UV complete field theory in (2+1)D with symmetry breaking at all temperatures

Michael M. Scherer

Ruhr University Bochum

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Outline

- **Introduction & key question**
  - Inverted symmetry breaking & phase diagrams
  - FRG 101: O(N) models and finite T

- **O(N)×Z_2 field theory at finite T**
  - Mechanism for high-temperature SB
  - Fixed-points and UV completion
  - Quantum critical point & phase diagram

- **Conclusions**

Bilal Hawashin, RUB

Junchen Rong, IHES

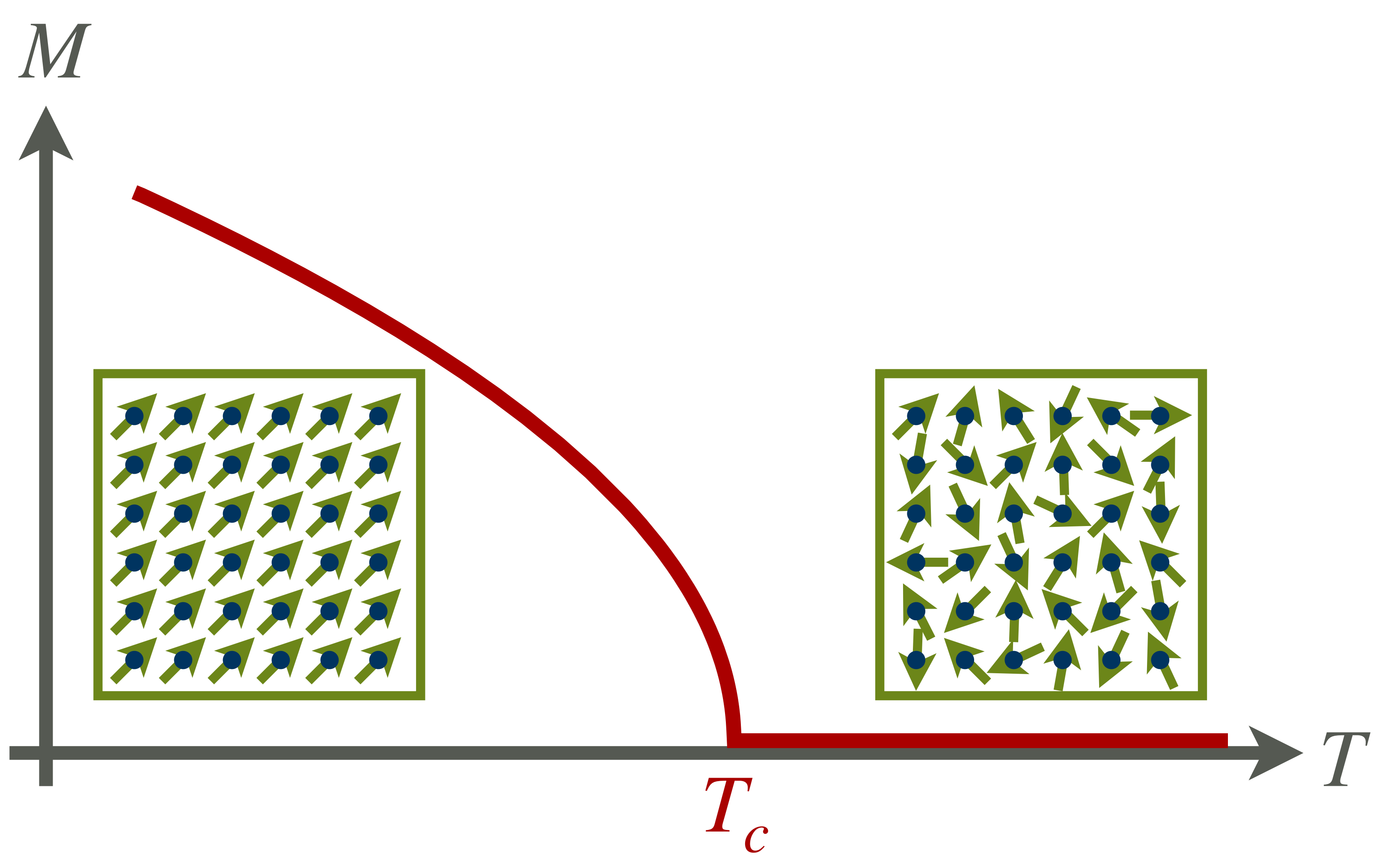
Heisenberg-Programm DFG Deutsche Forschungsgemeinschaft

MERCUR Mercator Research Center Ruhr

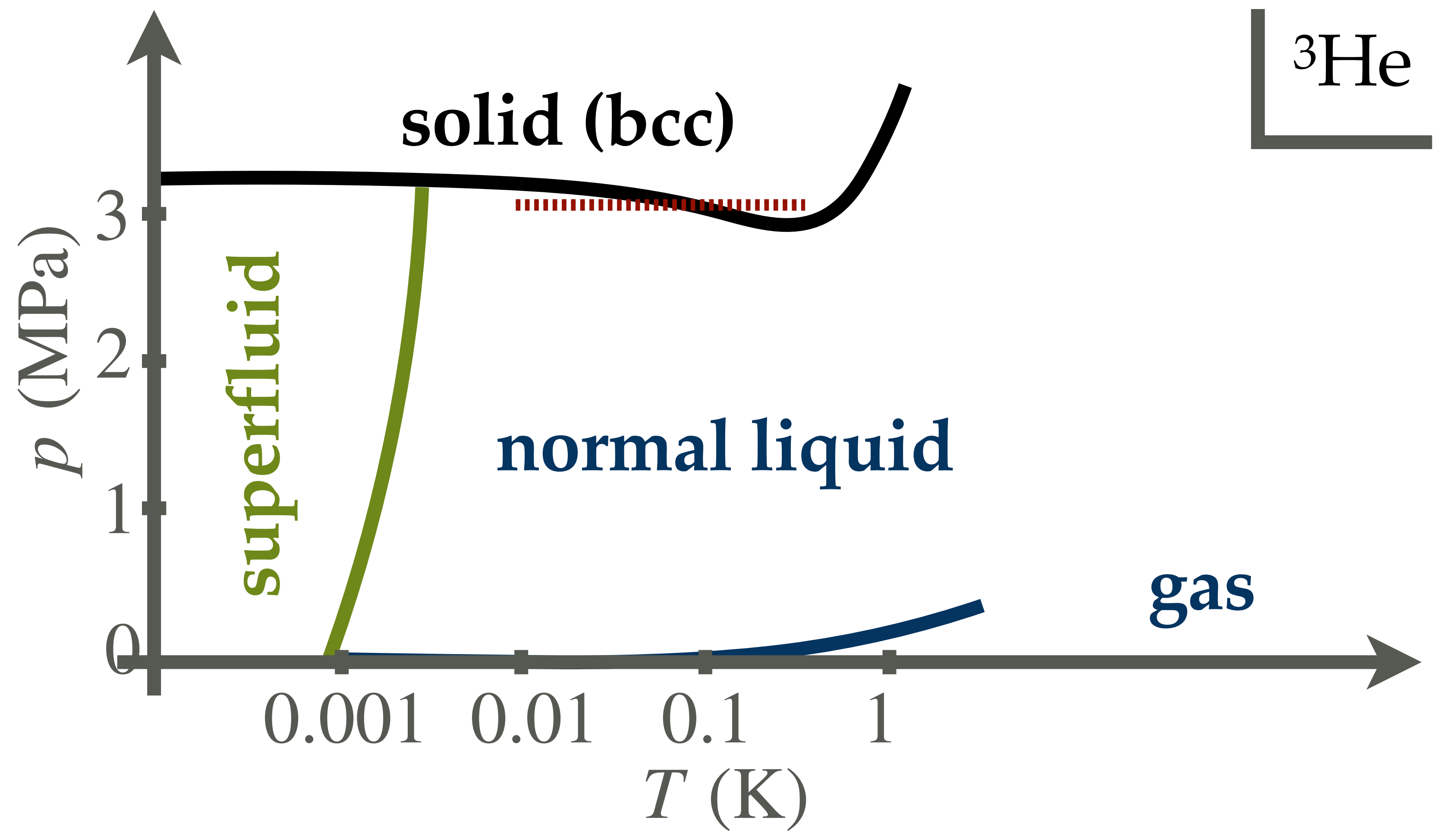
CRC 1238 Control and Dynamics of Quantum Materials

Symmetry breaking & temperature

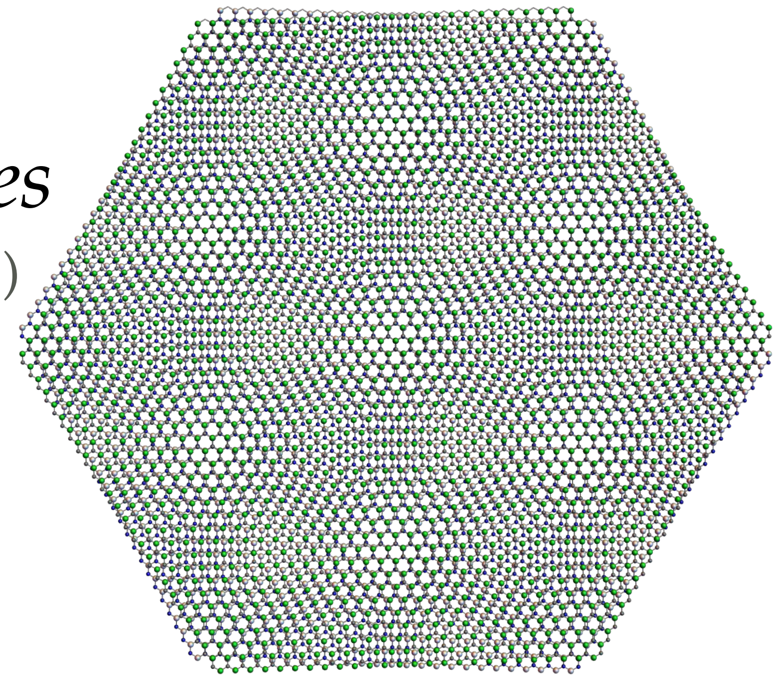
- Spontaneous *symmetry breaking* is usually a *low-temperature* phenomenon
- Towards *high temperatures symmetries* are *restored*
  - Consider **free energy** $F = E - T\,S$
  - F is minimized → *high-entropy* states dominate at high T
  - *high-entropy states* typically disordered → *symmetry restoration* for high T



- There are **exceptions**, e.g., **Pomeranchuk effect** Pomerantschuk, Zh. Eksp. Teor. Fiz.. (1950)
  - Liquid ^{3}He: obeys Fermi statistics → $S \propto T$
  - Solid ^{3}He: nucleon spins contribute excess entropy $S \propto$ const .
  - For $T \lesssim 0.3K$: $S_{\text{solid}} > S_{\text{liquid}}$ → ^{3}He *"freezes" when heated*
  - **Inverted phase diagram**!



Inverted phase diagrams



- **Pomeranchuk effect** also observed recently in *twisted bilayer graphene* and *moiré transition metal dichalcogenides*

  Rosen *et al.*, Nature 592 (2021)
  Saito *et al.*, Nature 592 (2021)
  Li *et al.*, Nature 597 (2021)

- *Also:* Structural transition in Rochelle salt, "order by disorder", ...

- *Application:* Pomeranchuk effect in ^{3}He can be used for *cooling* upon isentropic *compression*

- **Inverted phase diagrams** also exist in QFTs, e.g, with $O(N) \times O(M)$ symmetry

  Weinberg, PRD 9, 3357 (1974)
  Mohapatra & Senjanović, PRL 42, 1651 (1979)
  Pietroni, Rius, Tetradis, PLB 397, 119 (1997)
  ...

- *Applications* to domain wall and false vacuum problems, baryogenesis, inflation,...

  Senjanović, COSMO97 (1998)
  ...

- **Up to which temperatures T can this work?**

  - Lattice system w/ density matrix: $e^{-\beta H} \xrightarrow{T\to\infty} \mathbb{1}$

    ➡ All **sites** decouple → expectation values of local operators vanish → **symmetries restored at sufficiently high T**

    Kliesch, Gogolin, Kastoryano, Riera, Eisert, PRX 4, 031019 (2014)

  - **UV incomplete models** (e.g., due to Landau poles): phenomenon can only be verified up to T below UV cutoff

  - **UV complete models**: high-T limit could be nontrivial!

