Multipoint correlation functions: spectral representation, numerical evaluation, and improved estimators

Jan von Delft

Fabian Kugler, Seung-Sup Lee, Jan von Delft, Phys. Rev. X **11**, 041006 (2021)

Seung-Sup Lee, Fabian Kugler, Jan von Delft, Phys. Rev. X **11**, 041007 (2021)

Jae-Mo Lihm, Johannes Halbinger, Jeongmin Shim, Jan von Delft, Fabian Kugler, Seung-Sup Lee, Phys. Rev. B **109**, 125138 (2024)

Anxiang Ge, Nepomuk Ritz, Elias Walter, Santiago Aguirre, Jan von Delft, Fabian Kugler Phys. Rev. B **109**, 115128 (2024)

Jae-Mo Lihm (Seoul National University)

Johannes Halbinger (LMU Munich)

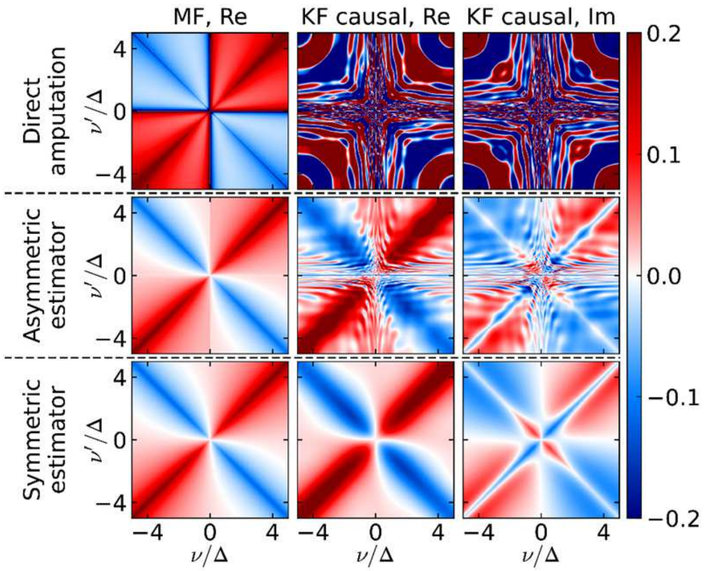
Jeongmin Shim (LMU Munich)

Anxiang Ge (LMU Munich)

Nepomuk Ritz (LMU Munich)

Fabian Kugler (CCQ, Flatiron)

Seung-Sup Lee (Seoul National University)



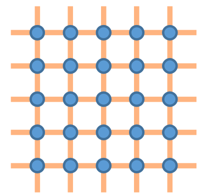
Acknowledgement: many slides are borrowed from Jae-Mo Lihm & Seung-Sup Lee

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN | ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS | CeNS Center for NanoScience | MUNICH QUANTUM CENTER | MCQST | Munich Quantum Valley | Unterstützt von / Supported by Alexander von Humboldt Stiftung/Foundation

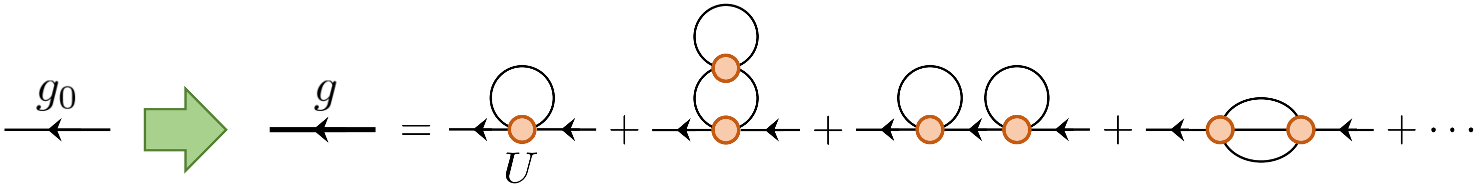
Outline

- Why multi-point functions?
- Limitations of purely field-theoretical approaches for strong-coupling problems
- Spectral representations
- NRG for local multi-point correlators
- Improved estimators for self-energy and 4-point vertices
- Outlook

Why multipoint correlation functions?

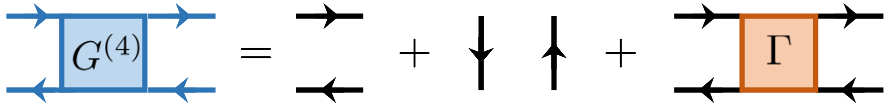


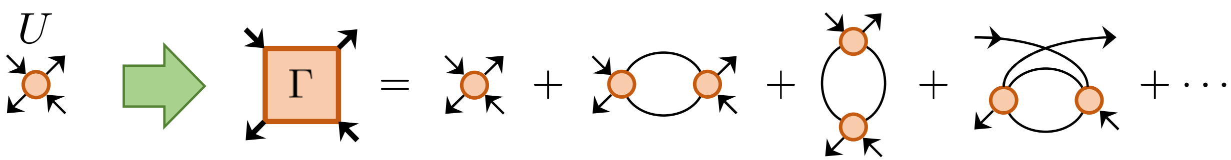
- Dynamical properties are encoded in correlation functions
- Two-point functions: local density of states, spectral function, …



- Four-point functions: magnetic susceptibility, conductivity, …

$$\langle \underbrace{c^\dagger_\sigma(\mathbf{x}_1, t_1^+) c_{\sigma'}(\mathbf{x}_1, t_1)}_{\mathbf{S}(\mathbf{x}_1, t_1)} \underbrace{c^\dagger_{\sigma'}(\mathbf{x}_2, t_2^+) c_\sigma(\mathbf{x}_2, t_2)}_{\mathbf{S}(\mathbf{x}_2, t_2)} \rangle \qquad \langle \underbrace{c^\dagger_\sigma(\mathbf{x}_1^+, t_1^+) c_{\sigma'}(\mathbf{x}_1, t_1)}_{\mathbf{j}(\mathbf{x}_1, t_1)} \underbrace{c^\dagger_{\sigma'}(\mathbf{x}_2^+) c_\sigma(\mathbf{x}_2, t_2)}_{\mathbf{j}(\mathbf{x}_2, t_2)} \rangle$$



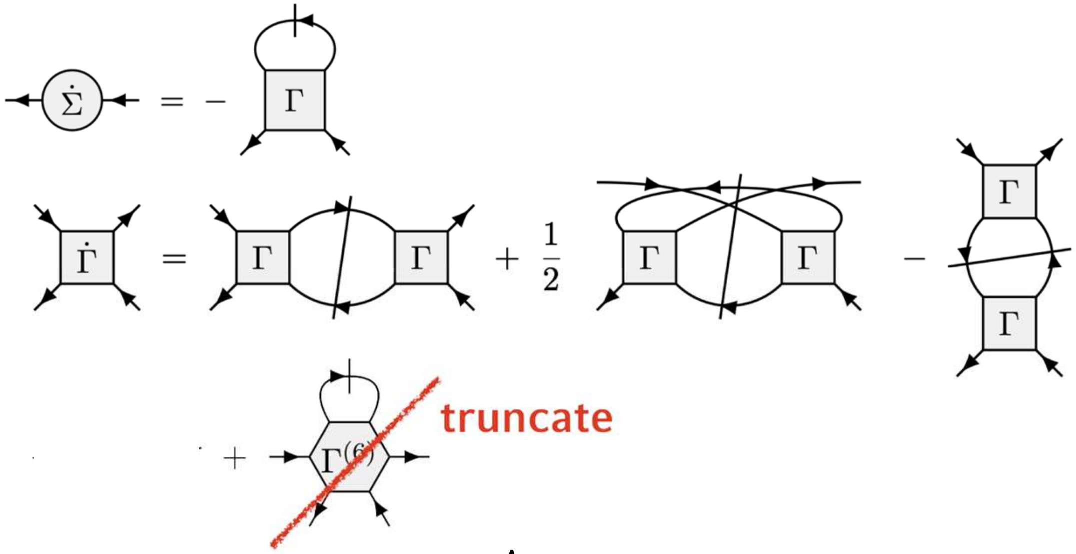


$F_{\sigma\sigma'}(\mathbf{k}_1, \omega_1; \mathbf{k}_2, \omega_2; \mathbf{k}_3, \omega_3)$: 4-point vertex: energy-dependent effective interaction (needed as function of <u>real</u> frequencies)

Field-theoretical approaches for fermionic problems

1-loop fRG

Review: Metzner Salmhofer, Honerkamp, Meden, Schönhammer, RMP (2012)

