Effective field theory at nuclear scales from the functional renormalization group

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Outline

❶ Context

Where does the EDF method stand within the landscape of nuclear structure theories ?

❷ Lessons from empirical EDFs

1st lesson : Effective (pseudo-)Hamiltonians with simple forms do the job

2nd lesson : Static correlations can be optimally grasped via SSBs + bosonic fluctuations of order parameters

❸ Towards a rigorous formulation of nuclear EDFs

WFT, DFT & EA perspectives

FRG

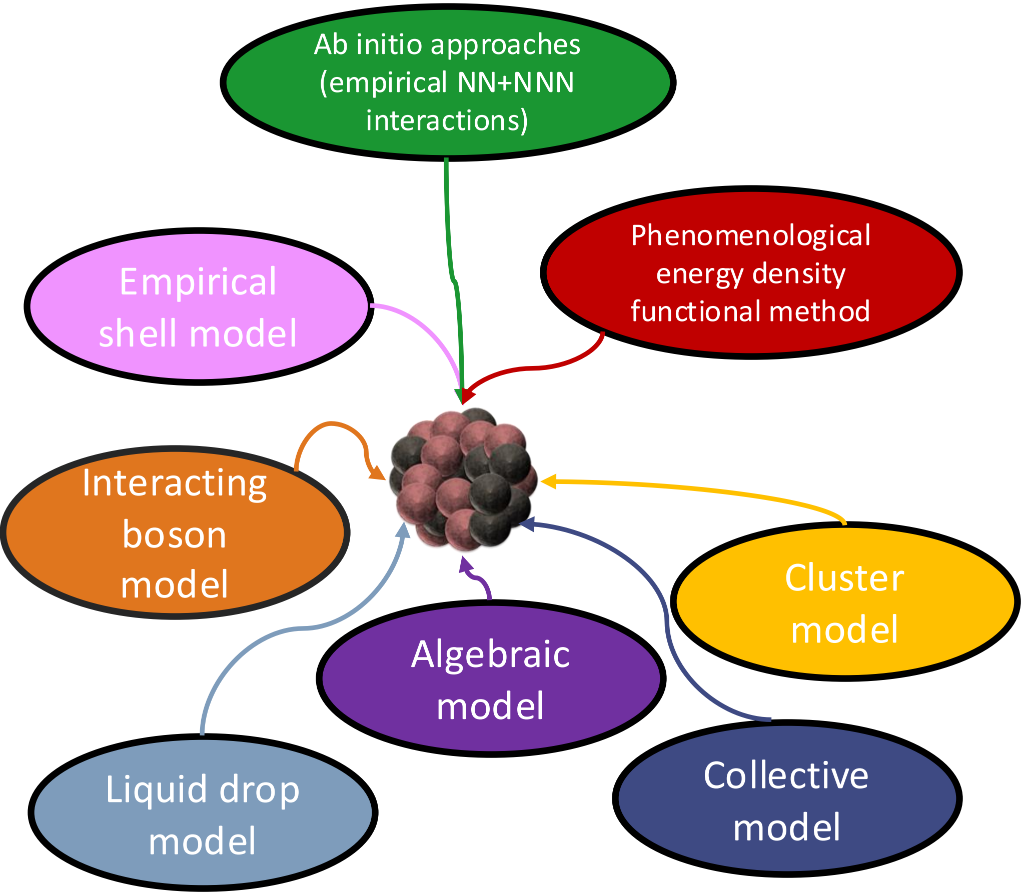
Application to symmetric nuclear matter

cea

❶ Context : Strategies

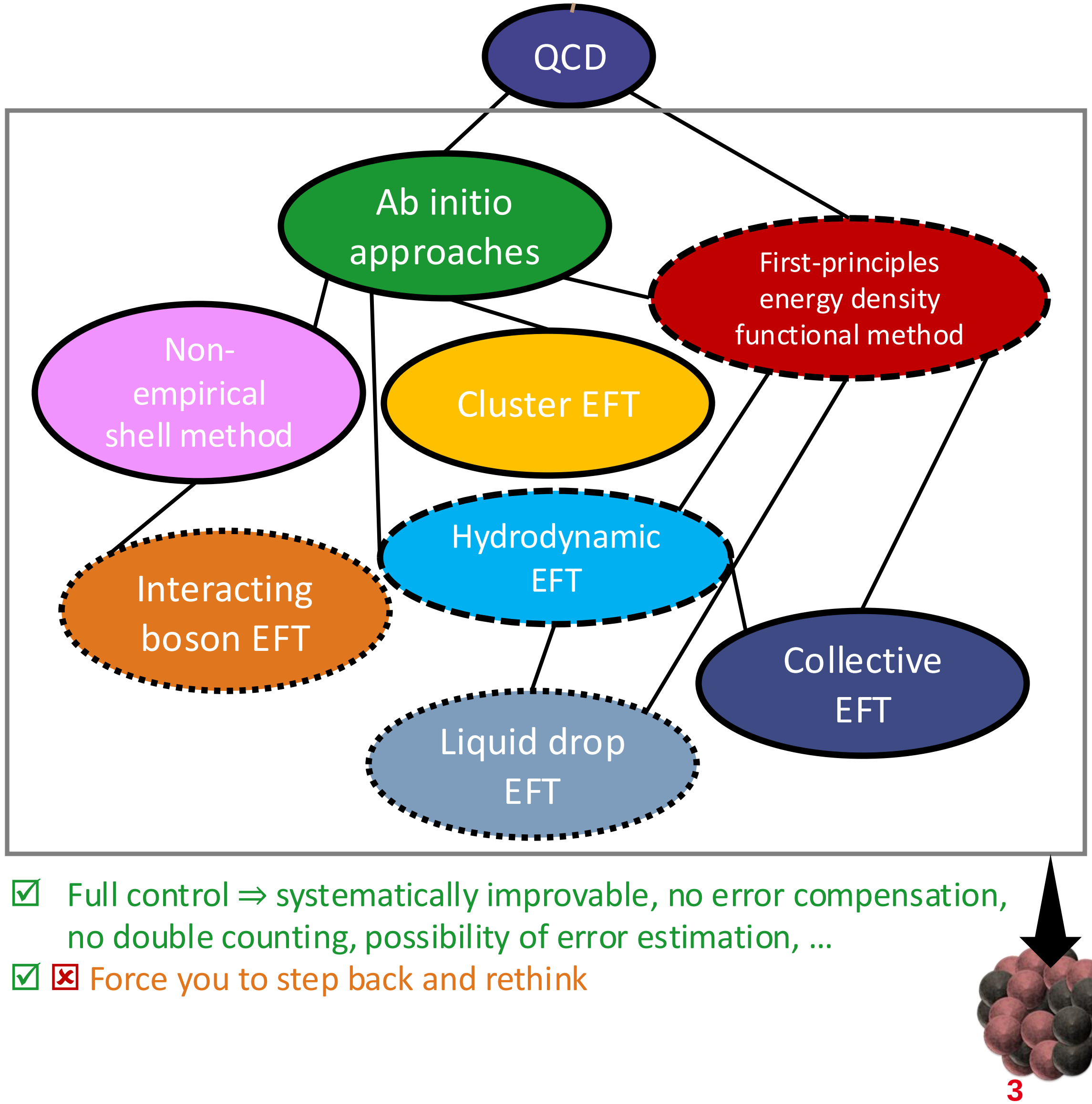
◉ Achieve a { *accurate* / *predictive* / *computationally affordable* } description ?

Era of models



☑ Gives insight about relevant scales/dofs
☑ Ready to be used
☒ Lack of control
⇒ double counting issues, error compensation, no error assessment

Era of effective (field) theories



☑ Full control ⇒ systematically improvable, no error compensation, no double counting, possibility of error estimation, …
☑ ☒ Force you to step back and rethink

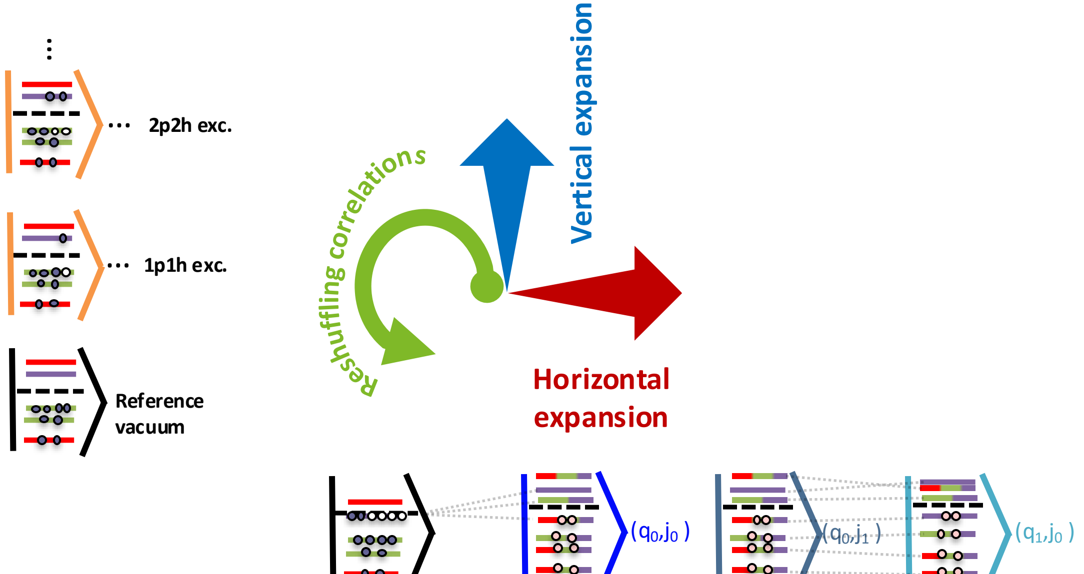
1 Context : Nuclear structure from a microscopic viewpoint

1) Nucleus: *A* interacting, structure-less nucleons
2) Structure & dynamic encoded in Hamiltonian, Functional, …
3) Solve *A*-nucleon Schrödinger/Dirac equation to desired accuracy

$$H(\ldots)|\Psi_{\mu,\sigma}\rangle = E_{\mu\tilde{\sigma}}\,|\Psi_{\mu,\sigma}\rangle$$

Strongly correlated WF

Rationale for grasping nucleon correlations



$U(\lambda)HU^{\dagger}(\lambda)$

Ab initio

- **Systematically improvable free-space Hamiltonian in χEFT**
- **Solving Schrödinger equation**
  - Pre-processing H
  - Refined many-body schemes with controlled uncertainties
    - --> *CI (full space diag.) : exponential scaling*
    - --> *Hybrids (valence space diag.) : mixed scaling*
    - --> ***Expansion methods (partition, expand and truncate) : polynomial scaling***