    Chai, Chaudhuri, Choi, Komargodski, Rabinovici, Smolkin, PRL 125, 131603 (2020)
    Bajc, Lugo, Sannino, PRD 103, 096014 (2021)

UV complete models and temperature

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- Short distance limit of UV complete field theory is a CFT / scale invariant #asymptotic-safety
- **What happens to CFT at finite T?**
  - CFT does not have inherent scale → any nonzero T is equivalent to any other nonzero T
  - ➡ If CFT shows **SSB at some finite T** → there is **SSB at all T!**
  - ➡ Question: **Are there unitary, local, nontrivial CFTs that do not restore SYM at infinite T?**

Polyakov, PLB 72, 477 (1978)
Komargodski, Sharon, Thorngren, Zhou, SciPost Phys 6, 003 (2019)

- **Conjecture**: $O(N) \times Z_2$ symmetric scalar field theory in $D = 2 + 1$ dimensions at *biconical fixed point*

Chai, Chaudhuri, Choi, Komargodski, Rabinovici, Smolkin, PRL 125, 131603 (2020)

$$S = \int d^D x \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + \frac{m_\phi^2}{2}\phi^2 + \frac{m_\chi^2}{2}\chi^2 + \frac{\lambda_\phi}{8}\phi^4 + \frac{\lambda_\chi}{8}\chi^4 + \frac{\lambda_{\phi\chi}}{4}\phi^2\chi^2 \right)$$

  - **Discrete SB at finite T**: $O(N) \times Z_2 \longrightarrow O(N)$
  - *Evidence from* 4-ϵ expansion (*unitarity violating for $D \notin \mathbb{N}$, no MWH for $\epsilon = 1$*), 2+1D in long-range PT (*non-local*)

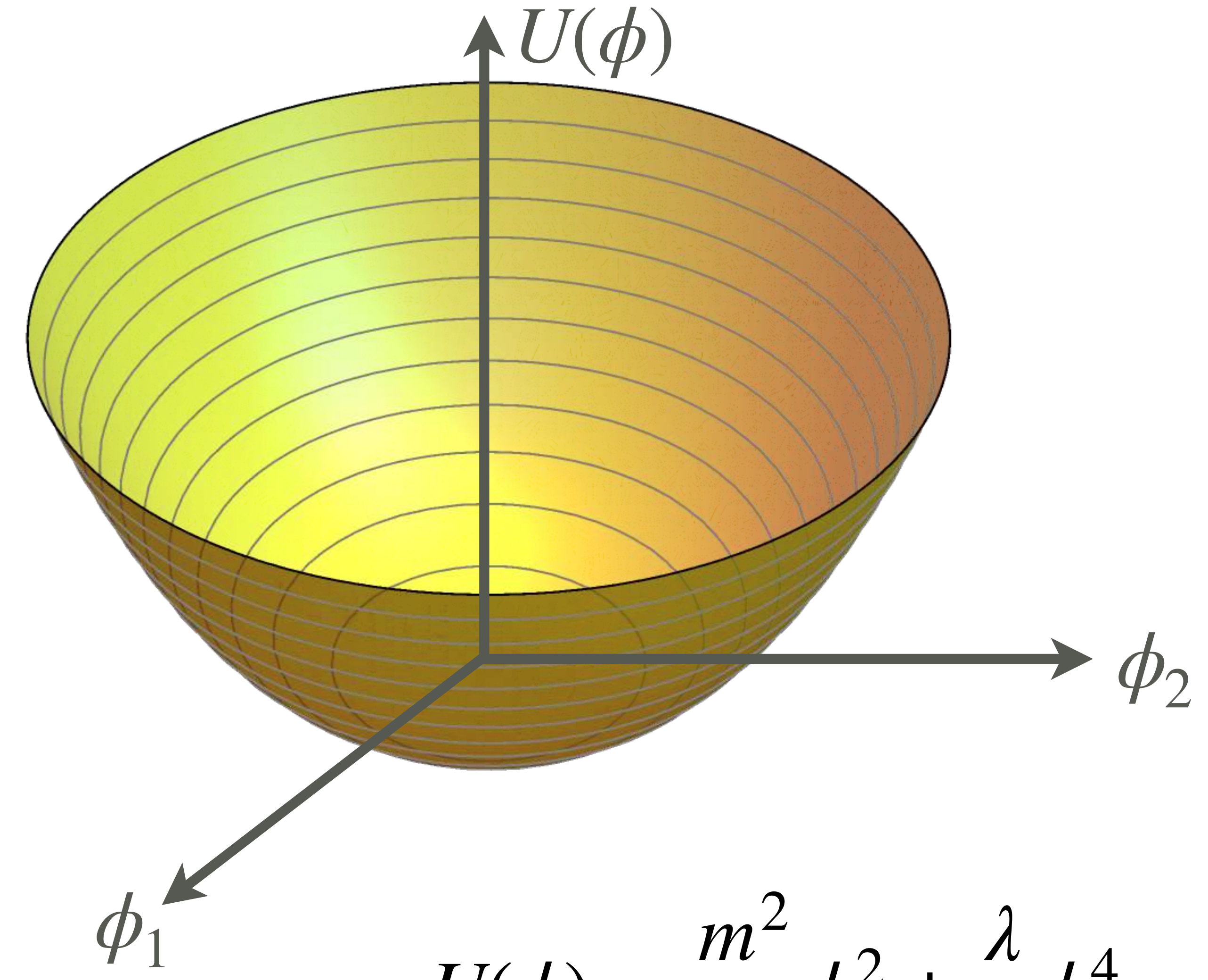
Hogervorst, Rychkov, van Rees, PRD 93, 125025 (2016)
Chai, Dymarsky, Smolkin, PRL 128, 011601 (2022)

FRG 101: O(N) model

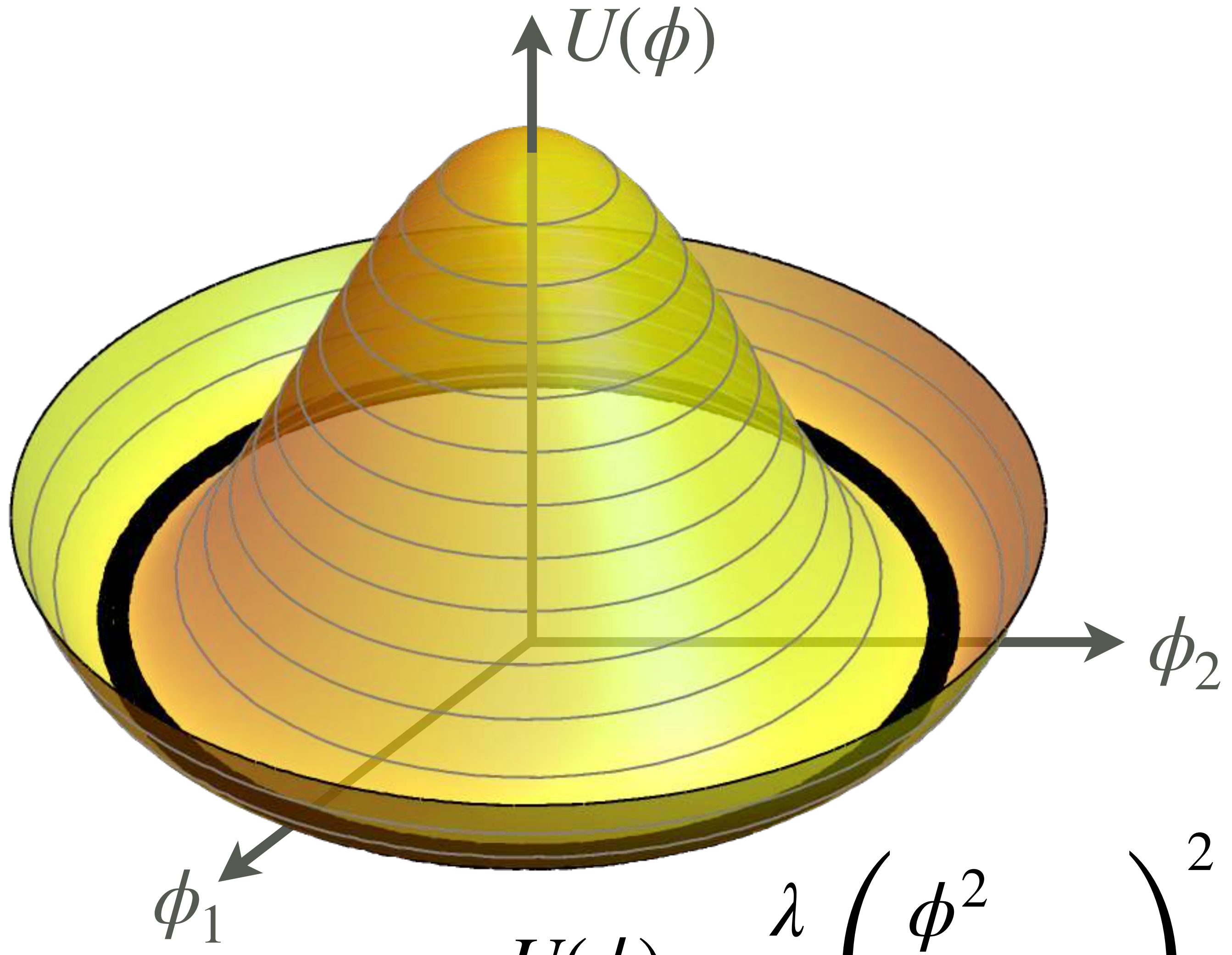
- Effective average action in LPA': $\Gamma_k = \int d^D x \left[\frac{Z_k}{2} \left(\partial\phi \right)^2 + U_k(\phi) \right]$

  - Consider D dimensional space *or* D=d+1 dimensional Euclidean spacetime

  - Effective potential maybe in symmetric (SYM) or symmetry broken (SSB) regime



$$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4, \qquad m^2, \lambda > 0$$



$$U(\phi) = \frac{\lambda}{2}\left(\frac{\phi^2}{2} - \kappa \right)^2, \qquad \kappa, \lambda > 0$$

"condensate"

  - Boundedness of potential requires $\lambda > 0$ (neglecting higher-order terms)

FRG 101: O(N) model

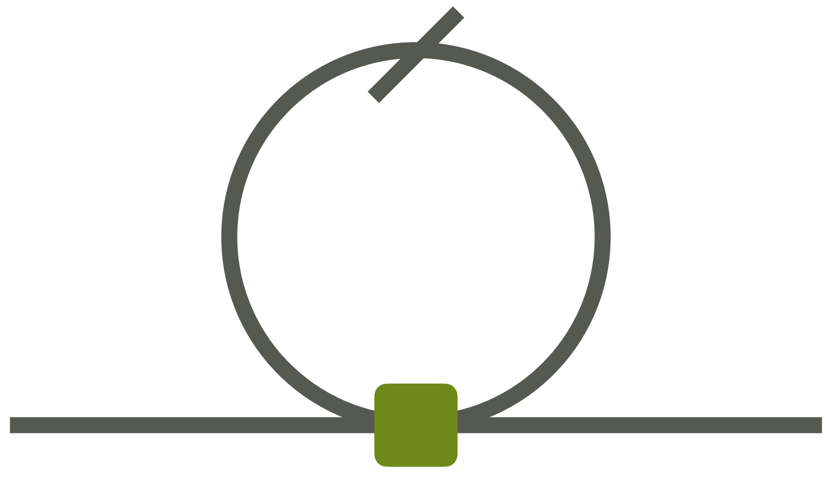
- Effective average action in LPA': $$\Gamma_k = \int d^D x \left[\frac{Z_k}{2} \left(\partial\phi\right)^2 + U_k(\phi) \right]$$

