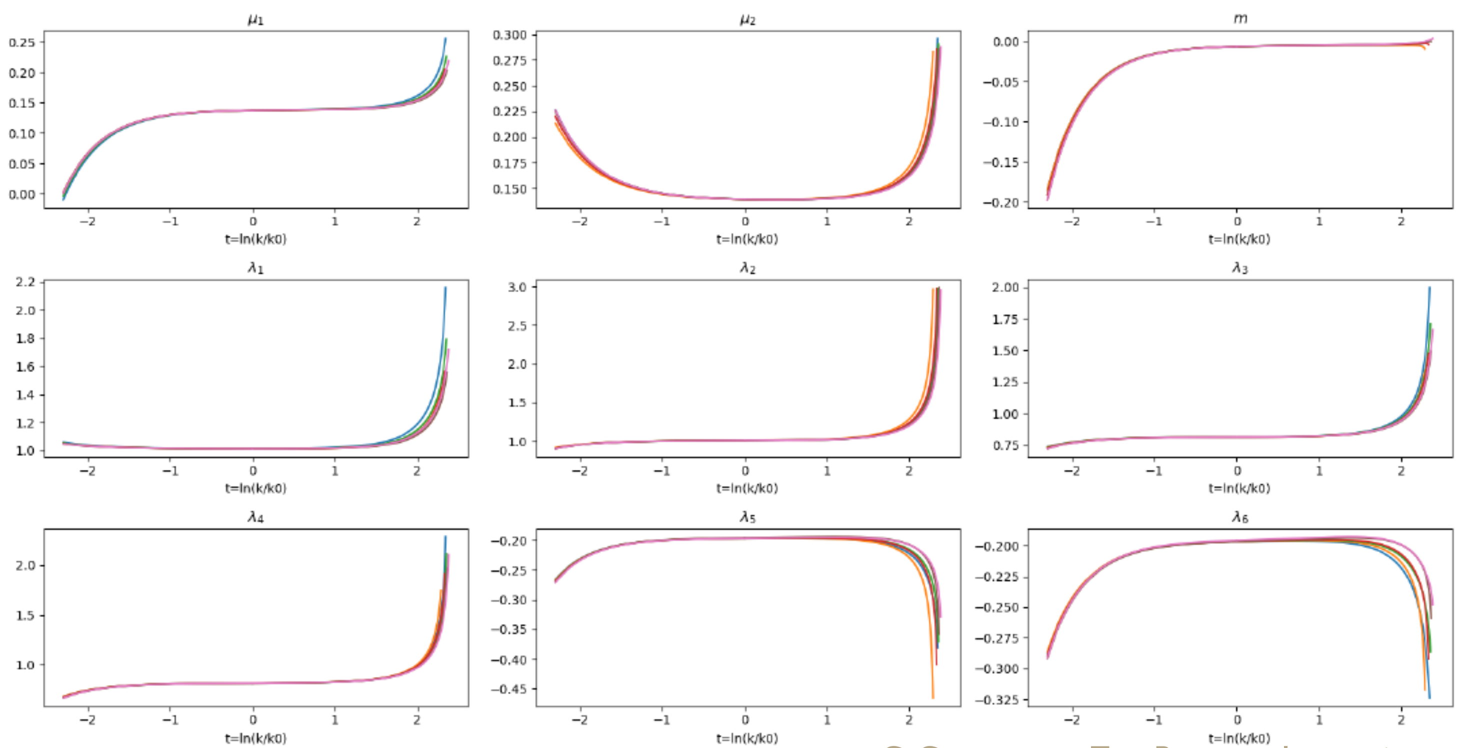
2.50	-2.23	-2.09	-2.03	1.94	1.41	0.36	-0.11	0.10	0.03

Linealizations of the Flow close to a FP:
$\theta_i > 0$ define a IR attractor – Critical Surface with relevant behaviour, where $\theta_i < 0$ is UV attractor