How to challenge ab initio frontiers

EDF

- **Effective pseudo-Hamiltonian**

Free-space interactions → *Effective in-medium interactions*

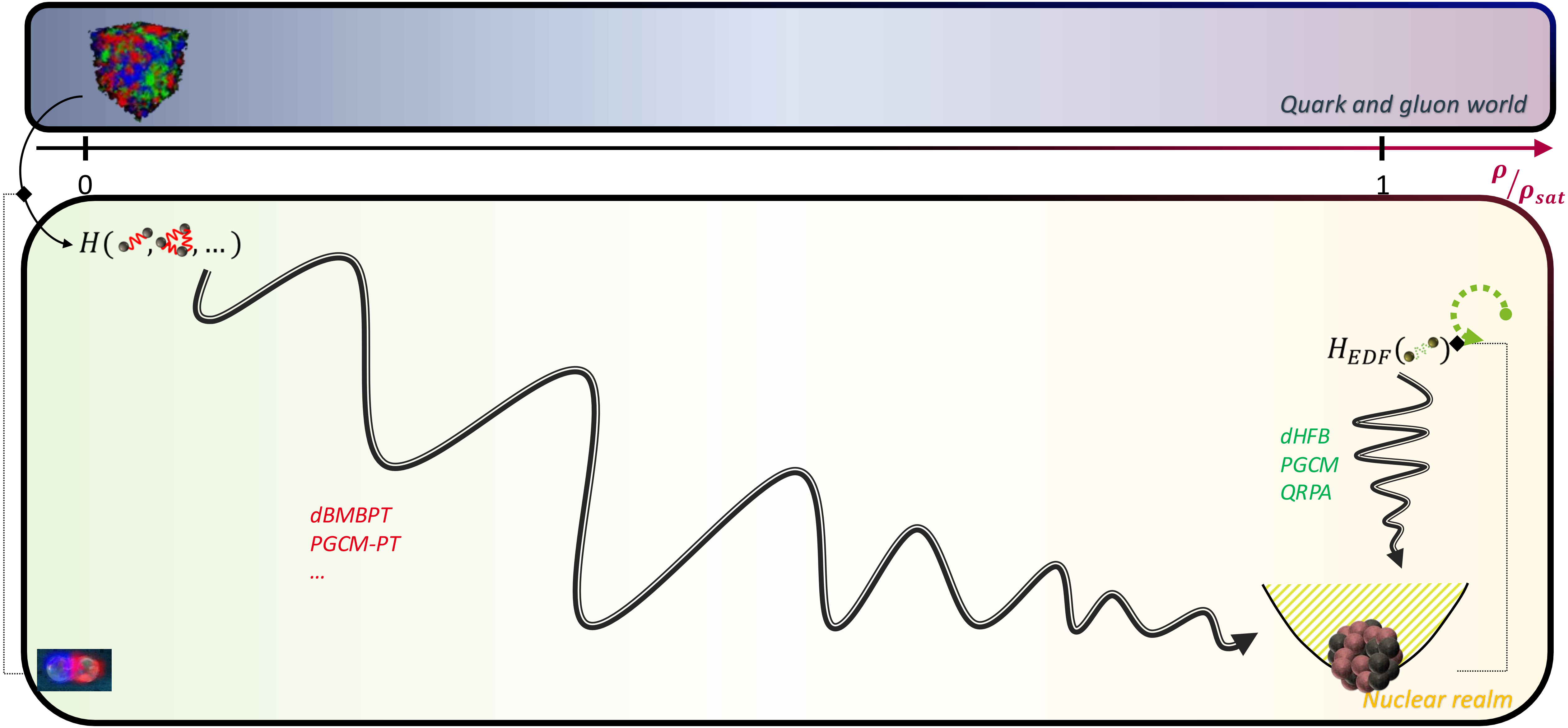
$|\Psi_{\mu,\sigma}\rangle$ *Complicated WF* → $|\Theta_{\mu\sigma}\rangle$ Simplified auxiliary WF

- **Various levels of realization**
  - --> *Hartree-Fock-Bogoliubov (HFB)*
  - --> *Projected Generator Coordinate Method (PGCM)*
  - --> *Quasiparticule Random Phase Approximation (QRPA)*

How to improve current EDFs

How to turn EDF in EFT ?

❶ Context : Nuclear structure from a microscopic viewpoint



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2 Lessons from empirical EDFs : main idea

- **Hamiltonian H acting in $\mathcal{H}_A$** and **Schrödinger equation**

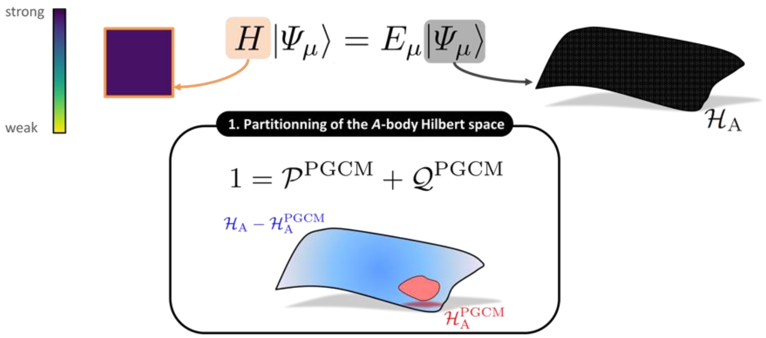
$$H = T + V + W + \cdots$$
$$= \frac{1}{(1!)^2}\sum_{\substack{a_1\\b_1}} t^{a_1}_{b_1} A^{a_1}_{b_1} + \frac{1}{(2!)^2}\sum_{\substack{a_1a_2\\b_1b_2}} v^{a_1a_2}_{b_1b_2} A^{a_1a_2}_{b_1b_2} + \frac{1}{(3!)^2}\sum_{\substack{a_1a_2a_3\\b_1b_2b_3}} w^{a_1a_2a_3}_{b_1b_2b_3} A^{a_1a_2a_3}_{b_1b_2b_3} + \cdots$$

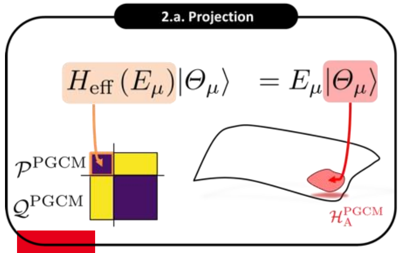
$$A^{a_1\ldots a_k}_{b_1\ldots b_k} \equiv c^\dagger_{a_1} \ldots c^\dagger_{a_k} c_{b_k} \ldots c_{b_1}$$

$$H|\Psi_\mu\rangle = E_\mu|\Psi_\mu\rangle$$

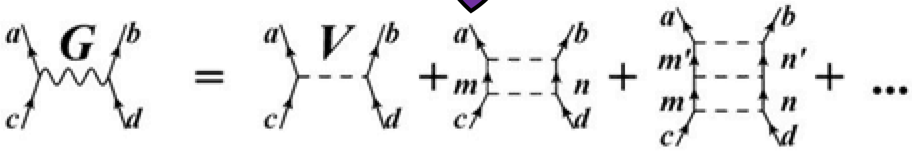
- EDF method postulates the existence of **H_{EDF} acting in $\mathcal{H}_A^{\mathrm{PGCM}}$** yielding the same low-energy observables than with H

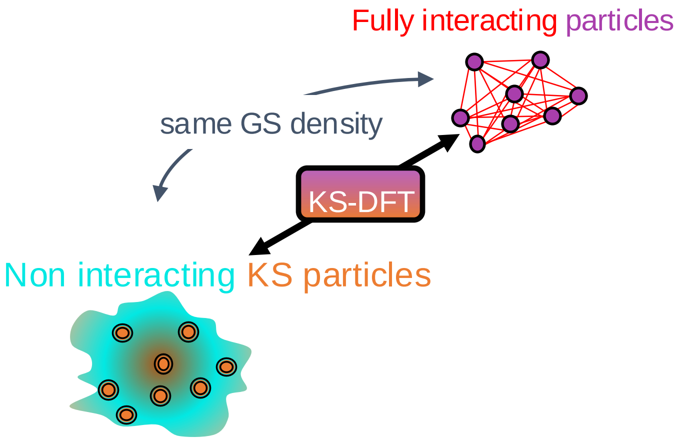
$$E_\mu = \frac{\langle\Psi_\mu|H|\Psi_\mu\rangle}{\langle\Psi_\mu|\Psi_\mu\rangle} = \frac{\langle\Theta_\mu|H_{EDF}|\Theta_\mu\rangle}{\langle\Theta_\mu|\Theta_\mu\rangle}, |\Theta_\mu\rangle \in \mathcal{H}_A^{\mathrm{PGCM}}$$





Projection **Resummation** **Mapping**





⇒ **Resulting objects are real monsters !!**

❷ Lessons from empirical EDFs : Lesson 1

◉ Empirical effective interactions with simple forms do the job !!

Galilean EDF — **Lorentzian EDF**

Explicit density-dependence

Galilean EDF: Gogny D1 vertex

$$V_{12} = \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)\, e^{-\frac{(\vec{r}_1-\vec{r}_2)^2}{\mu_i^2}}$$
$$+ t_0 (1 + x_0 P_\sigma)\, \delta(\vec{r}_1 - \vec{r}_2)\, \rho^\alpha\left(\frac{\vec{r}_1 - \vec{r}_2}{2}\right)$$
$$+ i W_{LS}\, \overleftarrow{\nabla}_{12}\, \delta(\vec{r}_1 - \vec{r}_2) \times \overrightarrow{\nabla}_{12} \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$$

Lorentzian EDF: DDME Lagrangians

$$\mathcal{L}_{NN} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - M - \sum_b g_b(\rho)\, \phi_b \mathcal{O}_b\right)\psi$$

$$g_i(\rho_v) = g_i(\rho_{sat}) f_i(\xi)\,,\ i = \sigma, \omega,$$
$$f_i(\xi) = a_i \frac{1 + b_i(\xi + d_i)^2}{1 + c_i(\xi + d_i)^2},$$
$$g_\rho(\rho_v) = g_\rho(0) e^{-a_\rho \xi},$$
$$f_\pi(\rho_v) = f_\pi(0) e^{-a_\pi \xi},$$