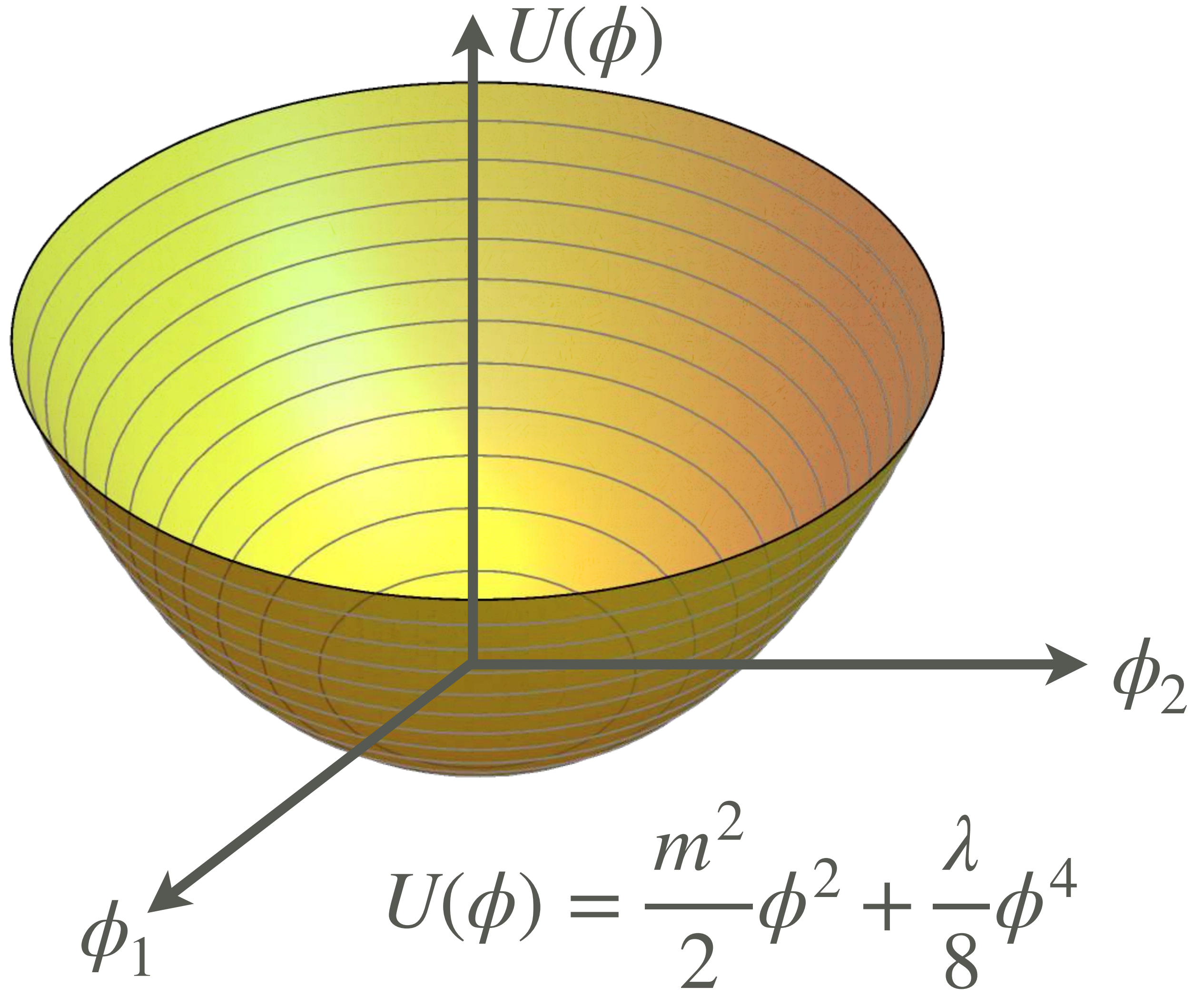
  - Flow of *dimensionless effective potential in D=3 with Litim regulator in LPA with* $\rho = \phi^2/2$

$$k\partial_k u(\rho) = \underbrace{-3u + \rho u'}_{\text{rescaling}} + \underbrace{\frac{1}{6\pi^2}\frac{1}{1+u'+2\rho u''}}_{\text{radial mode}} + \underbrace{\frac{N-1}{6\pi^2}\frac{1}{1+u'}}_{\text{Goldstones}}$$

  - Flow equation for *dimensionful mass in SYM regime:*

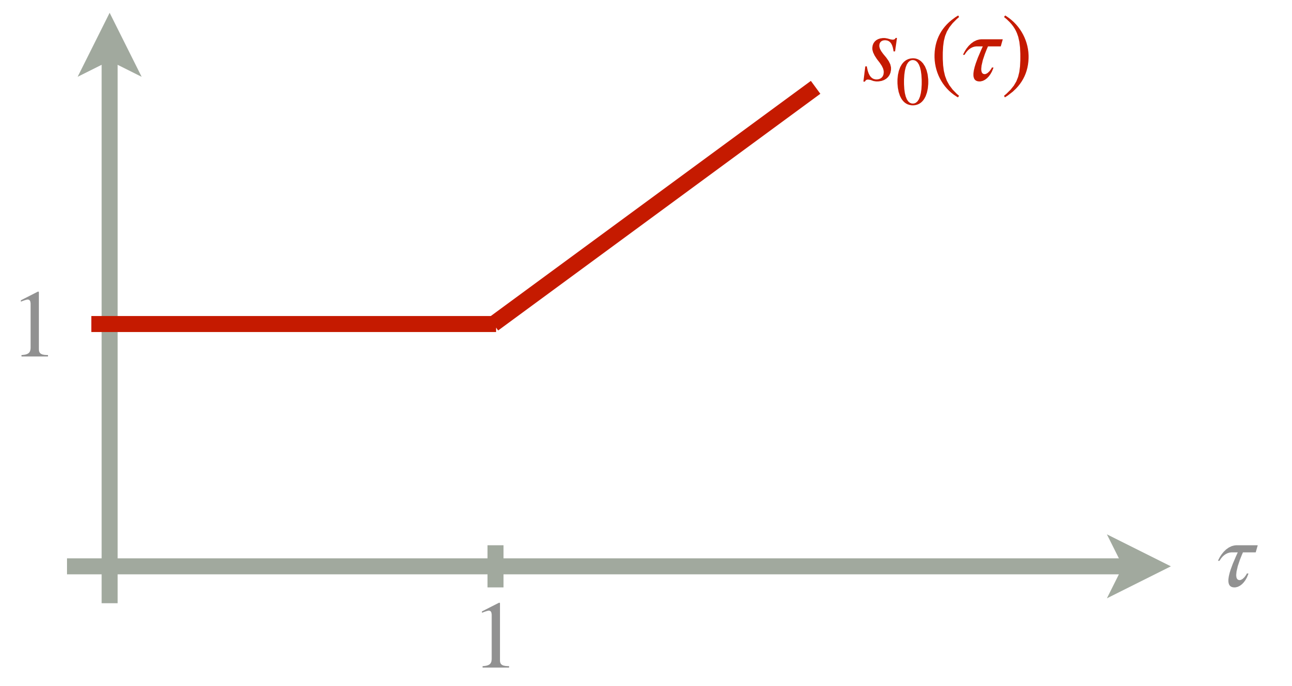
$$k\partial_k m^2 = -(N+2)\frac{a_D}{3\pi^2}k^{D+2}\frac{\lambda}{(k^2+m^2)^2} \sim$$





  - R.H.S. is negative ⇒ for $k \to 0$ the **mass² cannot decrease** ⇒ bosonic fluctuations keep system in SYM phase.

O(N) model at finite temperature

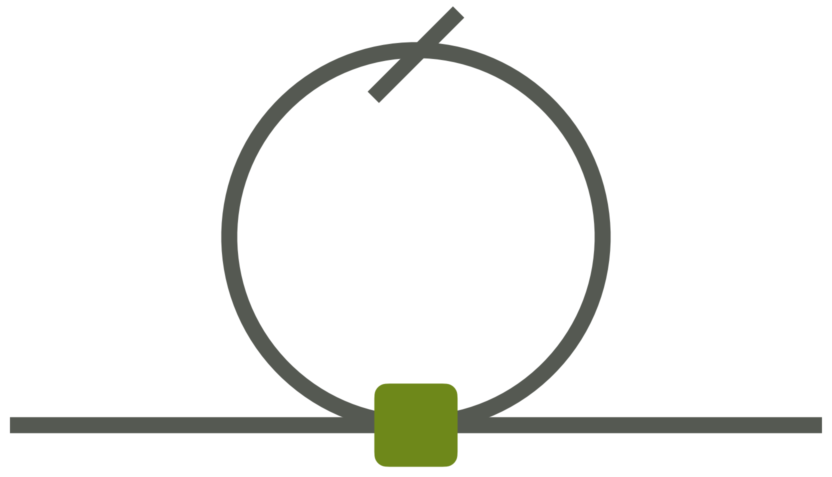


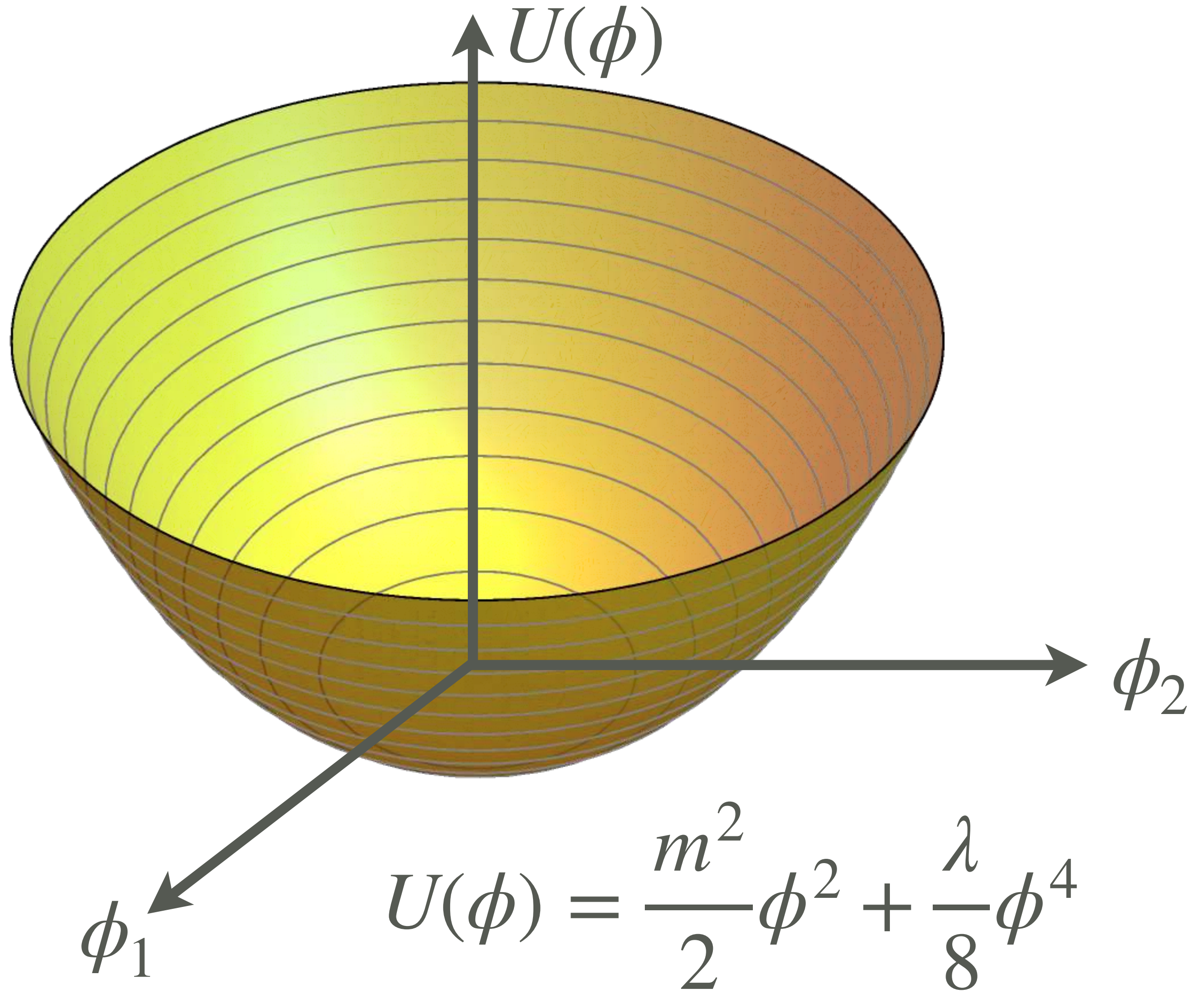
- Introduce finite T: $\quad q_0 \to i\omega_n = 2\pi n T, \quad \int \frac{d^D q}{(2\pi)^D} \to T \sum_{n\in\mathbb{Z}} \int \frac{d^d q}{(2\pi)^d}$
  - Use "covariant" Litim regulator and $\tau = 2\pi T/k$

$$k\partial_k u(\rho) = \underbrace{-3u + \rho u'}_{\text{rescaling}} + \underbrace{\frac{1}{6\pi^2}\frac{1}{1+u'+2\rho u''}}_{\text{radial mode}} s_0(\tau) + \underbrace{\frac{N-1}{6\pi^2}\frac{1}{1+u'}}_{\text{Goldstones}} s_0(\tau)$$

