# Generalized Hertz action for quantum criticality in Fermi systems

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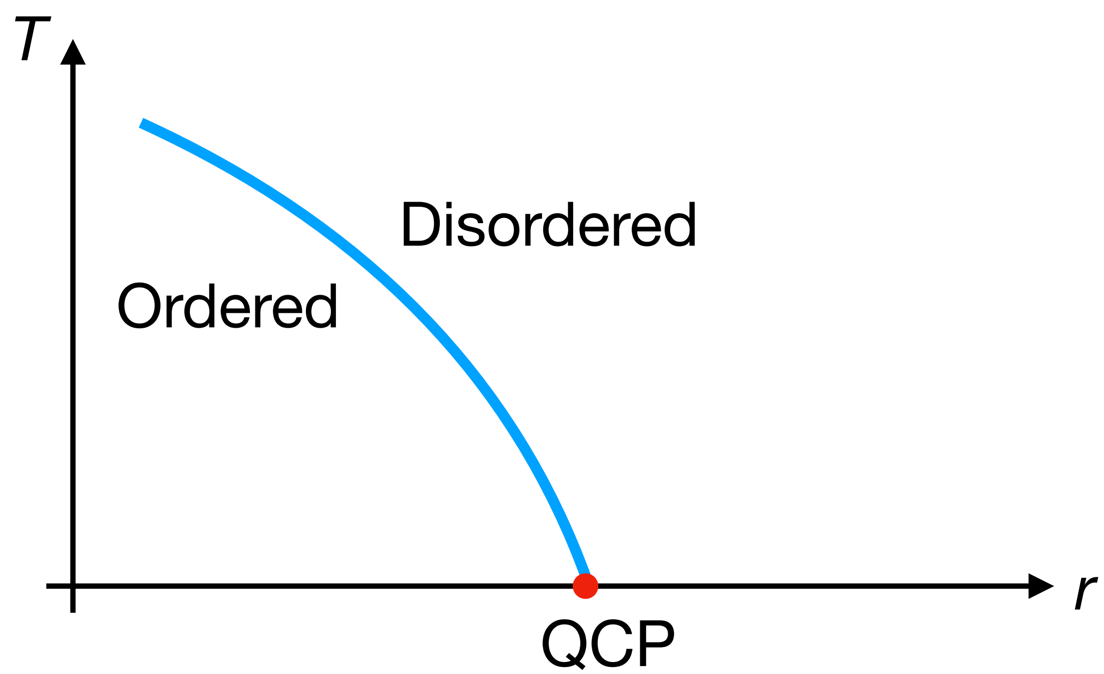
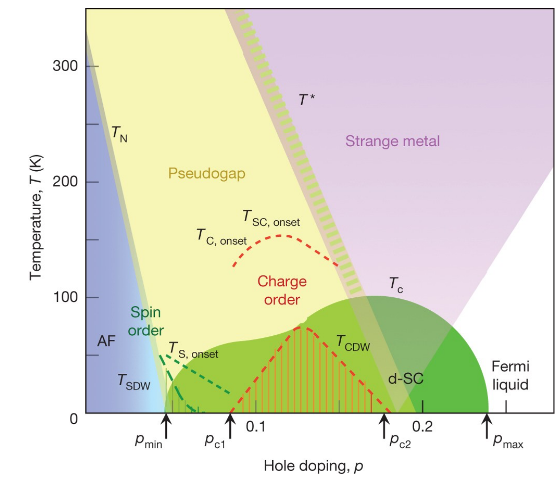
# Quantum criticality in clean electronic systems

General setup:




Keimer *et. al.*, Nature **518**, 179 (2015)

At $T > 0$ critical singularities controlled by the classical (Wilson-Fisher) F-P

What is the correct low-energy action to describe the QCP ?

