Asymptotic Safety and Vacuum Stability in the Litim-Sannino Model

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Outline

I. Motivation

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» Asymptotic Safety in d=4, renormalisable, perturbatively controlled ⟶ guaranteed UV fixed point

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	Dirac, (LiSa)	Majorana fermions

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» UV conformal window in LiSa entirely accessible in perturbation theory – lessions for IR FP in QCD?

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single trace

double trace

gauge sector (QCD)

$\downarrow \psi$

Yukawa (chiral!)

$\uparrow \phi$

scalar sector

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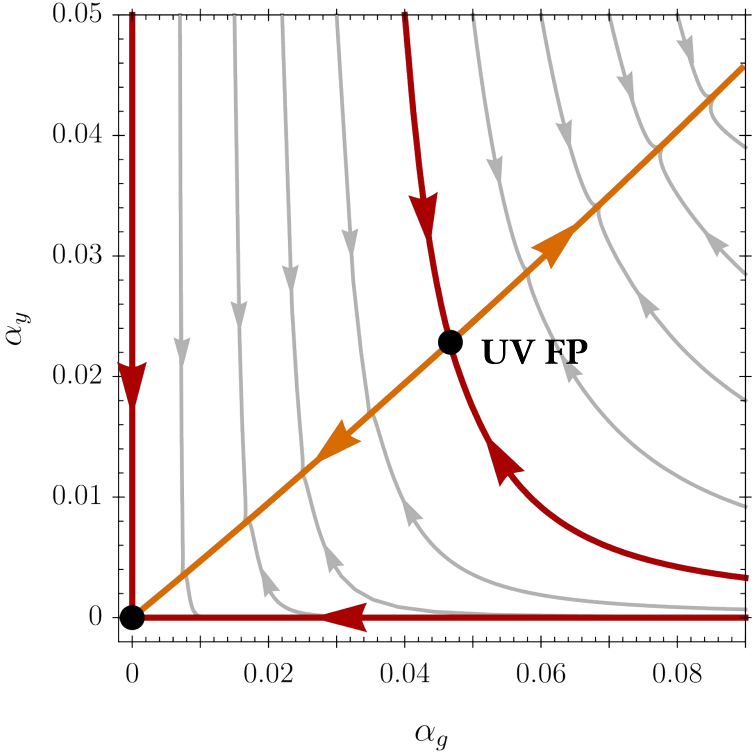
$\longrightarrow$ interacting fixed point $g^*, y^*, u^*, v^* \neq 0$

UV fixed point



1 relevant and 3 irrelevant directions

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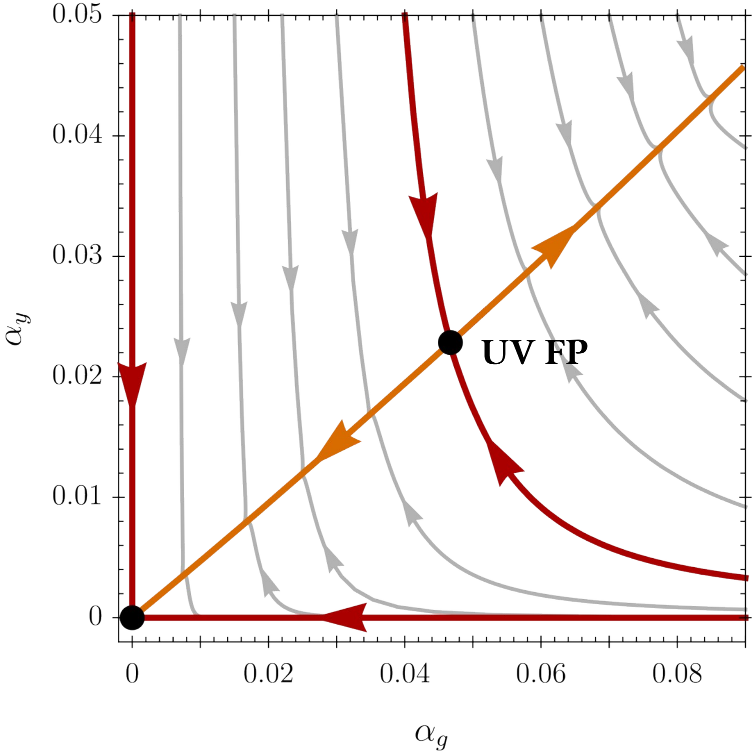