Non explicit density-dependence

Galilean EDF: Bennaceur et al semi-regularized vertex

$$\hat{V}(x_1, x_2; x_3, x_4) = \delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4) g_a(r_{12}) \hat{O}_j^{(n)}(\mathbf{k}_{12}, \mathbf{k}_{34})$$
$$\times \left\{W_\nu^{(n)} \hat{1}_\sigma \hat{1}_\tau + B_\nu^{(n)} \hat{P}_\sigma \hat{1}_\tau - H_\nu^{(n)} \hat{1}_\sigma \hat{P}_\tau - M_\nu^{(n)} \hat{P}_\sigma \hat{P}_\tau\right\}$$

$$\hat{V} = W_3\left(\hat{V}_1 + \hat{V}_2\right)$$

$$\hat{V}_1 = \hat{1}_r \hat{1}_q \hat{1}_\sigma g_{a_3}(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3)\,,$$
$$\hat{V}_2 = \hat{1}_r \hat{1}_q \hat{P}^\sigma_{23} g_{a_3}(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3)\,.$$

Lorentzian EDF: NL Lagrangians

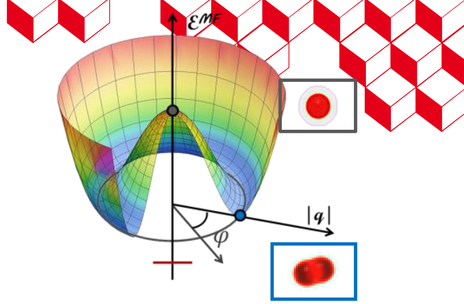
$$\mathcal{L}_{NN} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - M - \sum_b g_b \phi_b \mathcal{O}_b\right)\psi - U[\sigma]$$

$$U[\sigma] = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

--> Simple form ⇔ Fermi-liquid fixed point to be grasped via RG techniques ?

$$\mathcal{L}^\sigma_{NN} = [g_\sigma \bar{\psi}\sigma\psi](x), \qquad \mathcal{L}^\pi_{NN} = \left[\frac{f_\pi(\rho_v)}{m_\pi}\bar{\psi}\gamma^5\gamma^\mu \partial_\mu \vec{\pi} \star \vec{\tau}\psi\right](x)$$
$$\mathcal{L}^\omega_{NN} = [g_\omega \bar{\psi}\gamma^\mu \omega_\mu \psi](x),$$
$$\mathcal{L}^\rho_{NN} = [g_\rho \bar{\psi}\gamma^\mu \vec{\rho}_\mu \star \vec{\tau}\psi](x), \quad \mathcal{L}^{\omega+\rho;T}_{NN} = \left[\bar{\psi}\sigma^{\mu\nu}\left(-\frac{g^T_\omega}{2M}\Omega_{\mu\nu} - \frac{g^T_\rho}{2M}\vec{\mathcal{R}}_{\mu\nu} \star \vec{\tau}\right)\psi\right](x)$$

❷ Lessons from empirical EDFs : Lesson 2

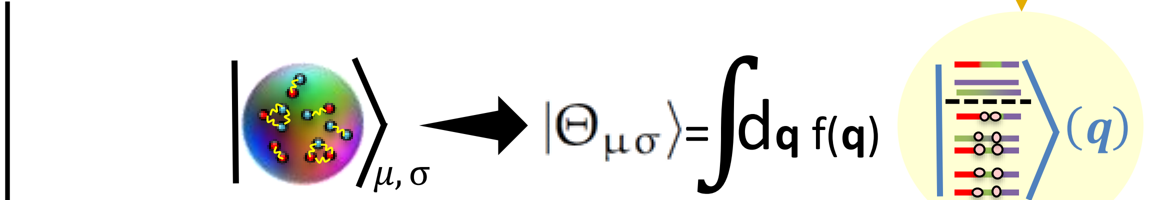


◉ Lesson n°2 : GS + low-lying collective excited states via horizontal expansion

◈ dHFB treatment

◈ Post-HFB treatment : PGCM

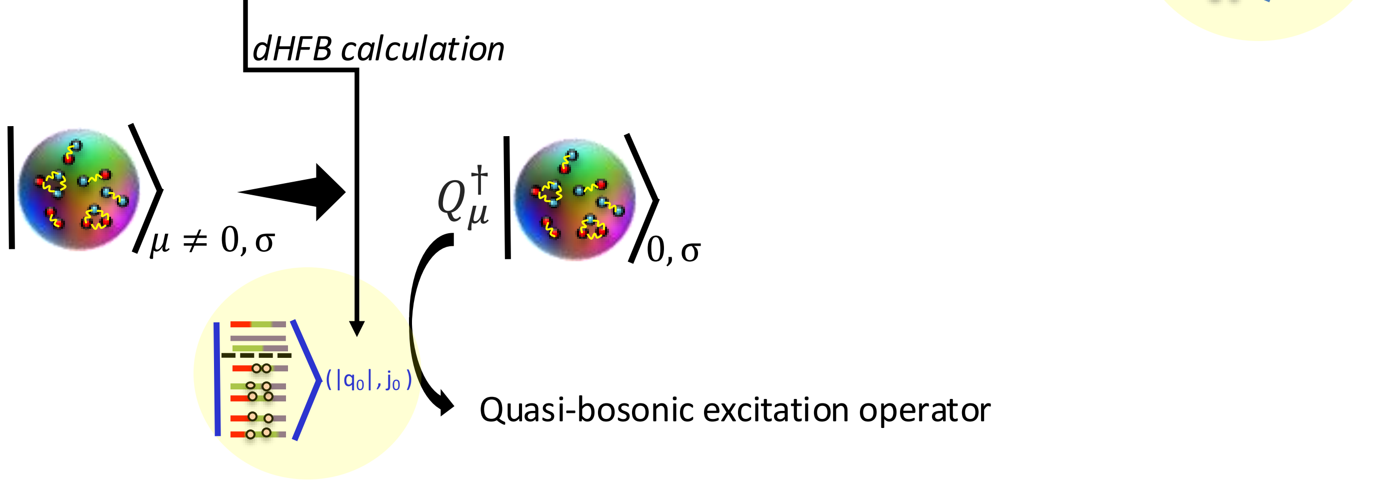
--> Symmetry-conserving (non orthogonal) mixture of symmetry-breaking HFB vacua



◈ Post-HFB : QRPA

--> Excitations = coherent mixture of 2-qp excitations

--> Harmonic limit of the GCM



--> Static correlations : fluctuations of bosonic order parameters

⇒ (Partially-)bosonizing the theory ?

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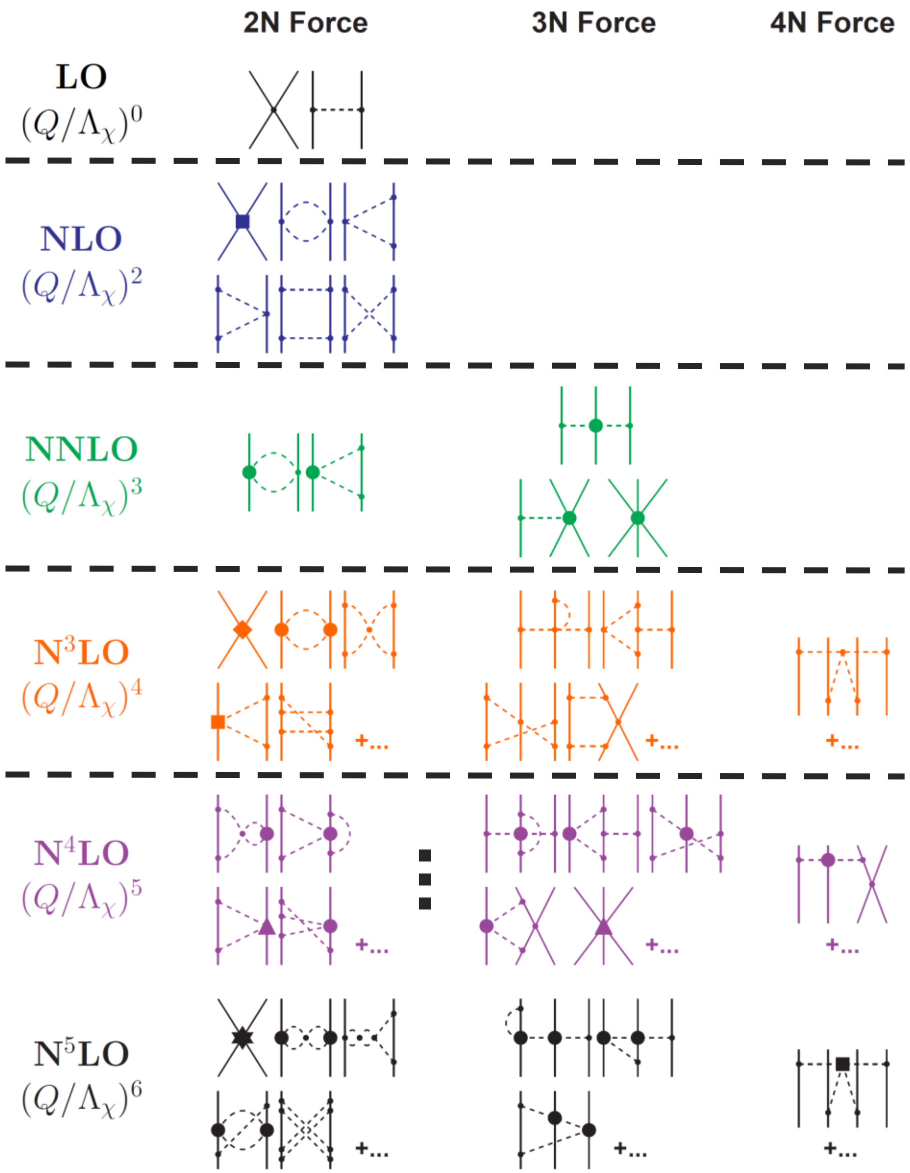
3 Towards a rigorous formulation of nuclear EDFS : WFT perspective

Chiral effective field theory = interactions expansion



$$H_{\rm LO} \equiv T + V^{\rm 2N}_{\rm LO}$$

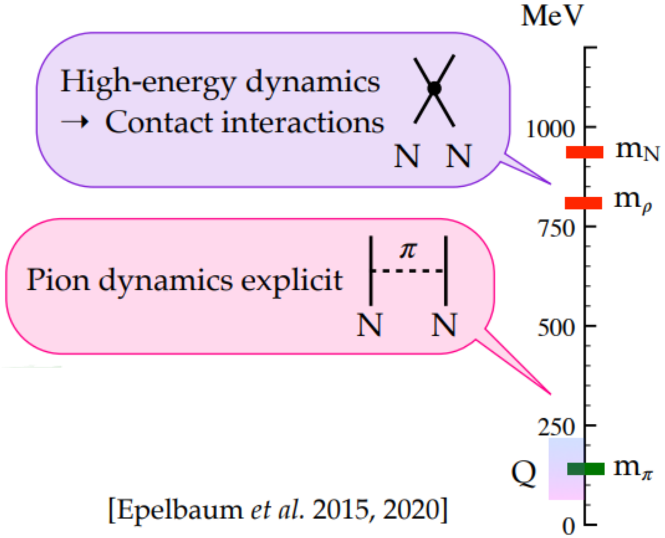
$$H_{\rm NLO} \equiv T + V^{\rm 2N}_{\rm NLO}$$

$$H_{\rm N^2LO} \equiv T + V^{\rm 2N}_{\rm N^2LO} + V^{\rm 3N}_{\rm N^2LO}$$

$$H_{\rm N^3LO} \equiv T + V^{\rm 2N}_{\rm N^3LO} + V^{\rm 3N}_{\rm N^3LO} + V^{\rm 4N}_{\rm N^3LO}$$

$$\vdots$$

$$H_{\rm N^kLO} \equiv T + V^{\rm 2N}_{\rm N^kLO} + V^{\rm 3N}_{\rm N^kLO} + \dots$$

