Global fixed point potential approach to frustrated magnets

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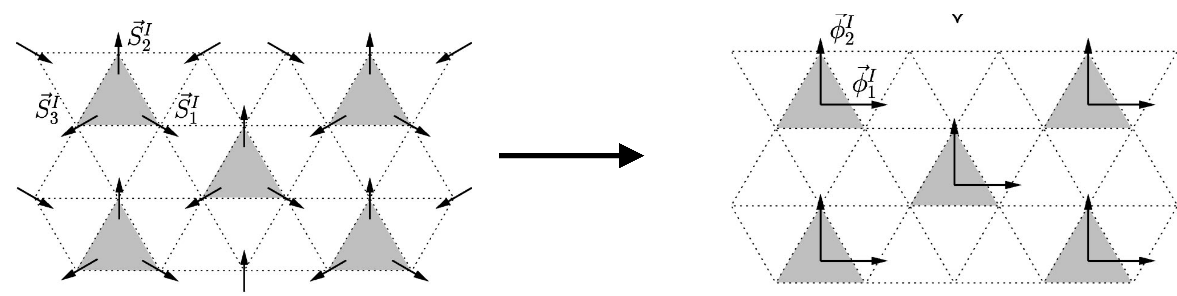
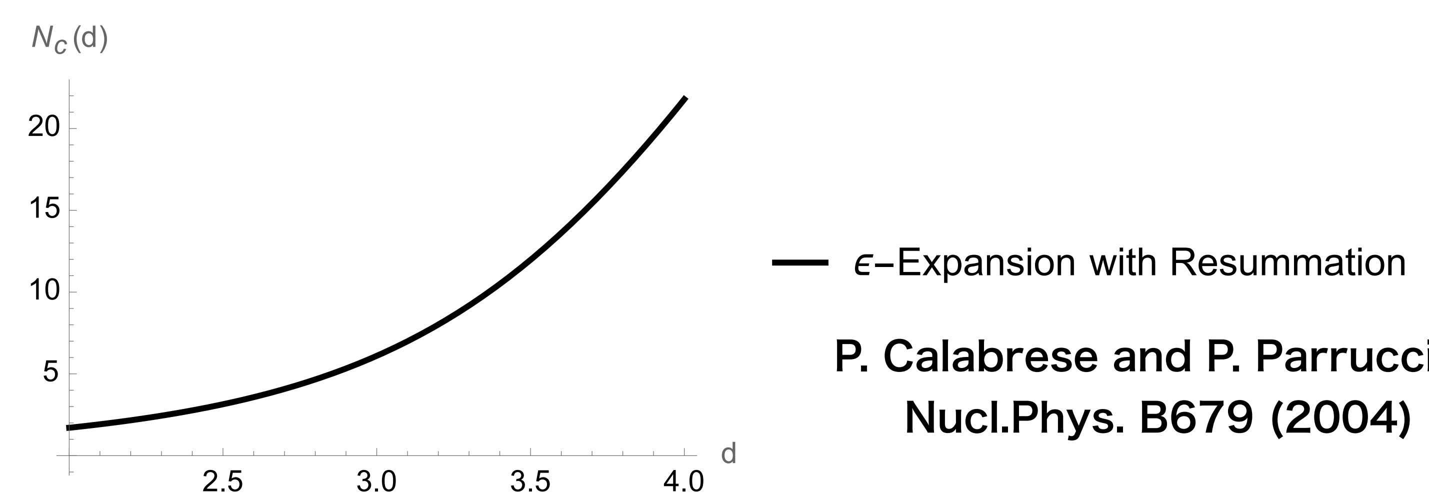
$O(N) \times O(2)$ models

- For Stacked Triangular Antiferromagnets (STA), the order parameter is $N \times 2$ matrix.



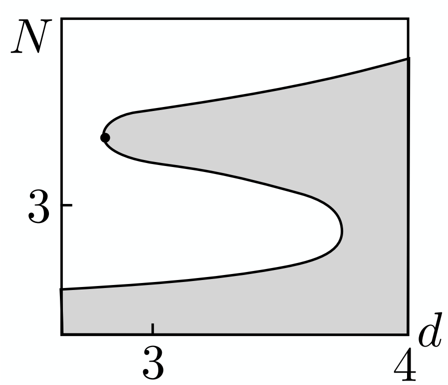
- For N above (below) $N_c(d)$, the transition is of second (first) order.