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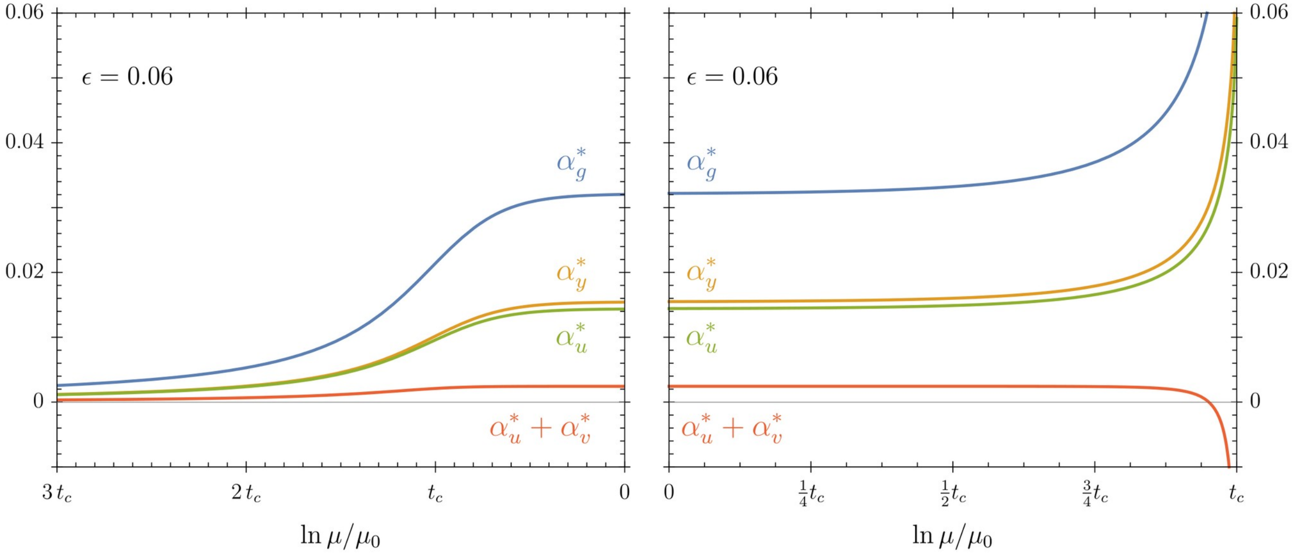

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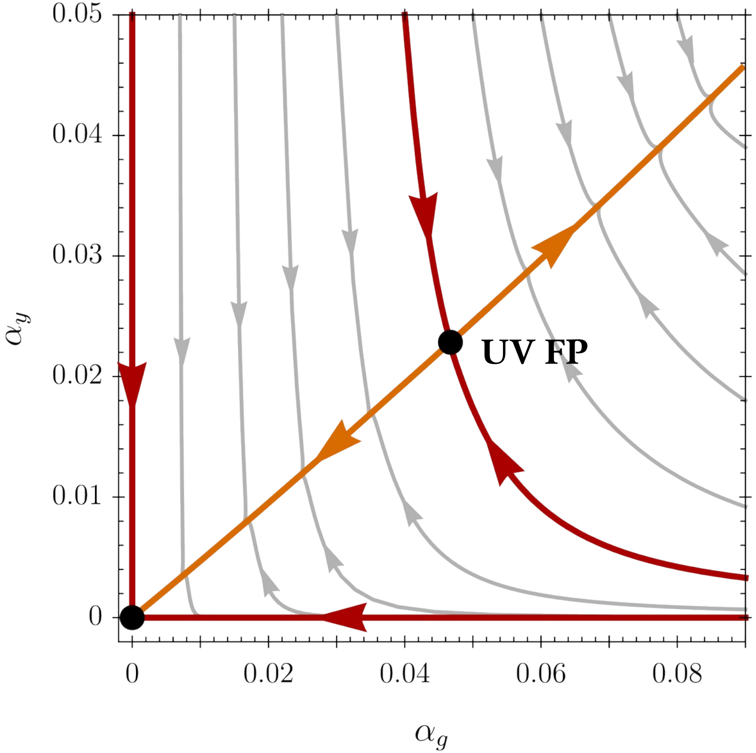

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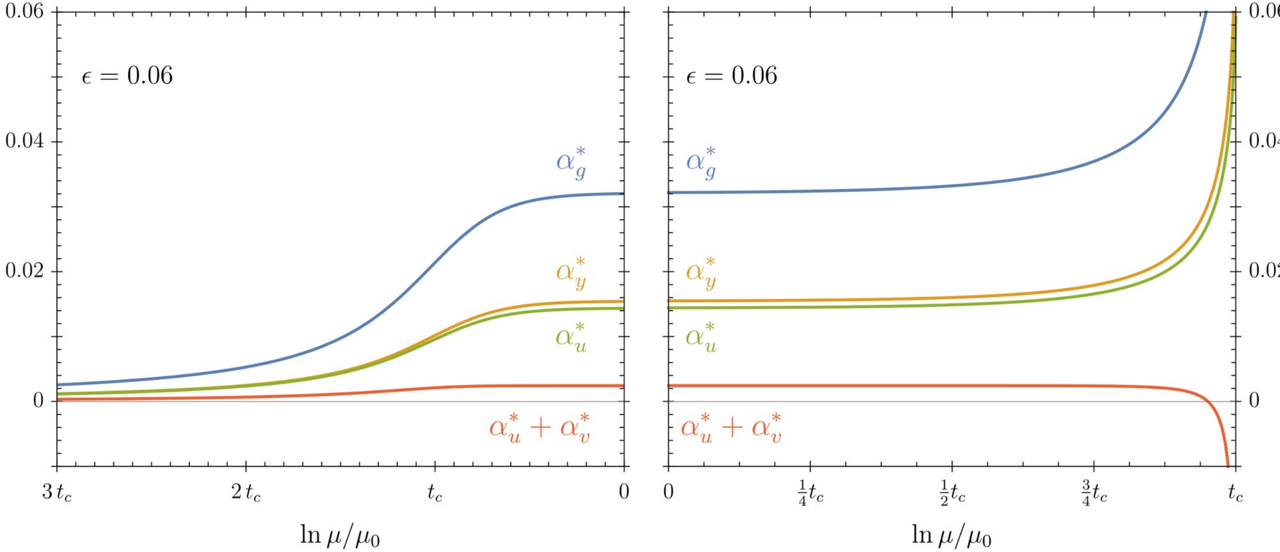