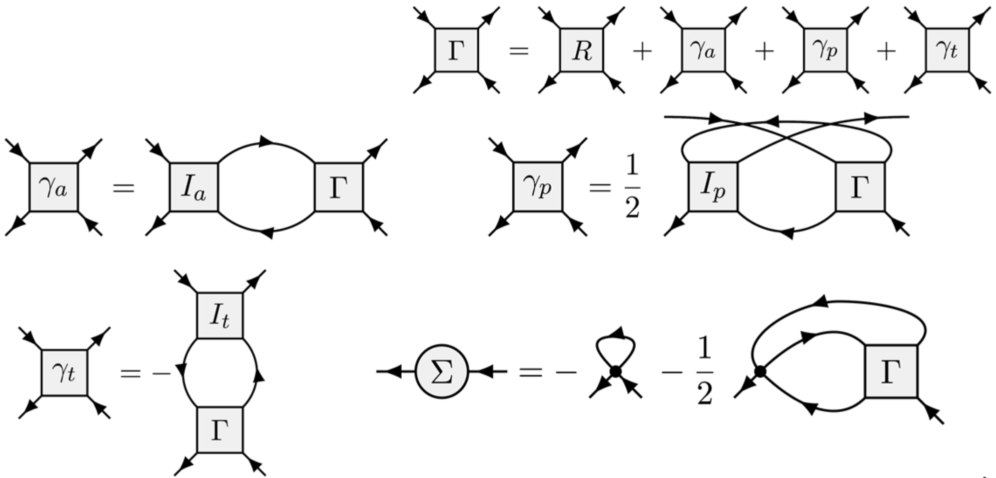


fRG flow: $G_0 \to G_0^{\Lambda}, \quad \Lambda_{\text{initial}} \to \Lambda_{\text{final}}$

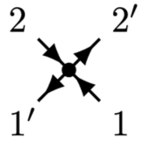
$\Sigma^R|_{\Lambda=\infty} = \Sigma_{\text{Hartree}}, \quad \Gamma|_{\Lambda=\infty} = \Gamma_0$

Parquet formalism

Review: Bickers (2004)



parquet approximation: $R = \Gamma_0 + \mathcal{O}[(\Gamma_0)^4] \approx \Gamma_0 =$



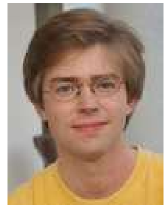
- **multi-loop fRG**: ensures that $\dot{\Gamma}$ = total derivative; integrating flow yields regulator-independent results; sums up all parquet diagrams in parquet approximation

Kugler, von Delft, PRL 2018, PRB 2018, NJP (2018)

Fabian Kugler

Benchmark study of 1-loop fRG vs. parquet approximation (PA) vs. NRG

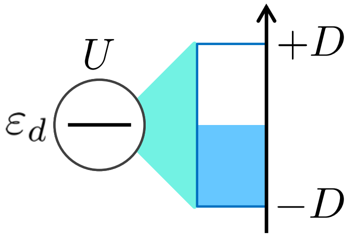
Ge, Ritz, Walter, Aguirre, von Delft, Kugler, PRB **109**, 115128 (2024)



talk by
Nepomuk Ritz
(LMU Munich)
Thursday, 14:50

Single-impurity Anderson model

$$H = \sum_\sigma \varepsilon_d d_\sigma^\dagger d_\sigma + U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow + \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k\sigma} v(d_\sigma^\dagger c_{k\sigma} + c_{k\sigma}^\dagger d_\sigma)$$

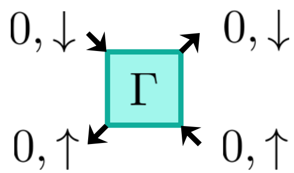


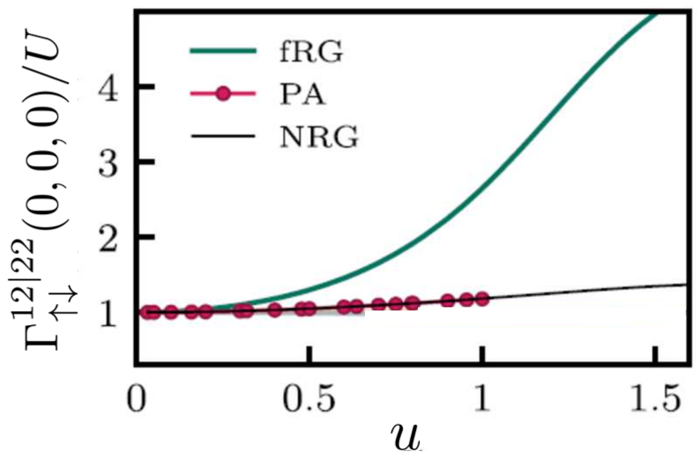
$\varepsilon_d = -U/2$

$\Delta =$ effective level width

Kondo temperature: $T_K = \sqrt{\Delta U/2}\exp\left[-\pi\left(\frac{U}{8\Delta} + \frac{\Delta}{U}\right)\right]$

$\Gamma_{\uparrow\downarrow}^{12|22}(0,0,0)$





vary $u = U/(\pi\Delta)$ at fixed $T/U = 0.01$

- Low−energy properties of this model can be computed essentially exactly using the numerical renormalization group (NRG)
- Use fRG / PA / NRG to compute $\Gamma_{\uparrow\downarrow}^{12|22}(\nu = 0, \nu' = 0, \omega = 0)$
- Conclusion: 1-loop fRG and parquet both fail for $u > 1$
- both 1-loop fRG and parquet approximation do not yield non−perturbative results

Approaches based on exact treatments of local correlations

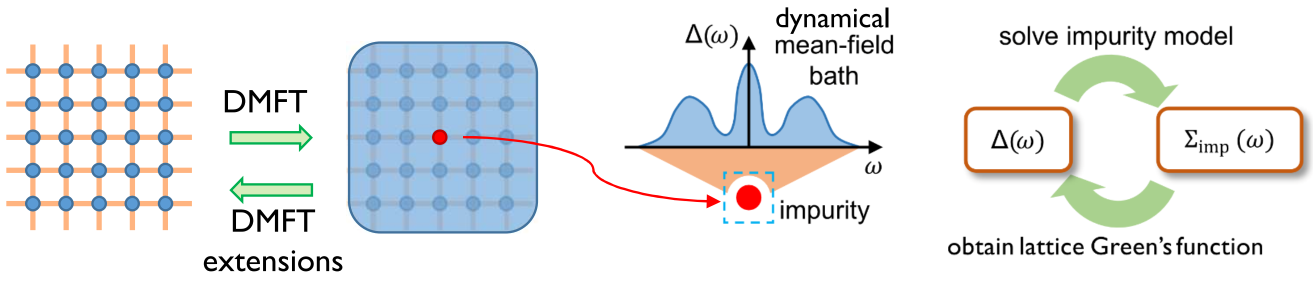
Dynamical mean field theory (DMFT)

Reviews: Georges et al. RMP (1996), Kotliar et al. RMP (2006)

treat local correlations exactly

by solving self-consistent impurity model

$$G(\mathbf{k},\omega) = \left[\omega - \varepsilon_{\mathbf{k}} + \mu - \Sigma(\not{\mathbf{k}},\omega)\right]^{-1}$$



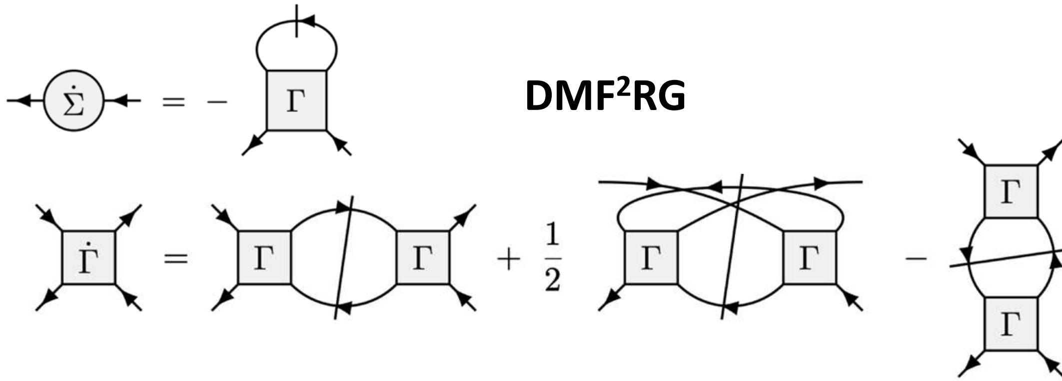
extensions of DMFT

Review: Rohringer et al, *RMP* (2018), **Benchmark:** Schäfer et al, *PRX* (2021)

include nonlocal correlations by field-theoretic methods

with DMFT input for local self-energy and 4-point vertex

$$G(\mathbf{k},\omega) = \left[\omega - \varepsilon_{\mathbf{k}} + \mu - \Sigma(\mathbf{k},\omega)\right]^{-1}$$



$\Sigma_{\text{initial}} = \Sigma_{\text{DMFT}}$ $\quad \Gamma_{\text{initial}} = \Gamma_{\text{DMFT}}$

Examples:

dynamical vertex approximation — Held et al. (2008)

dual fermions — Havermann et al. (2008)

DMF²RG — Taranto et. al (2014), Vilardi et al. (2019), poster by Marcel Schäfer

self-consistent parquet — Lihm et al. (to be published)

this talk →

needed: local 2- and 4-point correlators (full frequency dependence!)

