Functional renormalisation of UV-safe gauge theories coupled to matter

Daniele Rizzo

In collaboration with

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&
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During my visit at

Sussex University
Brighton, UK

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Gauge-Yukawa Theory

Litim, Sannino (2014)

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Gauge-Yukawa Theory

Gauge $F^a_{\mu\nu}\,(a = 1, ..., N_C^2 - 1)$

Fermions $Q_i\,(i = 1, ..., N_F)$

Scalars $H \in N_F \times N_F$

$$\epsilon \equiv \frac{N_F}{N_C} - \frac{11}{2}$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\,F_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}\left(\partial_\mu H^\dagger \partial^\mu H\right) + \mathrm{Tr}\left(\bar{Q}\, i\not{D}\, Q\right)$$
$$-y\,\mathrm{Tr}\left(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L\right) - u\,\mathrm{Tr}\left(H^\dagger H\right)^2 - v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

Litim, Sannino (2014)

Results from Perturbation Theory

Under perturbative expansion, the theory has an ultraviolet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$
$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

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Bond, Litim, Vazquez, Steudtner (2017)

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Litim, Sannino (2014)

Litim, Riyaz, Stamou, Steudtner (2023)

Bond, Litim, Vazquez, Steudtner (2017)

Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2$$

$$y\,\mathrm{Tr}(\bar{Q}HQ)$$

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \xrightarrow{\qquad\qquad} \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2$$

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$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \sum_n \gamma_n (\mathrm{Tr} H^\dagger H)^{n-2} (\mathrm{Tr} H^\dagger H)^2$$

$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \sum_m \alpha_m (\mathrm{Tr} H^\dagger H)^{m-2}\,\mathrm{Tr}(H^\dagger H)^2$$

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Beyond marginal operators

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Beyond marginal operators

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$$u\,\mathrm{Tr}\left(H^\dagger H\right)^2 \longrightarrow \underset{C(\mathrm{Tr}H^\dagger H)}{\boxed{\sum_m \alpha_m (\mathrm{Tr}H^\dagger H)^{m-2}}}\,\mathrm{Tr}(H^\dagger H)^2$$

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Beyond marginal operators

$$v\left(\mathrm{Tr}\,H^\dagger H\right)^2 \longrightarrow \underbrace{\boxed{\sum_n \gamma_n (\mathrm{Tr}H^\dagger H)^{n-2}(\mathrm{Tr}H^\dagger H)^2}}_{U(\mathrm{Tr}H^\dagger H)}$$

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$$y\,\mathrm{Tr}(\bar{Q}HQ) \longrightarrow \underbrace{\boxed{\sum_l Y_l\,(\mathrm{Tr}H^\dagger H)^l}}_{Y(\mathrm{Tr}H^\dagger H)}\,\mathrm{Tr}(\bar{Q}HQ)$$

Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$

$\partial_t C(\mathrm{Tr} H^\dagger H)$

$\partial_t Y(\mathrm{Tr} H^\dagger H)$

Functional Renormalization Group

$\partial_t U(\mathrm{Tr} H^\dagger H)$ $\partial_t C(\mathrm{Tr} H^\dagger H)$ $\partial_t Y(\mathrm{Tr} H^\dagger H)$

Wetterich equation

$$\partial_t \Gamma_k = \frac{1}{2}\,\mathrm{STr}\left[\partial_t R_k \cdot \left(\Gamma_k^{(2)} + R_k\right)^{-1}\right]$$

LPA approximation

Regulator

$$R_k = Z_k\,(k^2 - q^2)\,\Theta(k^2 - q^2)$$

Flow equations similar
to Fejos (2014)
and Fejos, Patkós (2020)

Flow

We define dimension-less couplings:

$$U = k^4 \, u \qquad\qquad \mathrm{Tr}\, H^\dagger H = \rho \, k^2$$

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Flow

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And compute the flow:

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

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Canonical dimension

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Anomalous dimension

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Quantum corrections from the scalar potential

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Canonical dimension

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Quantum corrections from the scalar potential

Quantum corrections from the yukawa

Flow

$$\partial_t u = -4u + (2+\eta_H)\rho u' + \frac{1}{2}\left(\frac{1}{1+u'+4\rho\, c} + \frac{1}{1+u'}\right) - \frac{2N_C}{N_F}\frac{1}{1+\rho\, y^2}$$

