Order of the SU(N_f)$\times$SU(N_f) chiral transition

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GF and T. Hatsuda, Phys. Rev. D**110**, 016021 (2024)
GF, Phys. Rev. D**105**, L071506 (2022)

Introduction

- Chiral phase transition at the physical point: crossover
- Quark mass dependence? Chiral limit? 1st order or 2nd order?
- QCD Lagrangian without quark masses:

$$\mathcal{L} \;=\; -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \bar{q}_i\big(i\gamma^\mu(D_\mu)_{ij}\big)q_j$$

$\longrightarrow$ $SU(3)$ gauge symmetry
$\longrightarrow$ exact $U_L(N_f) \times U_R(N_f)$ chiral symmetry
$\longrightarrow$ anomalous breaking of $U_A(1)$ axial symmetry

- Low temperature: spontaneous breaking
  $SU_L(N_f) \times SU_R(N_f) \longrightarrow SU_V(N_f)$

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- Low temperature: spontaneous breaking
  $SU_L(N_f) \times SU_R(N_f) \longrightarrow SU_V(N_f)$
- Ginzburg-Landau paradigm for second order (or weakly first order) transitions:

  i.) there exists a local order parameter Φ near the transition
  ii.) the (UV) free energy can be expanded in terms of Φ
  iii.) structure of the free energy $\longleftrightarrow$ symmetries

Ginzburg–Landau analysis of the chiral transition

- GL theory for the chiral transition:
  $\longrightarrow$ Hubbard-Stratonovich transformation: $(\Phi)_{ij} \leftrightarrow \bar{q}_L^i q_R^j$
  $\longrightarrow$ integrate out quarks and gluons
  $\longrightarrow$ perform dimensional reduction at finite T
- Chiral transformation: $\Phi \to L\Phi R^\dagger$
- The most general free energy functional (<u>no anomaly</u>):

$$\Gamma = \int d^3x \Big[m^2 \,\mathrm{Tr}\,(\Phi^\dagger\Phi) + g_1 \big(\mathrm{Tr}\,(\Phi^\dagger\Phi)\big)^2 + g_2\,\mathrm{Tr}\,(\Phi^\dagger\Phi\Phi^\dagger\Phi) + \ldots + \mathrm{Tr}\,(\partial_i\Phi^\dagger\partial_i\Phi) + \ldots \Big]$$

  $\longrightarrow$ $U_A(1)$ anomaly included via: $a(\det\Phi^\dagger + \det\Phi)$
- 2nd order transitions $\longleftrightarrow$ scale invariance (RG fixed point)
- Can the system show scaling behavior?
  $\longrightarrow$ Is there an RG fixed point with one relevant direction?

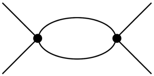
Ginzburg–Landau analysis of the chiral transition

- Pisarski & Wilczek analysis of the Ginzburg–Landau theory[1]:

  $\longrightarrow$ one-loop calculation of the β functions (no anomaly)

  $\longrightarrow$ counterterms for g_1, g_2:

$$\delta g_1, \delta g_2 \sim$$



- Results (ϵ-expansion, $\epsilon = 4 - d$):

$$\begin{aligned}
\beta_{g_1} &= -\epsilon g_1 + \frac{N_f^2 + 4}{4\pi^2} g_1^2 + \frac{N_f}{\pi^2} g_1 g_2 + \frac{3g_2^2}{4\pi^2} \\
\beta_{g_2} &= -\epsilon g_2 + \frac{3}{2\pi^2} g_1 g_2 + \frac{N_f}{2\pi^2} g_2^2
\end{aligned}$$