General multipoint functions $\langle O^1(t_1)\dots O^\ell(t_\ell)\rangle$: spectral representations

Kugler, Lee, von Delft, *PRX* **11**, 041006 (2021)

- General & unifying: any ℓ, all frequency formalisms

Matsubara zero-temperature Keldysh

- **Ex:** ℓ-point (ℓp) correlator, real-freq. **zero-temp.** formalism

$$\mathcal{G}(\boldsymbol{t}) = (-\mathrm{i})^{\ell-1}\left\langle \mathcal{T}\prod_{i=1}^{\ell}\mathcal{O}^i(t_i)\right\rangle$$

$$= \sum_p (\pm 1)^p \prod_{i=1}^{\ell-1}\left[-\mathrm{i}\theta(t_{p(i)} - t_{p(i+1)})\right]\left\langle \prod_{i=1}^{\ell}\mathcal{O}^{p(i)}(t_{p(i)})\right\rangle$$

convolution theorem: Fourier trafo.

$$\mathcal{G}(\boldsymbol{\omega}) = 2\pi\delta(\omega_{1\dots\ell})\sum_p (\pm 1)^p \int \mathrm{d}^{\ell-1}\omega'_p\, K(\boldsymbol{\omega}_p - \boldsymbol{\omega}'_p)\, S[\boldsymbol{\mathcal{O}}_p](\boldsymbol{\omega}'_p)$$

$$\omega_{1\dots\ell} = \omega_1 + \dots + \omega_\ell$$

Fabian Kugler

convolution kernel

formalism-dependent

system-independent

partial spectral function (PSF)

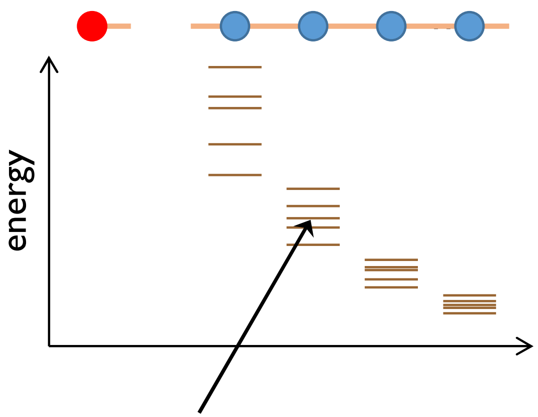
formalism-independent

system-dependent, has Lehmann representation through eigenstates, eigenenergies of Hamiltonian

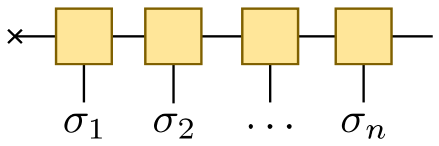
- Two-stage computation: (1) compute PSF; (2) convolve PSF with kernels

→ Different formalisms or frequencies: same PSF, only change kernels

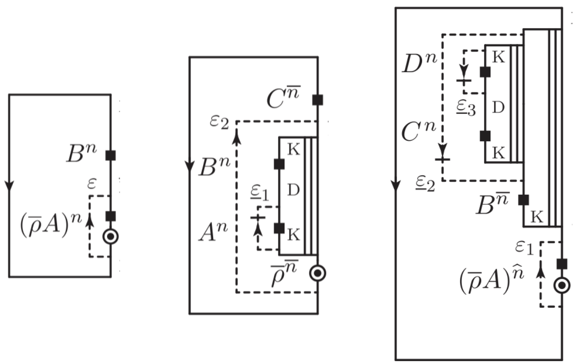
Quantum impurity models: Numerical renormalization group (NRG)



$$\sum_{\vec{\sigma}} \mathrm{Tr}\left(A^{\sigma_1}\cdots A^{\sigma_n}\right)|\vec{\sigma}\rangle =$$



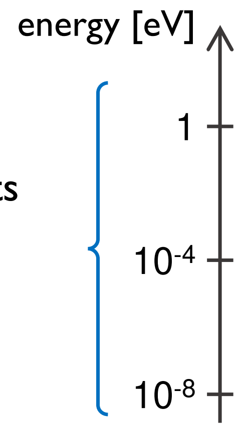
- Logarithmic discretization: Wilson chain — Wilson, *RMP* (1975); Bulla, Costi, Pruschke, *RMP* (2008)
- Iterative diagonalization: complete basis of energy eigenstates — Anders, Schiller, *PRL* (2005)
- Tensor network formulation: many-body states as matrix product states (MPS) — Weichselbaum, *PRB* (2012)
- (Partial) spectral functions:
  - two-point: Peters, Pruschke, Anders, *PRB* (2006); Weichselbaum, von Delft, *PRL* (2007)
  - multipoint: Lee, Kugler, von Delft, *PRX* 11, 041007 (2021)



ε_i : PSF arguments

ν, ν', ω : frequencies

T : temperature



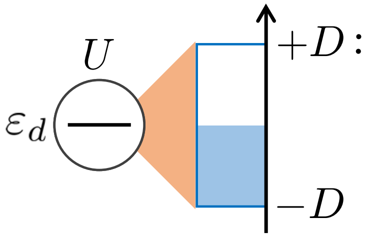
Andreas Weichselbaum

Seung-Sup Lee

Vertex of Anderson impurity model (AIM)

$$H = \sum_\sigma \varepsilon_d d_\sigma^\dagger d_\sigma + U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow + \sum_{k\sigma} \varepsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_{k\sigma} v(d_\sigma^\dagger c_{k\sigma} + c_{k\sigma}^\dagger d_\sigma)$$

Kugler, Lee, von Delft, *PRX* **11**, 041006 (2021)