$\longrightarrow$ fixed point is under perturbative control

Exact perturbative control

» Veneziano limit: $N_{f,c} \to \infty$ but $N_f/N_c = \text{const.}$

» introduce 't Hooft couplings:

$$\alpha_g = \frac{N_c\, g^2}{(4\pi)^2} \qquad \alpha_y = \frac{N_c\, y^2}{(4\pi)^2} \qquad \alpha_u = \frac{N_f\, u}{(4\pi)^2} \qquad \alpha_v = \frac{N_f^2\, v}{(4\pi)^2}$$

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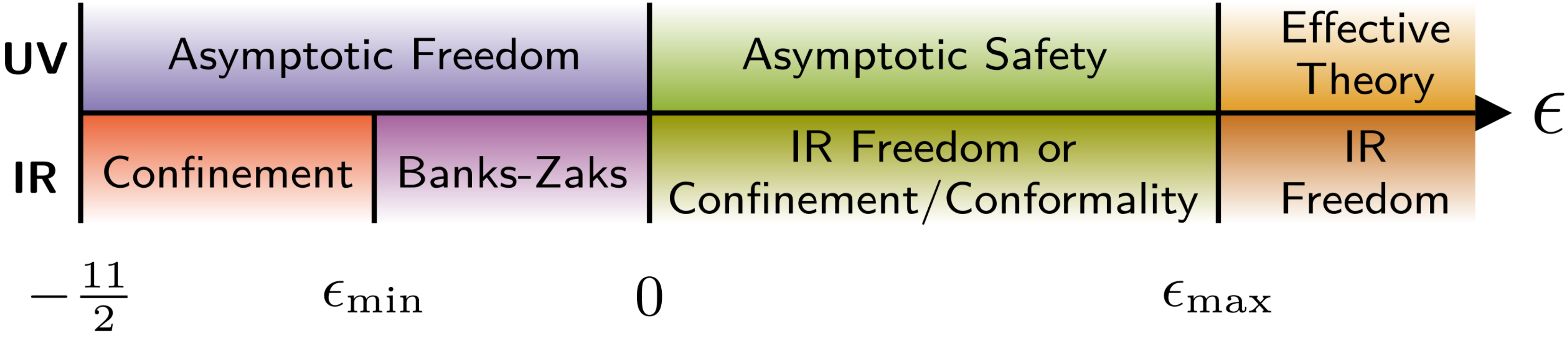
» conformal expansion: $\alpha^* = \epsilon\, a_{\text{LO}} + \epsilon^2\, a_{\text{NLO}} + \epsilon^3\, a_{\text{NNLO}} + \ldots$

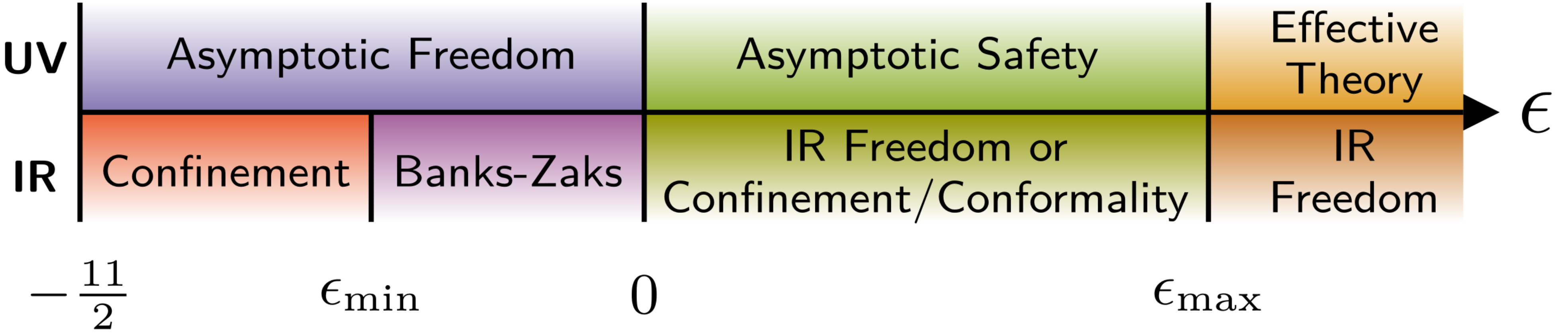
2-loop gauge 1-loop Yukawa 1-loop quartic	3-loop gauge 2-loop Yukawa 2-loop quartic	4-loop gauge 3-loop Yukawa 3-loop quartic
[Litim,Sannino, 2014]	[Bond, Medina, Litim, TS, 2017]	[Litim, Riyaz, Stamou, TS, *2023*] [Bednyakov, Mukhaeva, *2023*]