# Hertz - Millis theory (1976, 1993)

$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi] e^{-\mathcal{S}_{mic}[\bar{\psi}, \psi]}$$

known dominant instability - order parameter $\phi$

Hubbard - Stratonovich transformation

$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi, \phi] e^{-\mathcal{S}_{fb}[\bar{\psi}, \psi, \phi]}$$

(decoupled fermionic interactions)

integrate fermions out

$$\mathcal{Z} = \int \mathcal{D}\phi e^{-\mathcal{S}_b[\phi]}$$

( The problematic point ! )

attempt an expansion of $\mathcal{S}_b[\phi]$ in powers of $\phi$

and expansion of vertices in $q$ $\quad q := (q_0, \vec{q})$

$$\mathcal{S}_b[\phi] \longrightarrow \mathcal{S}_H[\phi]$$

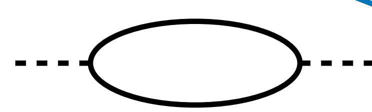
(Hertz action)

# The problem with H-M:

( Focus on *d=2* and *Q=0* instabilities at *T=0* )

$$\mathcal{S}_b[\phi] \longrightarrow \mathcal{S}_b^{(2)}[\phi] + \mathcal{S}_b^{(4)}[\phi] + \dots$$

$$\mathcal{S}_b^{(2)}[\phi] = \int_q \phi_{-q}\,[1 - \chi_0(q)]\,\phi_q$$



Discontinuous function of $q$ (at $q$=0)

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0}\left[m^2 + Z\vec{q}^2 + A\frac{|q_0|}{|\vec{q}|}\right]\phi_{\vec{q},q_0} + u\int_x \phi(x)^4 \qquad x := (\tau, \vec{x})$$

Landau damping

singular quartic vertex
replaced by a local interaction coupling

form valid for
$|q_0|/|\vec{q}| \ll 1$

Problem more severe for interaction vertices.

# The problem with H-M (ctd):

$$S_H[\phi] = \int_q \phi_{-\vec{q},-q_0}\left[m^2 + Z\vec{q}^2 + A\frac{|q_0|}{|\vec{q}|}\right]\phi_{\vec{q},q_0} + u\int_x \phi(x)^4$$

Landau damping

highly singular quartic vertex
replaced by a local interaction

form valid for $|q_0|/|\vec{q}| \ll 1$

Remarks: Problem arises because of integrating out the soft (fermionic) modes

Landau damping generated from fermions at the FS

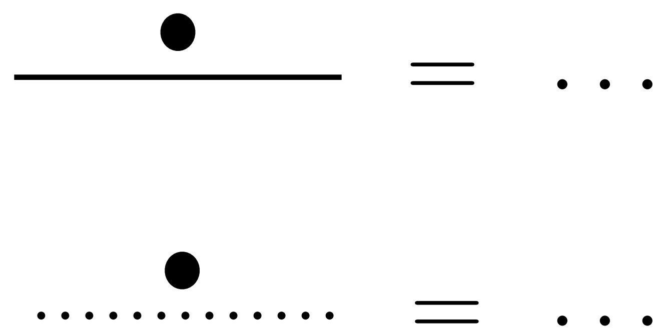
Necessity of keeping fermions well recognized in literature

Result of H-M: QCP governed by a classical-like F-P in effective dimensionality $D=d+z$, $z=3$.

*Still many interesting and important predictions!*

# Earlier work on coupled f-b flows

(Mostly F-T RG)



( *Lee, Mandal, Metlitski, Mross, Sachdev, Holder, Metzner, Drukier, Kopietz, Fitzpatrick, Raghu …* )

Typically use the H-M propagator to compute loop integrals

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**Wilsonian RG** $\Longrightarrow$ Conventional Landau damping *cannot* appear until all fermions become integrated out completely.

What replaced it at finite scales?

Present focus:

Set up RG where the Bose propagator involves only contributions from integrating out high energy fermions.