RG Trajectories at the IR Fixed point

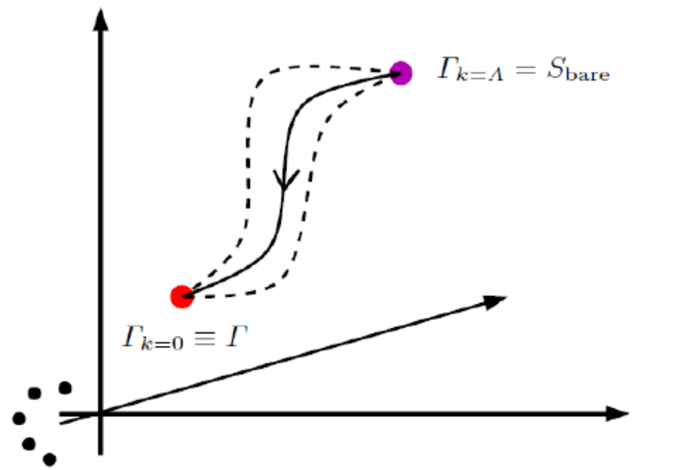
Numerical solution

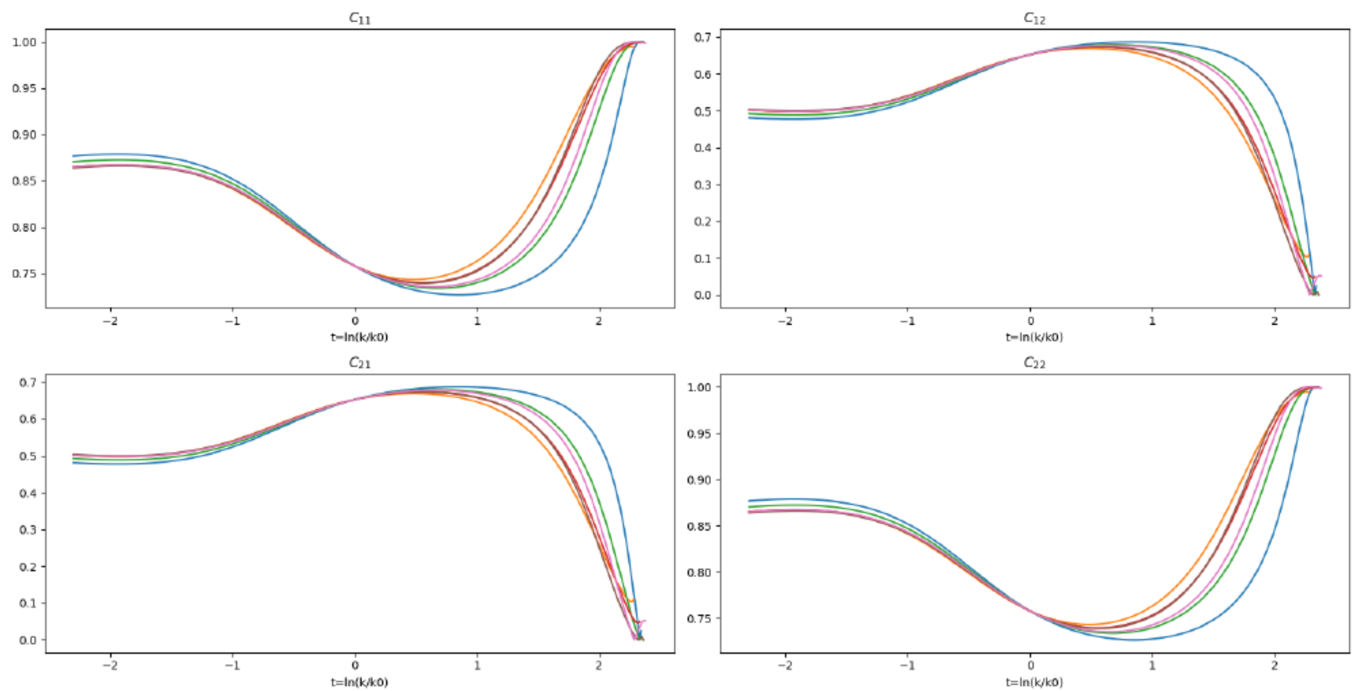




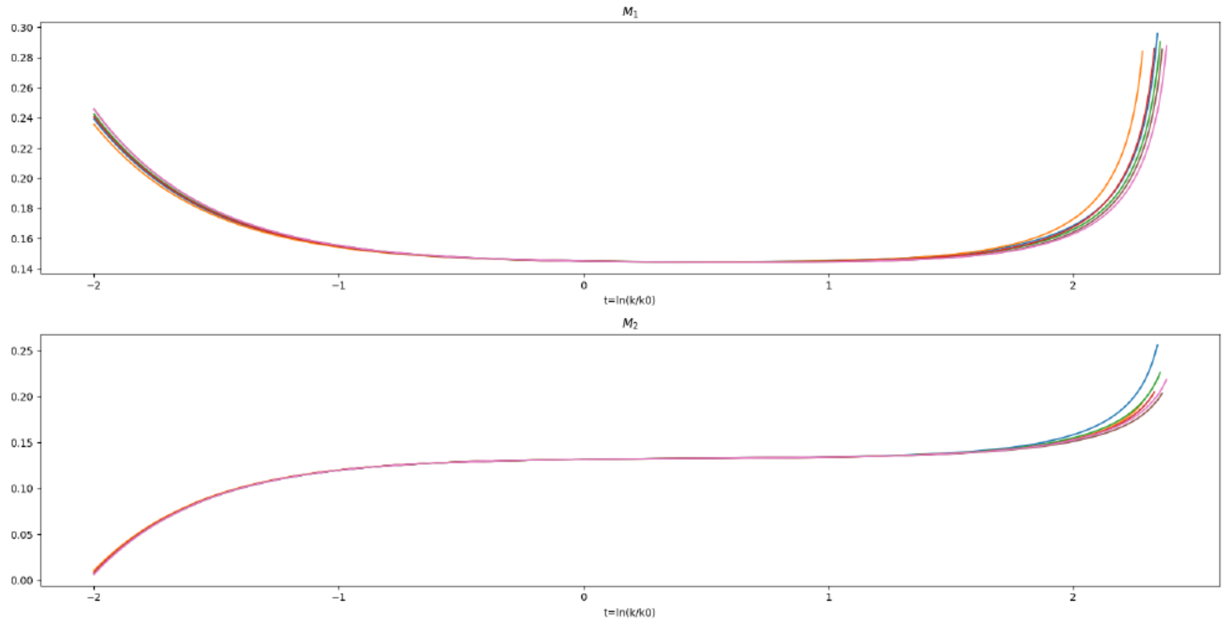
Results

$$\psi'_1 = C_{k,11}\psi_1 + C_{k,12}\psi_2$$
$$\psi'_2 = C_{k,21}\psi_1 + C_{k,22}\psi_2$$





Numerical evolution for the final Pomeron Intercepts:



Cancino and Contreras *Universe* 10 (2024) 3, 103

Analysis of ε-Expansion for RFT

Considering the RFT for N=2 Pomerons

$$S = \int d\tau d^D x \Big[\psi^{\dagger T} Z_\psi i\partial_\tau \psi + \psi^{\dagger T} Z_\psi^{\frac{1}{2}} Z_{\alpha'} \alpha' \nabla^2 Z_\psi^{\frac{1}{2}} \psi + \psi^{\dagger T} Z_\psi^{\frac{1}{2}} Z_\mu \tilde{\alpha}' \mu Z_\psi^{\frac{1}{2}} \psi - \frac{i}{2} \tilde{\alpha}' \left(\lambda_{i,jk} + \delta\lambda_{i,jk}\right) M^{\frac{\epsilon}{2}} \left(\psi_i^\dagger \psi_j \psi_k + \psi_i \psi_j^\dagger \psi_k^\dagger \right) \Big] ,$$

in D = 4 - ε, where the M denotes the renormalization scale, we can obtain the Beta function at the one loop:

$$\begin{aligned} \beta_{i,jk} = & -\frac{\epsilon}{2}\lambda_{i,jk} + \frac{1}{N}\frac{1}{8(4\pi)^2}\lambda_{l,mn}\lambda_{l,mn}\lambda_{i,jk} \\ & -\frac{1}{8}\frac{1}{(4\pi)^2}\left(\lambda_{i',lm}\lambda_{i,lm}\lambda_{i'jk} + \lambda_{j',lm}\lambda_{j,lm}\lambda_{i,j'k} + \lambda_{k',lm}\lambda_{k,lm}\lambda_{ijk'}\right) \\ & +\frac{1}{2}\frac{1}{(4\pi)^2}\left(\lambda_{i,ab}\lambda_{j,ac}\lambda_{b,ck} + \lambda_{i,ab}\lambda_{k,ac}\lambda_{b,cj}\right) , \end{aligned}$$

and the solution for the Fixed Point have the same structure.