Conformal Windows of LiSa



Conformal Windows of LiSa

Gaussian is UV FP



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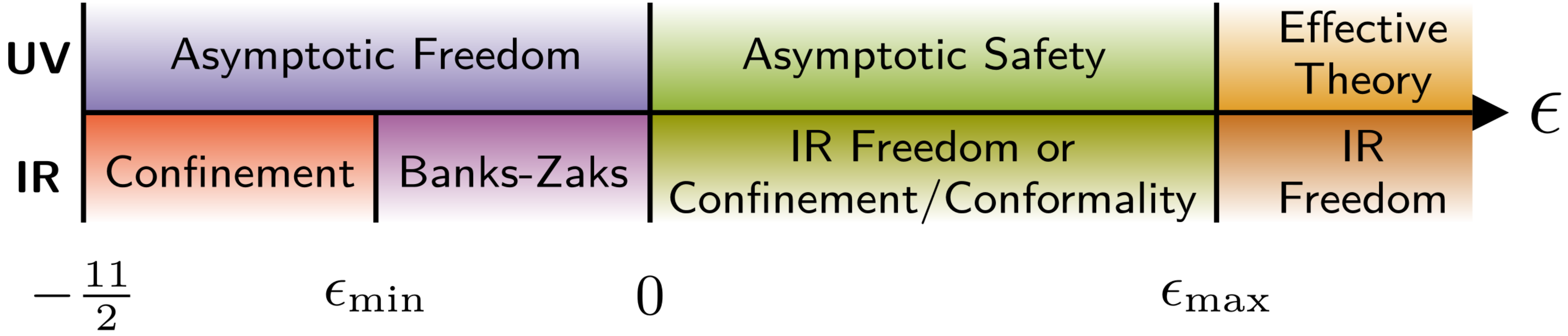

IR FP $\alpha_g^* \neq 0$

Conformal Windows of LiSa

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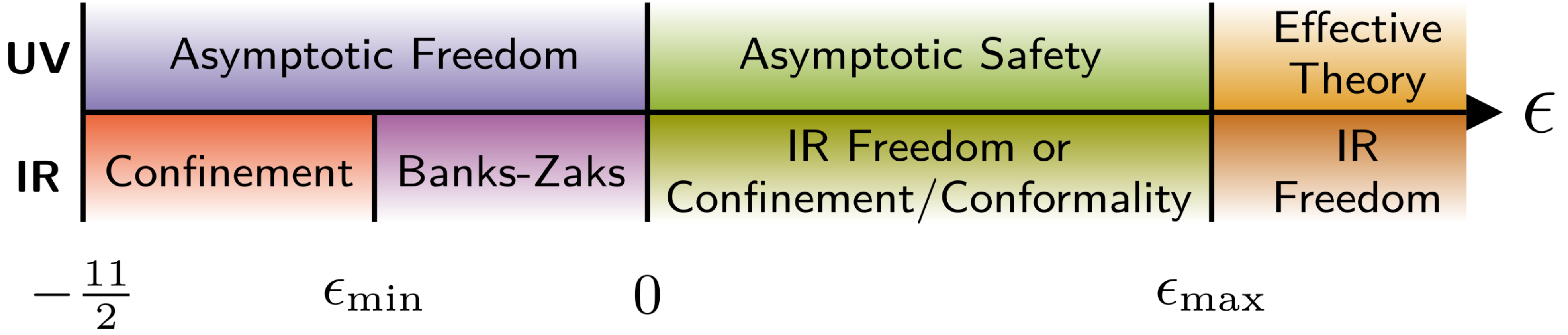
fully interacting UV FP

$\alpha^*_{g,y,u,v} \neq 0$



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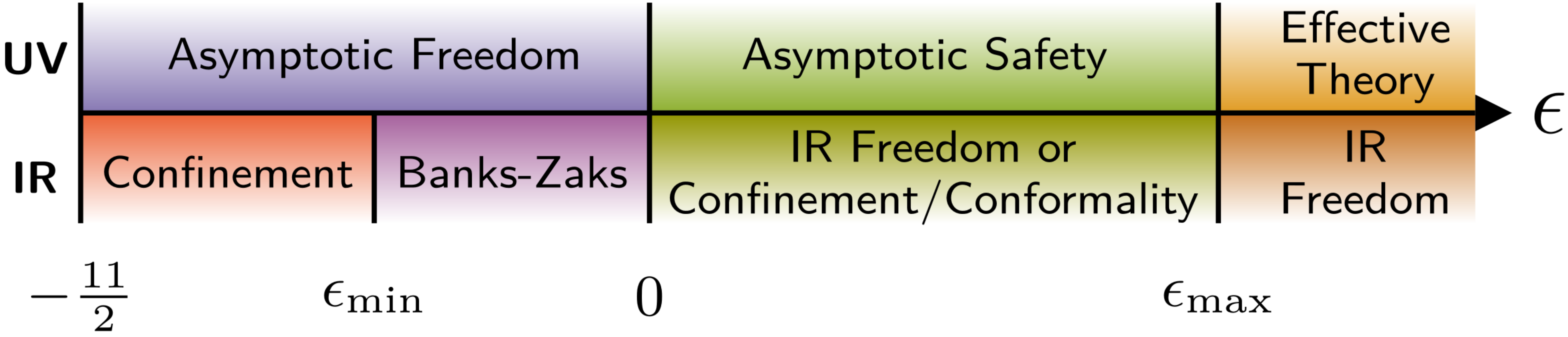

IR FP $\alpha^*_g \neq 0$

$\longrightarrow$ disappears outside of UV conformal window $[0,\ \epsilon_{\text{max}}]$

$\longrightarrow$ determine ϵ_{max} $\longrightarrow$ $(N_f,\ N_c)_{\text{min}}$

$\longrightarrow$ determine why

III. UV Conformal Window

How to probe the UV conformal window

» beta functions $\beta_{g,y,u,v}$ at **433**: β_g @ 4L, β_y @ 3L, $\beta_{u,v}$ @ 3L [Litim, Riyaz, Stamou, TS, *2023*]

$\longrightarrow$ fixed point values $\alpha^*_{g,y,u,v}(\epsilon)$ from $\beta_{g,y,u,v} = 0$ up to ϵ^3

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$$(\alpha_x - \alpha^*_x) = c_{x,i} \left(\frac{\mu}{\mu_0}\right)^{\vartheta_i} \qquad \vartheta_1 < 0 < \vartheta_{2,3,4}$$