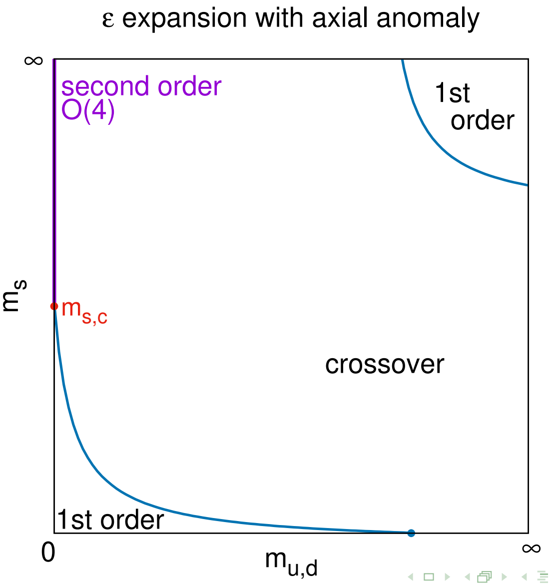
- No infrared stable fixed point at T_C if $N_f > \sqrt{3}$

  $\Longrightarrow$ 2nd order transition cannot occur!

- Inclusion of the anomaly: the transition might be 2nd order for $N_f = 2$ [$O(4)$ exponents]

[1] R. D. Pisarski and F. Wilczek, Phys. Rev. D**29**, 338 (1984)

Columbia plot:



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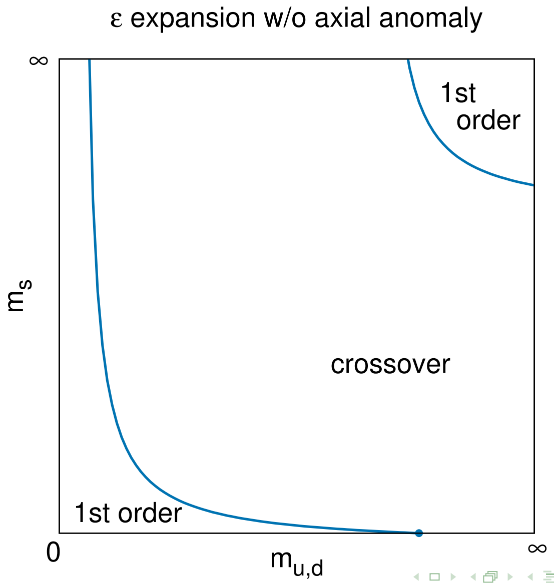
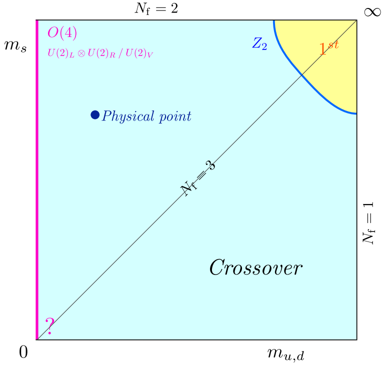


- F. Cuteri, O. Philipsen, and A. Sciarra, JHEP **11**, 141 (2021)
  $\longrightarrow$ chiral transition is second order for all N_f up to the conformal window



[2] L. Dini et al., Phys. Rev. D105, 034510 (2022)
[3] Y. Zhang et al., arXiv:2401.05066
[4] J. Bernhardt and C.-S. Fischer, Phys. Rev. D**108**,114018 (2023)
[5] S. R. Kousvos and A. Stergiou, SciPost Phys. **15**, 075 (2023)

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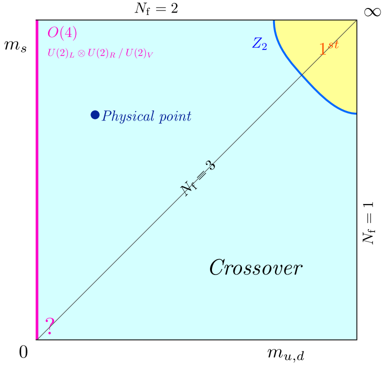

- Lattice QCD result with highly improved staggered fermions[2]
- Lattice QCD with Mobius domain wall fermions[3]
- Dyson-Schwinger approach[4]
- Conformal bootstrap approach[5]

Where is the corresponding IR fixed point?