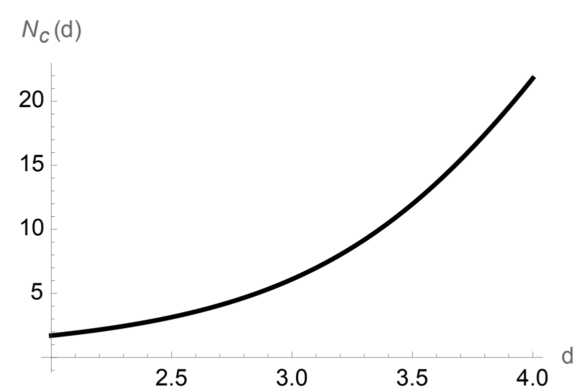
P. Calabrese and P. Parruccini, Nucl.Phys. B679 (2004)

$O(N) \times O(2)$ models

- $N_c(d=3)$ is found to be very close to 3 and its precise determination is crucial to know whether the transition is first or second order for the systems realized in nature.



Reehorst-Rychkov-Sirois-van Rees, arXiv 2024



- It is also interesting for physics in $d = 3$ to study the part of the curve $N_c(d)$ below $d = 3$ (However it was difficult to study it within previous approximations of NPRG).

Numerical simulations for STA

- Numerical simulations for XY and Heisenberg STA found first order transition (Loison-Schotte, Itakura, Thanh Ngo-Diep).

Numerical simulations for STA

- A recent simulation of Heisenberg STA with a very large lattice size 384^3 found second order transition corresponding to a focus FP with a complex-valued correction-to-scaling exponent (Nagano-Kawamura).

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Monte Carlo study of the critical properties of noncollinear Heisenberg magnets: $O(3) \times O(2)$ universality class

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The critical properties of the antiferromagnetic Heisenberg model on the three-dimensional stacked-triangular lattice are studied by means of a large-scale Monte Carlo simulation in order to get insight into the controversial issue of the criticality of the noncollinear magnets with the $O(3) \times O(2)$ symmetry. The maximum size studied is 384^3, considerably larger than the sizes studied by the previous numerical works on the model. Availability of such large-size data enables us to examine the detailed critical properties including the effect of corrections to the leading scaling. Strong numerical evidence of the continuous nature of the transition is obtained. Our data indicate the existence of significant corrections to the leading scaling. Careful analysis by taking account of the possible corrections yields critical exponents estimates, $\alpha = 0.44(3)$, $\beta = 0.26(2)$, $\gamma = 1.03(5)$, $\nu = 0.52(1)$, $\eta = 0.02(5)$, and the chirality exponents, $\beta_\kappa = 0.40(3)$ and $\gamma_\kappa = 0.77(6)$, supporting the existence of the $O(3)$ chiral [or $O(3) \times O(2)$] universality class governed by a "chiral" fixed point. We also obtain an indication that the underlying fixed point is of the focus type, characterized by the complex-valued correction-to-scaling exponent, $\omega = 0.1^{+0.4}_{-0.05} + i\,0.7^{+0.1}_{-0.4}$. The focus-like nature of the chiral fixed point accompanied by the spiral-like renormalization-group (RG) flow is likely to be the origin of the apparently complicated critical behavior. The results are compared and discussed in conjunction with the results of other numerical simulations, several distinct types of RG calculations including the higher-order perturbative massive and massless RG calculations and the nonperturbative functional RG calculation, and the conformal-bootstrap program.

Renormalization group studies on $O(N)\times O(2)$ models

- With ϵ-expansion, the value of $N_c(d=3)$ is systematically found larger than 3 as it is also the case for the NPRG calculations

- On the contrary, the perturbative calculation performed directly in $d=3$ at six loops yields a focus fixed point for $N=2,3$, suggesting the second order phase transition.

Conformal bootstrap studies on $O(N) \times O(2)$ models

- An early study found a critical FP for $N = 2{,}3$ in $d = 3$ and the critical exponents in good agreement with the focus FPs found with the perturbative fixed dimensional approach.
(Ohtsuki-Nakayama PRD 2014,2015)

- In a recent refined conformal bootstrap study, a lower bound for systems satisfying reflection positivity is $N_c(d) > 3.78$.
(Reehorst-Rychkov-Sirois-van Rees, arXiv 2024)

Experimental results

- Helimagnets are expected to belong to the same universal class.

Compound	Ref.	α	β	γ	ν
Tb	[102]	0.20(3)			
	[103]		0.23(4)		
	[104]		0.21(2)		
	[105]				0.53
Ho	[106]	$1^{\rm st}$order			
	[107]	0.27(2)			
	[95]	0.10-0.22			
	[108]		0.30(10)	1.24(15)	0.54(4)
	[108]		0.37(10)		
	[109]		0.39(3)		
	[110]		0.39(2)		
	[111]		0.39(4)		
	[112]		0.39(4)		
	[112]		0.41(4)		
	[113]			1.14(10)	0.57(4)
	[114]		0.38(1)		
Dy	[115]		0.335(10)		
	[116]		$0.39^{+0.04}_{-0.02}$		
	[110]		0.38(2)		
	[109]		0.39(1)		
	[113]			1.05(7)	0.57(5)
	[117]	0.24(2)			

TABLE II: The critical exponents of the XY helimagnets.