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» typical shape (all loop orders)

$$\beta_g = \alpha_g^2 \left[\tfrac{4}{3}\epsilon + b_g(\alpha_{g,y,u}, \epsilon)\right]$$

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&\quad\longrightarrow \text{quadratic shape, up to two solutions } \alpha_v^{*\pm} \text{ for each } \alpha^*_{g,y,u}
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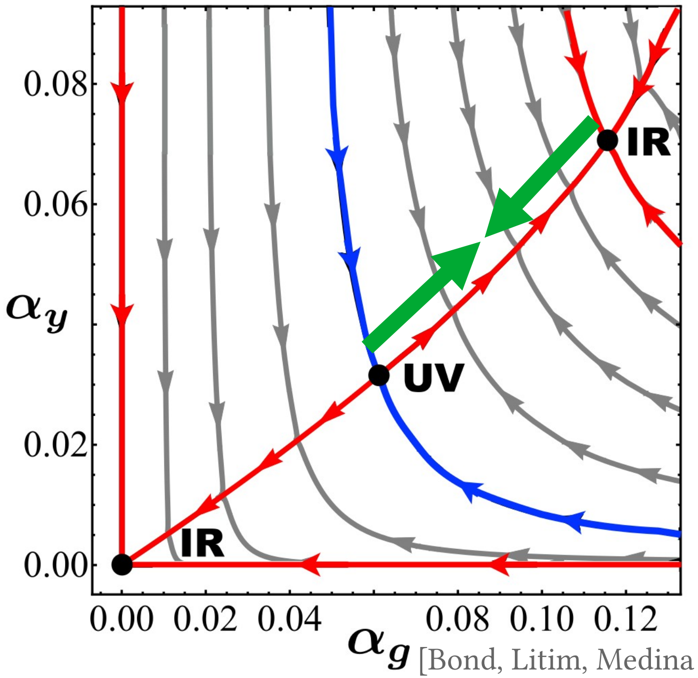

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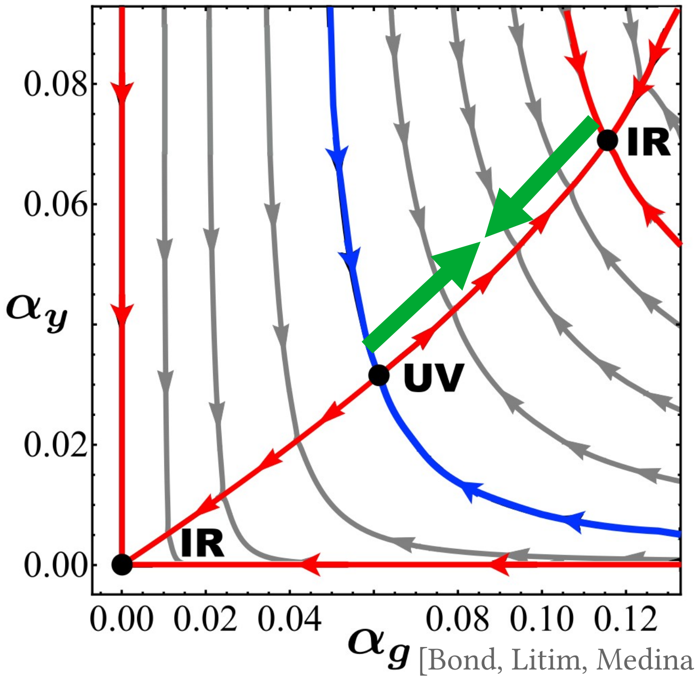

[Bond, Litim, Medina '20]

$\longrightarrow$ single trace merger in $\alpha_{g,y,u}^*$ system

$$\vartheta_1(\epsilon_{\max}) = 0$$

$\longrightarrow$ double trace merger: two solutions α_v^* for same $\alpha_{g,y,u}^*$

$$\vartheta_3(\epsilon_{\max}) = 0$$

Investigating the conformal window

» ϵ – expansion of $\alpha_i^*(\epsilon)$ and $\vartheta_i(\epsilon)$

$\longrightarrow$ series is exact up to third term

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$$\alpha_u^* + \alpha_v^{*+} \approx +0.0625\,\epsilon - 0.192\,\epsilon^2 - 1.62\,\epsilon^3 + \dots \qquad \epsilon_{\text{vac}} \approx 0.327$$

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→ series is exact up to third term

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$$\epsilon_{\rm vac} \approx 0.327 \xrightarrow{\text{Pade [2/1]}} 0.087$$