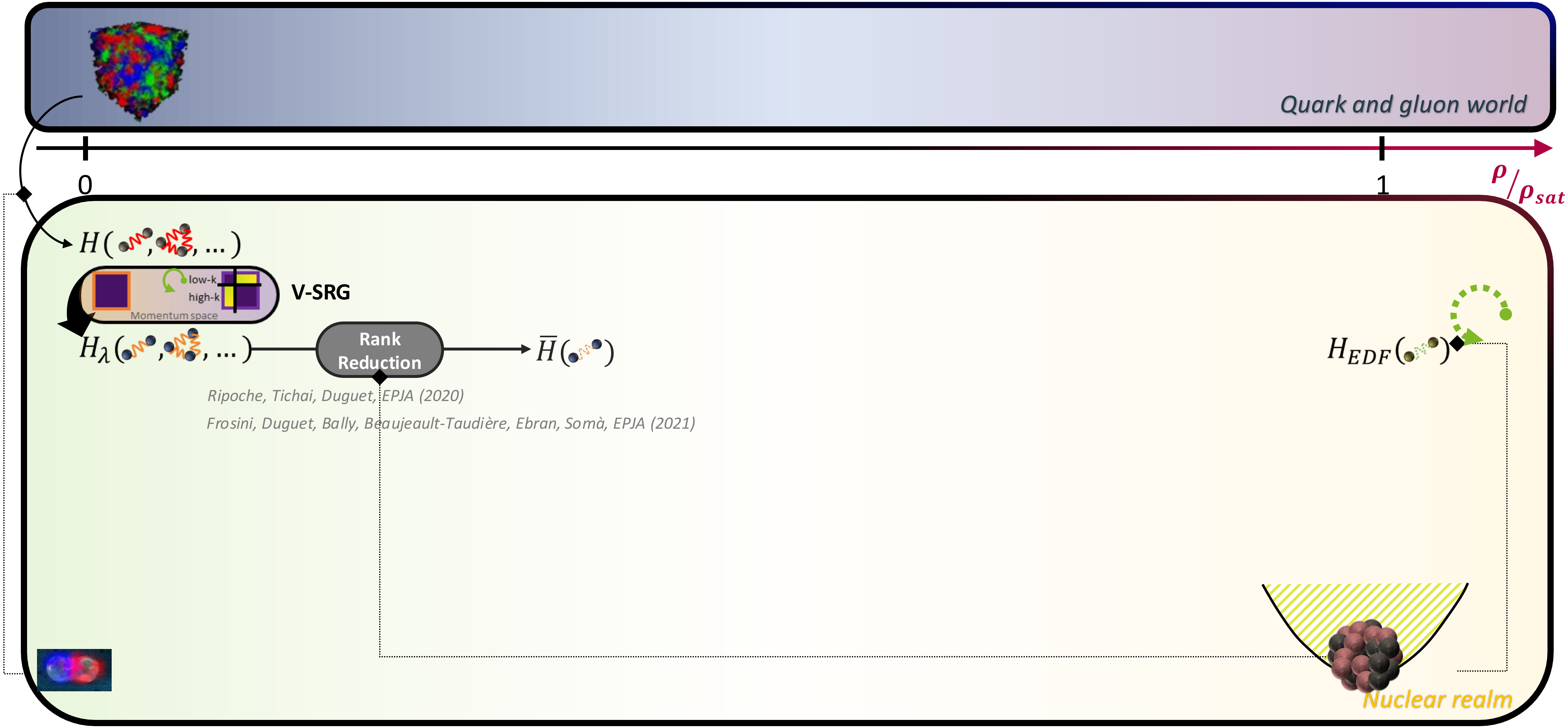


[Epelbaum *et al.* 2015, 2020]

Major challenges

- ▶Can k-body, k>3, be omitted in A>>3?
- ▶$N^{3/4}$LO 2N for high precision; 3N? 4N?
- ▶More profound issues…

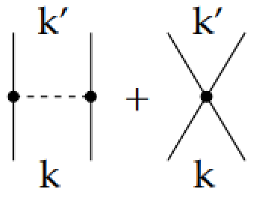
❸ Towards a rigorous formulation of nuclear EDFS : WFT perspective

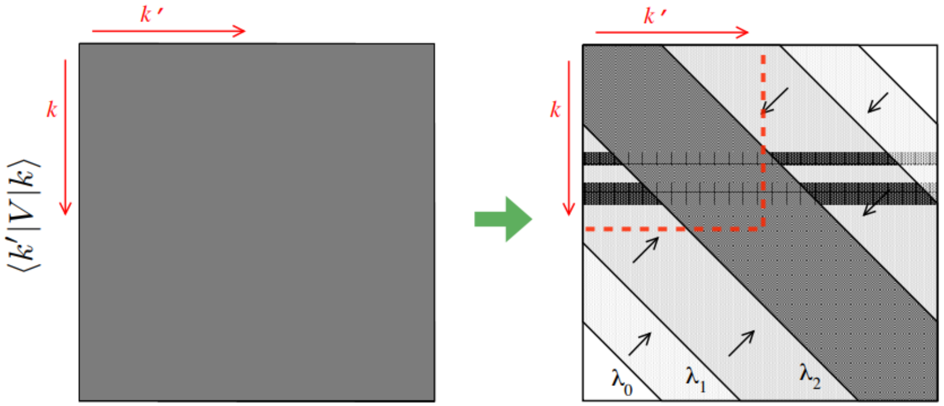


Similarity renormalization group transformation of H

►Need very large n_{dim} (e_{max}) due to **hard core of V^{2N}** → large ME between low and high momenta basis states

→ Unitary **Similarity Renormalization Group** (SRG) transformation of H to tame it down

$\langle k'|V|k\rangle =$  $+ \ldots$



[Bogner *et al.* 2010]

$$H_s = U_s H U_s^\dagger \equiv T_{\rm rel} + V_s$$

from which one finds the flow equation

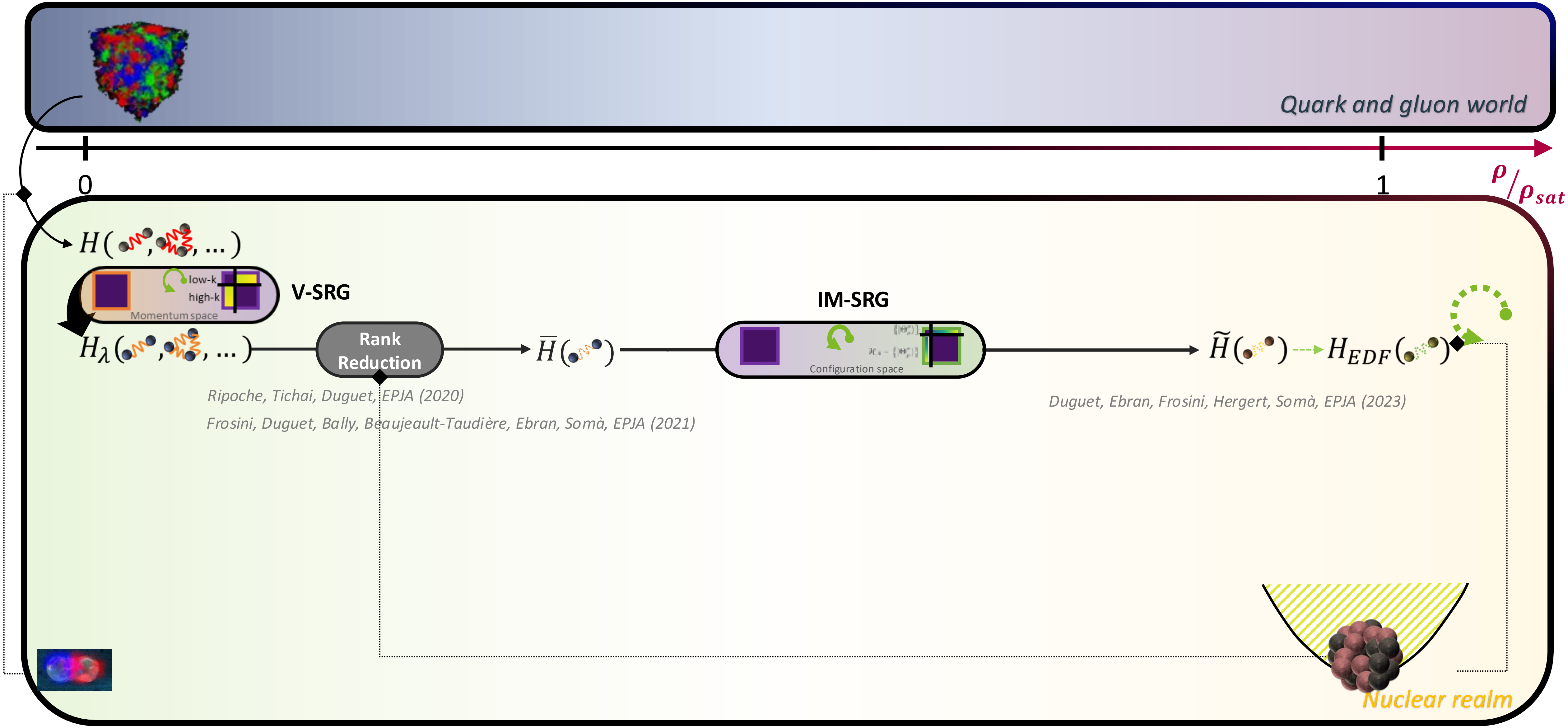
$$\frac{dH_s}{ds} = [\eta_s, H_s], \quad \eta_s = [T_{\rm rel}, H_s]$$

The flow parameter s is usually replaced with $\lambda = s^{-1/4}$ in units of fm^{-1} (a measure of the spread of off-diagonal strength).