Tuğba Büyükbeşe, PhD Thesis

$$\begin{aligned}\partial_t c = {}& 2\eta_H c + (2+\eta_H)\rho c' - \frac{2N_C}{N_F}\frac{y^4}{\left(1+\rho y^2\right)^3} \\ &+\frac{1}{2}\left(-\frac{128\rho^3 c^5}{\left(1+u'\right)^3\left(1+4\,\rho\, c+u'\right)^3} + \frac{64\rho^2 c^3\left(c-\rho c'\right)}{\left(1+u'\right)^2\left(1+4\,\rho\, c+u'\right)^3} - \frac{8\rho c c'}{\left(1+4\,\rho\, c+u'\right)^3}\right. \\ &\left.\qquad - \frac{48\rho^2 c^2 c'}{\left(1+u'\right)\left(1+4\,\rho\, c+u'\right)^3} + \frac{16c^2}{\left(1+4\,\rho\, c+u'\right)^3} - \frac{2c'}{\left(1+4\,\rho\, c+u'\right)^2}\right)\end{aligned}$$

Tuğba Büyükbeşe, PhD Thesis

$$\begin{aligned}\partial_t y = {}& -3\alpha_g y(0) + \frac{1}{2}(2\eta_\psi + \eta_H)y + (2+\eta_\phi)\rho y' - \frac{1}{2}\left(\frac{y'}{\left(1+4\,\rho\, c+u'\right)^2} + \frac{y'}{\left(1+u'\right)^2}\right) \\ &+\frac{y^3}{2\left(1+\rho y^2\right)\left(1+4\rho\, c+u'\right)}\left(\frac{1}{1+4\rho\, c+u'} + \frac{1}{1+\rho y^2}\right) - \frac{y^3}{2\left(1+u'\right)\left(1+\rho y^2\right)}\left(\frac{1}{1+\rho y^2} + \frac{1}{1+u'}\right)\end{aligned}$$

Fixed Point

$$\partial_t u = 0 \qquad \partial_t c = 0 \qquad \partial_t y = 0$$

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Fixed Point

$$\partial_t u = 0 \quad\Downarrow\quad u(\rho) = \sum_{n=0}^{N\to\infty} \alpha_n \rho^{n+1}$$

$$\partial_t c = 0 \quad\Downarrow\quad c(\rho) = \sum_{n=1}^{N\to\infty} \gamma_n \rho^{n-1}$$

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Fixed Point

$$\begin{tikzcd}
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Fixed Point & Power Counting in ε

Dim 4:

Fixed Point & Power Counting in ε

Dim 4: $\alpha_h^* \equiv \gamma_1^*$ $\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$ $\alpha_y \equiv y_0^{2,*}$ $\approx \epsilon$

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Dim 6:

Fixed Point & Power Counting in ε

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Dim 6:	γ_2^*	α_2^*	y_1^*	**?**

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$$\partial_t \gamma_2 = +2\gamma_2 + \gamma_2 \left(4\alpha_1 + 20\gamma_1 + 6y_0^2\right) + \left(\frac{363}{2} y_0^6 - 48\alpha_1\gamma_1^2 - 96\gamma_1^3\right) + \mathcal{O}(\epsilon^4)$$

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Canonical dimension

Anomalous dimension $\approx \epsilon$

ϵ^3

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Dim 2n:

Fixed Point & Power Counting in ε

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Dim 6: γ_2^* $\quad$ α_2^* $\quad$ y_1^* $\quad$ $\approx \epsilon^3$

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Canonical dimension

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Dim 2n: $\partial_t \lambda_{2n} = (2n-4)\lambda_{2n} + \lambda_{2n}\left(A\,\epsilon + \mathcal{O}(\epsilon^2)\right) + B\epsilon^n$

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Fixed Point & Power Counting in ε