  - Flow equation for *dimensionful mass* at high T or small k:

$$k\partial_k m^2 = -(N+2)\frac{b_D}{3\pi^2}k^{d+2}\frac{\lambda}{(k^2+m^2)^2}T \sim$$





$U(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{8}\phi^4$

  - Higher T can make fluctuations even stronger with same sign ⇒ system remains SYM phase.
  - Corresponds to *standard expectation*: higher temperature → *no transition into SSB regime*

O(N) model at finite temperature

- Introduce finite T: $$q_0 \to i\omega_n = 2\pi n T, \quad \int \frac{d^D q}{(2\pi)^D} \to T \sum_{n\in\mathbb{Z}} \int \frac{d^d q}{(2\pi)^d}$$

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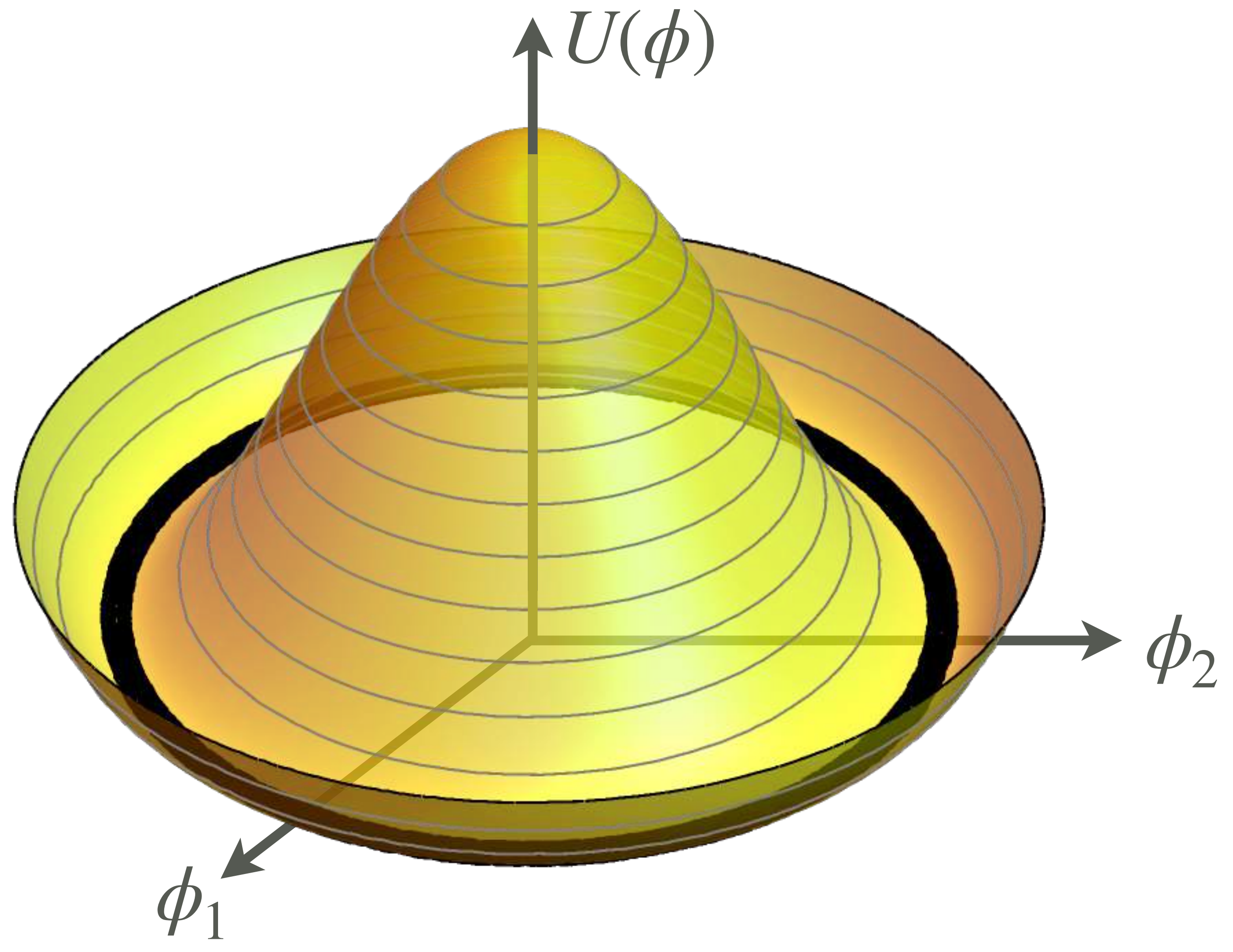


  - Flow equation for **condensate κ** at high T or small k:

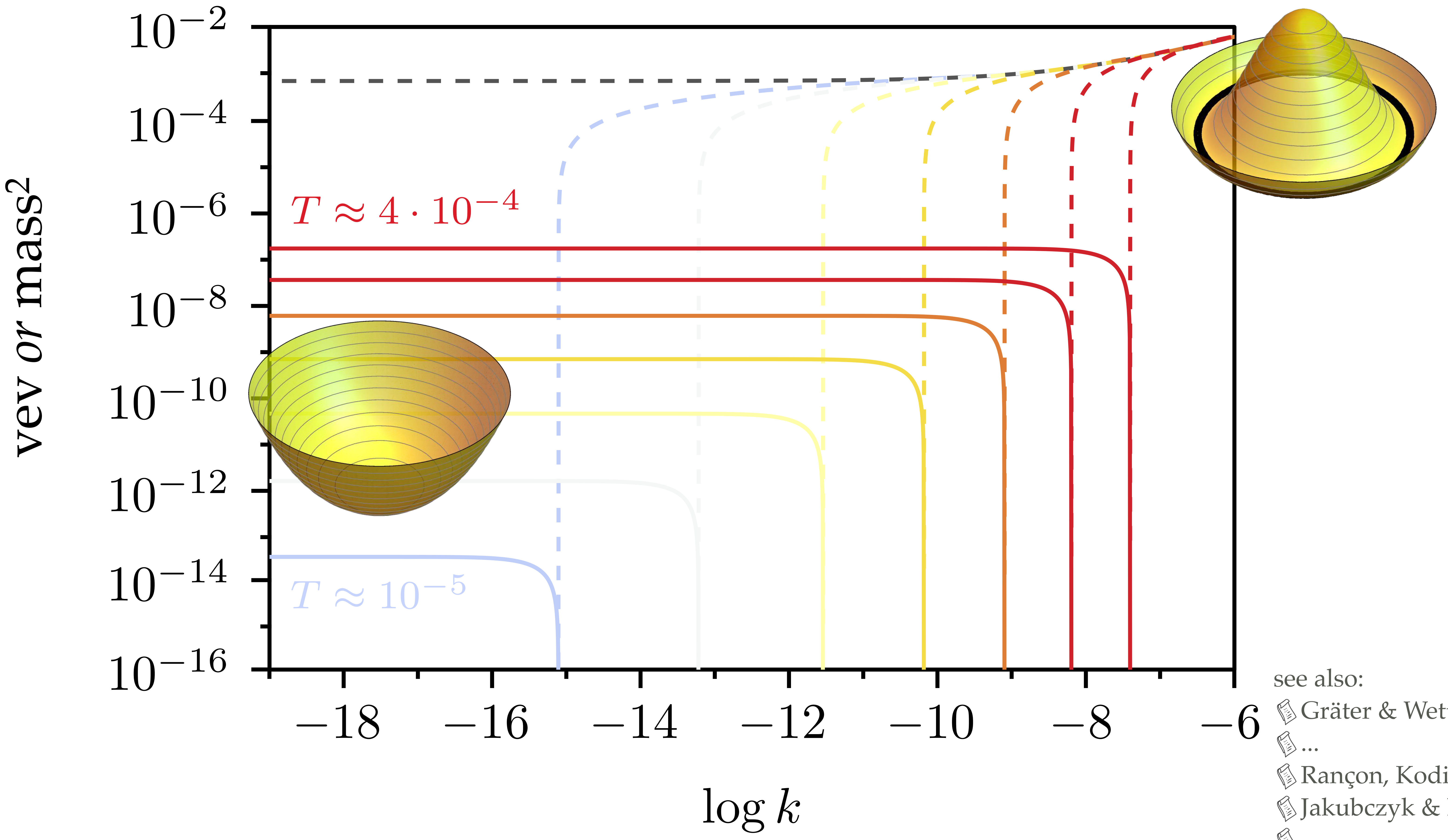
$$k\partial_k \kappa \quad = \quad c_D(N-1)k^{d-2}T \quad + \quad \mathcal{O}(k^{D+1})$$

  - For $N > 1$ and $d \leq 2$: **Condensate κ** melts down at finite k → **no SSB** → **Mermin-Wagner**!

  - For $N = 1$ or $d > 2$: Flow gets arbitrarily slow towards IR ⇒ **condensate** $\kappa_{\text{IR}} > 0$ is possible ⇒ SSB

O(N) model at finite temperature

- Numerical example for $N > 1$, $D = 2 + 1$
  - Start flow in SSB regime ($\kappa > 0$, dashed lines) → always flows into SYM regime ($m^2 > 0$, solid lines) for $T > 0$

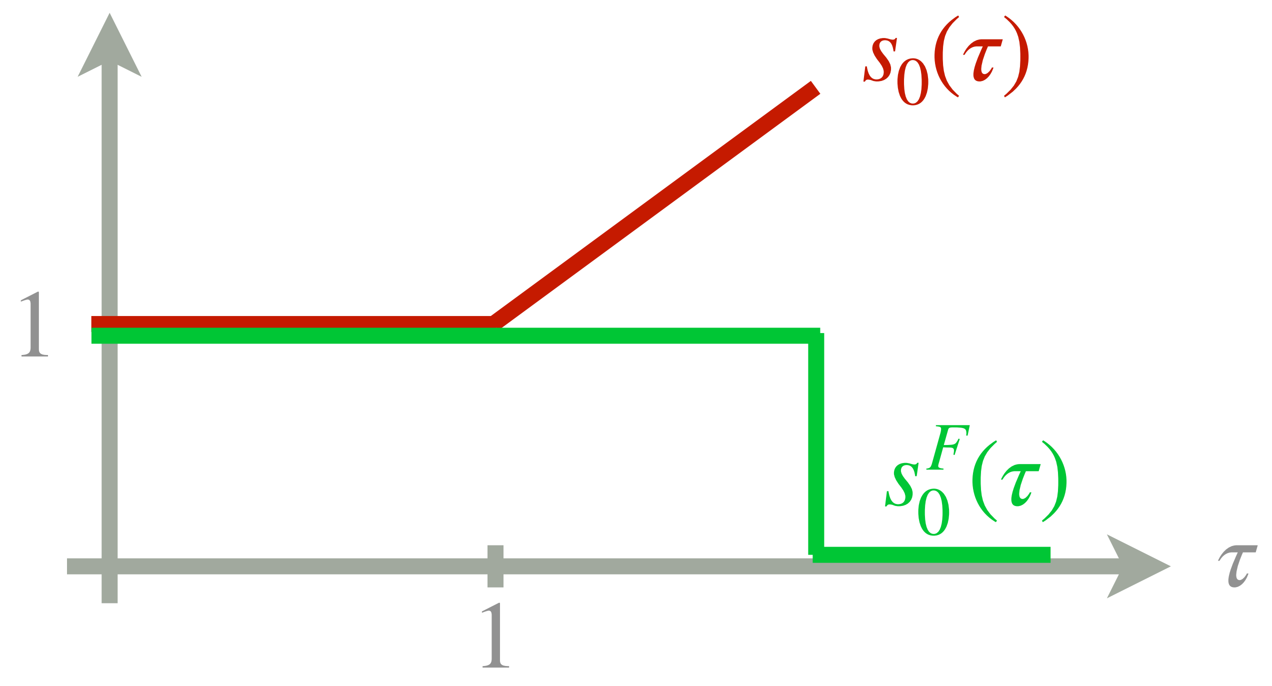


see also:
Gräter & Wetterich, PRL 75, 378 (1995)
...
Rançon, Kodio, Dupuis, Lecheminant, PRE 88, 012113 (2013)
Jakubczyk & Eberlein, PRE 93, 062145 (2016)
...