Compound	Ref.	α	β	γ	ν
VCl_2	[133]		0.20(2)	1.05(3)	0.62(5)
VBr_2	[134]	0.30(5)			
A	[135]		0.22(2)		
B	[87, 136, 137]		0.24(1)	1.16(3)	
	[138]	0.244(5)			
$CsNiCl_3$	[98, 139]	0.25(8)			
	[99]	0.23(4)			
	[100]		0.28(3)		
$CsMnI_3$	[98]	0.28(6)			
C	[140]	0.23(7)			
	[141]		0.29(1)	[0.75(4)]	[0.42(3)]
	[142]		0.28(2)		

TABLE IV: The critical exponents of the Heisenberg STA. The abbreviations A, B and C stand for $Cu(HCOO)_2 2CO(ND_2)_2 2D_2O$, $Fe[S_2CN(C_2H_5)_2]_2Cl$ and $CsMn(Br_{0.19}I_{0.81})_3$ respectively. The data in brackets are suspected to be incorrect. They are given for completeness.

B. Delamotte, D. Mouhanna, and M. Tissier
Phys. Rev. B (2004)

Revisiting $O(N) \times O(2)$ models with NPRG (after some refinement) might be useful.

Approximations in NPRG

- Field expansion of the local potential

- Derivative expansiton (DE) of effective action

Discrepancy in $d = 3$ between NPRG and some bootstrap studies may be due to the approximations used in NPRG.

In this study, we do not make any field expansion of the local potential (but do derivative expansions).

$O(N) \times O(2)$ models

- The order parameter for STA is a $N \times 2$ matrix $\Phi = (\phi_1, \phi_2)$ that satisfies $\phi_i \cdot \phi_2 = \delta_{i,j}$ for $i, j = 1{,}2.$ The constraint can be replaced by a soft potential $U\left(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2\right)$ whose minima are given by $\phi_i \cdot \phi_2 = const\, \delta_{i,j}$. The effective Hamiltonian is

$$H = \int d^d\mathbf{x} \left(\frac{1}{2} \left[\left(\partial \boldsymbol{\phi}_1\right)^2 + \left(\partial \boldsymbol{\phi}_2\right)^2 \right] + U\left(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2\right) \right)$$

Derivative expansion (DE) of the effective action

$$\Gamma_k = \int_x \left\{ U_k(\rho, \tau) + \frac{1}{2} Z_k \Big(\left(\partial \boldsymbol{\varphi}_1\right)^2 + \left(\partial \boldsymbol{\varphi}_2\right)^2 \Big) + \frac{1}{4} \omega_k \left(\boldsymbol{\varphi}_1 \cdot \partial \boldsymbol{\varphi}_2 - \boldsymbol{\varphi}_2 \cdot \partial \boldsymbol{\varphi}_1 \right)^2 \right\} .$$

- This approximation, that we call LPA' with ω_k, reproduces one-loop results by $\epsilon = 4 - d$ and $\epsilon = d - 2$ expansions. We call the approximation setting $\omega_k = 0$ here as LPA'. At criticality $Z_{k \to 0} \sim \left(\frac{k}{\Lambda} \right)^{-\eta}$.
- The approximation with $\eta = 0$ ($Z_k = const$) and $\omega_k = 0$ is called LPA.

Flow equation

$$\partial_t \Gamma_k[\boldsymbol{\varphi}_i] = \frac{1}{2}\mathrm{Tr}\int_{x,y} \partial_t R_k(x-y)\left(\frac{\delta^2\Gamma_k\left[\boldsymbol{\varphi}_i\right]}{\delta\varphi_i^{\alpha}\left(\mathbf{x}\right)\delta\varphi_{i'}^{\alpha'}\left(\mathbf{y}\right)} + R_k\left(\mathbf{x}-\mathbf{y}\right)\delta_{i,i'}\delta_{\alpha,\alpha'}\right)^{-1}$$

- $\alpha, \alpha' = 1,2,\ldots,N$ and $i, i' = 1,2$
- We employ a family of regulators $R_k\left(\mathbf{q}^2\right) = \beta Z_k k^2\left(1-\frac{\mathbf{q}^2}{k^2}\right)^{\alpha}\Theta\left(k^2-\mathbf{q}^2\right)$, which is useful for analytical treatment when $\alpha = \beta = 1$
- We evaluate η and ω_k at the minimum of the potential.