Dim 4:	$\alpha_h^* \equiv \gamma_1^*$	$\alpha_h^* + \alpha_v^* \equiv \alpha_1^*$	$\alpha_y \equiv y_0^{2,*}$	$\approx \epsilon$
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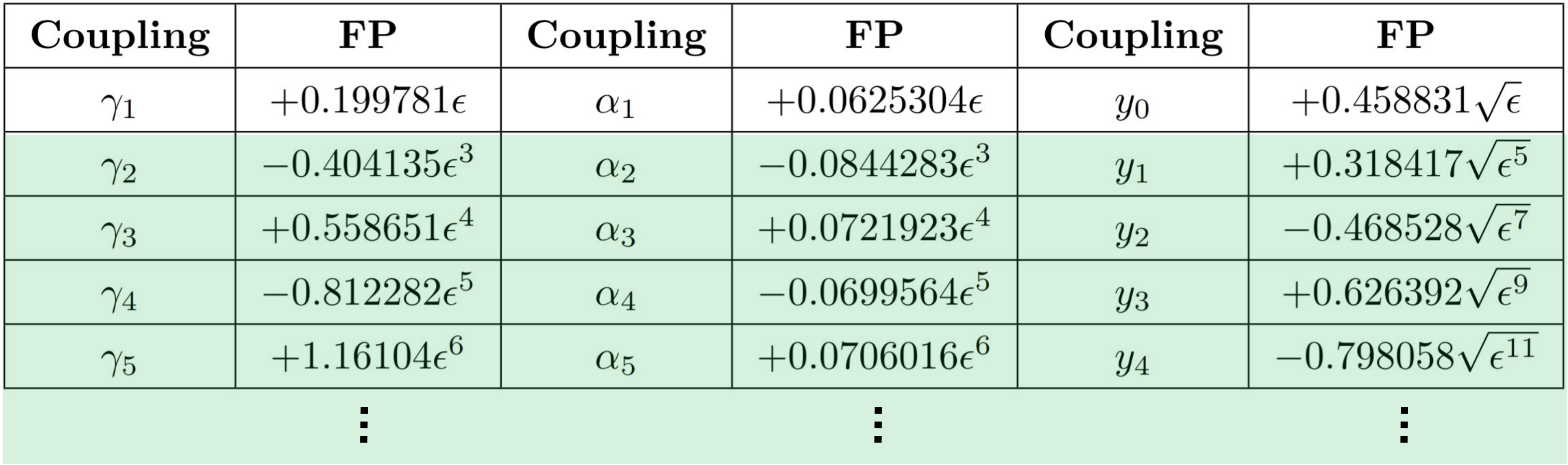
Fixed Point & Power Counting in ε



Coupling	FP	Coupling	FP	Coupling	FP
γ_1	$+0.199781\epsilon$	α_1	$+0.0625304\epsilon$	y_0	$+0.458831\sqrt{\epsilon}$
γ_2	$-0.404135\epsilon^3$	α_2	$-0.0844283\epsilon^3$	y_1	$+0.318417\sqrt{\epsilon^5}$
γ_3	$+0.558651\epsilon^4$	α_3	$+0.0721923\epsilon^4$	y_2	$-0.468528\sqrt{\epsilon^7}$
γ_4	$-0.812282\epsilon^5$	α_4	$-0.0699564\epsilon^5$	y_3	$+0.626392\sqrt{\epsilon^9}$
γ_5	$+1.16104\epsilon^6$	α_5	$+0.0706016\epsilon^6$	y_4	$-0.798058\sqrt{\epsilon^{11}}$
	$\vdots$		$\vdots$		$\vdots$

Fixed Point

$$\begin{tikzcd}
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\end{tikzcd}$$

Resummation

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Resummation

At leading order in ε a re-summation of the couplings can be performed:

$$u^*(\rho) = \alpha_1^* \rho^2 + \frac{A^2\,\rho^2}{4}\log(1 + A\,\rho) + \frac{B^2\,\rho^2}{4}\log(1 + B\,\rho) - \frac{N_c}{N_F} D^2\,\rho^2\log(1 + D\,\rho)$$