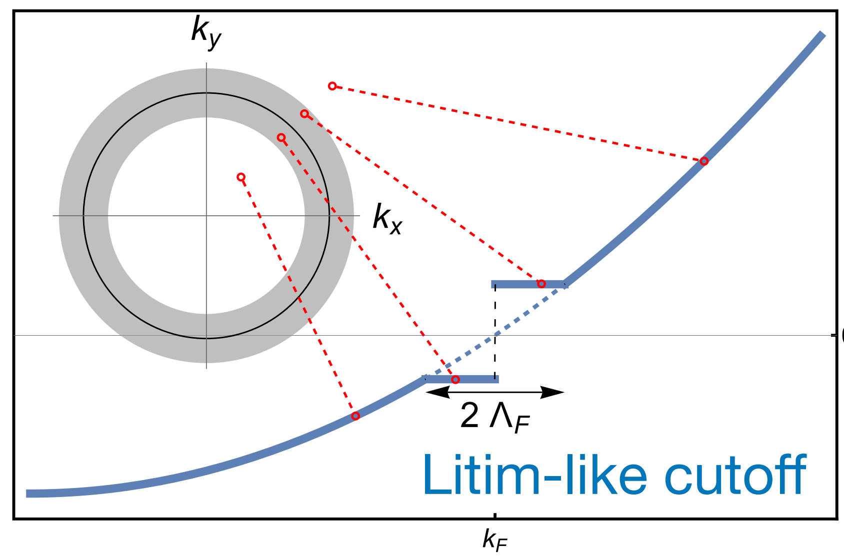
Integrate out  fermions and bosons ”in parallel”.

Use Wetterich framework.

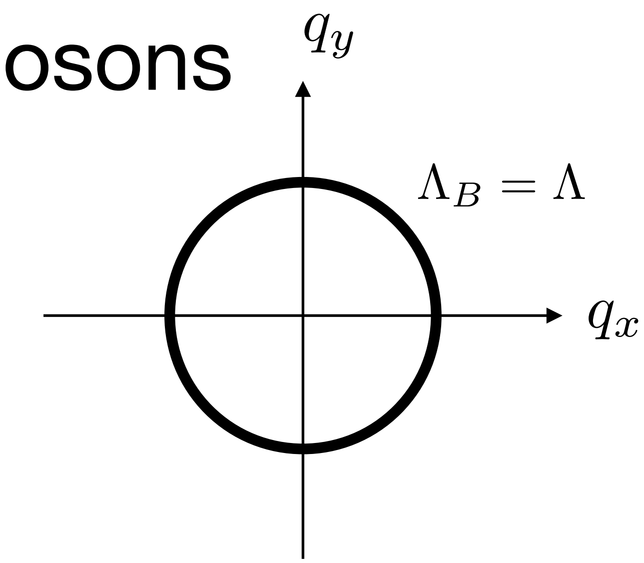
$$\mathcal{Z} = \int \mathcal{D}[\bar{\psi}, \psi, \phi] e^{-\mathcal{S}_{fb}[\bar{\psi}, \psi, \phi]}$$

Cutoff scale(s):

Fermions




Bosons




H-M: spirit: $\Lambda_F \to 0$ first, and only then $\Lambda \to 0$

$$\Lambda_F = (\Lambda - \Lambda_0)\theta(\Lambda - \Lambda_0) \qquad \Lambda_0 > 0$$

Alternative: $\Lambda_F = \Lambda$

(fermions and bosons integrated out "in parallel")