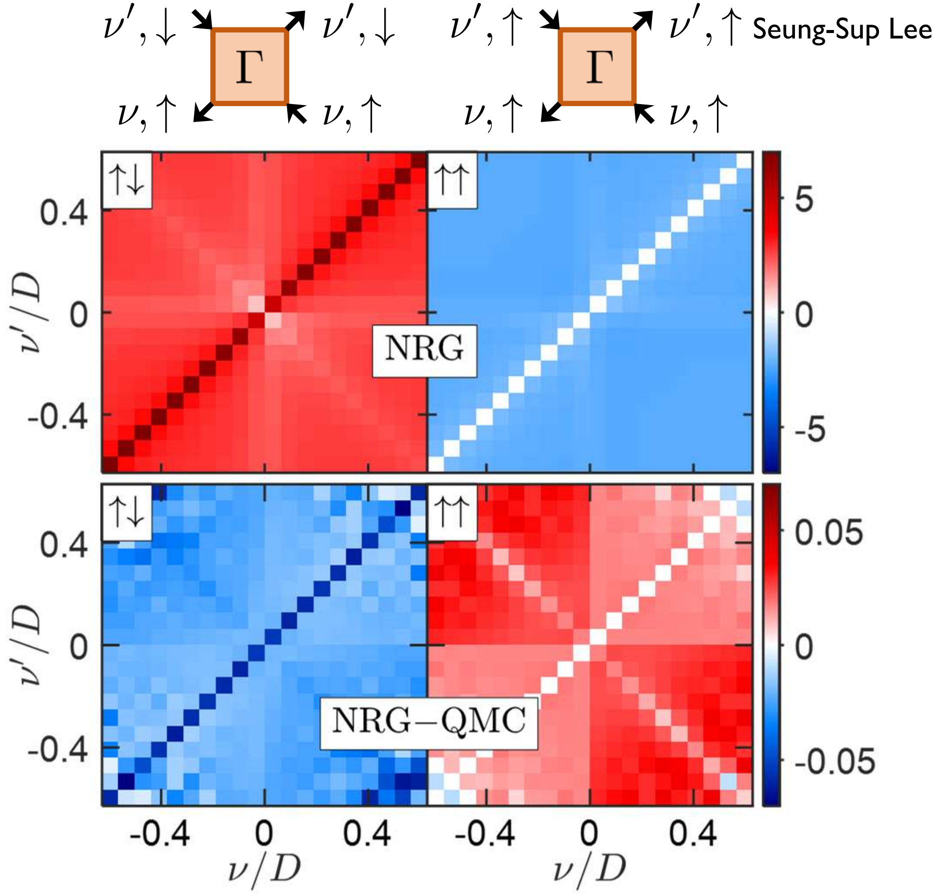
$\varepsilon_d = -U/2$

$U = 0.2D$

$\Delta = 0.04D$

Kondo temperature: $T_K \simeq 5 \times 10^{-3} D$

$\Gamma_{\sigma\sigma'}(\mathrm{i}\nu, \mathrm{i}\nu', \mathrm{i}\omega = 0)/U$



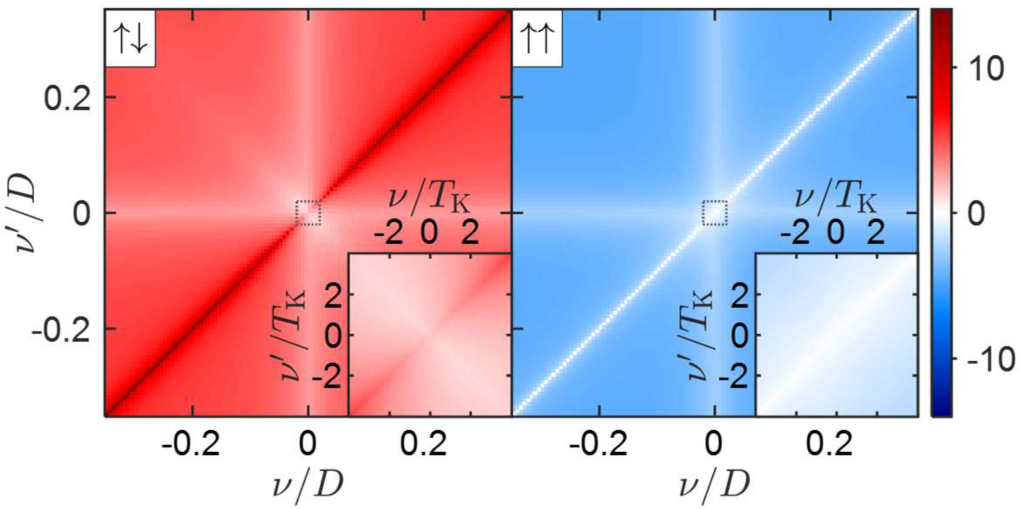
- Benchmark imaginary-frequency vertex against quantum Monte Carlo (QMC) by Toschi group (TU Wien) $T = 0.01D > T_K$

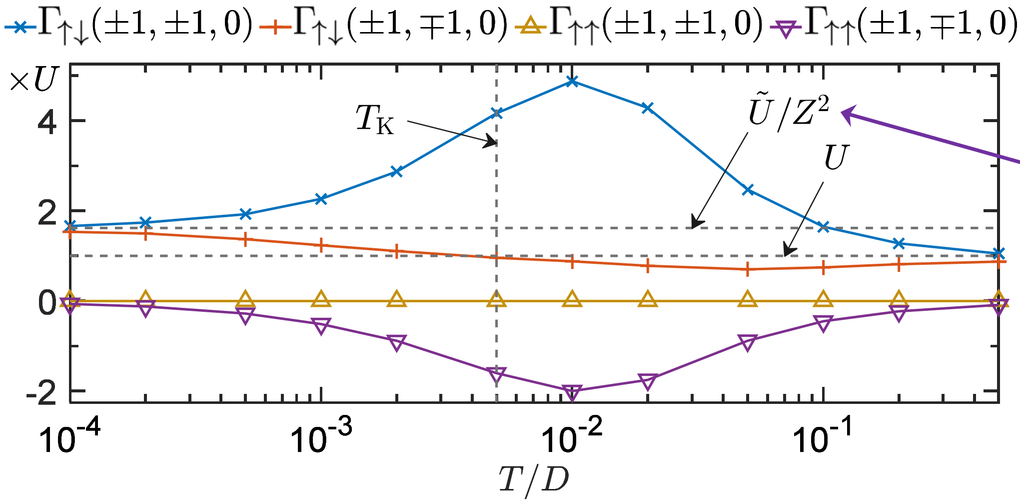
Chalupa et al, *PRB* (2018); *PRL* (2021)

	QMC	NRG
imaginary frequency	○	○
accuracy against statistical noise	△	○
low temperature	✕	○
real frequency	✕	○

Vertex of Anderson impurity model (AIM)

Lee, Kugler, von Delft, *PRX* 11, 041006 (2021)





$T = 10^{-4} D$

Kondo temperature: $T_K \simeq 5 \times 10^{-3} D$

- stronger features at higher freq.; reduced at lower freq.
- $T \gg U$: bare vertex $U\delta_{\bar{\sigma}\sigma'}$
- $T \simeq T_K$: most pronounced correlation effects
- $T \ll T_K$: Fermi liquid

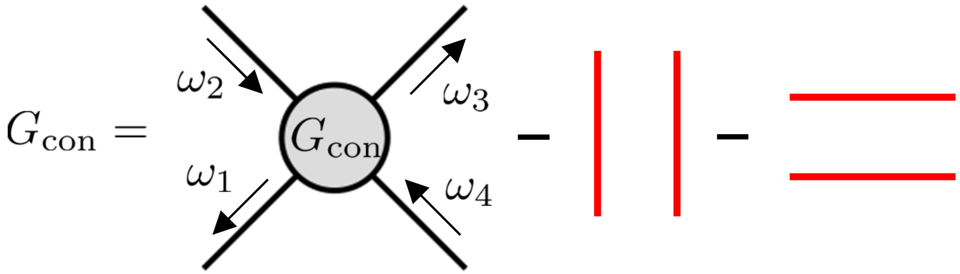
Renormalized perturbation theory (RPT) parameters:
$\tilde{U} \simeq 0.20U, Z \simeq 0.36$

From strongly interacting particles to weakly interacting quasiparticles!

Challenges when calculating 4-point vertex $G = \langle \mathcal{T}\ d(t_1) d^\dagger(t_2) d(t_3) d^\dagger(t_4) \rangle$

- Subtraction of the disconnected part

$$g(\omega) = G[d, d^\dagger](\omega)$$



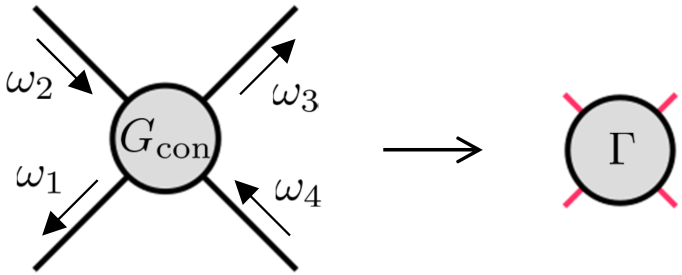
$$G_{\text{con}}(\omega_1, \omega_2, \omega_3, \omega_4) = G_{\text{tot}}(\omega_1, \omega_2, \omega_3, \omega_4) - ig(\omega_1)g(\omega_3)[\delta(\omega_1 + \omega_4) - \delta(\omega_1 + \omega_2)]$$

in the non-interacting limit: $G_{\text{tot}} = G_{\text{dis}}$

$$G_{\text{con}} = 0$$

requires numerically delicate cancellations

- Amputation of external legs



$$\Gamma(\omega_1, \omega_2, \omega_3, \omega_4) = \frac{G_{\text{con}}(\omega_1, \omega_2, \omega_3, \omega_4)}{g(\omega_1)g(-\omega_2)g(\omega_3)g(-\omega_4)}$$

large-frequency limit $\sim \dfrac{U/\omega^4}{1/\omega^4} = U \qquad (\omega \gg 1)$

requires numerically delicate cancellations

Remedy: use improved estimators!