***Bartels, Contreras and Vacca; JHEP* 05 (2024) 032**
Bartels, Contreras and Vacca to appear in arXiv

Summary and outlook

Using Numerical analysis of the RFT in the FRG

1. We can find different Fixed Points solutions.

2. We studied the interaction of two Pomeron with triple Pomeron vertex

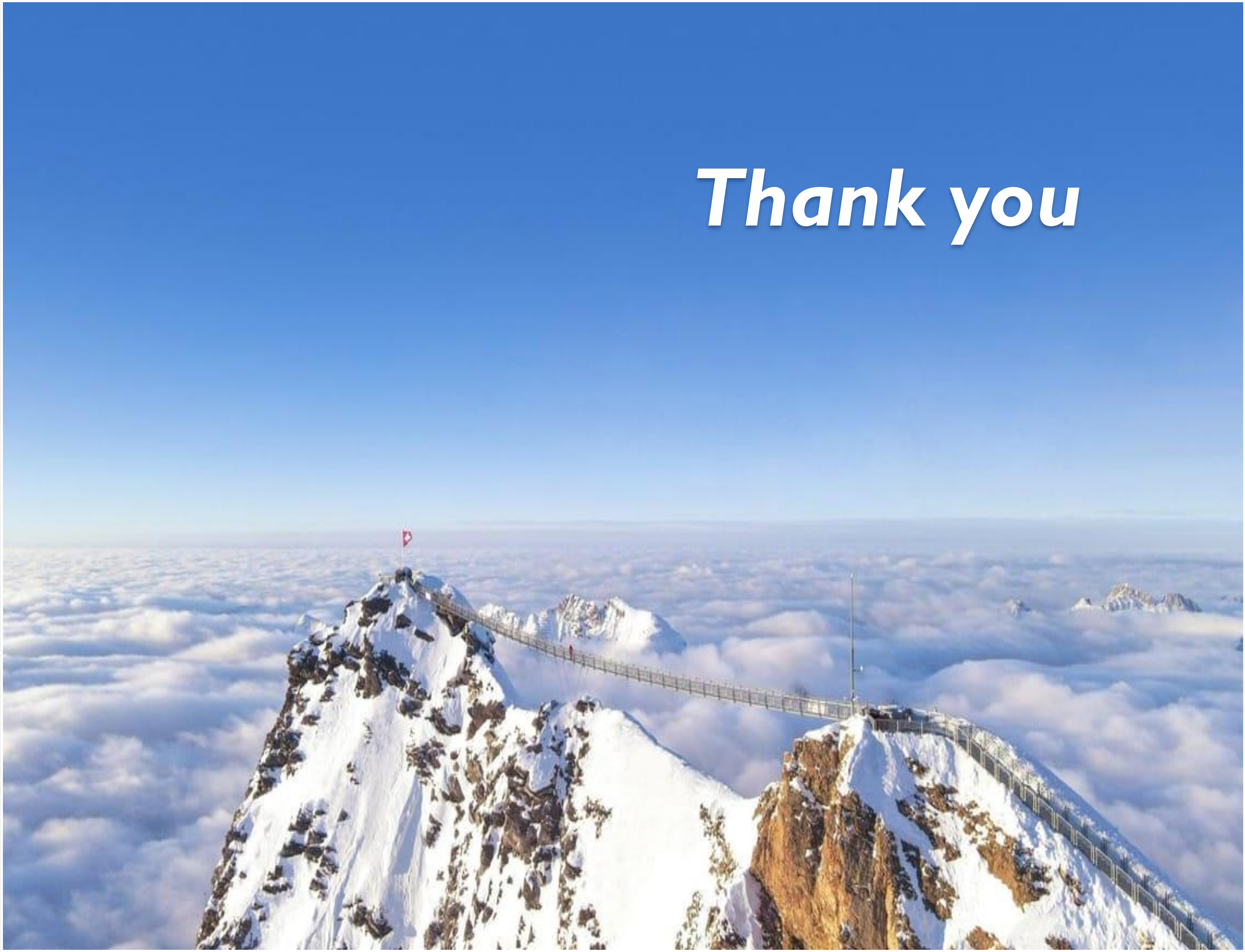
3. Can interpreted that the Pomeron really mixed between the two BFKL discrete Pomeron around the IR fixed point

4. The final states are intercept eigenstate of the 2 mixed discrete Pomeron

5. The intercept ω_n of the discrete N-Pomeron which have a significant contribution to the gluon density at HERA now has a k-dependence $\omega_{n.k}$, and we need to study its effect.

In the future:

Extension to high order Pomeron vertex
N-Pomeron Interaction in diagonal basis


Thank you