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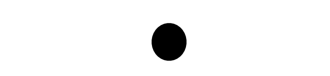
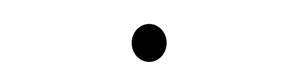
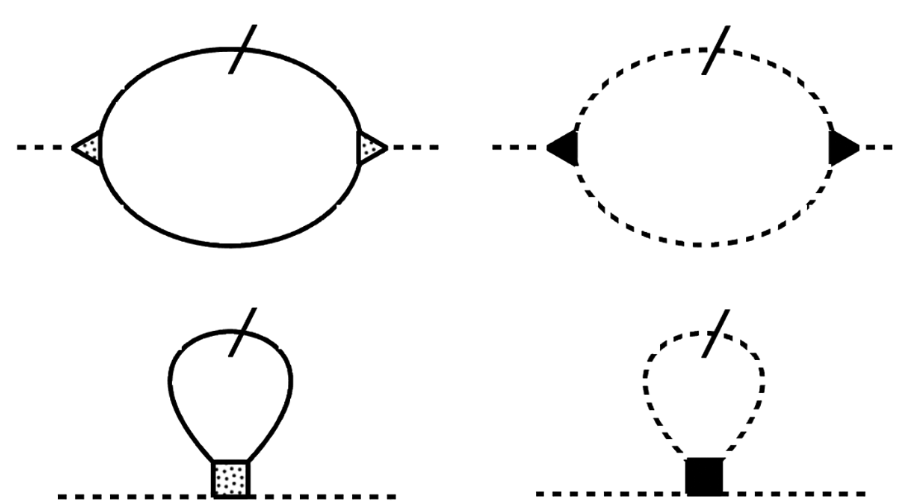
## Coupled flows of Fermi and Bose propagators

( From Wetterich eq. )

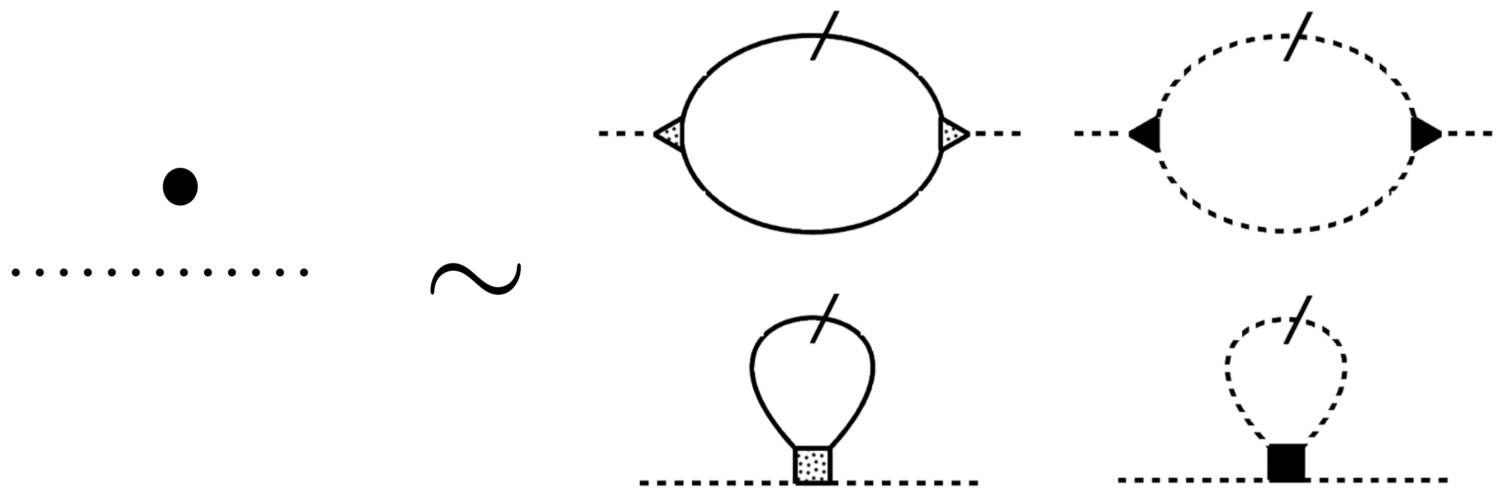
 $=$ $\ldots$

 $=$ $\ldots$



(Exact)

Boson propagator flow:

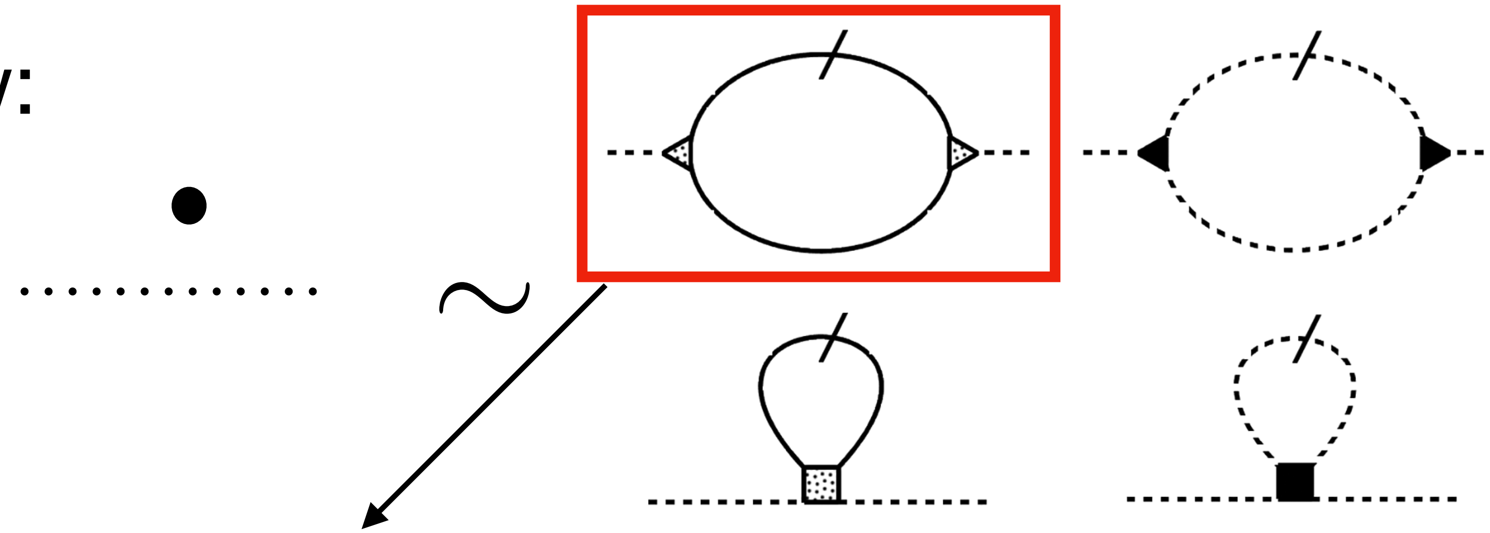


<u>Present truncation:</u>

Disregard the fermion self-energy and all the generated interaction vertices involving fermions; neglect Yukawa flow.

Fully encompasses H-M ( for $\Lambda_F \to 0$ taken first ), but also allows to take $\Lambda_F = \Lambda$.

Boson propagator flow:



$$\mathcal{X}(q,\Lambda_F) = -2g^2 \int_k \partial_\Lambda R_f(\vec{k}) \tilde{G}_0(k)^2 (\tilde{G}_0(k+q) + \tilde{G}_0(k-q))$$

$$\tilde{G}_0(k)^{-1} = [-ik_0 + \xi_{\vec{k}} + R_f(\vec{k})].$$

$$B(\vec{q}, q_0, \Lambda_F(\Lambda)) := \int_{\Lambda_u}^{\Lambda} d\Lambda' \mathcal{X}$$