Lihm, Halbinger, Shim, von Delft, Kugler, Lee, PRB **109**, 125138 (2024)

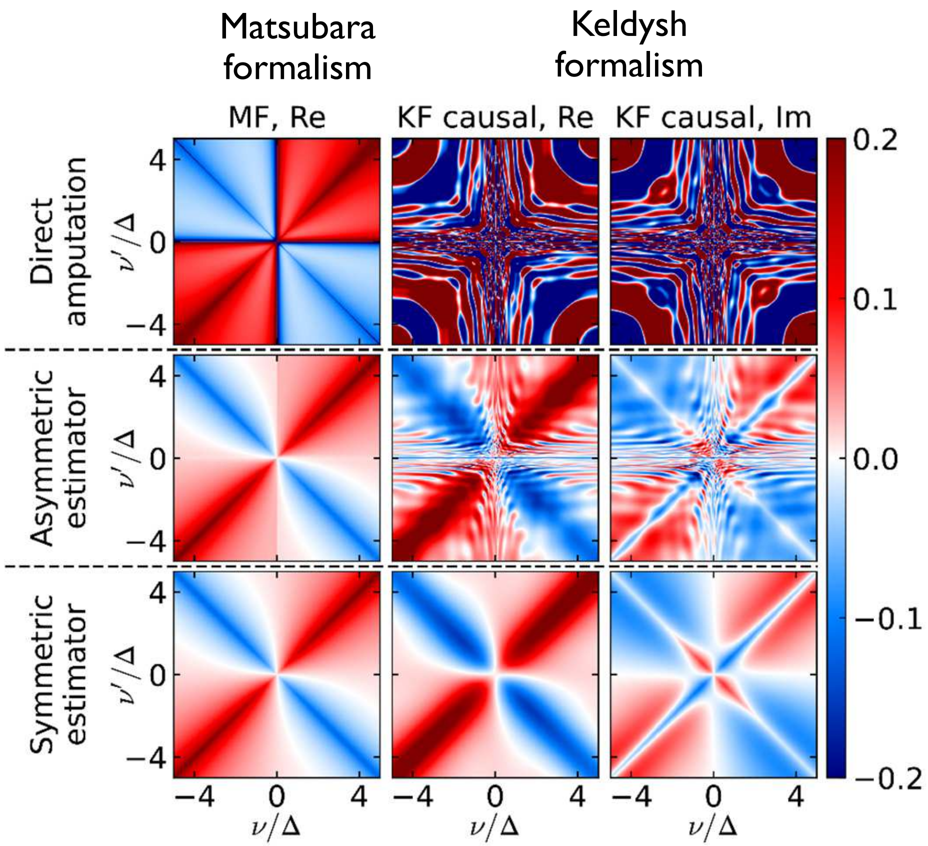
"improved estimator" is formally equivalent to desired function, but yields <u>better cancellation of numerical artifacts</u>;

can be derived using equations of motion (EOM)

amputate external legs by division:

use equation of motion for one operator:

use equations of motion for four operators:



Seung-Sup Lee

Jae-Mo Lihm

EOM for 2-point propagator

$$G[d_1, d_2^\dagger](t_1, t_2) = -i\left\langle \mathcal{T} d_1(t_1) d_2^\dagger(t_2)\right\rangle$$

Matsubara zero-temperature Keldysh

- EOM of a time-ordered correlator: $i\frac{\partial}{\partial t_1} G[d_1, d_2^\dagger](t_1, t_2) = \delta(t_1 - t_2)\delta_{12} + G\big[[d_1, H], d_2^\dagger\big](t_1, t_2)$

$$\{d_1, d_2^\dagger\} = \delta_{12}$$

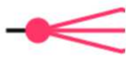
- EOM in frequency domain: $\omega G[d_1, d_2^\dagger](\omega) = \delta_{12} + G\big[[d_1, H], d_2^\dagger\big](\omega)$

$$H_0 = \sum_\sigma \varepsilon_d d_\sigma^\dagger d_\sigma + \sum_{k\sigma} \varepsilon_k c_{b\sigma}^\dagger c_{k\sigma} + \sum_{k\sigma} (v_k d_\sigma^\dagger c_{k\sigma} + \mathrm{H.c.}), \qquad H_{\mathrm{int}} = U d_\uparrow^\dagger d_\uparrow d_\downarrow^\dagger d_\downarrow$$

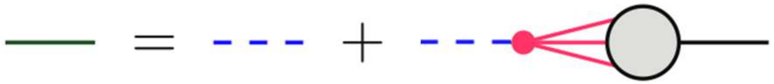
$$\Delta(\omega) = \sum_k \frac{|v_k|^2}{\omega - \varepsilon_k}$$

$$\underbrace{\big(\omega - \varepsilon_d - \Delta(\omega)\big)}_{(g^0)^{-1}} G[d_\sigma, d_{\sigma'}^\dagger](\omega) = \delta_{\sigma\sigma'} + G[q_\sigma, d_{\sigma'}^\dagger](\omega)$$

$$q_\sigma = [d_\sigma, H_{\mathrm{int}}] = U d_\sigma d_{-\sigma}^\dagger d_{-\sigma}$$



$$G[d_\sigma, d_{\sigma'}^\dagger](\omega) = g^0 \delta_{\sigma\sigma'} \quad + \quad g^0 \; G[q_\sigma, d_{\sigma'}^\dagger](\omega)$$

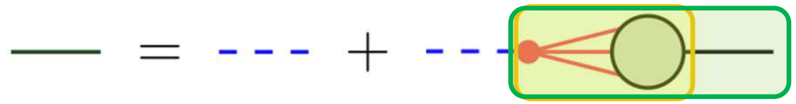


Asymmetric estimator for self-energy

- 1st-order EOM

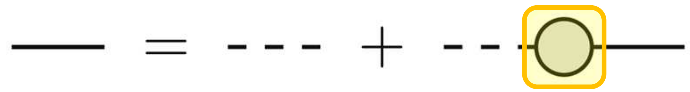
$$g(\omega) = G[d_\sigma, d^\dagger_{\sigma'}](\omega) = g^0 \delta_{\sigma\sigma'} \quad + \quad g^0 \, G[q_\sigma, d^\dagger_{\sigma'}](\omega)$$

$$q_\sigma = [d_\sigma, H_{\mathrm{int}}] = U d_\sigma d^\dagger_{-\sigma} d_{-\sigma}$$

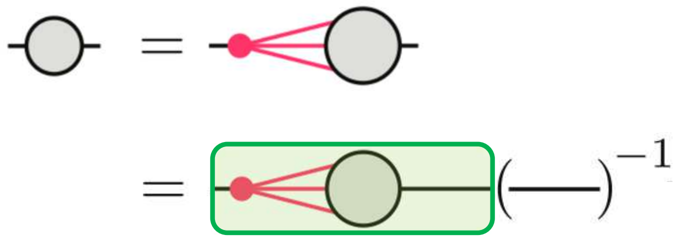


- Dyson equation

$$g = g^0 + g^0 \; \Sigma \; g$$



compare:



- Asymmetric improved estimator $\quad \Sigma = G[q_\sigma, d^\dagger_\sigma] \quad g^{-1}$ known as "Bulla trick" in NRG community

(equivalent to the Schwinger-Dyson equation)

Improved estimators for multi-point functions

- We want the improved estimators to be
  - (A) **Symmetric**: symmetric in all indices of the vertex, and
  - (B) **Full**: Involving only full correlators (not the bare ones)

	2-point self-energy $\Sigma(\nu)$	**4-point vertex $\Gamma(\nu, \nu', \omega)$**
asymmetric, **full**	Bulla et al. (1998)	Hafermann et al. (2012)
symmetric, bare	Kaufmann et al. (2019)	Kaufmann et al. (2019)
symmetric, full	Kugler (2022)	**Lihm et al. (2024)**

← **our work**

Jae-Mo Lihm

Bulla, Hewson, Pruschke, J. Phys.: Condens. Matter **10**, 8365 (1998); Hafermann, Patton, Werner, PRB **85**, 205106 (2012);
Kaufmann, Gunacker, Kowalski, Sangiovanni, Held, PRB 100, 075119 (2019); Kugler, PRB **105**, 245132 (2022)