Evolution of the potential

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2 dq (k^2 + k'^2 - 2q^2) V_s(k,q) V_s(q,k')$$

3 Towards a rigorous formulation of nuclear EDFS : WFT perspective



Ripoche, Tichai, Duguet, EPJA (2020)

Frosini, Duguet, Bally, Beaujeault-Taudière, Ebran, Somà, EPJA (2021)

Duguet, Ebran, Frosini, Hergert, Somà, EPJA (2023)

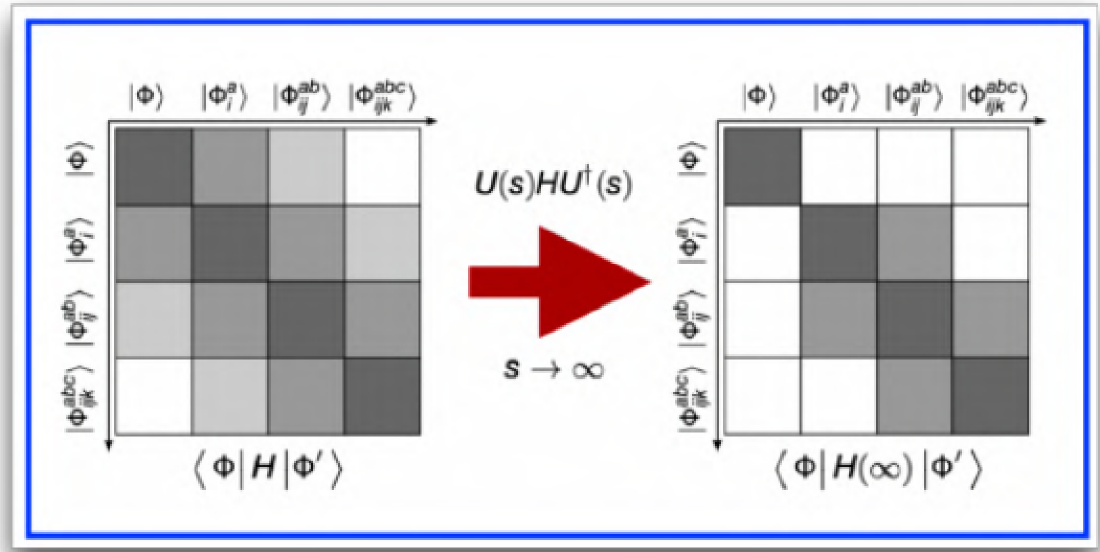
Apply unitary transformations to $\hat{H}$ in the configuration space to obtain ground state

$$\hat{H}(s) = \hat{U}(s)\hat{H}_0\hat{U}^\dagger(s)$$

- Flow equation

$$\frac{d\hat{H}(s)}{ds} = [\hat{\eta}(s), \hat{H}(s)]$$

- The generator $\eta(s)$ is chosen to decouple a given reference state from its excitations.
- Not necessary to construct the whole H matrix in the configuration space.



H. Hergert et al., Phys. Rep. 621, 165 (2016)

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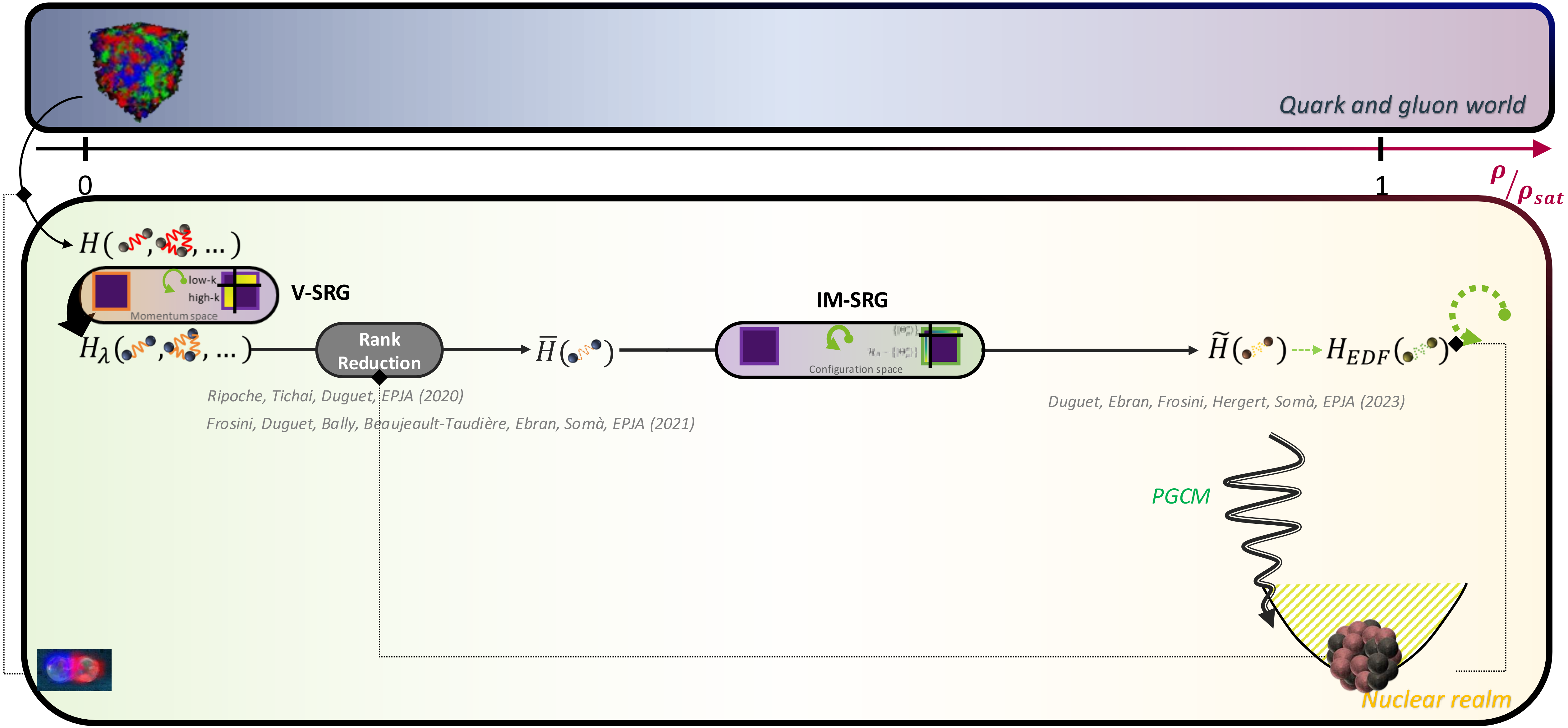
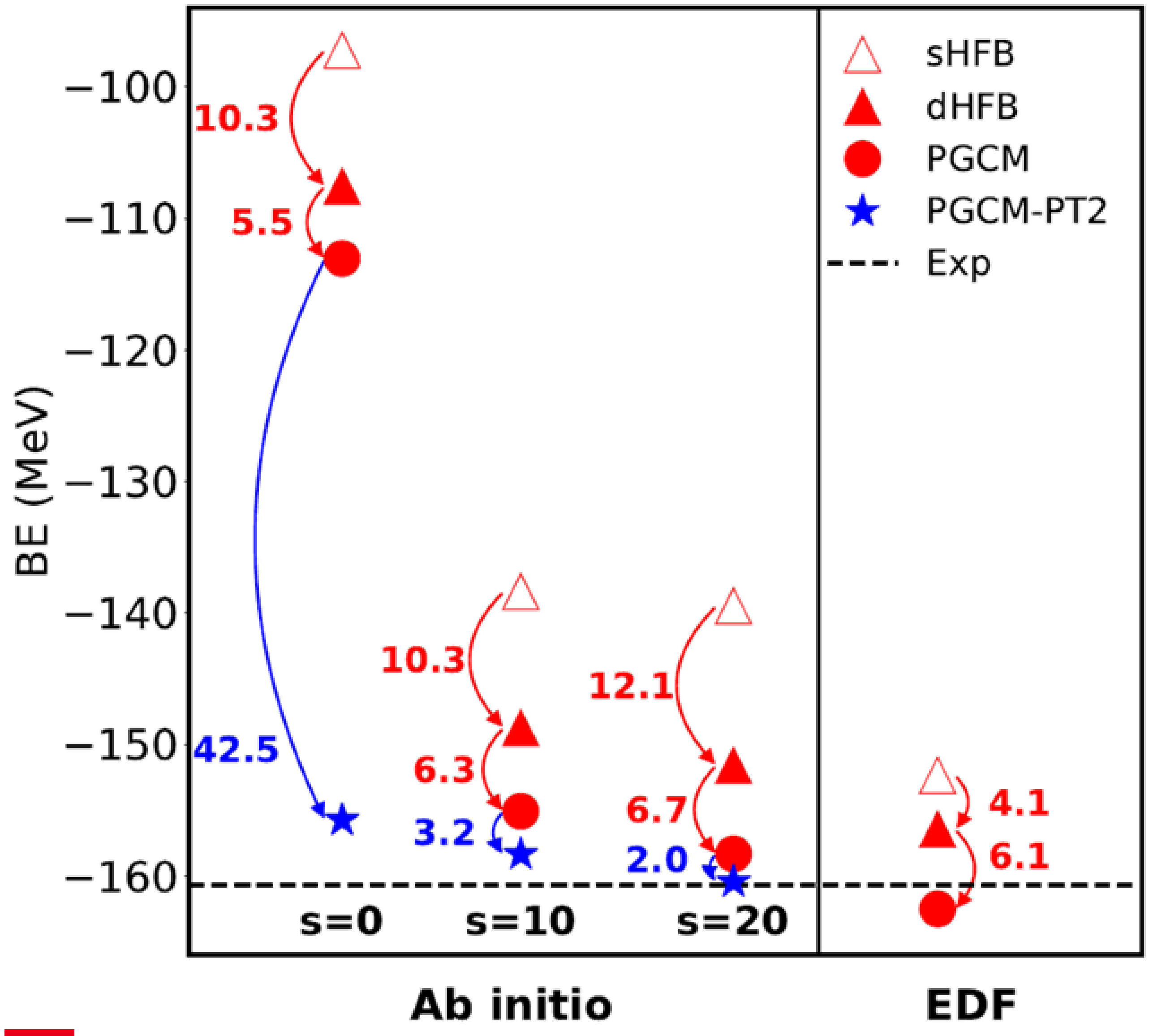
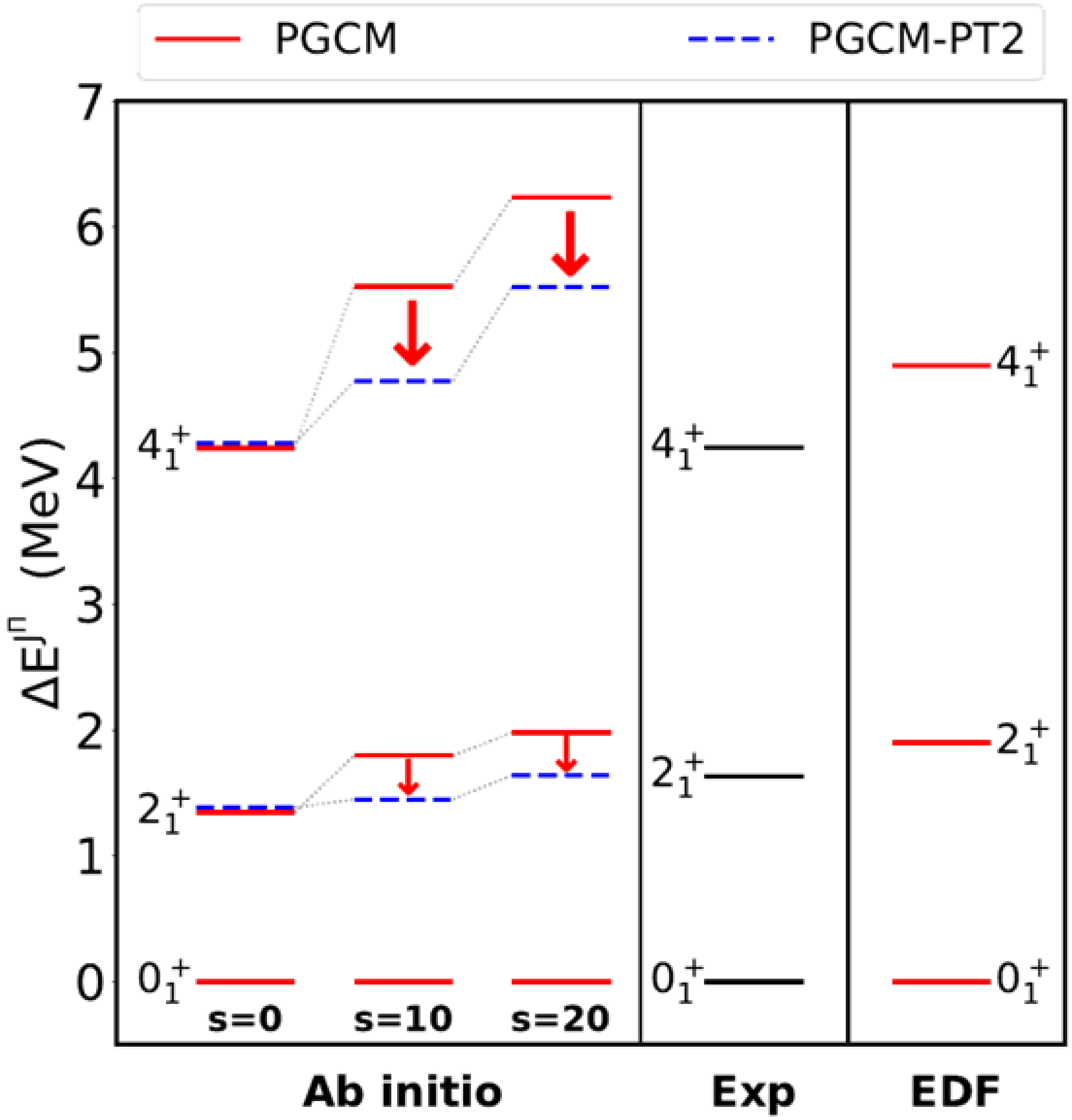
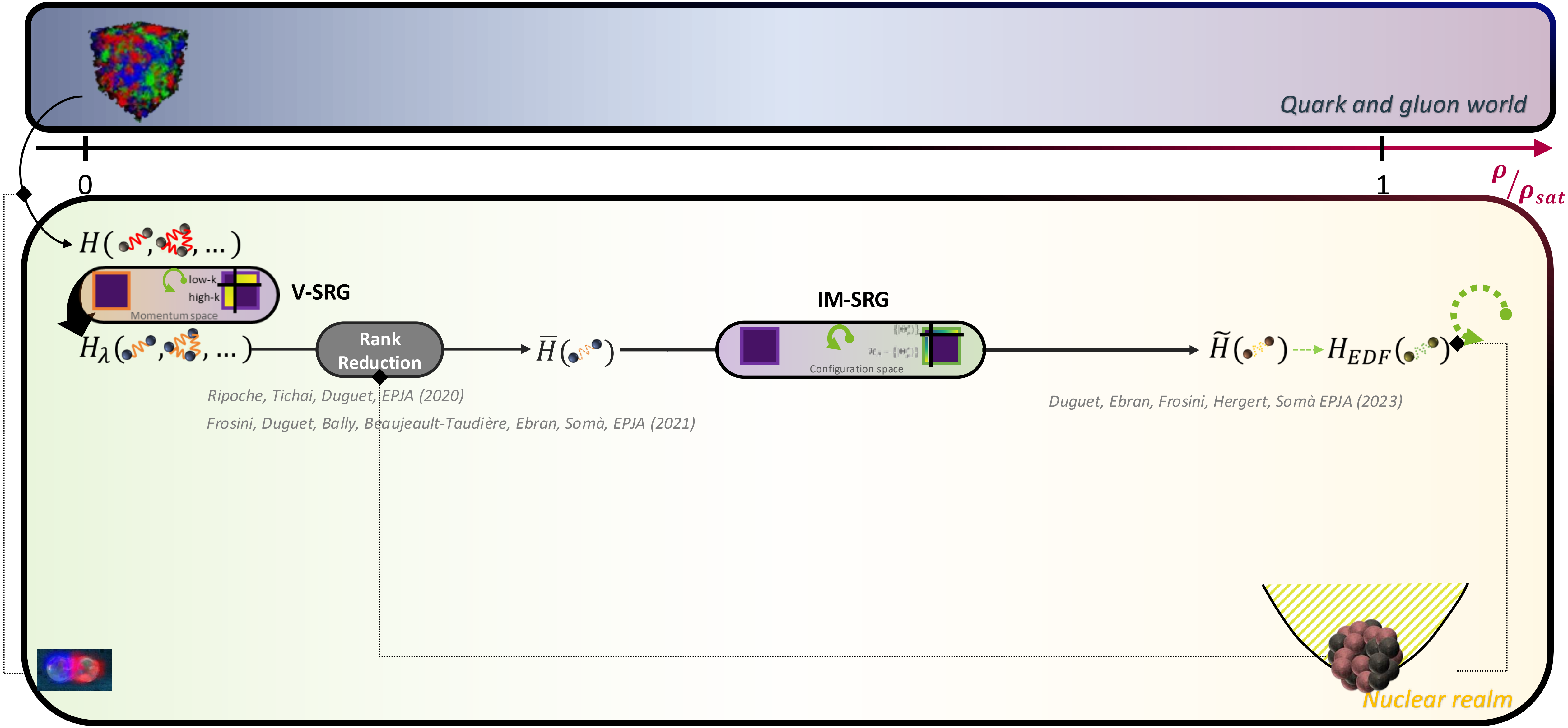