$$A \equiv 2\alpha_1^* \qquad B \equiv 2\alpha_1^* + 4\gamma_1^*$$

$$D \equiv \frac{N_F}{N_C}\alpha_y^*$$

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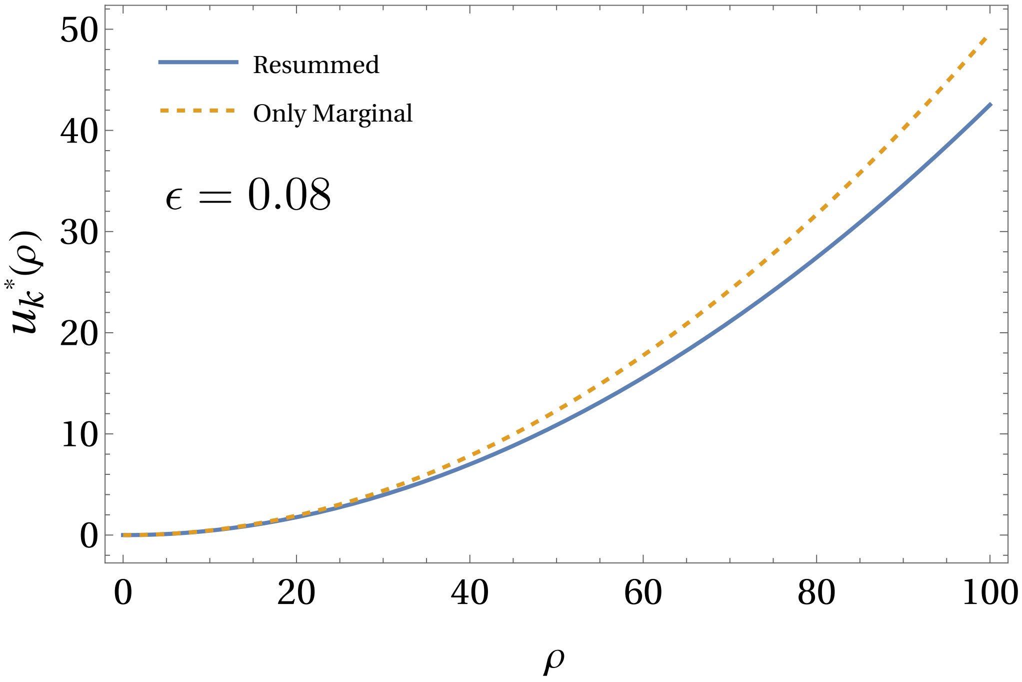

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$$\begin{aligned} c^*(\rho) = \gamma_1^* &- \frac{1}{32}(3A^2 + 8AB + B^2)\log(1 + A\rho) + \frac{1}{32}(11A^2 - 8AB + 9B^2)\log(1 + B\rho) \\ &+ \frac{(A-B)(6A^2 + 48AB - 30B^2)\rho}{64(1 + A\rho)^2(1 + B\rho)^2} + \frac{(A-B)(7A^3 + 103A^2B - 19AB^2 - 19B^3)\rho^2}{64(1 + A\rho)^2(1 + B\rho)^2} \\ &+ \frac{(A-B)(58A^3B + 48A^2B^2 - 34AB^3)\rho^3}{64(1 + A\rho)^2(1 + B\rho)^2} + \frac{(A-B)(40A^3B^2 - 16A^2B^3)\rho^4}{64(1 + A\rho)^2(1 + B\rho)^2} \end{aligned}$$

$$\begin{aligned} y^*(\rho) = y_0^* \Bigg(1 &- \frac{AD}{2(A-D)}\log(1 + A\rho) + \frac{BD}{2(B-D)}\log(1 + B\rho) + \frac{(A-B)D^2}{2(A-D)(D-B)}\log(1 + D\rho) \\ &- \frac{(A-B)D\rho}{4(1 + A\rho)(1 + B\rho)(1 + D\rho)}\Bigg) \end{aligned}$$

$$A \equiv 2\alpha_1^* \qquad B \equiv 2\alpha_1^* + 4\gamma_1^*$$

$$D \equiv \frac{N_F}{N_C}\alpha_y^*$$

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

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Eigenvalues of M → Critical Exponents θ_i