(1-leg) EOM for multipoint correlator

2-point correlator: $(g^0)^{-1}G[d_1, d_2^\dagger] = \delta_{12} + G[q_1, d_2^\dagger]$ $\{d_1, d_2^\dagger\} = \delta_{12}$

from differentiation time-ordering theta functions

Jae-Mo Lihm

ℓ-point correlator: $G[d_1, O^2, \cdots, O^\ell](\boldsymbol{t}) = (-i)^{\ell-1}\left\langle \mathcal{T} d_1(t_1) O^2(t_2) \cdots O^\ell(t_\ell)\right\rangle$

$$(g^0)^{-1}G[d_1, O^2, \cdots, O^\ell] = \sum_{n=2}^{\ell} G[O^2, \cdots, [d_1, O^n]_{\zeta_n}, \cdots, O^\ell] + G[q_1, O^2, \cdots, O^\ell]$$

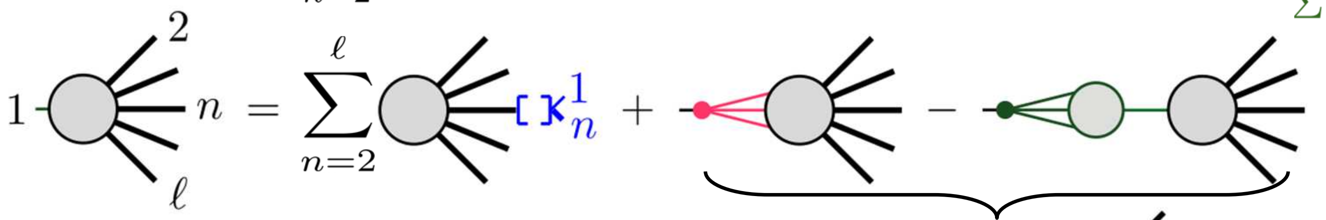
from differentiation time-ordering theta functions

$(g^0)^{-1} = g^{-1} + \Sigma$

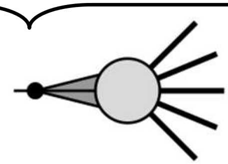
$$g^{-1}G[d_1, \cdots] = \sum_{n=2}^{\ell} G[\cdots [d_1, O^n]_{\zeta_n}, \cdots] + G[q_1, \cdots](\omega) - \Sigma G[d_1, \cdots]$$

$\Sigma = G[q_\sigma, d_\sigma^\dagger]g^{-1}$

amputation of one external leg on left side! (without using $(g^0)^{-1}$ or g^{-1} on right side!)

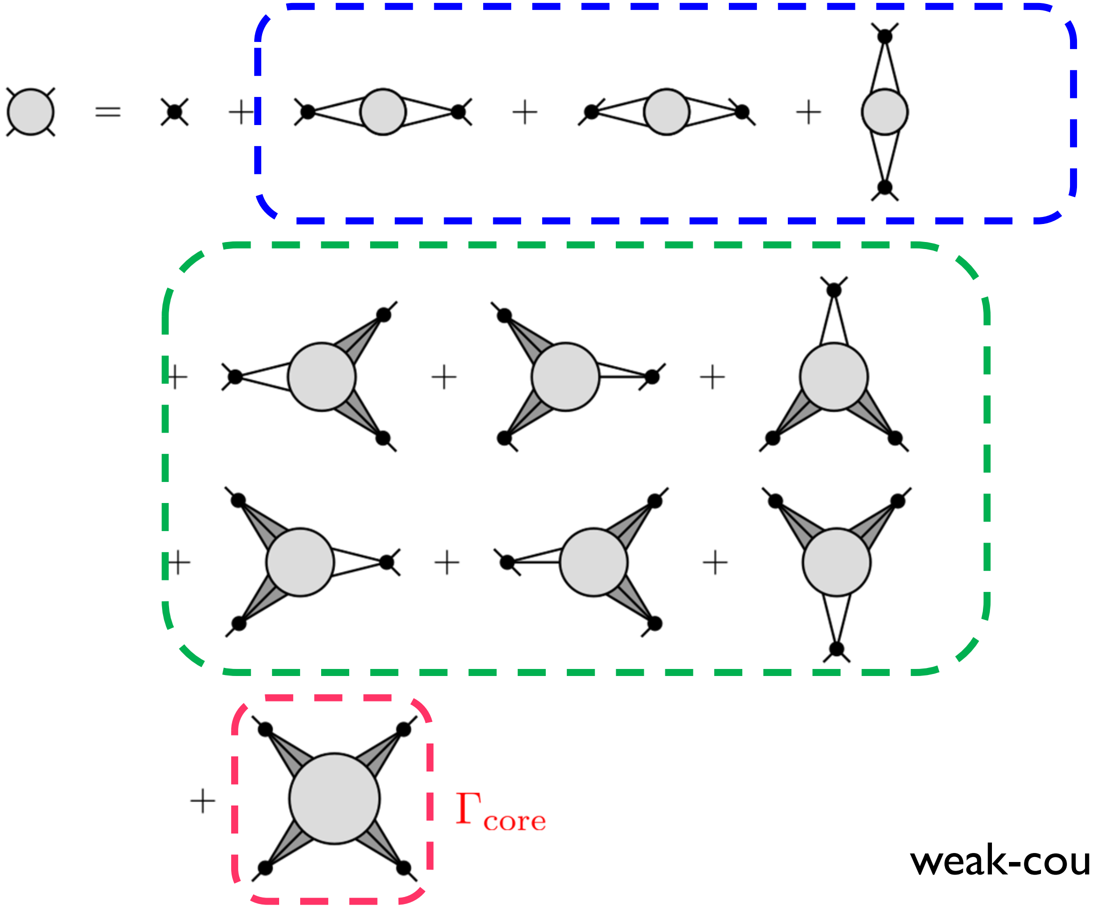


subtraction of reducible diagrams yields <u>1-particle irreducible vertex</u> for leg 1



Symmetric improved estimator for 4-point vertex

$$\Gamma = g_1^{-1} g_3^{-1} G[d_1, d_2^\dagger, d_3, d_4^\dagger] g_2^{-1} g_4^{-1}$$



$$\Gamma(\nu,\nu';\omega) = \Gamma_{\text{bare}} + \sum_{r=\text{ph},\text{pp},\overline{\text{ph}}} \mathcal{K}_1^r(\omega_r)$$
$$+ \sum_{r=\text{ph},\text{pp},\overline{\text{ph}}} \mathcal{K}_2^r(\nu_r;\omega_r) + \mathcal{K}_{2'}^r(\nu_r';\omega_r)$$
$$+ \Gamma_{\text{core}}(\nu,\nu';\omega)$$

Jae-Mo Lihm

asymptotic classes [Wentzell et al. (2020)]

emerge naturally and are computed separately

$$\lim_{|\omega|\to\infty} \mathcal{K}_1(\omega) = 0$$

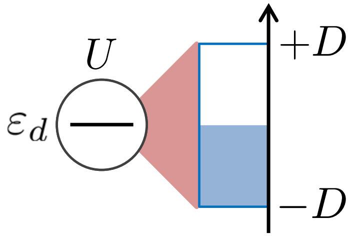
$$\lim_{|\nu|\to\infty} \mathcal{K}_2(\nu;\omega) = \lim_{|\omega|\to\infty} \mathcal{K}_2(\nu;\omega) = 0$$

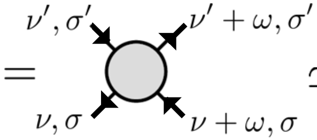
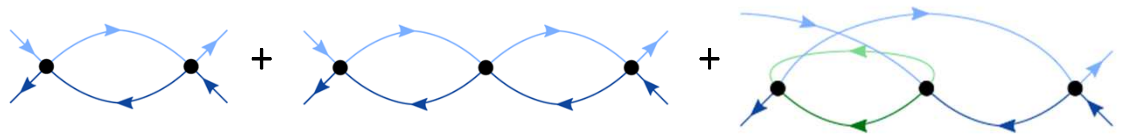
$$\lim_{|\nu|\to\infty} \Gamma_{\text{core}} = \lim_{|\nu'|\to\infty} \Gamma_{\text{core}} = \lim_{|\omega|\to\infty} \Gamma_{\text{core}} = 0$$

weak-coupling limit: $\mathcal{K}_1 = \mathcal{O}(U^2),\ \mathcal{K}_2 = \mathcal{O}(U^3),\ \Gamma_{\text{core}} = \mathcal{O}(U^4)$