→ boson propagator generated from integrating fermions down to the scale $\Lambda$

$$B(\vec{q}, q_0, \Lambda_F) = B_< \theta(-|\vec{q}| + \Lambda_F) + B_> \theta(|\vec{q}| - \Lambda_F)$$

$$B_< = -\mathcal{N}_< \frac{|\vec{q}|\Lambda_F}{q_0^2 + 4v_F^2 \Lambda_F^2}$$

New term

( Negative sign )

$$B_> \approx -\mathcal{N}_< \frac{\vec{q}^2}{q_0^2 + 4v_F^2 \vec{q}^2} + \mathcal{N}_> \frac{q_0}{|\vec{q}|} \left[ \arctan \frac{2v_F|\vec{q}|}{q_0} - \arctan \frac{2v_F \Lambda_F}{q_0} \right]$$

Recovers Landau damping

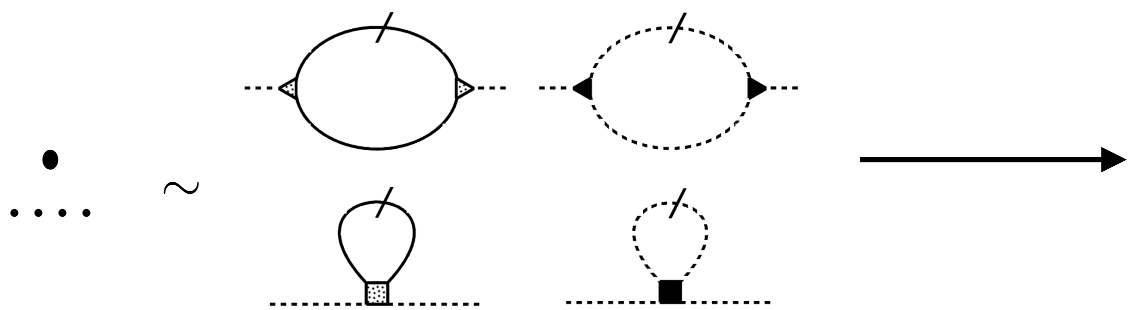
+ analytical terms

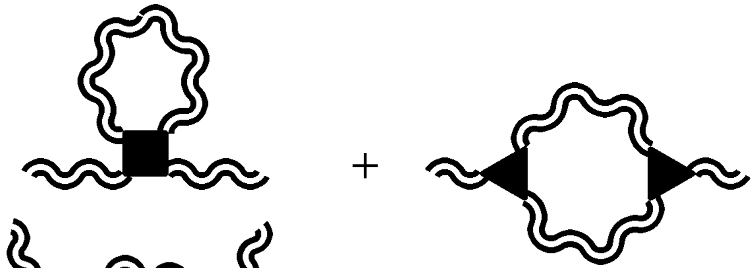
$B(\vec{q}, q_0, \Lambda_F)$ has minimum at $(q_0, |\vec{q}|) = (0, \Lambda_F)$

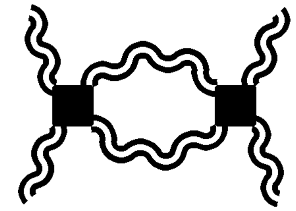
Ordering wavevector flows.

# Standard $\phi^4$ truncation for bosons

(with structureless vertex!)