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Ginzburg–Landau analysis of the chiral transition

- Potential problems with the Pisarski & Wilczek analysis:
  $\longrightarrow$ it uses the field theoretical RG
  (β functions from UV divergences $\Rightarrow$ massless)
  $\longrightarrow$ number of (perturbatively) relevant operators are restricted at $d \approx 4$
- $d = 4$: operators up to $\mathcal{O}(\phi^4)$ are not irrelevant
- $d = 3$: operators up to $\mathcal{O}(\phi^6)$ are not irrelevant

  $\longrightarrow$ $SU(N_f) \times SU(N_f)$ symmetry allows a richer structure of the free energy in $d = 3$
- Results of the ϵ expansion at LO are insensitive to the introduction of higher order terms
  $\longrightarrow$ an inherently $d = 3$ approach is important
  $\longrightarrow$ functional renormalization group (FRG)

- Local potential approximation (LPA):

$$\Gamma_k[\Phi] = \int_x \left(\frac{1}{2}\,\mathrm{Tr}\,[\partial_i \Phi^\dagger \partial_i \Phi] + V_k(\Phi)\right)$$

- How to build up the most general potential for N_f flavors?
  $\longrightarrow$ for $d = 3$ we need $\mathcal{O}(\phi^6)$!

- Independent chiral invariants for N_f flavors:

$$\begin{aligned} I_1 &= \mathrm{Tr}\,[\Phi^\dagger \Phi] \\ I_2 &= \mathrm{Tr}\,[\Phi^\dagger \Phi \Phi^\dagger \Phi] \\ I_3 &= \mathrm{Tr}\,[\Phi^\dagger \Phi \Phi^\dagger \Phi \Phi^\dagger \Phi] \\ &\ldots \\ I_{N_f} &= \mathrm{Tr}\,[(\Phi^\dagger \Phi)^{N_f}] \end{aligned}$$

$\longrightarrow$ only I_1, I_2 and I_3 enters the potential
(for $N_f = 2$, I_3 is not independent)

Chiral transition with the FRG

- The most general chirally symmetric renormalizable potential:

$$
\begin{aligned}
V_{ch}[\Phi] &= m^2\,\mathrm{Tr}\,[\Phi^\dagger\Phi] + g_1\big(\mathrm{Tr}\,[\Phi^\dagger\Phi]\big)^2 + g_2\,\mathrm{Tr}\,[\Phi^\dagger\Phi\Phi^\dagger\Phi] \\
&+ \lambda_1\big(\mathrm{Tr}\,[\Phi^\dagger\Phi]\big)^3 + \lambda_2\,\mathrm{Tr}\,[\Phi^\dagger\Phi]\cdot\,\mathrm{Tr}\,[\Phi^\dagger\Phi\Phi^\dagger\Phi] \\
&+ g_3\,\mathrm{Tr}\,[\Phi^\dagger\Phi\Phi^\dagger\Phi\Phi^\dagger\Phi]
\end{aligned}
$$

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- Possible $U_A(1)$ breaking terms:

$$I_{\mathrm{det}} = \det\Phi^\dagger + \det\Phi, \qquad \cancel{\tilde{I}_{\mathrm{det}} = \det\Phi^\dagger - \det\Phi}$$

$\longrightarrow$ $\tilde{I}^2_{\mathrm{det}}$ and $\det\Phi^\dagger\cdot\det\Phi$ are not independent

- If Φ is too large, I_{det} becomes perturbatively irrelevant!

$\longrightarrow$ $I_{\mathrm{det}} \sim \mathcal{O}(\phi^6)$

- For $N_f > 6$ the potential does not contain the anomaly

Chiral transition with the FRG

- $\mathbf{N_f = 5, 6}$:

$$V_A = a \cdot (\det \Phi^\dagger + \det \Phi)$$

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- $\mathbf{N_f = 4}$:

$$V_A = a \cdot (\det \Phi^\dagger + \det \Phi) + b \cdot \mathrm{Tr}\,[\Phi^\dagger \Phi](\det \Phi^\dagger + \det \Phi)$$

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- **$\mathbf{N_f = 3}$:**

$$\begin{aligned} V_A \;=\; & a \cdot (\det \Phi^\dagger + \det \Phi) + b \cdot \mathrm{Tr}\,[\Phi^\dagger \Phi](\det \Phi^\dagger + \det \Phi) \\ & + \; a_2 \cdot (\det \Phi^\dagger + \det \Phi)^2 \end{aligned}$$