Local potential of $O(N) \times O(2)$ models

- For any $\boldsymbol{\varphi}_1$ and $\boldsymbol{\varphi}_2$, we can "diagonalize" the matrix $\Psi = (\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2)$ as $M \equiv O_1 \Psi O_2$ so that

$$M \equiv \begin{pmatrix} \psi_1 & 0 \\ 0 & \psi_2 \\ \vdots & \vdots \\ 0 & 0 \end{pmatrix}.$$

- Because of $O(N) \times O(2)$ symmetry $U_k(\Psi) = U_k(M)$.

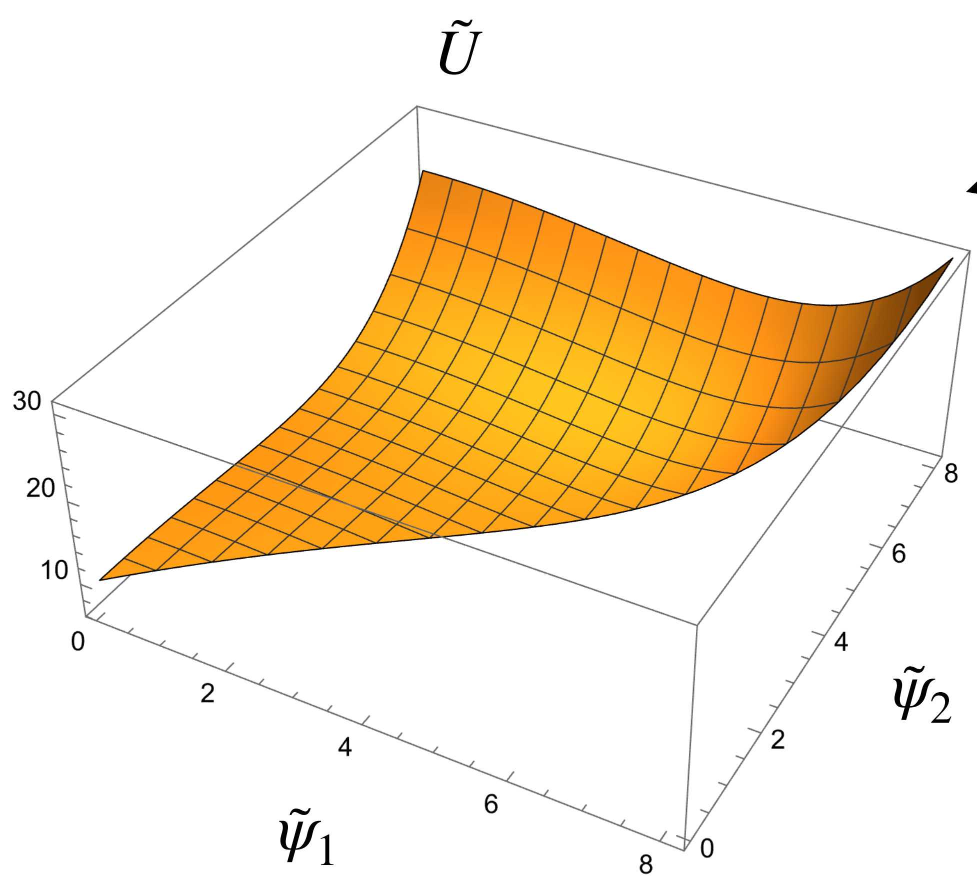
- The potential is a function of the $O(N) \times O(2)$ invariants $\rho = \psi_1^2 + \psi_2^2$ and $\tau = \frac{1}{4}\left(\psi_1^2 - \psi_2^2\right)^2$, which satisfies that

$$U_k\left(\psi_1, \psi_2\right) = U_k\left(-\psi_1, \psi_2\right) = U_k\left(\psi_1, -\psi_2\right) = U_k\left(\psi_2, \psi_1\right)$$

Dimensionless variables and FP potential

- In order to find FP solutions, we employ the following dimensionless quantities

$$\tilde{\psi}_i = \left(Z_k k^{2-d}\right)^{1/2} \psi_i, \quad \tilde{U}_k(\tilde{\psi}_i) = k^{-d} U_k\left(\psi_i\right).$$