O(N) model with Yukawa-coupled fermions

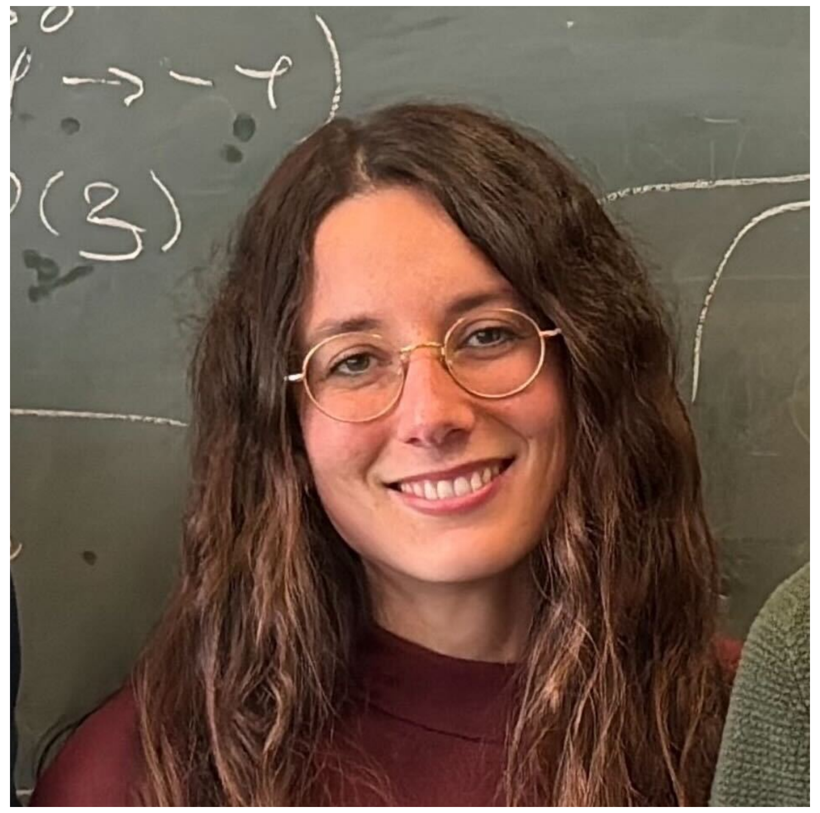
- How to drive system into SSB regime?
  - Flow equation for *dimensionful mass* with *massless fermions*

$$k\partial_k m^2 \quad \sim \quad -(N+2)\,\lambda\, s_0(\tau) \quad + \quad h^2 s_0^F(\tau)$$

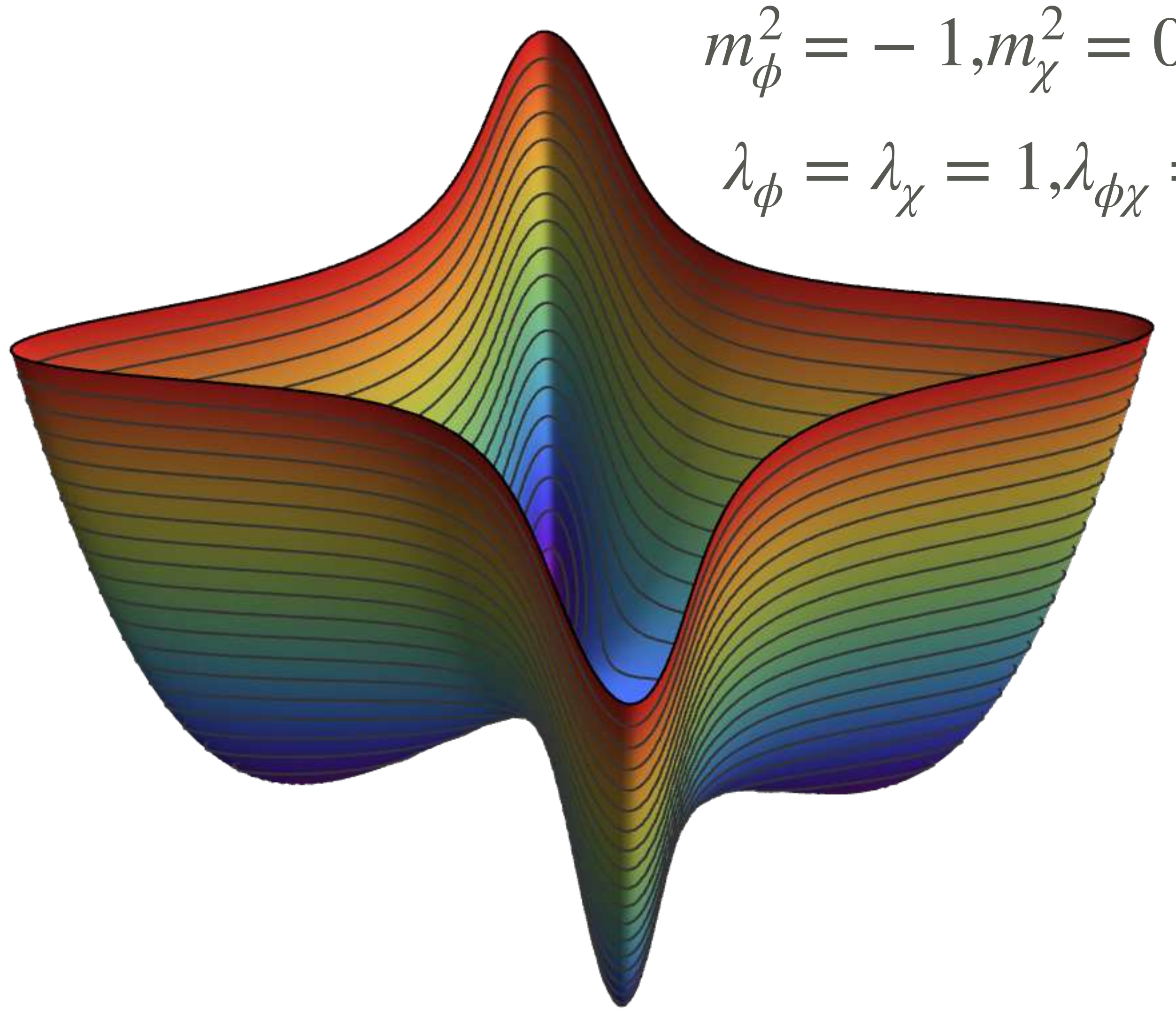


  - Fermion loop has *opposite sign* → can drive mass to zero and into SSB regime
  - *But:* for large $\tau \sim T/k$ it gets switched off → cannot drive high-T symmetry breaking

→ *Talk by*
Mireia Tolosa-Simeón



O(N) model coupled to Z_2 model



- Microscopic action with $O(N) \times \mathbb{Z}_2$ symmetry and $\rho_\phi = \phi_a\phi_a/2$ and $\rho_\chi = \chi^2/2$

$$S = \int d^D x \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + m_\phi^2\rho_\phi + m_\chi^2\rho_\chi + \frac{\lambda_\phi}{2}\rho_\phi^2 + \frac{\lambda_\chi}{2}\rho_\chi^2 + \lambda_{\phi\chi}\rho_\phi\rho_\chi \right)$$

  - Bound from below if $\lambda_\phi, \lambda_\chi > 0$ and $\lambda_\phi\lambda_\chi \geq \lambda_{\phi\chi}^2$ $\Rightarrow$ $\lambda_{\phi\chi}$ **can be negative**!

- *FRG flow of effective potential* $U(\rho_\phi, \rho_\chi)$ can be written down in *closed form* in LPA' for abitrary T

$$\partial_t u = -du + (d-2+\eta_\phi)\bar\rho_\phi u^{(1,0)} + (d-2+\eta_\chi)\bar\rho_\chi u^{(0,1)} + \left[I_R^d(\omega_\chi, \omega_\phi, \omega_{\phi\chi}) + (N-1) I_G^d(u^{(1,0)}) \right] S_\phi(\tau) + I_R^d(\omega_\phi, \omega_\chi, \omega_{\phi\chi}) S_\chi(\tau)$$

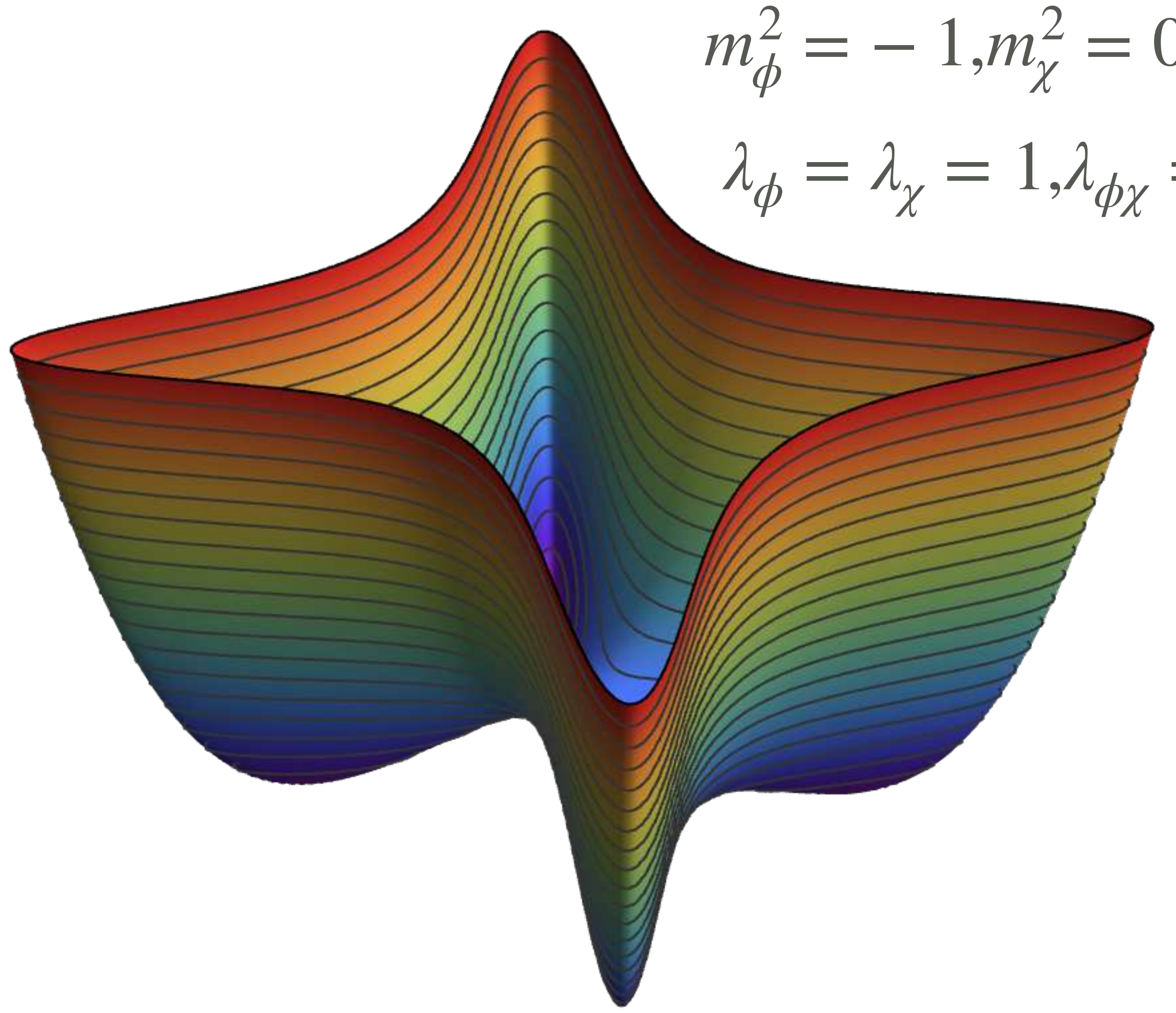
$$\omega_\phi = u_k^{(1,0)} + 2\bar\rho_\phi u_k^{(2,0)}, \omega_\chi = u_k^{(0,1)} + 2\bar\rho_\chi u_k^{(0,2)}, \omega_{\phi\chi}^2 = 4\bar\rho_\phi\bar\rho_\chi (u_k^{(1,1)})^2$$

Hawashin, Rong, Scherer, arXiv:2409.10606 (2024)
see also Pietroni, Rius, Tetradis, PLB 397, 119 (1997)