→ unstable second solution

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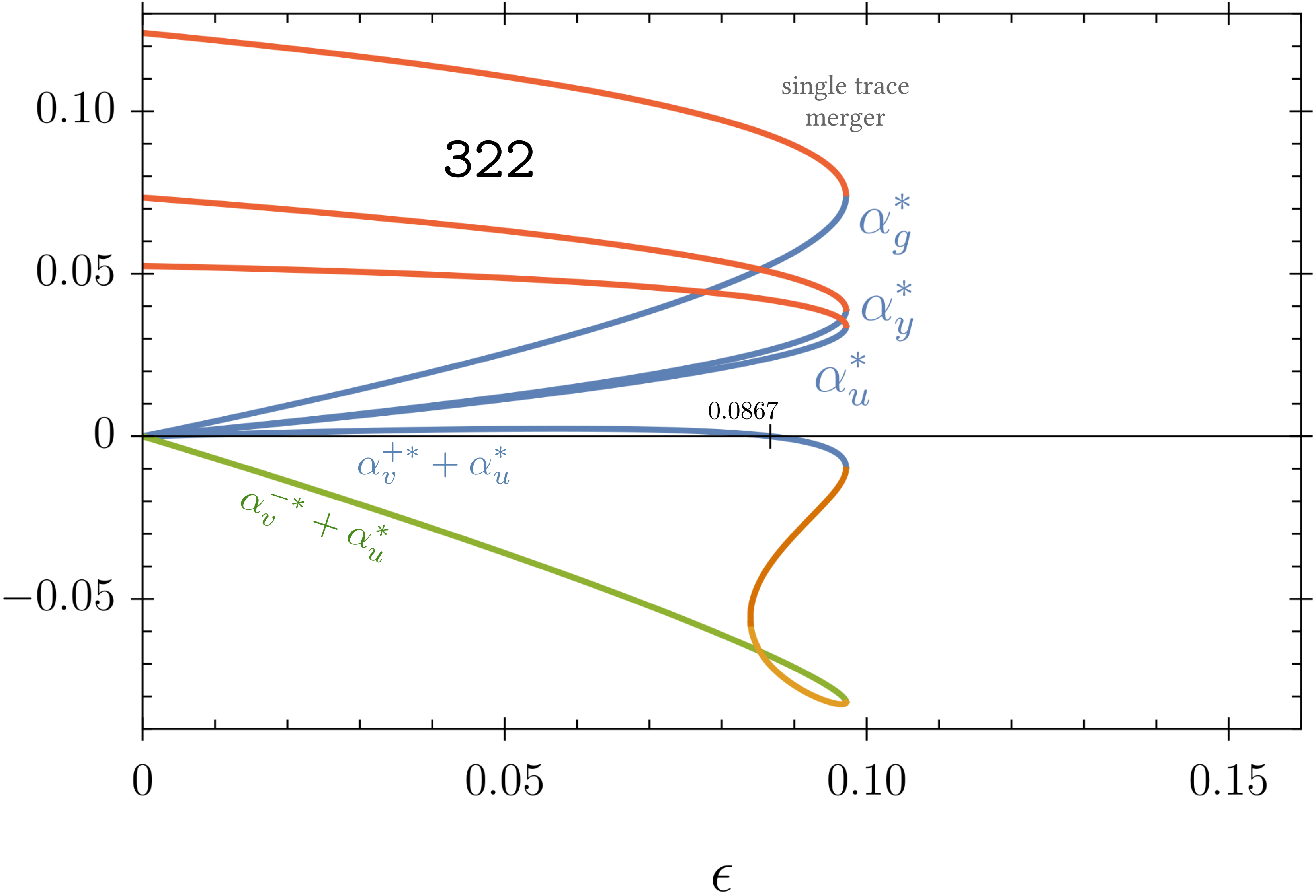
» $\epsilon -$ expansion is short! Higher order estimates:

$\longrightarrow$ Padè resummation of $\epsilon -$ expansion

$\longrightarrow$ Numerical solution $\beta_{g,y,u,v} = 0$

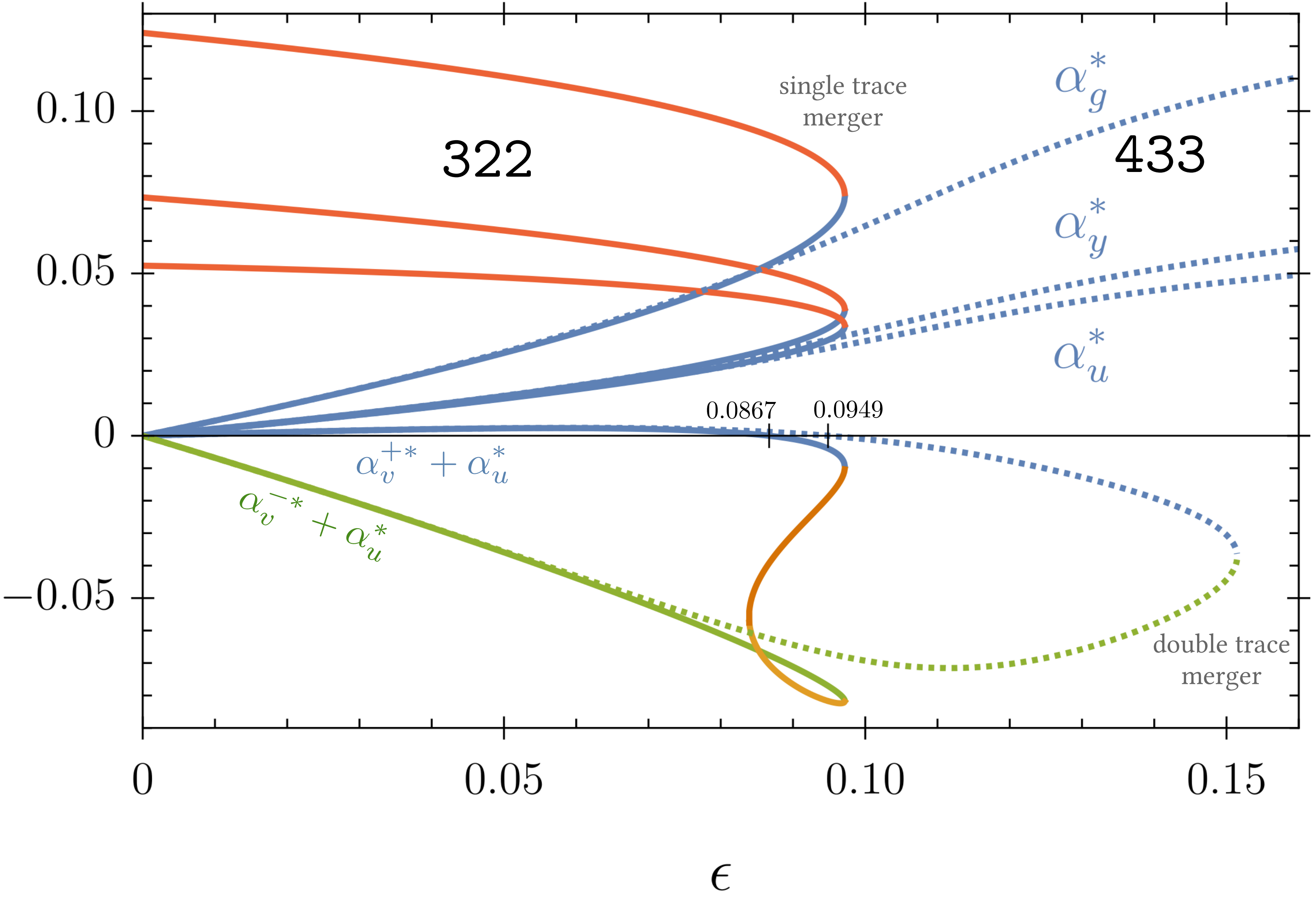
$\longrightarrow$ Separatrix expansion $\beta_{\text{sep}}(\alpha, \epsilon) = 0$ and resummations in α [$\longrightarrow$ Nahzaan Riyaz Poster]