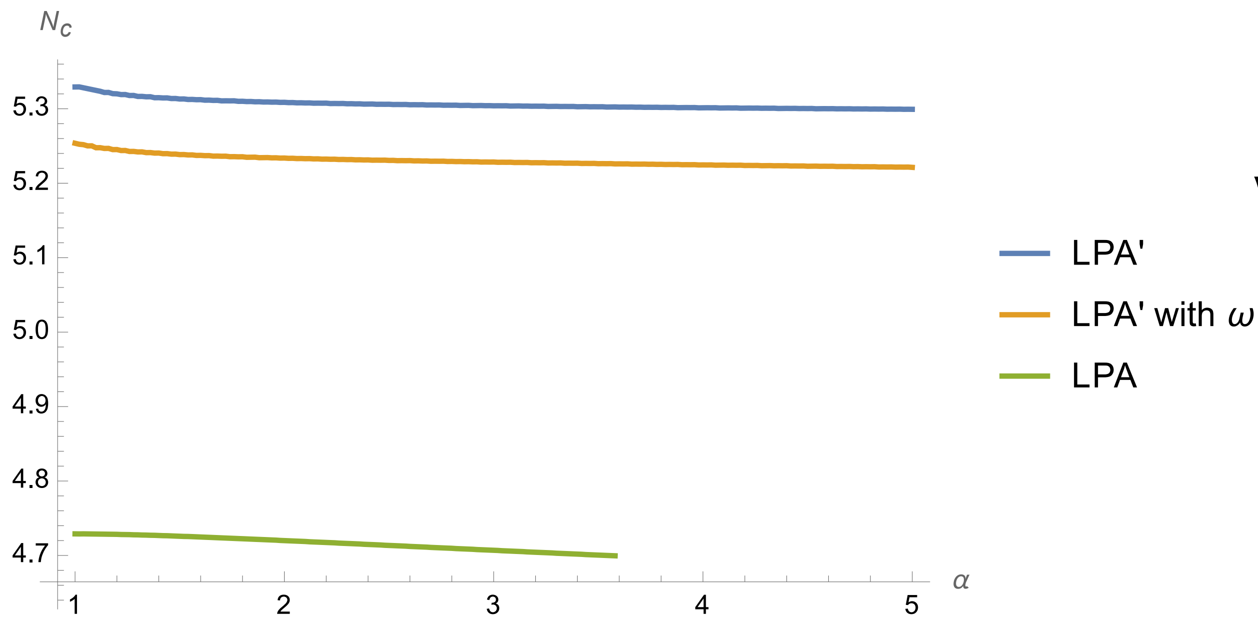


FP potential for C_+

(1-unstable critical FP)

C_+ exists for $N \geq N_c(d)$ and it vanishes at $N = N_c(d)$ by colliding with C_- (2-unstable multicritical FP)

$N_c(d=3)$



We fix $\beta = 1$

- The variations of $N_c(d=3)$ with respect to the parameter α seem much smaller than the uncertainty associated with the truncation of the DE (LPA, LPA', LPA' with ω_k).

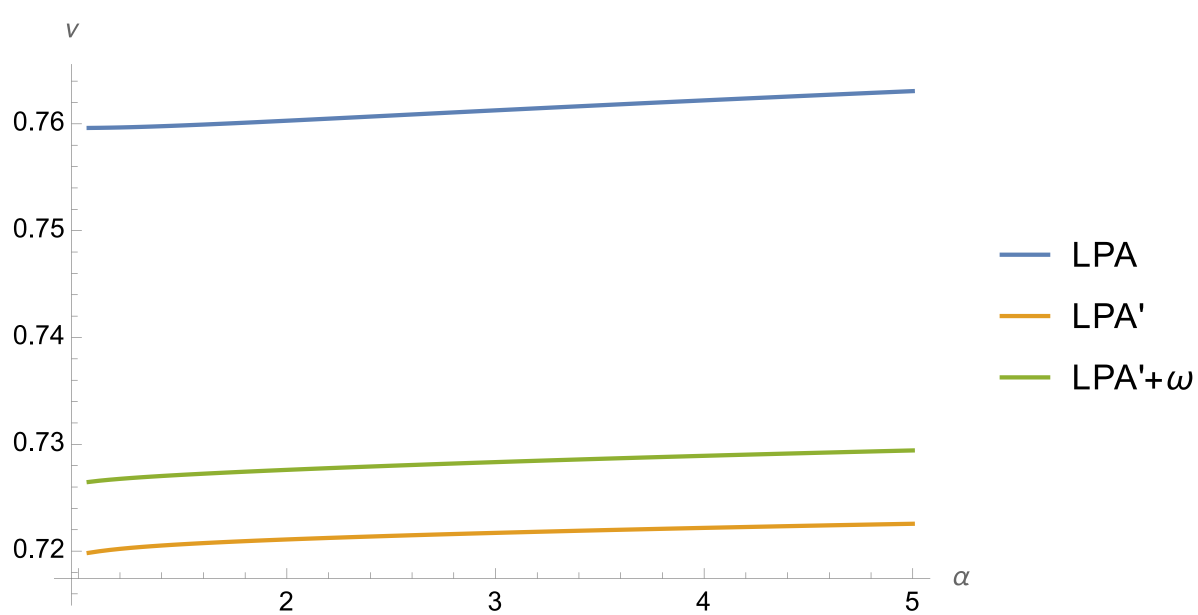
- This confirms previous NPRG results with "semi-expansion" $N_c^{\mathrm{semi}}(d=3) \simeq 4.68(2), 5.24(2)$ at LPA, LPA' with ω_k.