O(N) model coupled to Z_2 model

$m_\phi^2 = -1, m_\chi^2 = 0.5$

$\lambda_\phi = \lambda_\chi = 1, \lambda_{\phi\chi} = -0.9$



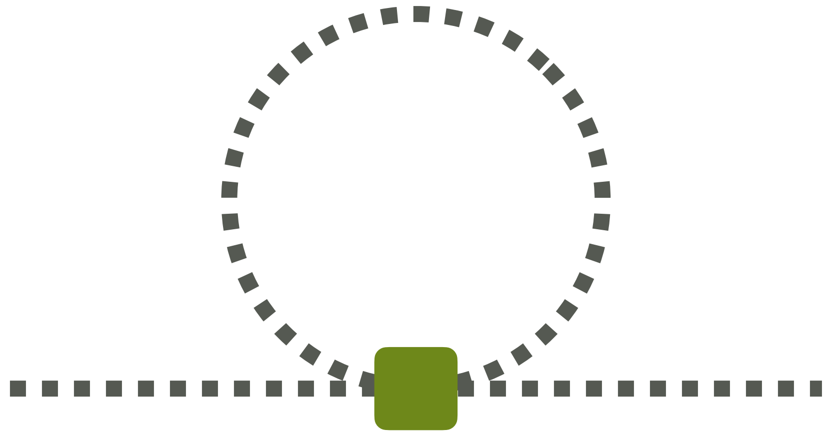
- Microscopic action with $O(N) \times \mathbb{Z}_2$ symmetry and $\rho_\phi = \phi_a\phi_a/2$ and $\rho_\chi = \chi^2/2$

$$S = \int d^D x \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + m_\phi^2\rho_\phi + m_\chi^2\rho_\chi + \frac{\lambda_\phi}{2}\rho_\phi^2 + \frac{\lambda_\chi}{2}\rho_\chi^2 + \lambda_{\phi\chi}\rho_\phi\rho_\chi \right)$$

  - Bound from below if $\lambda_\phi, \lambda_\chi > 0$ and $\lambda_\phi\lambda_\chi \geq \lambda_{\phi\chi}^2 \quad \Rightarrow \quad$ $\lambda_{\phi\chi}$ **can be negative!**

- *FRG flow of dimensionful mass* of Z_2 field χ at high T **for negative $\lambda_{\phi\chi}$** in D=2+1:

$$k\partial_k m_\chi^2 = \frac{k^4 a_D}{3\pi^2} \left(- \frac{3\lambda_\chi}{(k^2 + m_\chi^2)^2} + \frac{N|\lambda_{\phi\chi}|}{(k^2 + m_\phi^2)^2} \right) T \quad = \quad -$$



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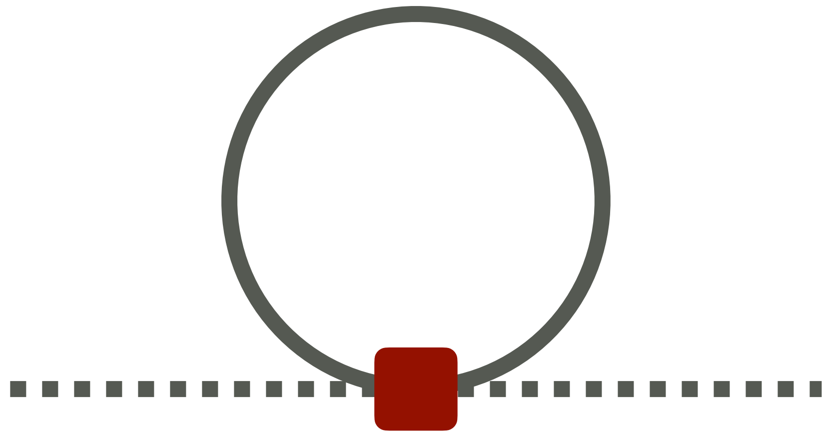

  - Boson loop with mixed coupling has *opposite sign* → **can potentially drive mass to zero and into SSB phase!**
  - *And:* for large $\tau \sim T/k$ it *won't be switched off* → **can drive SSB towards high T!**

Fixed points in $O(N) \times Z_2$ model in D=3 (T=0)

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- How can we get **SSB at all temperatures**?
  - Need **UV completion** of $O(N) \times \mathbb{Z}_2$ field theory with **sufficiently negative** $\lambda_{\phi\chi}$ contribution
  - **Fixed-point analysis!**

- Microscopic action generalized to $O(N) \times O(M)$

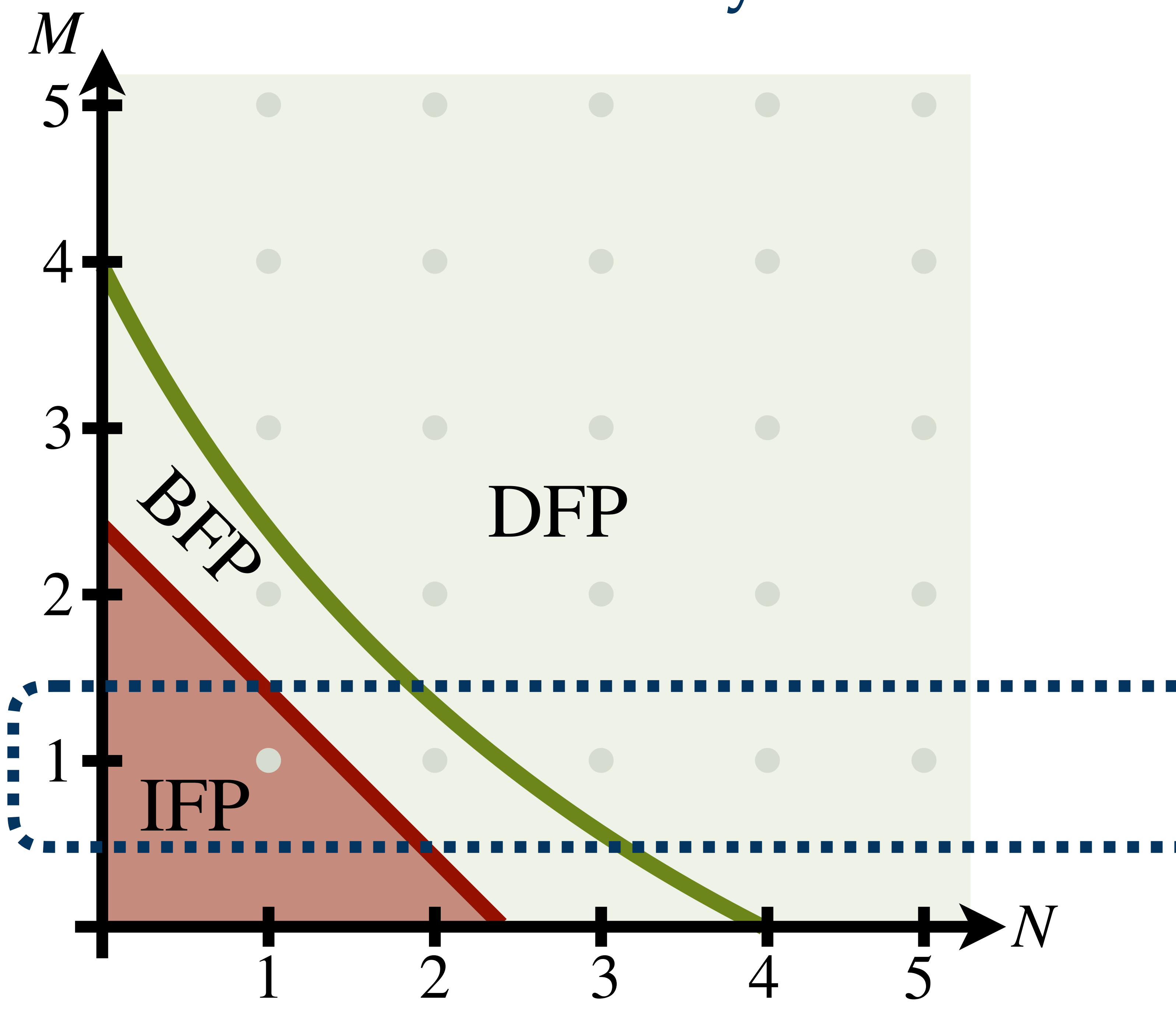
$$S = \int d^D x \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 + m_\phi^2 \rho_\phi + m_\chi^2 \rho_\chi + \frac{\lambda_\phi}{2}\rho_\phi^2 + \frac{\lambda_\chi}{2}\rho_\chi^2 + \lambda_{\phi\chi}\rho_\phi\rho_\chi \right)$$

(1) **Decoupled Wilson-Fisher FPs**: $\lambda_\phi^* > 0,\ \lambda_\chi^* > 0,\ \lambda_{\phi\chi}^* = 0$

(2) **Isotropic FP** w/ emergent $O(N{+}M)$: $\lambda_\phi^* = \lambda_\chi^* = \lambda_{\phi\chi}^* > 0$

(3) **Biconical fixed point**: $\lambda_\phi^* > 0,\ \lambda_\chi^* > 0,\ \lambda_{\phi\chi}^* \neq 0$

Infrared stability:



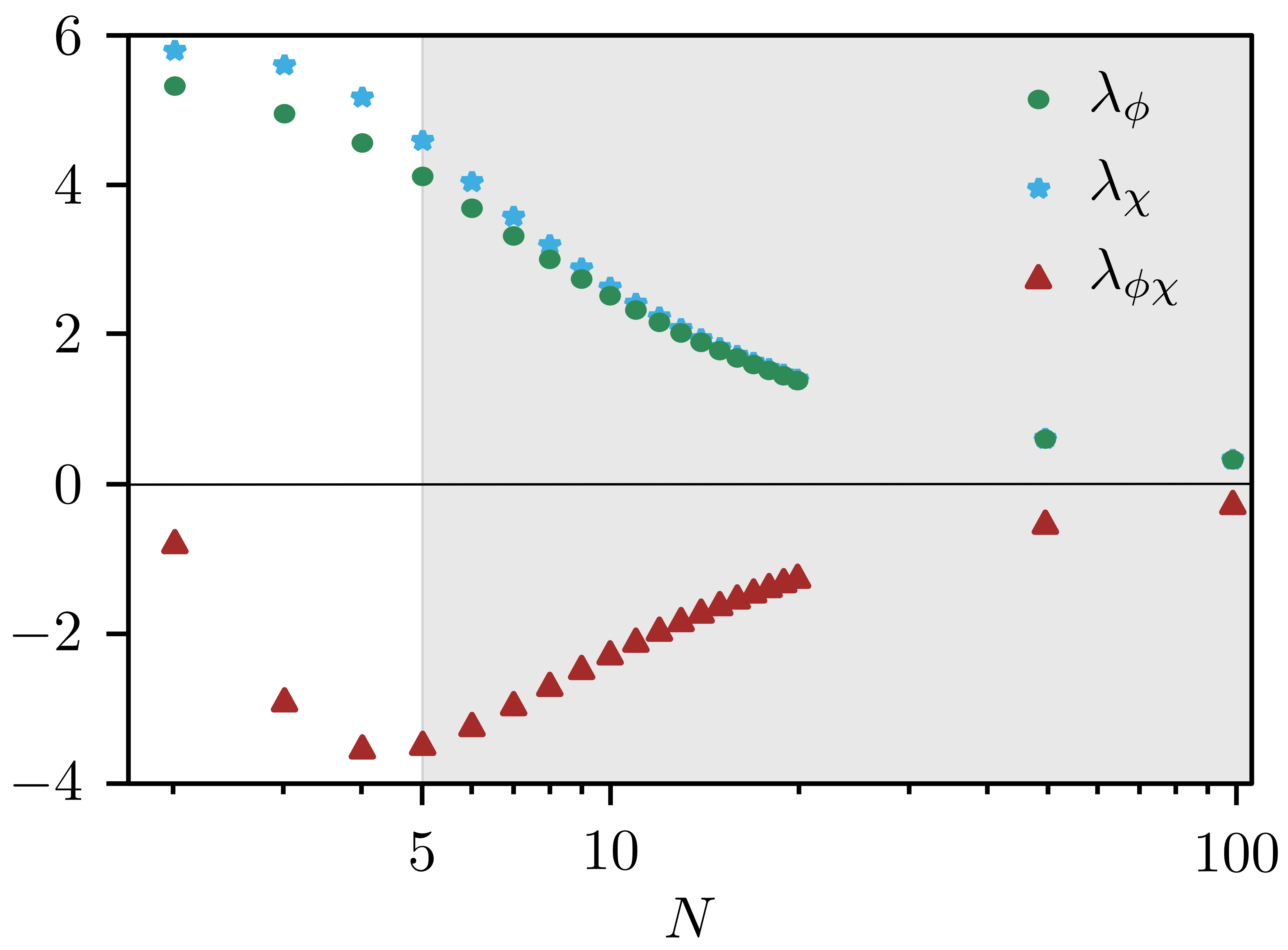
Nelson, Kosterlitz, Fisher (1974)
...
Calabrese, Pelissetto, Vicari (2003)
Eichhorn, Mesterhazy, Scherer (2013)
...