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- $\underline{\mathbf{N_f = 2}}$:

$$\begin{aligned} V_A = & \, a \cdot (\det \Phi^\dagger + \det \Phi) + b_1 \cdot \operatorname{Tr}[\Phi^\dagger \Phi](\det \Phi^\dagger + \det \Phi) \\ & + a_2 \cdot (\det \Phi^\dagger + \det \Phi)^2 + a_3 \cdot (\det \Phi^\dagger + \det \Phi)^3 \\ & + b_2 \cdot \left(\operatorname{Tr}[\Phi^\dagger \Phi] \right)^2 (\det \Phi^\dagger + \det \Phi) \\ & + b_3 \cdot \left(\operatorname{Tr}[\Phi^\dagger \Phi] \right)^3 (\det \Phi^\dagger + \det \Phi) + b_4 \cdot \operatorname{Tr}(\Phi^\dagger \Phi)^2 (\det \Phi^\dagger + \det \Phi) \end{aligned}$$

Chiral transition with the FRG

- Optimized flow equation:

$$k\partial_k V_k = \frac{k^5}{6\pi^2}\,\mathrm{Tr}\,[k^2 + V_k^{(2)}]^{-1}$$

- Identification of the scale dependencies:

$$\sum_n k\partial_k g_n \cdot \mathcal{O}_n = \sum_n \frac{k^5}{6\pi^2}[\ldots] \cdot \mathcal{O}_n$$

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- Problem:

  $\longrightarrow$ $V_k^{(2)}$ depends on the fields, not invariants!

  $\longrightarrow$ $[k^2 + V_k^{(2)}]$: $2N_f^2 \times 2N_f^2$ matrix, in practice cannot be inverted for a general field configuration

- Specific background:

$$\Phi = s_0 \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ldots & \\ & & & 1 \end{pmatrix} + s_L \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & \ldots & \\ & & & -(N_f - 1) \end{pmatrix}$$

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- The $\mathcal{O}_n$ operators become linear combinations:

$$\mathcal{O}_n = \sum_{\alpha+\beta=n} c^{\alpha\beta} s_0^{\alpha} s_L^{\beta}$$

$\longrightarrow$ at each order matching *rhs* and *lhs* leads to coupling flows

- β functions: $(g_n = k^{(6-n)/2}\bar{g}_n)$

$$\beta_n \equiv k\partial_k \bar{g}_n = -\frac{1}{2}(6-n)\bar{g}_n + k\partial_k g_n / k^{(6-n)/2}$$

Chiral transition with the FRG

- β functions without anomaly:

$$\begin{aligned}
\beta_{m^2} &= -2\bar{m}_k^2 - 2\frac{\bar{g}_{1,k}N_f(N_f^2+1)+\bar{g}_{2,k}(N_f^2-1)}{3\pi^2 N_f(1+\bar{m}_k^2)^2},\\
\beta_{g_1} &= -\bar{g}_{1,k} + 4\frac{\bar{g}_{1,k}^2 N_f^2(N_f^2+4)+2\bar{g}_{1,k}\bar{g}_{2,k}N_f(N_f^2-1)+2\bar{g}_{2,k}^2(N_f^2-1)}{3\pi^2 N_f^2(1+\bar{m}_k^2)^3} - \frac{3\bar{\lambda}_{1,k}N_f(N_f^2+2)+2\bar{\lambda}_{2,k}(N_f^2-1)}{3\pi^2 N_f(1+\bar{m}_k^2)^2},\\
\beta_{g_2} &= -\bar{g}_{2,k} + 8\frac{3\bar{g}_{1,k}\bar{g}_{2,k}N_f+\bar{g}_{2,k}^2(N_f^2-3)}{3\pi^2 N_f(1+\bar{m}_k^2)^3} - \frac{3\bar{g}_{3,k}(N_f^2-4)+\bar{\lambda}_{2,k}N_f(N_f^2+4)}{3\pi^2 N_f(1+\bar{m}_k^2)^2},\\
\beta_{\lambda_1} &= 4\frac{\bar{g}_{1,k}N_f^2(3\bar{\lambda}_{1,k}N_f(N_f^2+7)+2\bar{\lambda}_{2,k}(N_f^2-1))+\bar{g}_{2,k}N_f(N_f^2-1)(3N_f\bar{\lambda}_{1,k}+4\bar{\lambda}_{2,k})}{3\pi^2 N_f^3(1+\bar{m}_k^2)^3}\\
&- 4\frac{2\bar{g}_{1,k}^3 N_f^3(N_f^2+13)+6\bar{g}_{1,k}^2\bar{g}_{2,k}N_f^2(N_f^2-1)+12\bar{g}_{1,k}\bar{g}_{2,k}^2 N_f(N_f^2-1)+8\bar{g}_{2,k}^3(N_f^2-1)}{3\pi^2 N_f^3(1+\bar{m}_k^2)^4},\\
\beta_{\lambda_2} &= 4\frac{\bar{g}_{1,k}N_f\left(\bar{\lambda}_{2,k}N_f(N_f^2+19)+3\bar{g}_{3,k}(N_f^2-4)\right)+\bar{g}_{2,k}\left(15\bar{g}_{3,k}(N_f^2-4)+N_f(18\bar{\lambda}_{1,k}N_f+\bar{\lambda}_{2,k}(5N_f^2-1))\right)}{3\pi^2 N_f^2(1+\bar{m}_k^2)^3}\\
&- 4\frac{72N_f^2\bar{g}_{1,k}^2\bar{g}_{2,k}+6\bar{g}_{1,k}\bar{g}_{2,k}^2 N_f(2N_f^2+3)+\bar{g}_{2,k}^3(24N_f^2-90)}{3\pi^2 N_f^2(1+\bar{m}_k^2)^4},\\
\beta_{g_3} &= 4\frac{5N_f\bar{g}_{1,k}\bar{g}_{3,k}+4N_f\bar{g}_{2,k}\bar{\lambda}_{2,k}+(2N_f^2-17)\bar{g}_{2,k}\bar{g}_{3,k}}{\pi^2 N_f(1+\bar{m}_k^2)^3} - 4\frac{54\bar{g}_{1,k}\bar{g}_{2,k}^2 N_f+\bar{g}_{2,k}^3(4N_f^2-54)}{3\pi^2 N_f(1+\bar{m}_k^2)^4}.
\end{aligned}$$

- Fixed points: $\beta_i = 0 \ \forall \ i$

$\longrightarrow$ solve for marginal couplings analytically

$\longrightarrow$ substitute to the relevant couplings

$\longrightarrow$ find fixed points numerically

$\longrightarrow$ check stability matrix $(\partial\beta_i/\partial g_j)$ at the fixed points

$\mathbf{N_f}$	FP	$\bar{m}^2$	$\bar{g}_1$	$\bar{g}_2$	RD#
50	$O(2N_f^2)$	-0.33342	0.0017538	0	2
″	B_2^{50}	0.040303	-0.0029448	0.12152	2
″	C_1^{50}	-0.37509	0.0019579	-0.011198	1
″	$\tilde{C}_1^{50}$	-0.33342	0.0017556	-0.000088291	1
20	$O(2N_f^2)$	-0.33385	0.010939	0	2
″	B_2^{20}	0.043192	-0.018915	0.31043	2
″	C_1^{20}	-0.38411	0.012287	-0.030728	1
″	$\tilde{C}_1^{20}$	-0.33393	0.011010	-0.0014253	1
10	$O(2N_f^2)$	-0.33492	0.043430	0	2
″	B_2^{10}	0.059163	-0.086421	0.68317	2
″	C_1^{10}	-0.43356	0.048876	-0.082581	1
″	$\tilde{C}_1^{10}$	-0.33641	0.044669	-0.012667	1
6	$O(2N_f^2)$	-0.33516	0.11855	0	2
″	B_2^{6}	0.40276	-1.23414	3.80527	2
″	C_1^{6}	1.09084	-6.45942	16.76628	1
″	$\tilde{C}_1^{6}$	-0.34848	0.12934	-0.069536	1