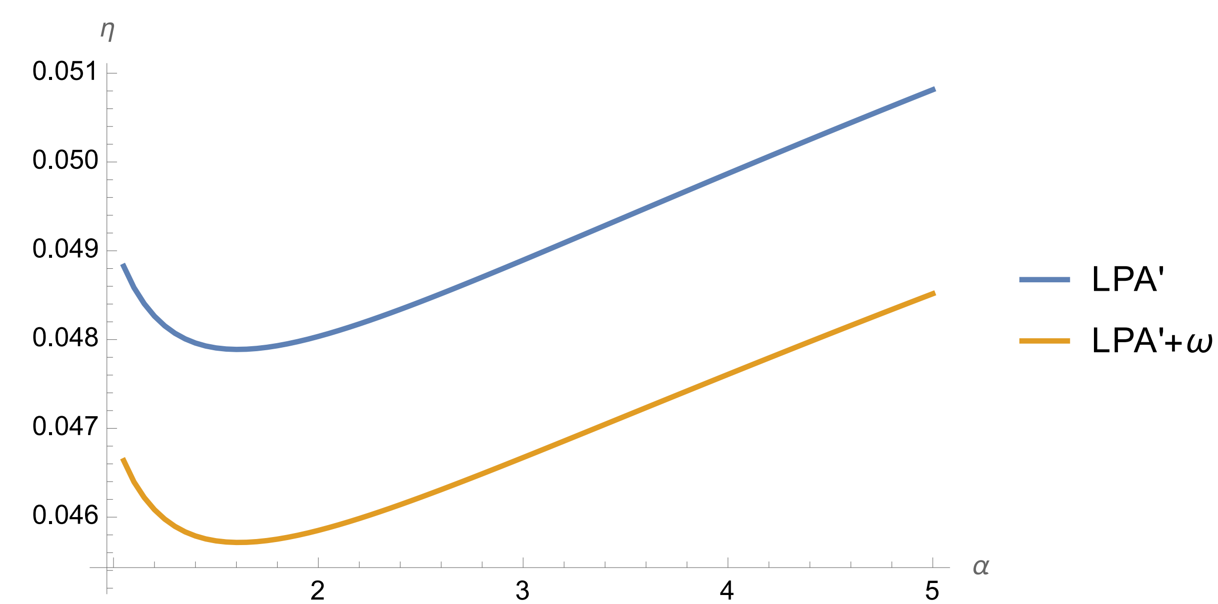
- NPRG results seem to converge well within each approximation (LPA, LPA', LPA' with ω_k).