Biconical fixed point in D=3

- BFP is not IR stable for $N > 2$ but can still define UV completion
  - $\lambda_{\phi\chi}$ is indeed **negative**!
  - For $N \geq 5$ we even find: $N|\lambda_{\phi\chi}| > 3\lambda_\chi$
  - For large enough N: *good convergence* within LPA(')n

$N = 10$	κ_ϕ	κ_χ	λ_ϕ	λ_χ	$\lambda_{\phi\chi}$	θ_1	θ_2	θ_3
LPA6	0.25	0.10	2.62	2.54	-2.34	2.02	1.06	0.61
LPA8	0.24	0.09	2.59	2.84	-2.43	1.98	1.07	0.61
LPA′6	0.24	0.10	2.50	2.61	-2.30	1.99	1.09	0.56
LPA′8	0.24	0.09	2.47	2.81	-2.34	1.95	1.09	0.57

negative, with $U(\rho_\phi, \rho_\chi)$ bounded from below!



  - Comparison w/ 5-loop at small N:

N=2	ν	η_ϕ	η_χ
5-loop	0.70(3)	0.037(5)	0.037(5)
FRG, LPA'6	0.68	0.04	0.04

Calabrese, Pelissetto, Vicari (2003)

RG flows & inverted phase diagram at finite T

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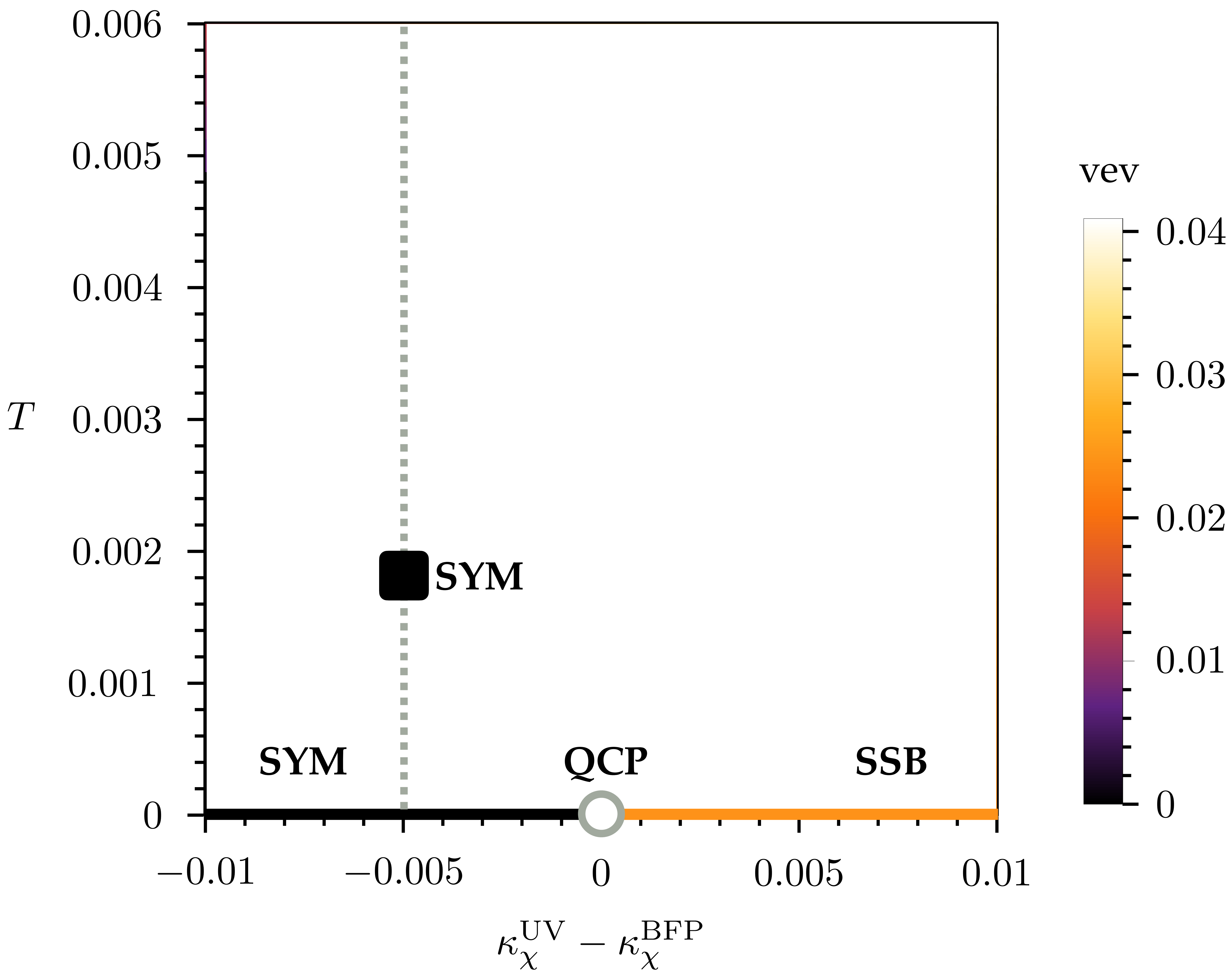
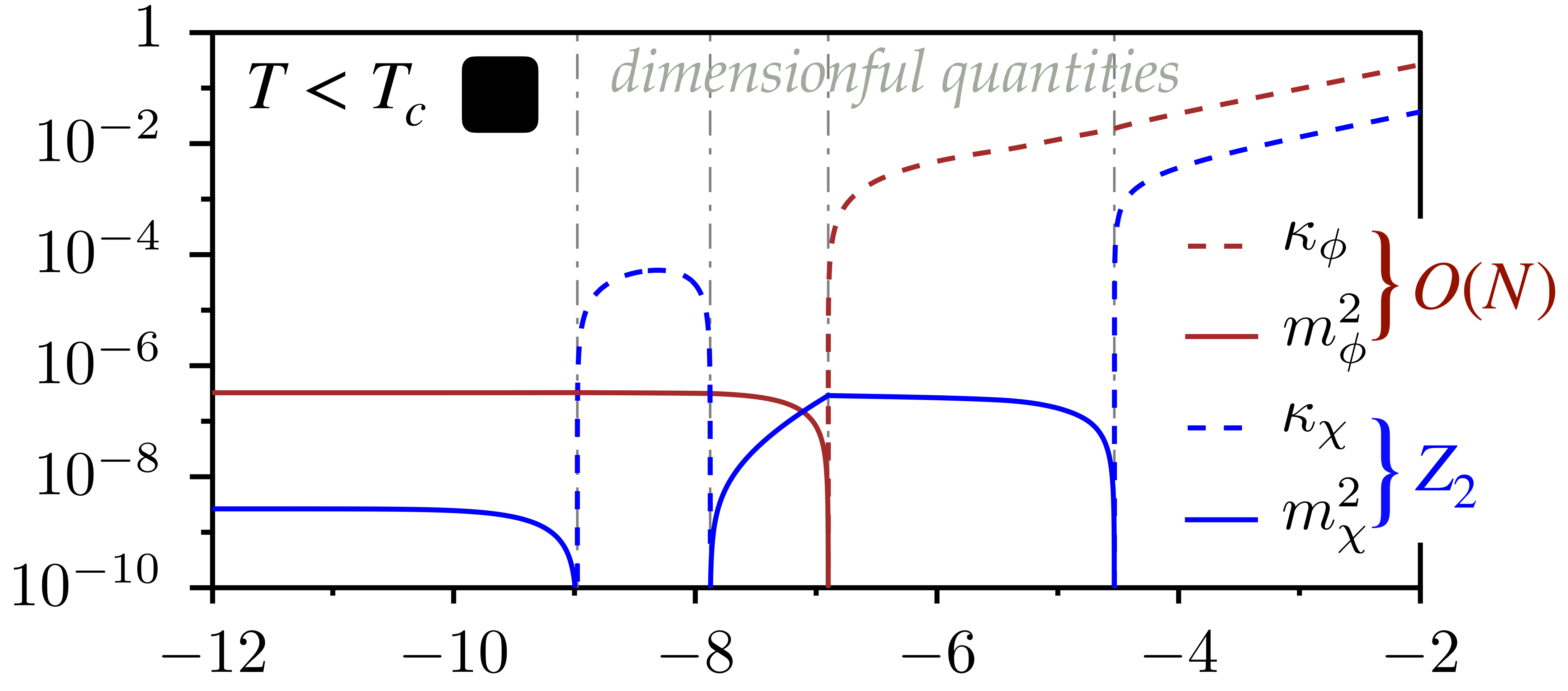
- Microscopic starting point is BFP in *SSB-SSB regime* ($T = 0$) for $N = 100$

(finite-order LPA'6, single Z_i per sector)



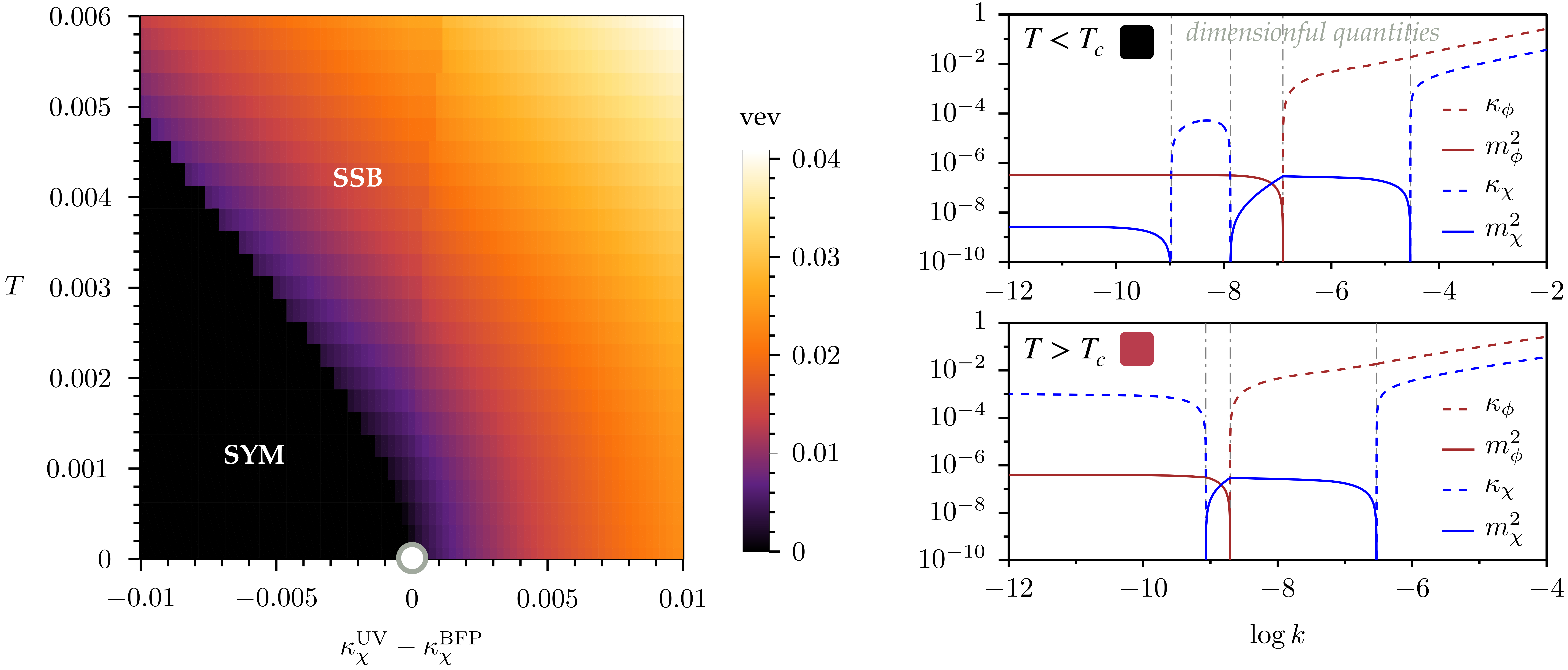


- At T=0:

QCP separates $O(N-1) \times Z_2$ from $O(N-1)$ phase

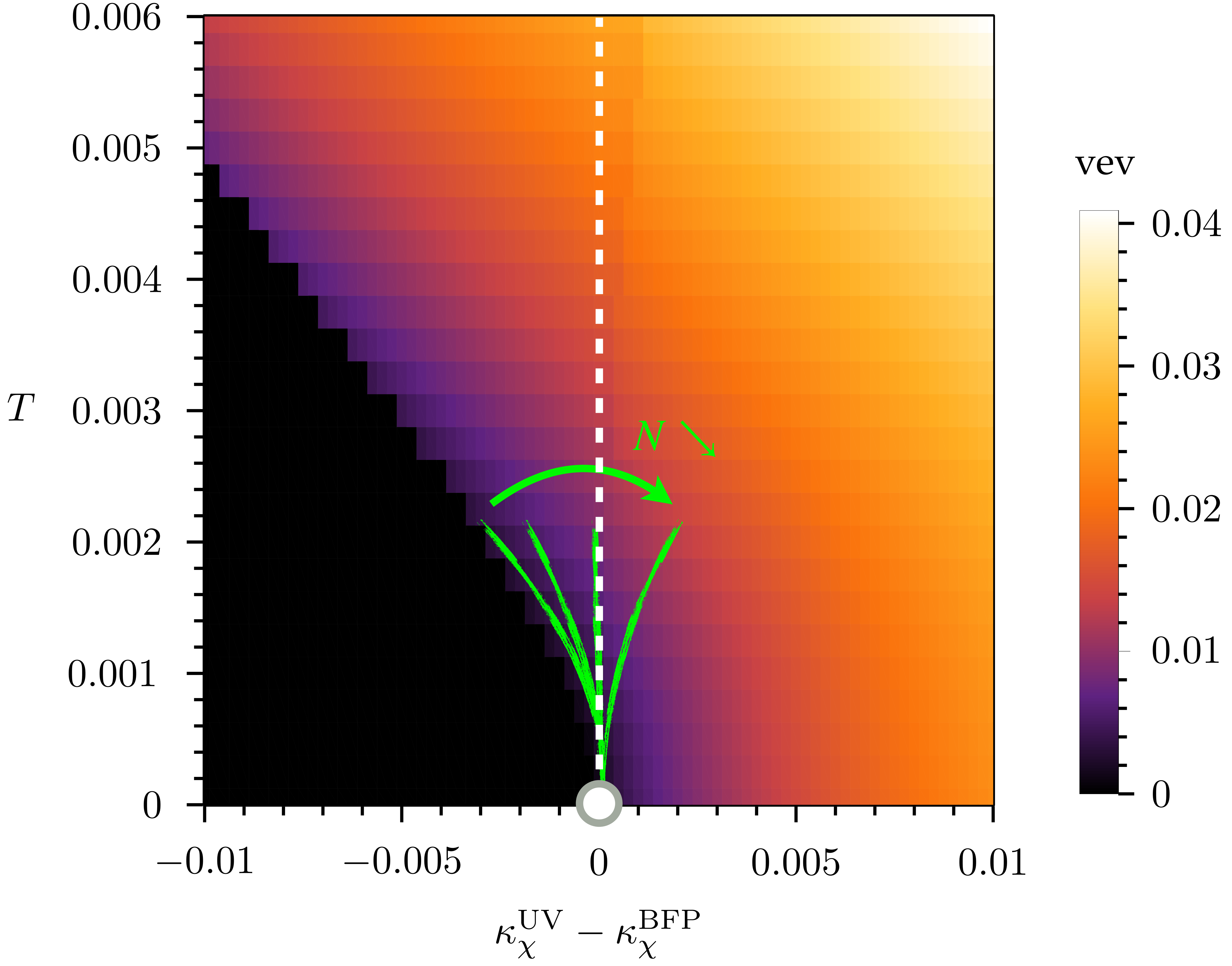
RG flows & inverted phase diagram at finite T

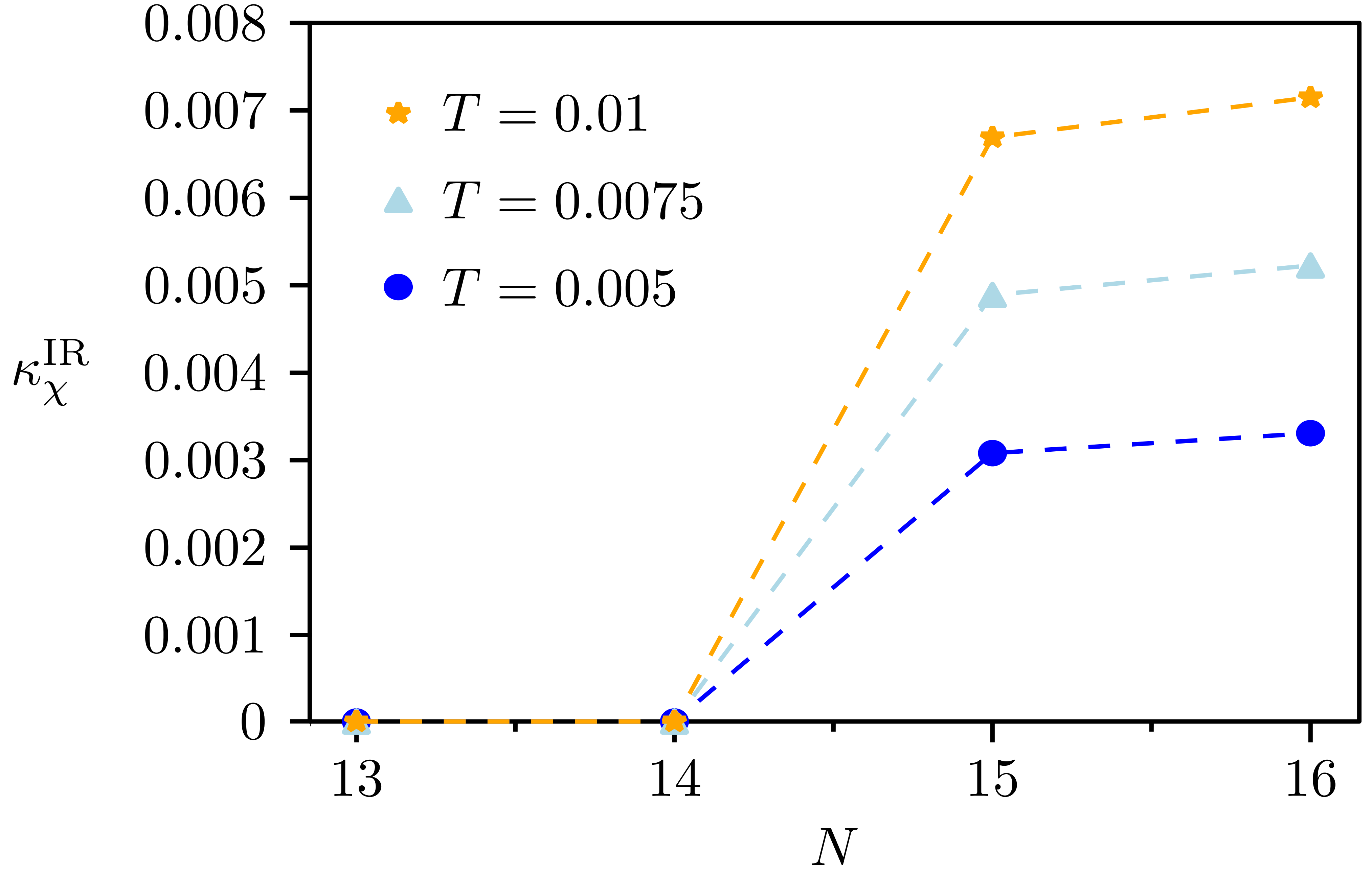
- Microscopic starting point is BFP in *SSB-SSB regime* ($T = 0$) for $N = 100$



RG flows & phase diagram at finite T

- Evolution of phase diagram with N





- Inverted phase diagram only for $N > N_c$
- For $N < N_c$ phase diagram is back to normal
- We find $N_c \sim 15$

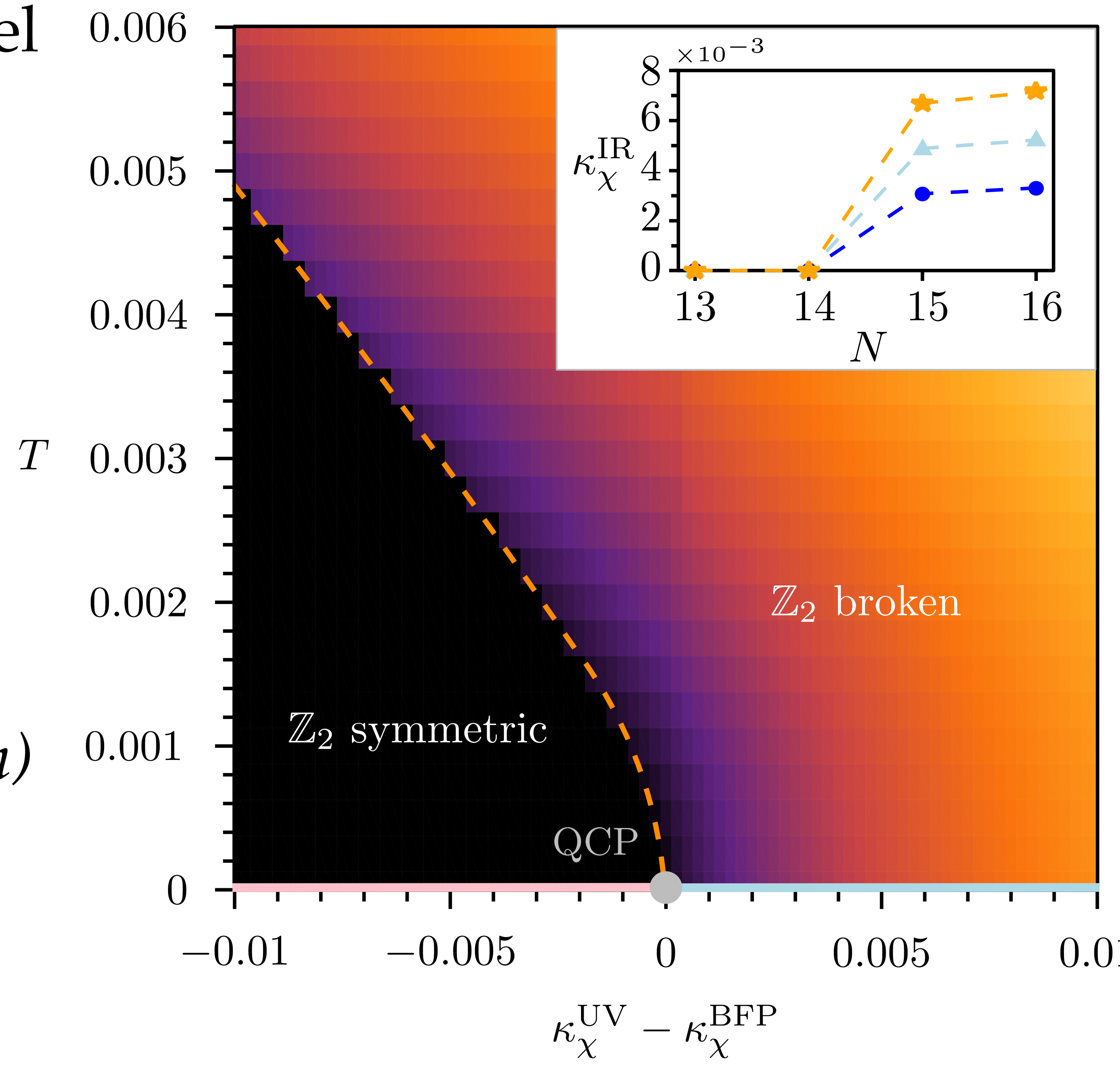
Conclusions

RUB

- **Inverted SB** occurs in various physical systems #pomeranchuk-effect #rochelle-salt
- Symmetry typically restored at higher T
- In **UV complete** QFTs **inverted SB** can potentially occur at all T due to **scale invariance**
- **Conjecture**: $O(N) \times Z_2$ symmetric field theory in D=2+1 at **BFP** *(evidence from D=4-ϵ or non-local models)*

- **Here**: finite-T phase diagram directly in D=2+1 for **UV complete, unitary, and local** model
  - Phase diagram is inverted for $N > N_c \sim 15 \rightarrow$ **SB at all temperatures**

    Hawashin, Rong, Scherer, arXiv:2409.10606 (2024)



- **Outlook**:
  - Solve for *full potential $U(\rho_\phi, \rho_\chi)$, velocity renormalization,... (quantitative 2D Ising transition)*
  - Circumvent no-go theorem: *UV complete single-field model* with $\lambda < 0$ but $\lambda_6 > 0$?