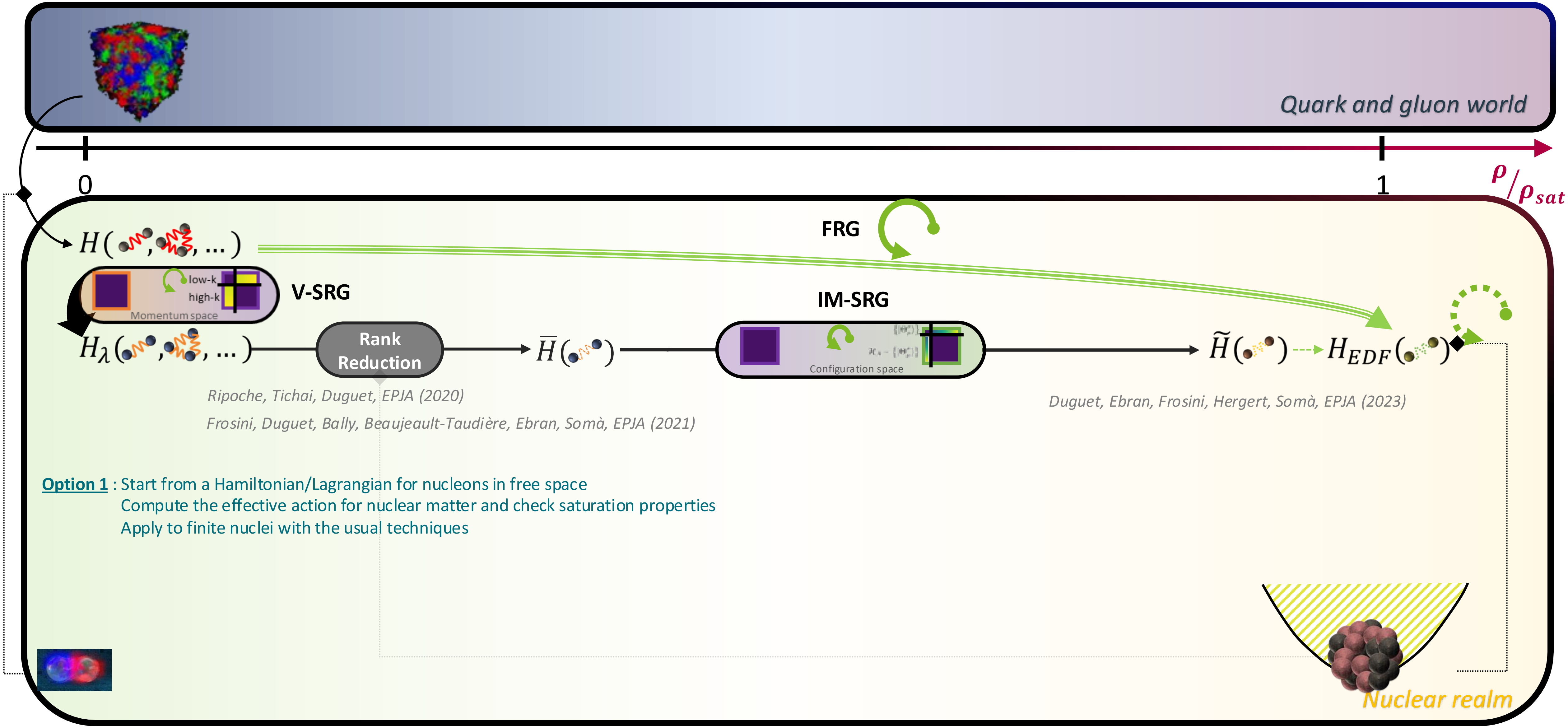


Duguet, Ebran, Frosini, Hergert, Somà, EPJA (2023)

