Quantum field theories of relativistic Luttinger fermions

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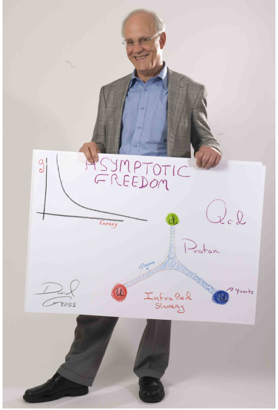
Looking for UV complete QFTs

- Quantum field theories exhibit remarkable predictive power regarding their scale of validity.
- They can be valid up to arbitrarily high energy scale through asymptotic freedom/safety: UV complete.
- We don't know any theory based on pure fermionic matter that is also UV complete in 4 dimensions.
- Luttinger fermions as new ingredient for UV complete QFTs.



Picture: David Gross who shared the Nobel Prize in 2004 with D. Politzer and F. Wilczek for discovering asymptotic freedom.

What is a Luttinger fermion?

- Effective degrees of freedom in solid-state physics.
- Discovered by J.M. Luttinger while looking for the most general form of the Hamiltonian of a semiconductor excitation in a magnetic field: J. M. Luttinger, Phys. Rev. 102, 1030 (1956)

PHYSICAL REVIEW VOLUME 102, NUMBER 4 MAY 15, 1956

Quantum Theory of Cyclotron Resonance in Semiconductors: General Theory*

J. M. Luttinger
University of Michigan, Ann Arbor, Michigan
(Received November 17, 1955)

The most general form of the Hamiltonian of an electron or hole in a semiconductor such as Si or Ge, in the presence of an external homogeneous magnetic field, is given. Two methods of obtaining the corresponding energy levels are discussed. The first should yield very accurate values for the magnetic field in the (111) direction for either Si or Ge. The second is a perturbation method and is expected to give good results only for Ge.

$$H = G_{ij}(\partial_i - ieA_i)(\partial_j - ieA_j)$$

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- Description of Quadratic Band Touching/Crossing (QBT/QBC) points:

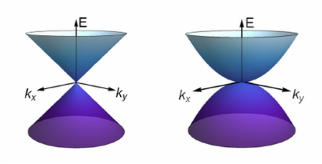

Figure: doi.org/10.1103/PhysRevB.101.161111

$$H = \sum_{i,j=1}^{d} G_{ij} p_i p_j, \qquad H^2 = p^4 \mathbb{1}$$

L. Janssen and I. F. Herbut, Phys. Rev. B 92, 045117 (2015)

What is a Luttinger fermion?

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Inspired by the diverse set of structures emerging from Luttinger fermions in solid-state physics, we generalized these d.o.f. to the relativistic case.

H. Gies, P. Heinzel, J. Laufkötter, and M. Picciau, Phys. Rev. D 110, 065001 (2024)

Relativistic Luttinger fermions

Kinetic action for relativistic theory of massless Luttinger fermions

$$S = \int d^d x [\bar{\psi} G_{\mu\nu} (i\partial^\mu)(i\partial^\nu)\psi]$$

$G_{\mu\nu}$ are $d_\gamma \times d_\gamma$ matrices (symmetric and traceless in Lorentz indices) which satisfy the Clifford algebra

$$\{G_{\mu\nu}, G_{\kappa\lambda}\} = -\frac{2}{d-1} g_{\mu\nu} g_{\kappa\lambda} + \frac{d}{d-1}(g_{\mu\kappa} g_{\nu\lambda} + g_{\mu\lambda} g_{\nu\kappa})$$

Properties of the algebra

Span $G_{\mu\nu}$ by an Euclidean Clifford algebra

$$G_{\mu\nu} = a^A_{\mu\nu}\gamma_A, \;\; \{\gamma_A, \gamma_B\} = 2\delta_{AB} \qquad a^A_{\mu\nu} \in \mathbb{C} \quad A, B = 1\ldots d_e$$

- Number of linearly independent elements to span the algebra
  $d_e = \frac{1}{2}d(d-1) + d - 1$
- Dimension of the Clifford algebra
  $d_{\gamma,irr} = 2^{\lfloor d_e/2 \rfloor}$

Define the conjugate spinor: $\bar{\psi} = \psi^\dagger h$ $\quad$ h spin metric H. Gies and S. Lippoldt, Phys. Rev. D 89, 064040 (2014)
Reality of the kinetic action implies $d_e = 11 \rightarrow d_\gamma = 32$

Mass terms

In order to classify different possibilities of gap formation, we look at the different mass terms that can be constructed for Luttinger fermions.