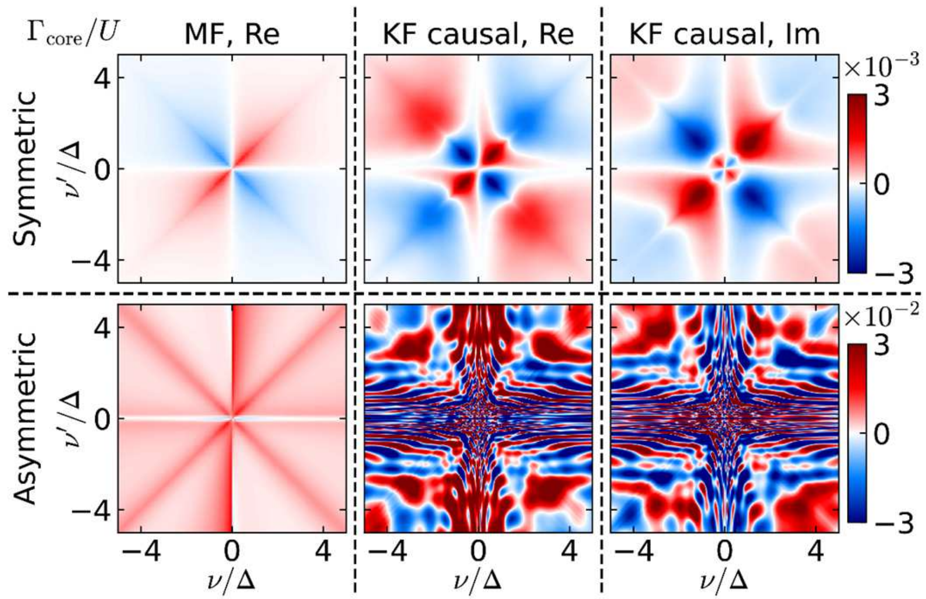
Wentzell, Li, Tagliavini, Taranto, Rohringer, Held, Toschi, Andergassen, PRB **102**, 085106 (2020)

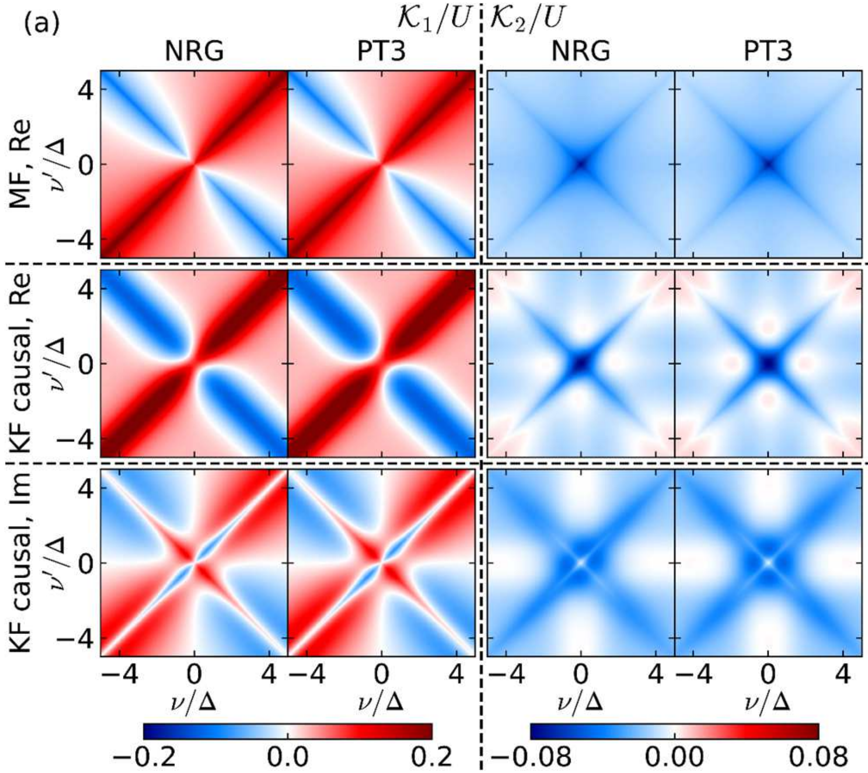
SIAM at weak coupling: benchmark NRG vs. 3rd-order perturbation theory (PT3)



$$\Gamma_{\sigma\sigma'}(\nu,\nu';\omega) =$$  $\simeq$ 

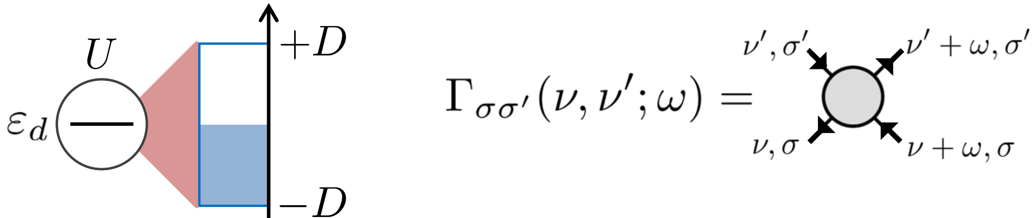
$$u = U/(\pi\Delta) = 0.16, \quad \Delta/D = 0.1, \quad T/U = 0.01$$





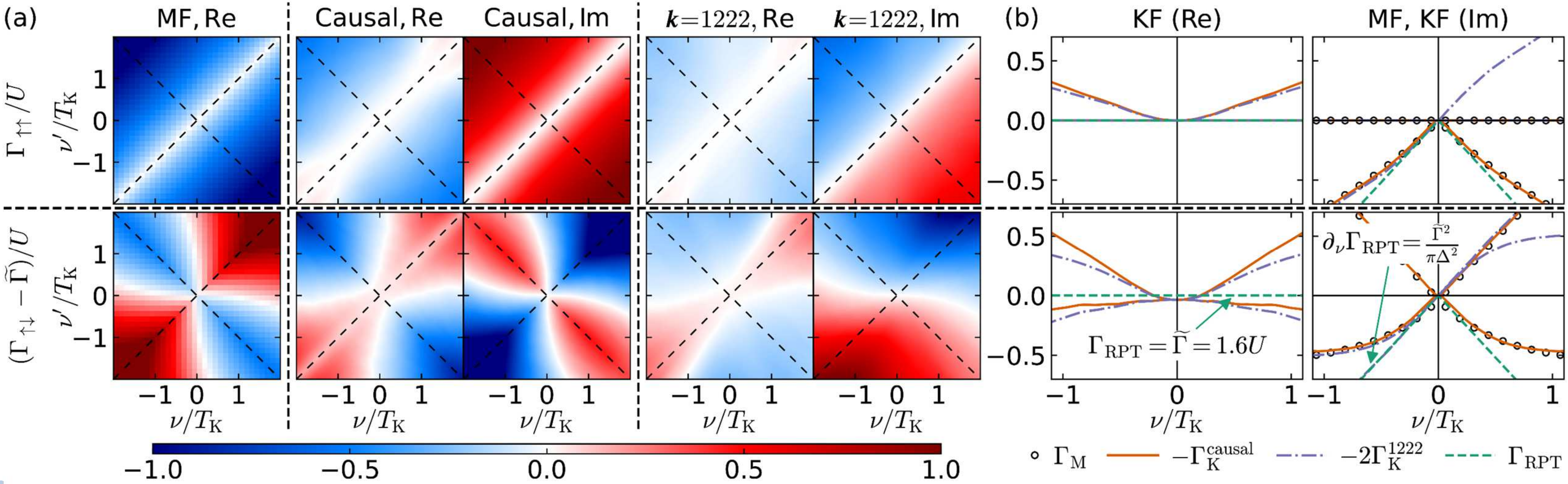
NRG is based on diagonalization, so getting correct small-*U* limit is nontrivial

SIAM at strong coupling: benchmark NRG vs. renormalized perturbation theory (RPT)

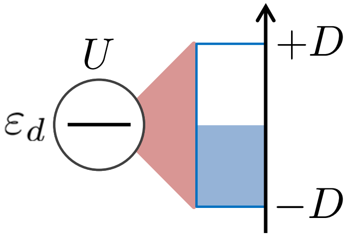


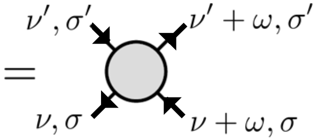
$u = U/(\pi\Delta) = 1.6, \quad \Delta/D = 0.04, \quad T/U = 0.0025$