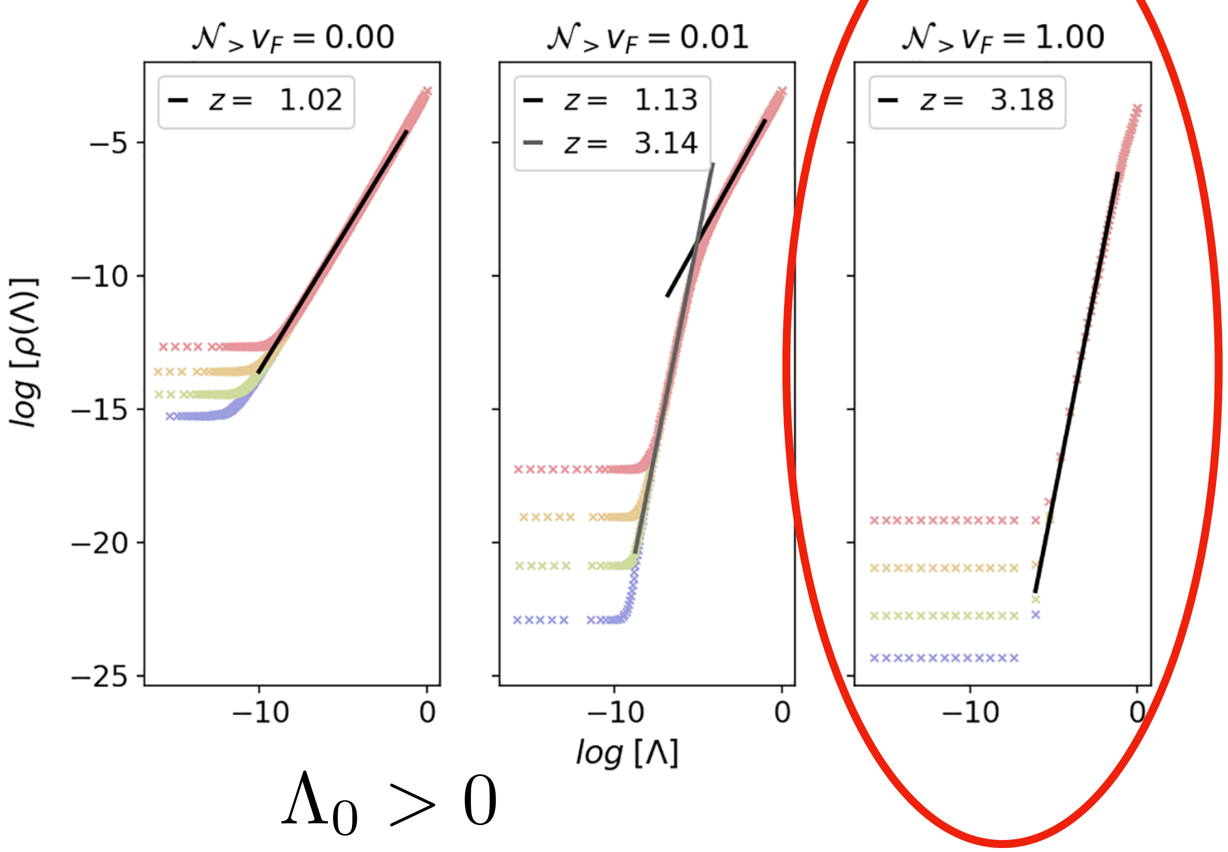
$\partial_\Lambda m^2 \sim$ 

$\partial_\Lambda u \sim$ 

Conventional Landau damping term replaced by $B(\vec{q}, q_0, \Lambda_F)$

## Most interesting result - o.p. flow at QCP $\phi_0^\Lambda \sim \Lambda^{z/2}$

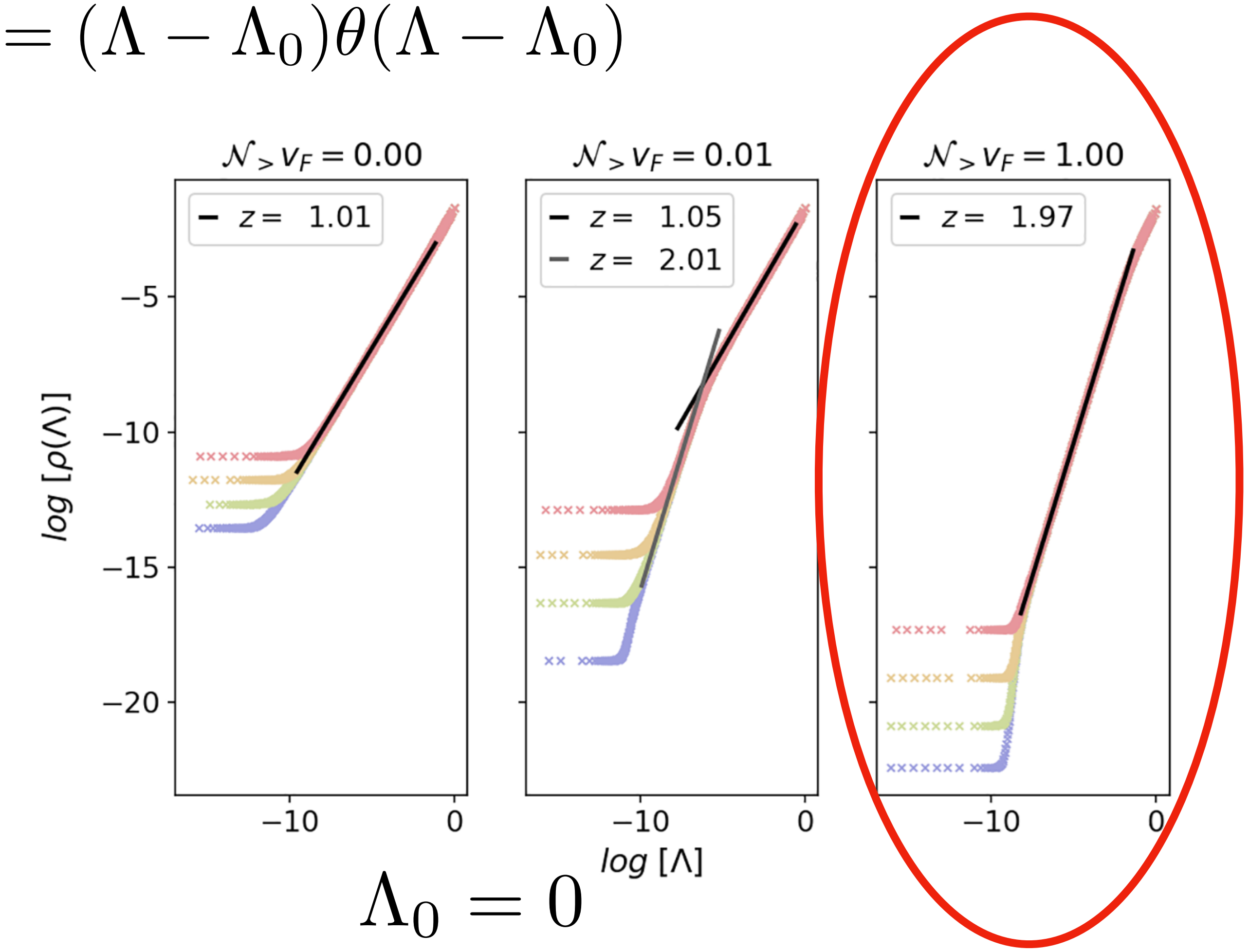
$$\Lambda_F = (\Lambda - \Lambda_0)\theta(\Lambda - \Lambda_0)$$




$\Lambda_0 > 0$

(back to the H-M paradigm)

$z = 3$ recovered




$\Lambda_0 = 0$

$z \approx 2$ (due to $B_<$)

Bosonic NPRG ctd:

$z = 2$ at odds with ”broadly expected” value $z = 3$

( Here recovered by a questionable procedure in the H-M spirit )

$\Lambda_0 > 0$

$z = 2$ seen in QMC simulations of fermionic QCPs with $\vec{Q} = 0$

Shattner *et al* PRX **6**, 031028 (2016)
Liu *et al* PRB **105**, L041111 (2022)

( Here obtained by the procedure of
integrating out fermions and bosons in parallel )

$\Lambda_0 = 0$
i.e. $\Lambda_F = \Lambda$

Further work, in progress:

- ***fermion self-energy*** → see poster of Mateusz

- boson vertex

- Yukawa flow

- finite $T$