$\mathbf{N_f}$	FP	$\bar{m}^2$	$\bar{g}_1$	$\bar{g}_2$	$\bar{a}$	RD#
5	$O(2N_f^2)$	-0.33386	0.16871	0	0	2
"	$\tilde{C}_1^5$	-0.36068	0.19128	-0.12675	0	1
"	A_3^5	-0.17023	0.14387	-0.056313	-2.79735	3

$\mathbf{N_f}$	FP	$\bar{m}^2$	$\bar{g}_1$	$\bar{g}_2$	$\bar{a}$	RD#
4	$O(2N_f^2)$	-0.32940	0.25800	0	0	3 (2)
"	$\tilde{C}_2^4$	-0.38129	0.31042	-0.25480	0	2 (1)
"	A_2^4	-0.34949	0.63992	-1.73326	-3.82052	2
"	$\tilde{A}_2^4$	-0.40273	0.21168	0.17473	-0.73657	2

$\mathbf{N_f}$	FP	$\bar{m}^2$	$\bar{g}_1$	$\bar{g}_2$	$\bar{a}$	$\bar{b}$	RD#
3	$O(2N_f^2)$	-0.31496	0.43763	0	0	0	3 (2)
"	$\tilde{C}_2^3$	-0.38262	0.59725	-0.62042	0	0	2 (1)
"	A_4^3	-0.01786	0.091631	-0.14148	-0.11900	0.39087	4
"	A_{1*}^3	-0.41126	0.73099	-0.88199	-0.46585	-0.91131	1*

Fixed points and stability

- Anomaly free fixed points for $N_f = 2$:

$\mathbf{N_f}$	FP	$\bar{m}^2$	$\bar{g}_1$	$\bar{g}_2$	RD#
2	$O(2N_f^2)$	-0.27094	0.85280	0	4 (3)
"	$\tilde{C}_2^2$	-0.20599	1.33367	-1.88211	2 (1)
"	$\hat{C}_2^2$	-0.26318	0.33093	1.71728	2 (1)

- Anomalous fixed points for $N_f = 2$?

$\longrightarrow$ numerically challenging

$\longrightarrow$ $a = -\infty$, $m^2 = \infty$ with $m^2 + a =$ finite

$\longrightarrow$ half of the modes decouple $\Rightarrow$ $O(4)$ FP

$\longrightarrow$ infrared stable at the critical temperature

Fixed points and stability

- Flavor continuity conjecture:
  The chiral transition is governed by the $\tilde{C}^{N_f}$ fixed points; other fixed points (if exist) do not have an influence.

- For $N_f \geq 5$, irrespectively of the $U_A(1)$ anomaly
  $\longrightarrow$ second order transition

- For $N_f = 2, 3, 4$ with disappearing $U_A(1)$ anomaly
  $\longrightarrow$ second order transition
  $\longrightarrow$ $\nu_{N_f=3} \approx 0.829$ [close $O(7)$ univ. class]
  $\longrightarrow$ $\nu_{N_f=2} \approx 0.638$ [close to Ising univ. class]

- For $N_f = 2, 3, 4$ with not disappearing $U_A(1)$ anomaly
  $\longrightarrow$ first order transition
  $\longrightarrow$ $U_A(1)$ breaking controls the strength of the transition
  $\longrightarrow$ weak anomaly $\Leftrightarrow$ weak first order transition