Standard mass term

$$\mathcal{L} = -\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - m^2 \bar{\psi}\psi$$

Equation of motion for ψ:

$$(G_{\mu\nu} p^\mu p^\nu - m^2)\psi = 0$$
$$\rightarrow (p^2 - m^2)(p^2 + m^2)\psi = 0$$

- spin-base & Lorentz invariant
- real
- regular mass poles ($p^2 = m^2$)
- tachyons ($p^2 = -m^2$)

Bilinear with γ_{10}

$$\mathcal{L} = -\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - i m_{10}^2 \bar{\psi}\gamma_{10}\psi$$

- same kind of solutions (regular mass and tachyons)

Mass terms

Bilinear with γ_{11}

$$\mathcal{L} = -\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - m_{11}^2 \bar{\psi} \gamma_{11} \psi$$

Equation of motion: $(p^4 + m^4)\psi = 0$

- two complex conjugate poles ($p^2 = \pm i m_{11}^2$)

Bilinear with γ_{01}

$$\mathcal{L} = -\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - i m_{01}^2 \bar{\psi} \gamma_{01} \psi$$

- $\gamma_{01} := -i\gamma_{10}\gamma_{11} \quad \gamma_{01} = \gamma_{01}^\dagger$
- again two complex conjugate poles ($p^2 = \pm i m_{01}^2$)

Self-interacting Luttinger fields

- Kinetic term $\sim G_{\mu\nu}\partial^\mu\partial^\nu \overset{d=4}{\rightarrow}$ canonical mass dimension of the fermions $[\psi] = 1$
- Quartic self interactions $\sim (\bar{\psi}\psi)^2$ are perturbatively renormalizable (RG marginal)
- 1024 fermion bilinears $\bar{\psi}\Gamma\psi$ (Γ element of the Clifford algebra)

We consider a set of massless theories of self-interacting Luttinger fermions with interactions defined in terms of the spinor bilinears we just mentioned.

We investigate the high-energy behaviour $\rightarrow \beta$ functions.

Self-interacting Luttinger fields: the zoo

Luttinger Gross-Neveu model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \frac{\bar{\lambda}}{2} (\bar{\psi}\psi)^2 \right]$$

γ_{10} model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - \frac{\bar{\lambda}}{2} (\bar{\psi}\gamma_{10}\psi)^2 \right]$$

γ_{11} model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - \frac{\bar{\lambda}}{2} (\bar{\psi}\gamma_{11}\psi)^2 \right]$$

γ_{01} model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \frac{\bar{\lambda}}{2} (\bar{\psi}\gamma_{01}\psi)^2 \right]$$

Luttinger NJL model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \frac{\bar{\lambda}}{2} \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_{10}\psi)^2 \right] \right]$$

- the model features a continuous U(1) symmetry

Self-interacting Luttinger fields

The masslessness of the models is protected by either a $\mathbb{Z}_2$ or a $U(1)$ symmetry.

Using FRG techniques $\rightarrow \beta$ function in the large N_f limit

$$\partial_t \lambda = -\frac{4N_f}{\pi^2}\lambda^2$$

- asymptotically free for $\lambda > 0$
- UV complete purely fermionic theories in $d = 4$.

Mean-field theory - GN model

In order to investigate the possible occurrence of gap formation, we use mean-field theory (Minkowski)

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi - \frac{\bar{\lambda}}{2} (\bar{\psi}\psi)^2 \right]$$

Hubbard-Stratonovich transformation

$$S_{\mathrm{FB}} = \int d^4x \left[-Z_\psi \bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \bar{h} \phi \bar{\psi} \psi - \frac{1}{2} \bar{m}^2 \phi^2 \right]$$

Matching condition

$$\bar{\lambda} = -\frac{\bar{h}^2}{\bar{m}^2}$$

Investigate the $\bar{\lambda} < 0$ branch.

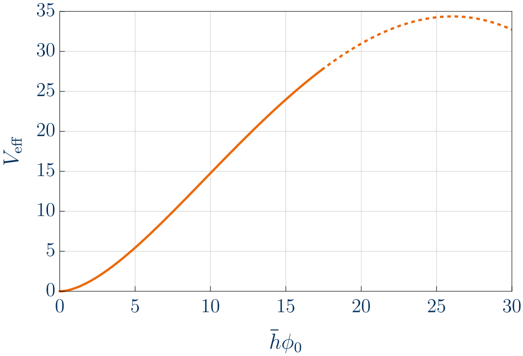
Mean-field theory - GN model

One-loop effective action

$$\Gamma_{1l} = -i \ln \det \left[-G_{\mu\nu} \partial^\mu \partial^\nu + \bar{h}\phi_0 \right]$$

- imaginary part $\propto \phi_0^2$: decay rate
- $\phi_0 = 0$ global minimum
- no mass and no tachyons

Identical discussion for the γ_{10} and the LNJL models.