❸ Towards a rigorous formulation of nuclear EDFS : WFT perspective



3 Towards a rigorous formulation of nuclear EDFS : FRG perspective



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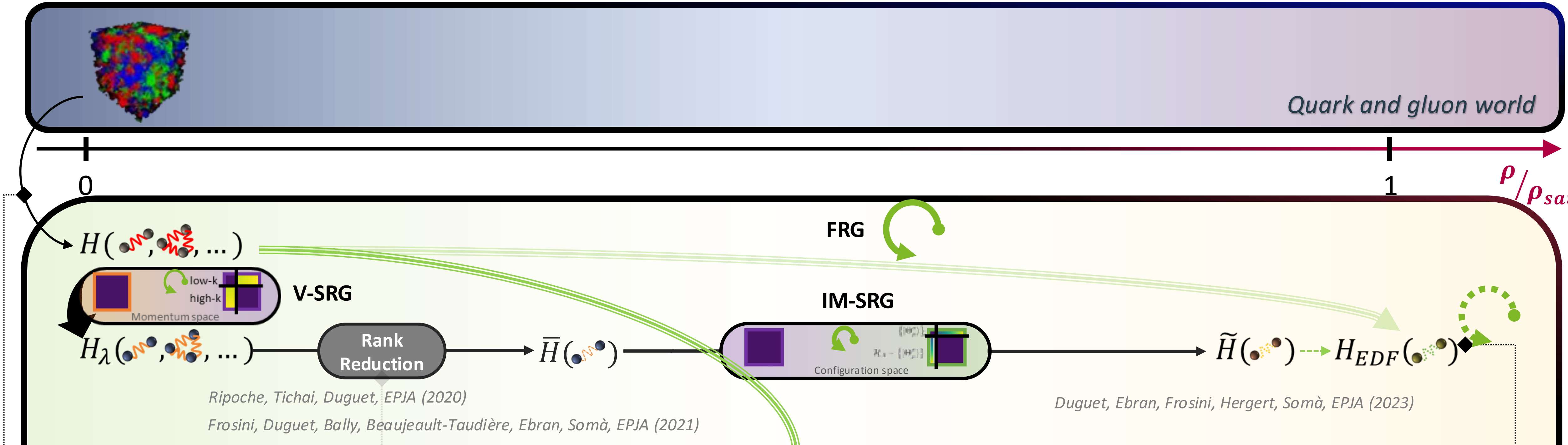
Frosini, Duguet, Bally, Beaujeault-Taudière, Ebran, Somà, EPJA (2021)

Duguet, Ebran, Frosini, Hergert, Somà, EPJA (2023)

Option 1 : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for nuclear matter and check saturation properties
Apply to finite nuclei with the usual techniques

❸ Towards a rigorous formulation of nuclear EDFS : FRG perspective



Option 1 : Start from a Hamiltonian/Lagrangian for nucleons in free space
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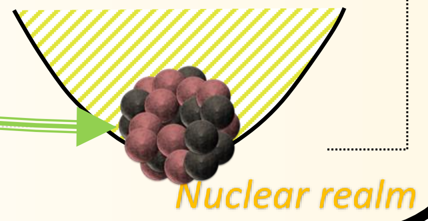
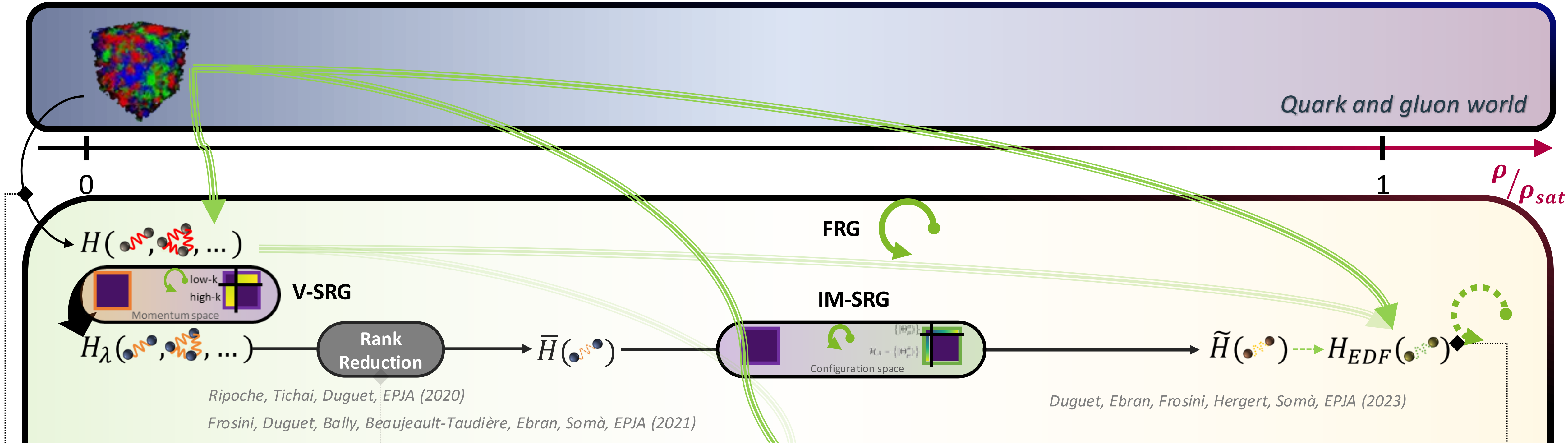
Apply to finite nuclei with the usual techniques

Option 2 : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for finite nuclei by accounting for all types of fluctuation
Compute all the desired correlation functions



3 Towards a rigorous formulation of nuclear EDFS : FRG perspective



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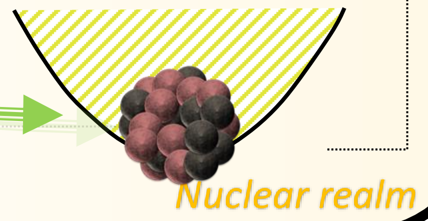
Option 2 : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for finite nuclei by accounting for all types of fluctuation
Compute all the desired correlation functions

Option 3 : Start from QCD
Compute the effective action for nucleons in free space/ infinite nuclear matter / finite nuclei



Global Strategy

Free space
N-N Lagrangian

FRG flow

Effective
N-N Lagrangian
in medium

Present work

$$\mathcal{L} = \mathcal{L}_{\mathrm{Bonn}}$$

LPA

FRG flow

?

Realistic N-N interaction
Not derived from QCD
Fitted on N-N scattering data

J. W. Negele & Erich Vogt. (1989) *Advances in Nuclear Physics.*

Preliminary

Benchmark
Properties of nuclear matter

Compare with other Beyond Mean-Field approaches :

Peter Ring *et al* 2023
J. Phys.: Conf. Ser. **2453** 012031

Bare N-N Lagrangian

$$\mathcal{L}_{\text{Bonn,int}} = \sum_{\text{mesons}} g_m \bar{\psi} D_m \psi$$

Lesson from empirical EDF

$$\mathcal{L}_{\text{Bonn,int}} \sim \mathcal{L}_{\text{NL3,int}}$$

Same analytical form for interaction

*NL3 :
common EDF interaction

Main structural difference

$$\mathcal{L}_{\text{Bonn}} \not\ni g_2 \frac{\sigma^3}{3} + g_3 \frac{\sigma^4}{4} \subset \mathcal{L}_{\text{NL3}}$$

Generated by FRG flow ?

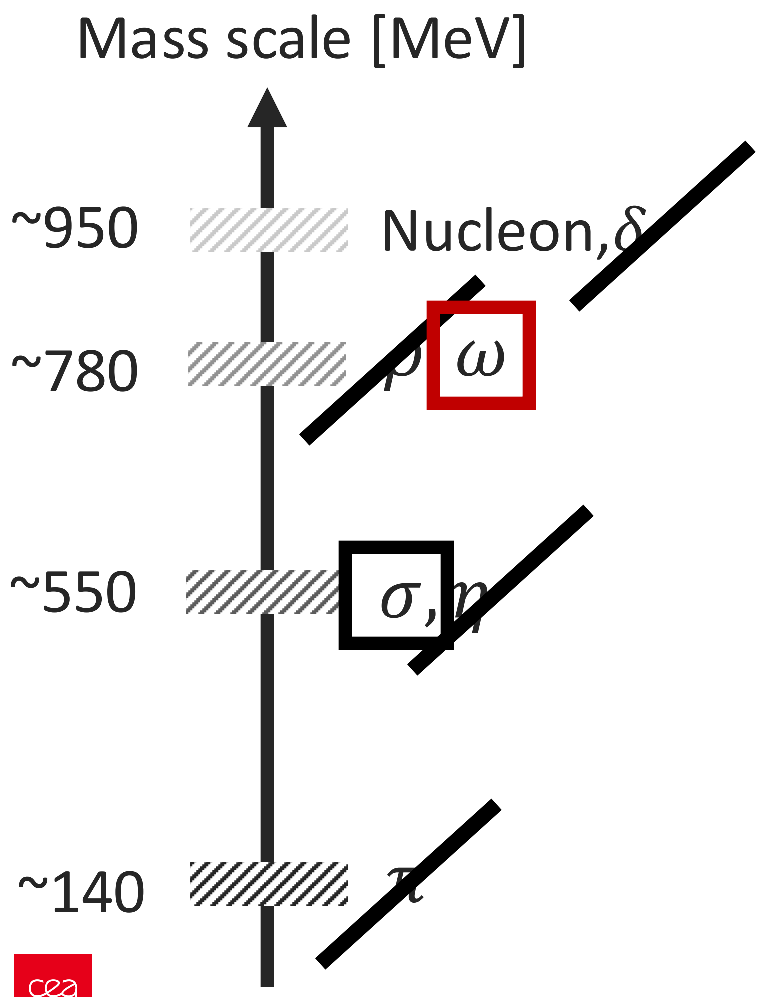
Bare N-N Lagrangian

Scalar

Pseudo-scalar

$$\mathcal{L}_{\text{int}} = \bar{\psi}\left[\left(M + g_\sigma \sigma + g_\delta \tau \cdot \delta\right) + \gamma_\mu \left(\omega^\mu + \tau \cdot \rho^\mu\right) + \gamma^5 \gamma^\mu \left(g_\eta \partial_\mu \eta + g_\pi \tau \cdot \partial_\mu \pi\right)\right]\psi$$

Vector



First calculations

Symmetric nuclear matter

+ (some) mesons @Mean-Field*

$$\rho = \delta = \eta = \pi = 0$$

ω_0 as external parameter

Flowing $U_k(\sigma)$

Preliminary ansatz

$$\Gamma_k = \int \left[U_{k,\sigma} - \frac{1}{2} m_\omega^2 \omega_0^2 + \mathcal{L}_N - \bar{\psi}(M + g_\sigma \sigma)\psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma \right]$$