Critical exponents

ν, η for $N = 6, d = 3$

We fix $\beta = 1$

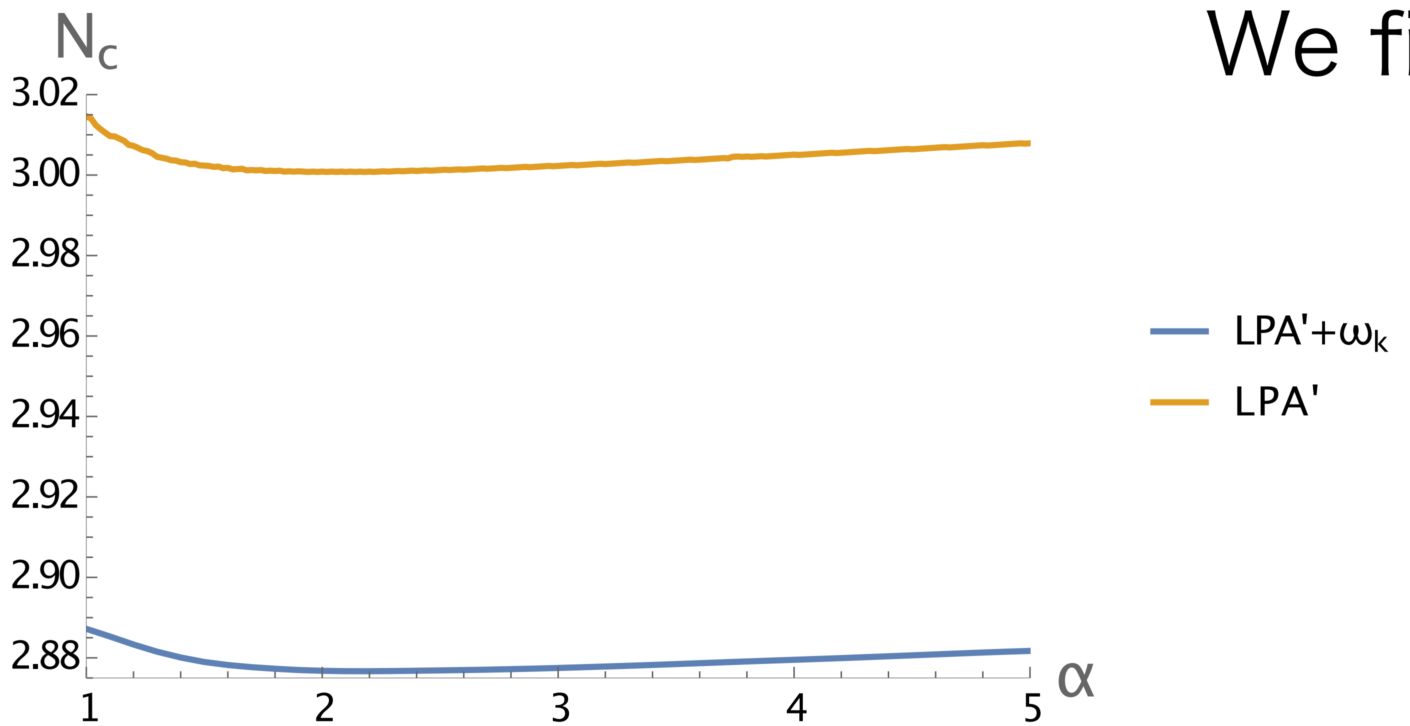




- Previous NPRG ("semi-expansion" with a different regulator) results give $\nu(N=6) = 0.695(5)$ and $\eta(N=6) = 0.042(2)$. Assuming that these results are converging, the dependence on the regulator is not small contrary to $N_c(d=3)$.

- The correction-to-scaling exponent (not shown) is real.