- $T = 0.029 T_{\mathrm{K}}$: Fermi-liquid regime
- Low-frequency vertex is consistent with RPT (a Fermi-liquid theory) for both Γ(0,0,0) and slope



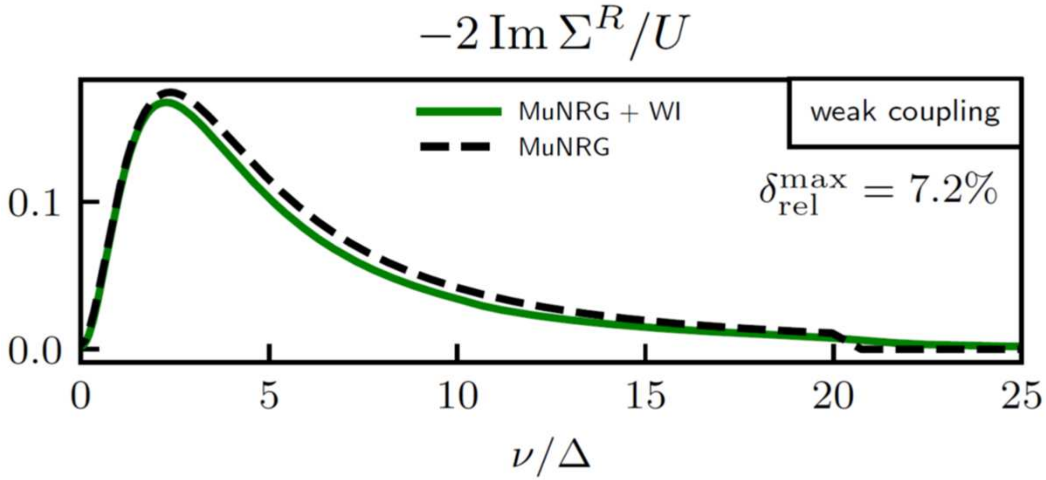
Ward identity



$\Gamma_{\sigma\sigma'}(\nu, \nu'; \omega) =$ 

$$u = U/(\pi\Delta) = 0.5, \quad \Delta/D = 0.04, \quad T/U = 0.01$$

Ward identity:
$$2\,\mathrm{Im}\Sigma^R(\nu) = \frac{1}{i\pi}\int_{-\infty}^{\infty} d\tilde{\nu}\,\mathrm{Im}\,\Delta^R(\tilde{\nu})G^R(\tilde{\nu})G^A(\tilde{\nu})\Big\{\Gamma^{12|21}_{\uparrow\downarrow+\uparrow\uparrow}(\tilde{\nu},\nu|\nu,\tilde{\nu}) - [1 - 2\,n_F(\tilde{\nu})]\Big[\Gamma^{12|22}_{\uparrow\downarrow+\uparrow\uparrow}(\tilde{\nu},\nu|\nu,\tilde{\nu}) - \Gamma^{22|21}_{\uparrow\downarrow+\uparrow\uparrow}(\tilde{\nu},\nu|\nu,\tilde{\nu})\Big]\Big\}$$



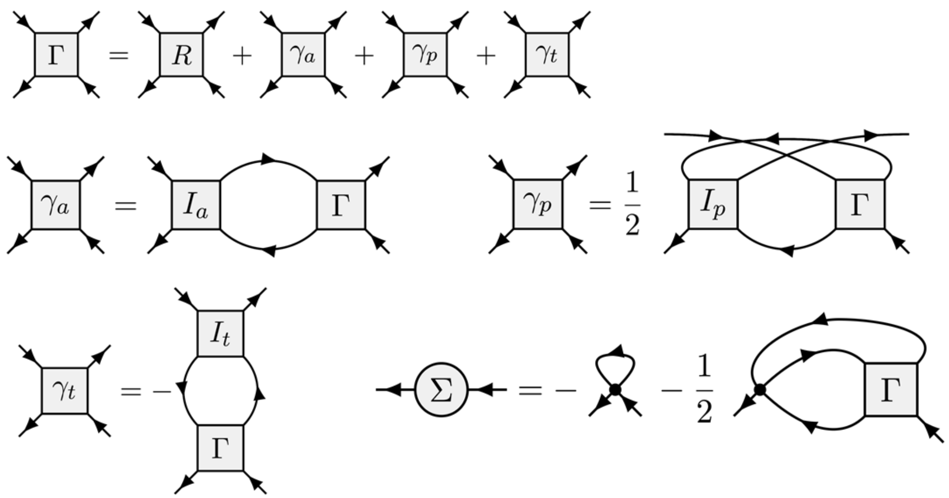
NRG results for self-energy and vertex satisfy Ward identity to within 7% !

This is highly non-trivial consistency check, since Ward identity involves integral over entire frequency axis.

Outlook: Going beyond impurity model – non-local extensions of DMFT

- Main methodological result: full frequency dependence of impurity models is accessible !

- Next steps: tackle non-local extensions of DMFT



- diagrammatic vertex approximation (DΓA)
- or DMF²RG
- parquet formalism

- Input: 4-point vertex of the impurity Hamiltonian

$$R = R_{\text{DMFT}}$$

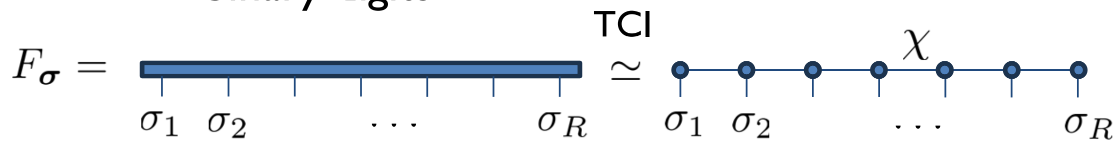
- Major challenge: vertex is "big object", with huge memory costs. Can it be compressed?

Outlook: Compression via Tensor Cross Interpolation (TCI)

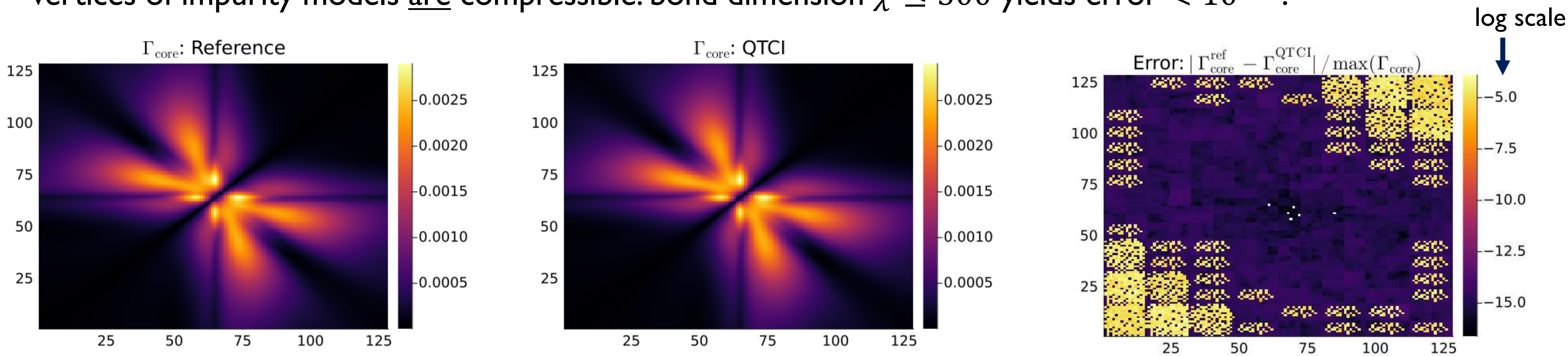
- Discretized functions can be expressed as tensors
- Sometimes, these tensors are compressible
- TCI is efficient scheme for finding compressed representation in form of a tensor train

$$f(x) = f(\sigma_1, \ldots, \sigma_R) = F_{\sigma_1, \ldots, \sigma_R} \qquad \sigma_\ell \in \{0, 1\}$$

binary digits



- Vertices of impurity models <u>are</u> compressible: bond dimension $\chi \leq 500$ yields error $< 10^{-5}$!



- Outlook for DMFT extensions: use TCI representation of vertices !

Fernández, Ritter, Jeannin, Li, Kloss, Louvet, Terasaki, Parcollet, von Delft, Shinaoka, Waintal, arXiv:2407.02454
Roshap, Ritter, von Delft, Kauch, Wallerberger, Shinaoka, in progress