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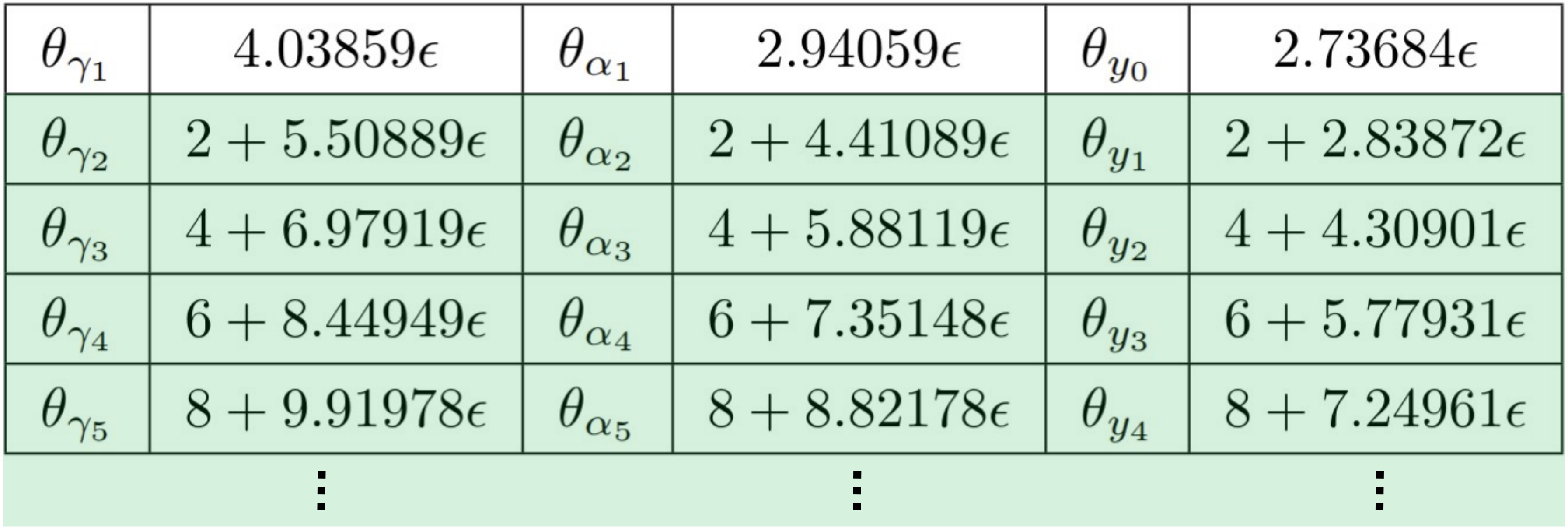
Eigenvalues of M → Critical Exponents θ_i



θ_{γ_1}	4.03859ϵ	θ_{α_1}	2.94059ϵ	θ_{y_0}	2.73684ϵ
θ_{γ_2}	$2+5.50889\epsilon$	θ_{α_2}	$2+4.41089\epsilon$	θ_{y_1}	$2+2.83872\epsilon$
θ_{γ_3}	$4+6.97919\epsilon$	θ_{α_3}	$4+5.88119\epsilon$	θ_{y_2}	$4+4.30901\epsilon$
θ_{γ_4}	$6+8.44949\epsilon$	θ_{α_4}	$6+7.35148\epsilon$	θ_{y_3}	$6+5.77931\epsilon$
θ_{γ_5}	$8+9.91978\epsilon$	θ_{α_5}	$8+8.82178\epsilon$	θ_{y_4}	$8+7.24961\epsilon$
	⋮		⋮		⋮

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

$$\alpha_{n-1}\,(\mathrm{Tr} H^\dagger H)^n$$

$$\gamma_{n-1}\,(\mathrm{Tr} H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2$$

$$y_n\,(\mathrm{Tr} H^\dagger H)^n\,\mathrm{Tr}(\bar{Q} H Q)$$

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Couplings

Critical Exponents

$$\alpha_{n-1}\,(\mathrm{Tr} H^\dagger H)^n$$

$$\theta_{\alpha_{n-1}} = (2n-4)\; +\; n\gamma_M$$

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Eigenvalues of M $\longrightarrow$ Critical Exponents θ_i

Couplings

Critical Exponents

$$\alpha_{n-1}\,(\mathrm{Tr}H^\dagger H)^n \qquad \theta_{\alpha_{n-1}} = (2n-4)\,+\,n\gamma_M$$

$$\gamma_{n-1}\,(\mathrm{Tr}H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2 \Longrightarrow \theta_{\gamma_{n-1}} = (2n-4)\,+\,(n-2)\gamma_M + \gamma_m$$

$$y_n\,(\mathrm{Tr}H^\dagger H)^n\,\mathrm{Tr}(\bar{Q}HQ)$$

Scalar potential close to the FP

Stability Matrix

$$M^i_k \equiv \left[\frac{\partial \beta^i(g)}{\partial g^k}\right]_{g=g^*}$$

Eigenvalues of M → Critical Exponents θ_i

Couplings

Critical Exponents

$\alpha_{n-1}\,(\mathrm{Tr} H^\dagger H)^n$ — $\theta_{\alpha_{n-1}} = (2n-4) + n\gamma_M$

$\gamma_{n-1}\,(\mathrm{Tr} H^\dagger H)^{n-2}\mathrm{Tr}(H^\dagger H)^2$ → $\theta_{\gamma_{n-1}} = (2n-4) + (n-2)\gamma_M + \gamma_m$