Mean-field theory - γ_{11} model

$$S = \int d^4x \left[-\bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \frac{\bar{\lambda}}{2} (\bar{\psi} \gamma_{11} \psi)^2 \right]$$

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$$S_{\mathrm{FB}} = \int d^4x \left[-Z_\psi \bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + \bar{h} \phi \bar{\psi} \gamma_{11} \psi - \frac{1}{2} \bar{m}_{11}^2 \phi^2 \right]$$

Matching condition

$$\bar{\lambda} = \frac{\bar{h}^2}{\bar{m}^2}$$

Investigate the $\bar{\lambda} > 0$ branch.

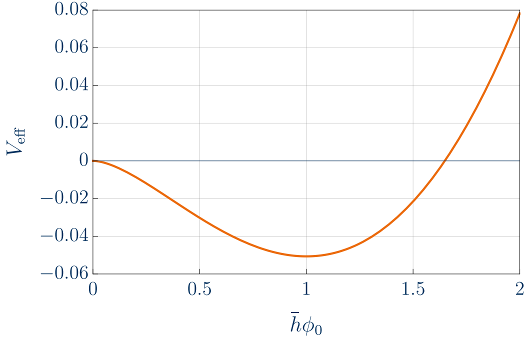
Mean-field theory - γ_{11} model

One-loop effective action

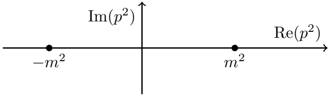
$$\Gamma_{1l} = -i \ln \det\left[-G_{\mu\nu}\partial^{\mu}\partial^{\nu} + \bar{h}\phi_0\gamma_{11}\right]$$

- discrete $\mathbb{Z}_2$ symmetry is spontaneously broken
- no imaginary part: stable ground state
- the effective potential has a non-trivial minimum at $\phi_0 = v$
- $\bar{m}_{11}^2 = \bar{h}v$

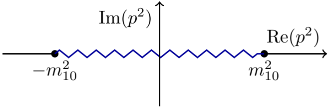
Same conclusions for the γ_{01} model.



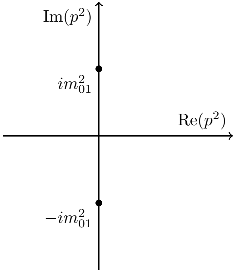
Structure of the massive propagators

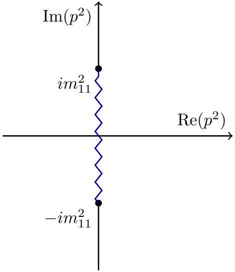


- standard mass & tachyon
- higher derivative theory: the tachyon is also a ghost



- branch cut
- no Källén–Lehmann spectral representation





- complex poles & complex branch cut
- no spectral representation
- $V''(\phi_0 = v) = \frac{2N_f}{\pi^2}\bar{h}^2$

Luttinger fermions are not asymptotic states. Context of confinement (QCD).

M. Stingl, Phys. Rev. D 34, 3863 (1986)

M. Stingl, Z.Phys.A 353, 423-445 (1996)

Recap and outlook

- Generalized Luttinger fermions to the relativistic case;
- Constructed self-interacting QFT with Luttinger fermions: UV complete QFT based on pure matter degrees of freedom;
- Investigated the IR region with the aid of mean-field theory: gap formation and SSB.

Recap and outlook

- Generalized Luttinger fermions to the relativistic case;
- Constructed self-interacting QFT with Luttinger fermions: UV complete QFT based on pure matter degrees of freedom;
- Investigated the IR region with the aid of mean-field theory: gap formation and SSB.

- Go beyond mean-field, with the inclusion of scalar fluctuations: FRG
  - $\Gamma_k = \int -Z_\psi \bar{\psi} G_{\mu\nu} \partial^\mu \partial^\nu \psi + h_k \phi (\bar{\psi} \gamma_{11} \psi) + \frac{Z_\phi}{2} \partial_\mu \phi \partial^\mu \phi + U_k(\phi)$;
  - Renormalized mass of the ϕ field?
  - Running of the Yukawa coupling;
  - Flow of the non-perturbative effective potential.
- Study phase transitions
  - Thermal PT: at finite temperature the fermion condensate will melt away (order of the PT?);
  - Quantum PT: inclusion of a fundamental scalar. UV complete Yukawa sector?