$\beta_{g,y,u,v} = 0$



0.10
0.05
0
−0.05
0
0.05
0.10
0.15
ϵ
single trace
merger
322
α_g^*
α_y^*
α_u^*
0.0867
$\alpha_v^{+*} + \alpha_u^*$
$\alpha_v^{-*} + \alpha_u^*$

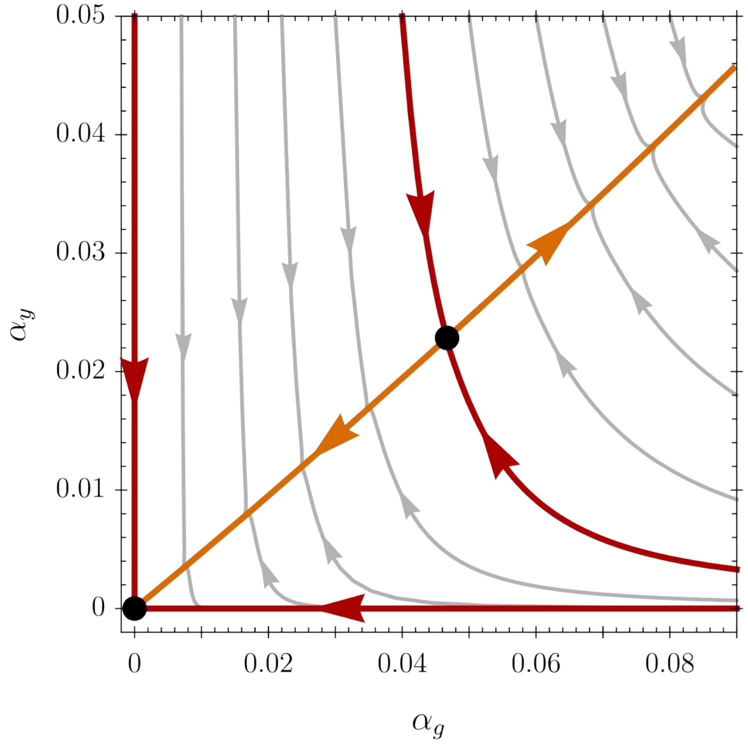
$\beta_{g,y,u,v} = 0$



single trace merger
322
433
α_g^*
α_y^*
α_u^*
0.0867
0.0949
$\alpha_v^{+*} + \alpha_u^*$
$\alpha_v^{-*} + \alpha_u^*$
double trace merger
0.10
0.05
0
−0.05
0
0.05
0.10
0.15
ϵ

Separatrix Expansion

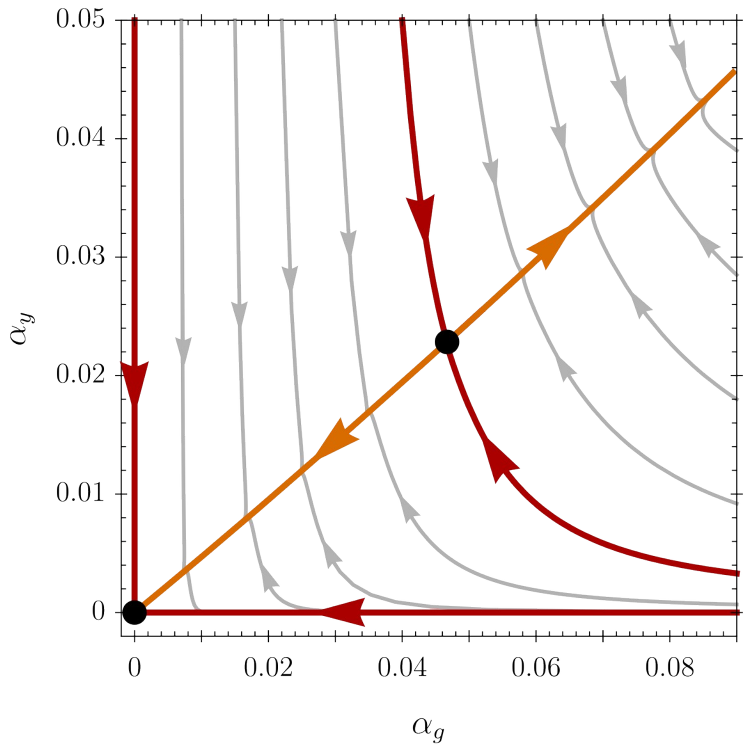
[⟶ Nahzaan Riyaz Poster]