$y_n\,(\mathrm{Tr} H^\dagger H)^n\,\mathrm{Tr}(\bar{Q} H Q)$ — $\theta_{y_n} = 2n + n\gamma_M + \left(\frac{\eta_H}{2} + \eta_Q\right)$

Results from Perturbation Theory

Under perturbative expansion, the theory has an ultraviolet Fixed Point:

$$g^* = +0.456\epsilon + 0.781\epsilon^2 + 6.610\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$y^* = +0.211\epsilon + 0.508\epsilon^2 + 3.322\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$u^* = +0.200\epsilon + 0.440\epsilon^2 + 2.693\epsilon^3 + \mathcal{O}(\epsilon^4)$$

$$v^* = -0.137\epsilon - 0.632\epsilon^2 - 4.313\epsilon^3 + \mathcal{O}(\epsilon^4)$$

Litim, Sannino (2014)

Litim, Riyaz, Stamou, Steudtner (2023)

Bond, Litim, Vazquez, Steudtner (2017)

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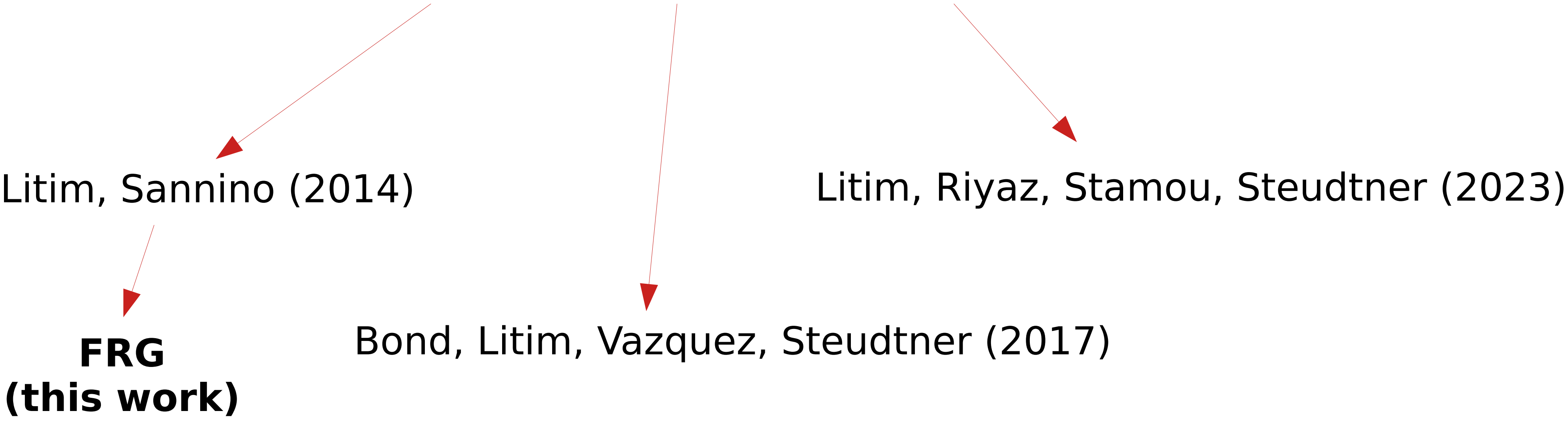
\end{aligned}
$$



Litim, Sannino (2014)

Litim, Riyaz, Stamou, Steudtner (2023)

FRG
(this work)

Bond, Litim, Vazquez, Steudtner (2017)

Conclusion

- Gauge-Yukawa theories are the only 4D QFT in which an ultraviolet fixed point is under perturbative control, with a precision of NNLO in the Veneziano parameter.

- In perturbation theory, the assumption is that beyond marginal operators stay at a Gaussian fixed point and are not generated by quantum fluctuations.

- In this work, for the first time, we study an infinite tower of beyond marginal operators in gauge-Yukawa theories with the functional Renormalization Group.

- The ultraviolet fixed point of such higher dimensional operators was computed at leading order in the Veneziano parameter and a power counting argument was described. Specifically, the fixed point of a coupling of dimension *2n* has a leading order in the Veneziano parameter given by *\epsilon^n*.

- Because of such power counting argument, it was possible to re-sum the power series expansions and obtain closed forms for the scalar potential and the Yuwaka in terms of a finite number of couplings.

THANKS FOR THE ATTENTION!

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