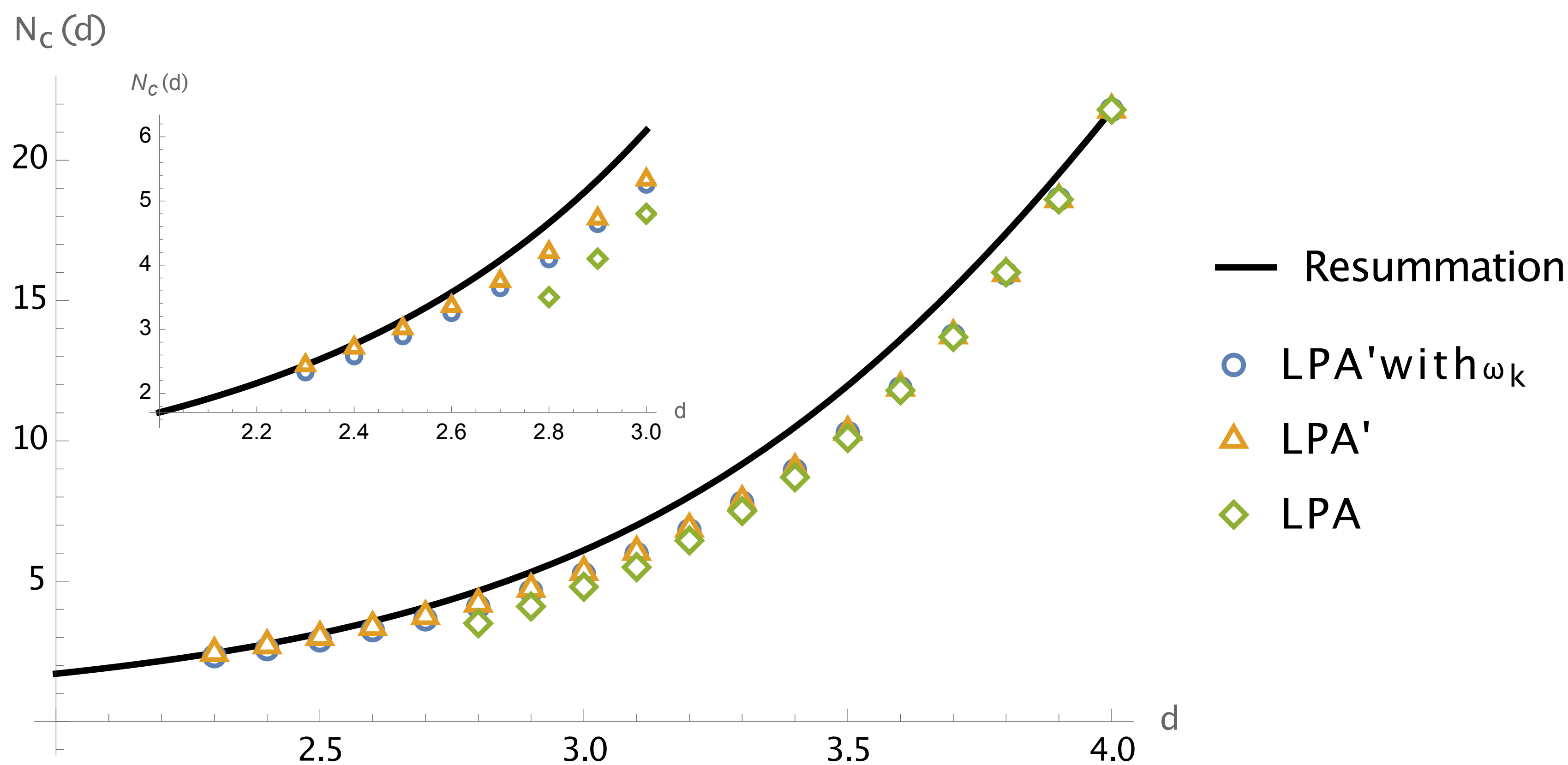
$N_c(d = 2.5)$



We fix $\beta = 1$

- In lower dimensions, the roles of η and ω_k are important.
- We recall that $N_c(d \leq 3)$ is interesting for physics in $d = 3$
- The solutions are numerically under control.

$N_c(d)$ in $2.3 \leq d \leq 4$

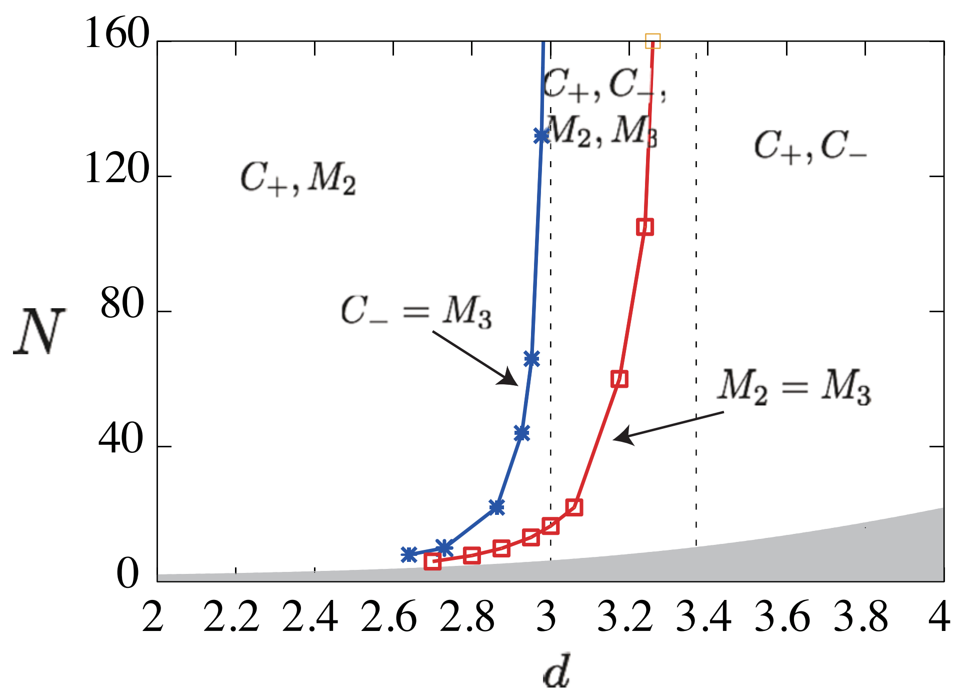


- The spreading of $N_c(d, \alpha, \beta)$ when α is varied between 1 and 5 and β between 1 and 2 is almost invisible on this scale.

- This suggests that S-shape behavior of $N_c(d)$ around $d = 3$ is not probable.

- As for the approach to $d = 2$, we have found good agreement with the ϵ expansion results after resummation.

Multicritical problems



M_2, M_3 **…2,3-unstable**

nonperturbative FP found with field expansion

S. Yabunaka and B. Delamotte, PRL 2017

- For multicritical FPs in $O(N)$ models, we found boundary layer behavior at Large-N, which cannot be captured by field expansions.

- Therefore it may be interesting to study the multicritical FP in $O(N) \times O(2)$ models without any field expansion (in progress).

Summary

- We have obtained $N_c(d)$ in $2.3 \le d \le 4$ solving the FP equation without any field expansion (but with derivative expansion).
- In $d = 3$, our results confirm what was found with field expansions of NPRG and recent conformal bootstrap results.
- As for the approach to $d = 2$, we have found good agreement with the ϵ expansion after resummation.