Associated Wetterich equation (T=0)

$$\mu_{\text{eff}} = \mu - g_\omega \omega_0$$

$$\partial_t U_{t,\sigma} = -A_t \left[\frac{1}{\sqrt{k^2 + U''_{t,\sigma}}} - 8 \frac{\sum_{\epsilon=\pm 1} \theta(\mathbf{E_N} + \epsilon \mu_{\text{eff}}) - 1}{\sqrt{\mathbf{k^2} + (\mathbf{M} + \mathbf{g}_\sigma \sigma)^\mathbf{2}}} \right]$$

Numerical details

Grid for σ

no Taylor expansion

Change of variable

For stability

$$\varpi = \log\left(\frac{k^2 + U''}{\Lambda^2}\right)$$

Time integration

Fully Implicit RK

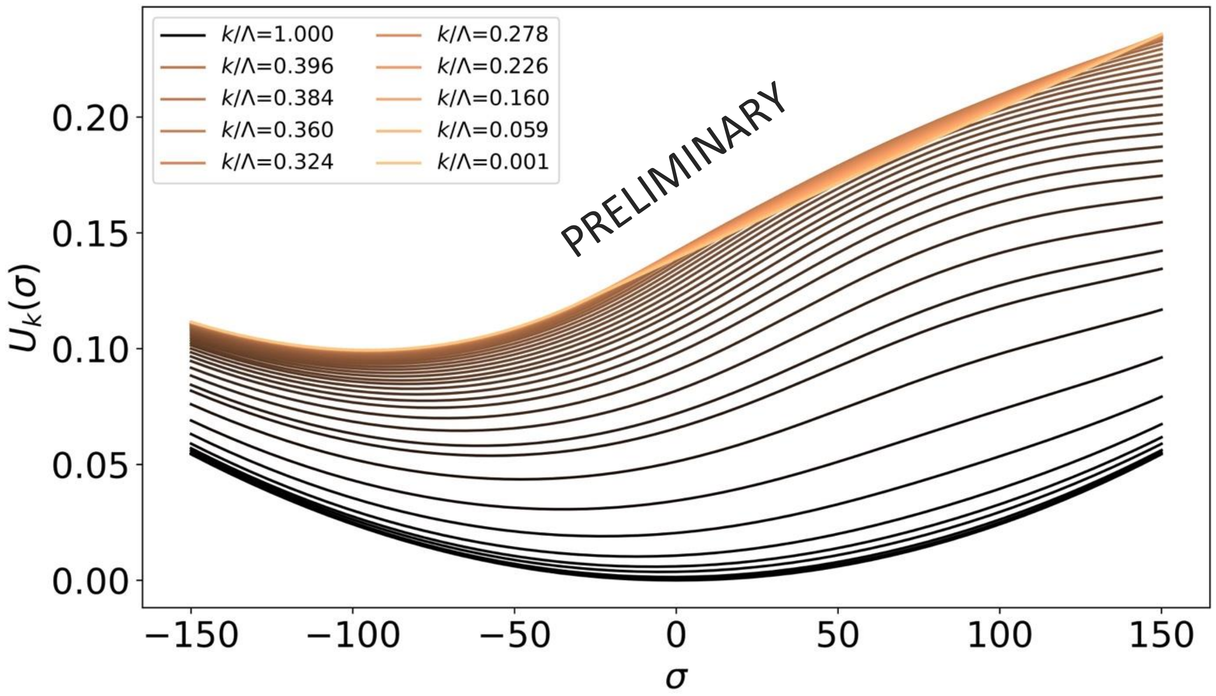
Integration constants

ODE for $U_k(\sigma_0), u_k(\sigma_0)$

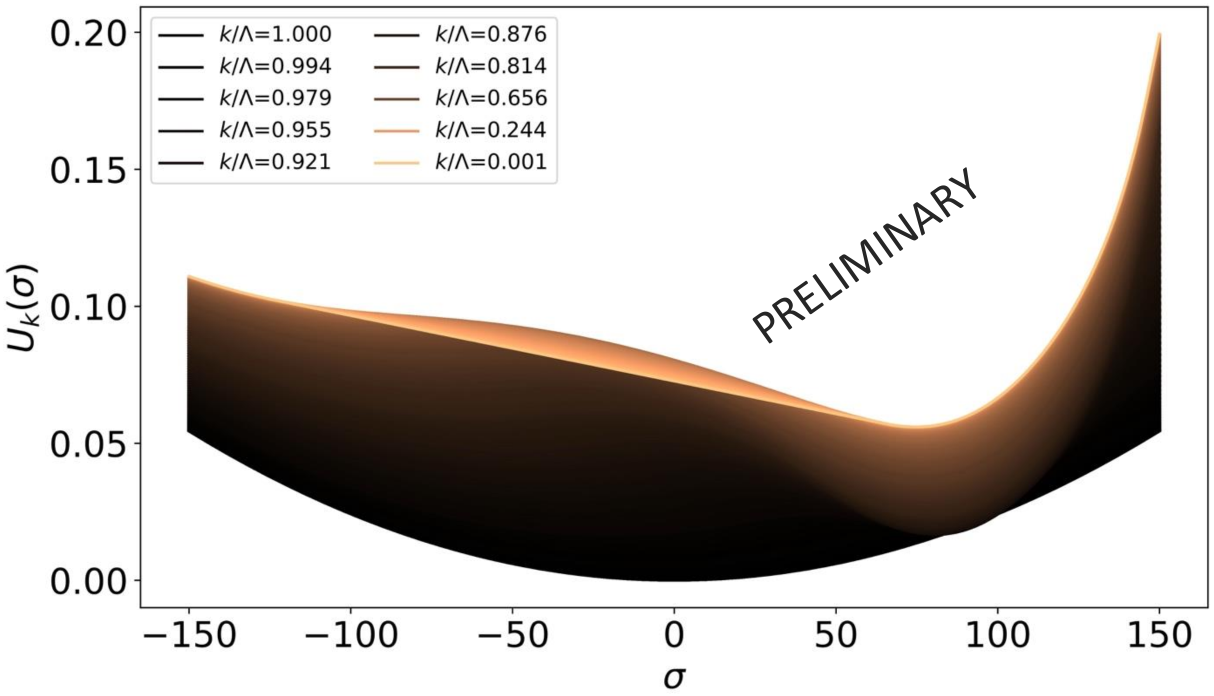
Ihssen, F., Sattler, F. R., & Wink, N. (2023). *Phys. Rev. D*, 107, 114009.

Preliminary results

$\mu < \mu_c$



$\mu > \mu_c$



Liquid-Gas Transition seems to be qualitatively captured
→ relax approximations to get quantitative results

Conclusion & outlook

Next steps

Full LPA

Include **all mesons consistently**

+ **flow** of **Yukawa couplings**

+ **WF renormalisation**

Adapt UV input

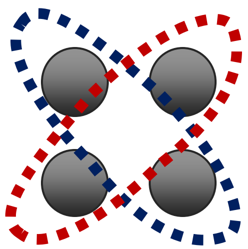
Chiral/QCD Lagrangian

Or directly with FRG ?

Prespectives

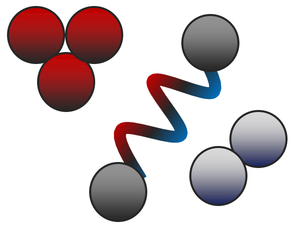
Pairing

Include a **dynamical** pairing field
-> **in-medium pairing** force



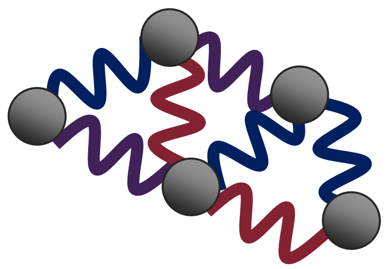
Clusters

Include **light nuclei** (tritium, Helium, deuteron,…) as **explicit dofs**



Finite nuclei

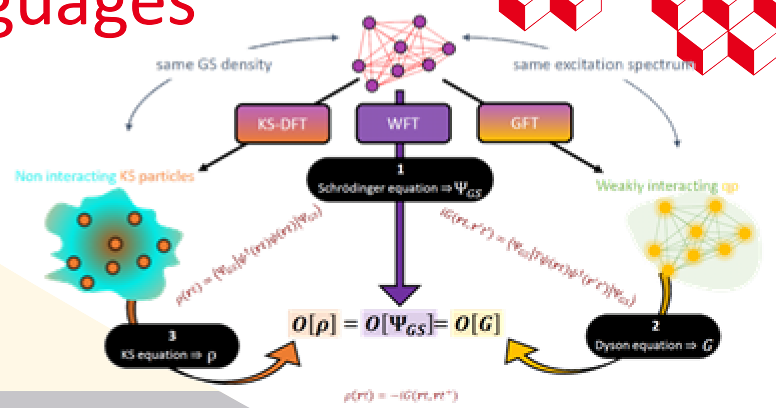
Use FRG as a Many-Body method

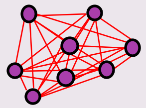


Thank you for your attention !

Back up

❸ Towards a rigorous formulation of nuclear EDFS : Languages



	Wave Function theories	Functional theories		
				
Based on	wave function $|\Psi\rangle$	reduced quantity ϱ		
Observables	$O[|\Psi\rangle] = \langle\Psi|O|\Psi\rangle$	$F[\varrho]$		
$\boldsymbol{\varrho}$		$G(\boldsymbol{r},\boldsymbol{r}';t-t')$	$\gamma(\boldsymbol{r},\boldsymbol{r}') = G(\boldsymbol{r},\boldsymbol{r}';t-t^{+})$	$\rho(\boldsymbol{r}) = \gamma(\boldsymbol{r},\boldsymbol{r})$
Functional		$\Phi_{LW}[G]$ or $\Sigma = \frac{\delta\Phi_{LW}}{\delta G}$	$E_{xc}[\gamma]$	$E_{xc}[\rho]$ or $v_{xc} = \frac{\delta E_{xc}}{\delta\rho}$
Approx.		"easy"	difficult	very difficult
Computationally		heavy	moderate	light

1) Nucleus: *A* interacting, structure-less nucleons

2) Structure & dynamic encoded in Hamiltonian, Functional, …

3) Solve master equation to desired accuracy

$$H(\ldots)|\Psi_{\mu,\sigma}\rangle = E_{\mu\tilde{\sigma}}|\Psi_{\mu,\sigma}\rangle$$

$$G^{-1}(x,x') = G_0^{-1}(x,x') - \Sigma(x,x')$$

$$h(\mathbf{r})f_\alpha(x) + \int dx'\Sigma(x,x';\varepsilon_\alpha)f_\alpha(x') = \varepsilon_\alpha f_\alpha(x)$$

$$E_{gs} = \min_{\gamma\in N-\mathrm{rep}} E[\gamma]$$

$$\left\{-\frac{\nabla^2}{2m} + v_{KS}(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$