Fixed points and stability

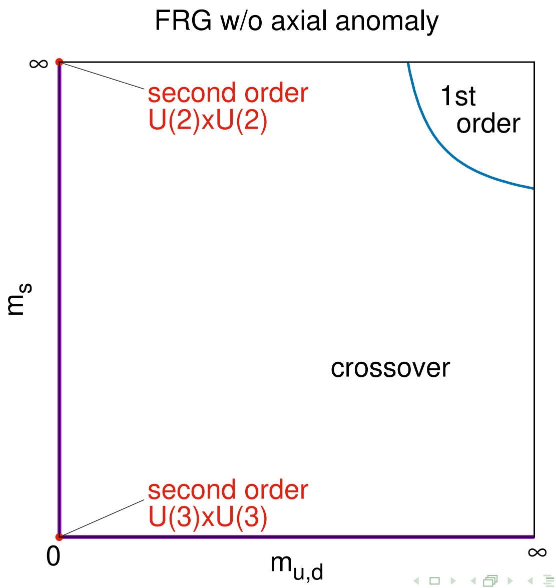
- For $N_f = 2$ and $N_f = 3$ the situation is more subtle
- $\underline{N_f = 3}$:

  $\longrightarrow$ there exist a fixed point with nonzero anomaly with one relevant direction

  $\longrightarrow$ however: the stability matrix has complex eigenvalues $\Rightarrow$ unnatural
- $\underline{N_f = 2\ }$:

  $\longrightarrow$ there also exist a fixed point with nonzero (infinite) anomaly with one relevant direction [$O(4)$ FP]
- Problem 1.) we do not know the domain of attraction of the fixed points
- Problem 2.) we do not know where the bare (UV) parameters lie in the parameter space

Columbia plot with flavor continuity



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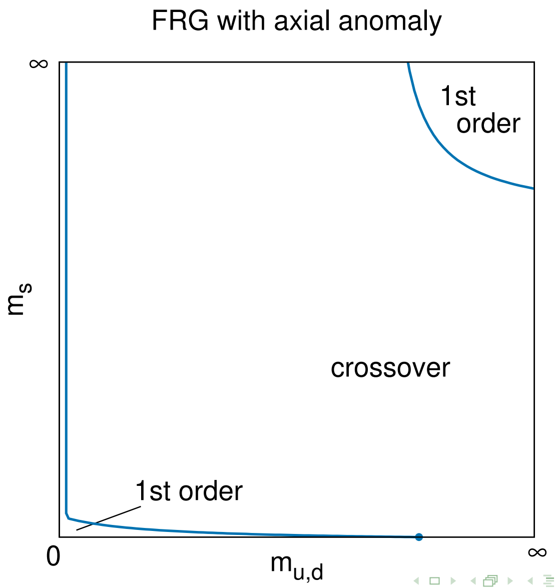

Transition orders without anomaly:

	$N_f = 2$	$N_f = 3$	$N_f = 4$	$N_f \geq 5$
ϵ expansion ($\epsilon = 1$)	1st order	1st order	1st order	1st order
FRG ($d = 3$)	2nd order	2nd order	2nd order	2nd order

Transition orders with anomaly:

	$N_f = 2$	$N_f = 3$	$N_f = 4$	$N_f \geq 5$
ϵ expansion ($\epsilon = 1$)	2nd order*	1st order	1st order	1st order
FRG ($d = 3$)	1st order (Case I) 2nd order (Case II) 2nd order (Case III)	1st order (Case I) 1st order (Case II) 2nd order (Case III)	1st order	2nd order

*:only with strong anomaly

Summary

- Re-analysis of the RG flows of the Ginzburg-Landau potential of chiral transition
  - $\longrightarrow$ scale evolution is obtained directly at $d = 3$ using the Functional Renormalization Group method
  - $\longrightarrow$ Local Potential Approximation + $\mathcal{O}(\phi^6)$ truncation: inclusion of all relevant and marginal interactions
- Results can be made consistent with recent lattice QCD simulations [i.e. chiral transition is second order]
  - $\longrightarrow$ there exist new classes of fixed points spanned in the entire N_f range
  - $\longrightarrow$ they are IR stable at T_C for $N_f \geq 5$
  - $\longrightarrow$ they are IR stable at T_C for $N_f = 2, 3, 4$ only if $U_A(1)$ is restored
- Future:
  - $\longrightarrow$ improve truncation (irrelevant operators, wavefunction renormalization, higher derivatives)
  - $\longrightarrow$ establishing fully non-perturbative fixed point potentials