» describe running along relevant separatrix

Separatrix Expansion

[$\longrightarrow$ Nahzaan Riyaz Poster]



» describe running along relevant separatrix

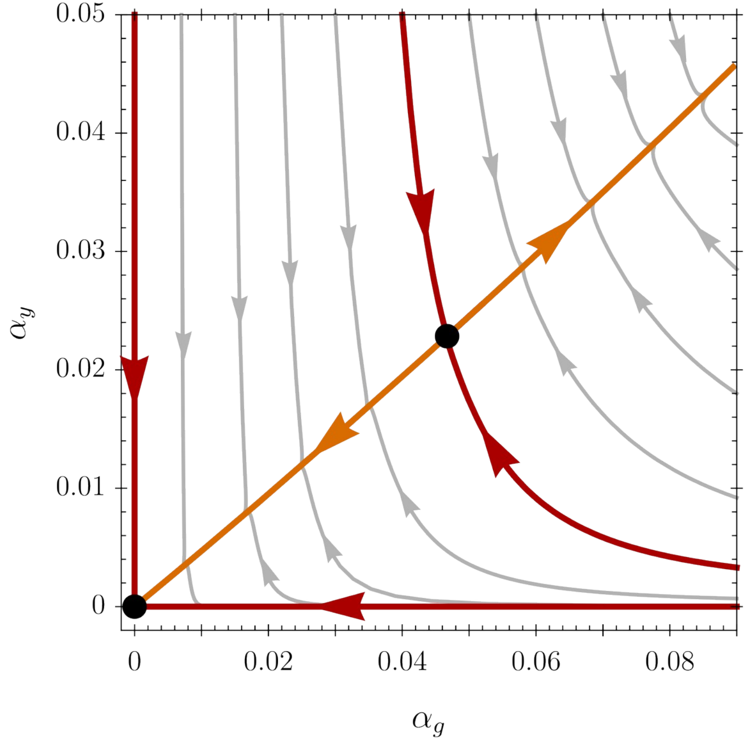
$$\longrightarrow \quad \beta_{\text{sep}} = \beta_g \quad \text{with} \ \ \alpha \equiv \alpha_g, \ \ \alpha_{y,u,v} = \sum_{\ell=1}^{\infty} c^{(\ell)}_{y,u,v}(\epsilon)\, \alpha^{\ell}$$

» Expansion in α around Gaussian (weak branch)

$$\beta_{y,u,v} = \frac{\mathrm{d}\alpha_{y,u,v}(\alpha)}{\mathrm{d}\alpha} \beta_g$$

Separatrix Expansion

[⟶ Nahzaan Riyaz Poster]



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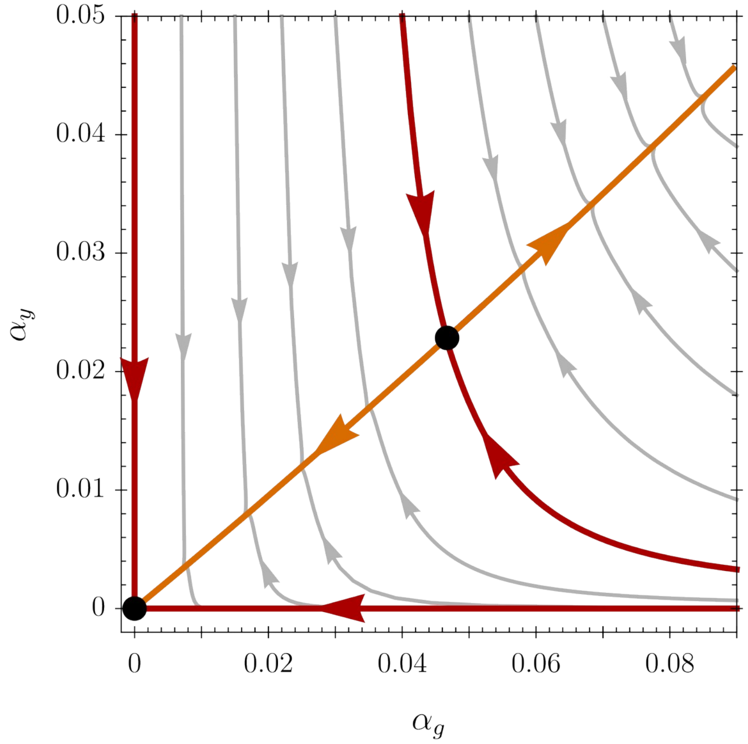
» effective 4L beta function along separatrix, single coupling

$$\beta_{\text{sep}} = \sum_{\ell=1}^{\infty} A_{\ell}(\epsilon)\, \alpha^{\ell+1}$$

⟶ complete ϵ dependence up to A_4

Separatrix Expansion

[$\longrightarrow$ Nahzaan Riyaz Poster]



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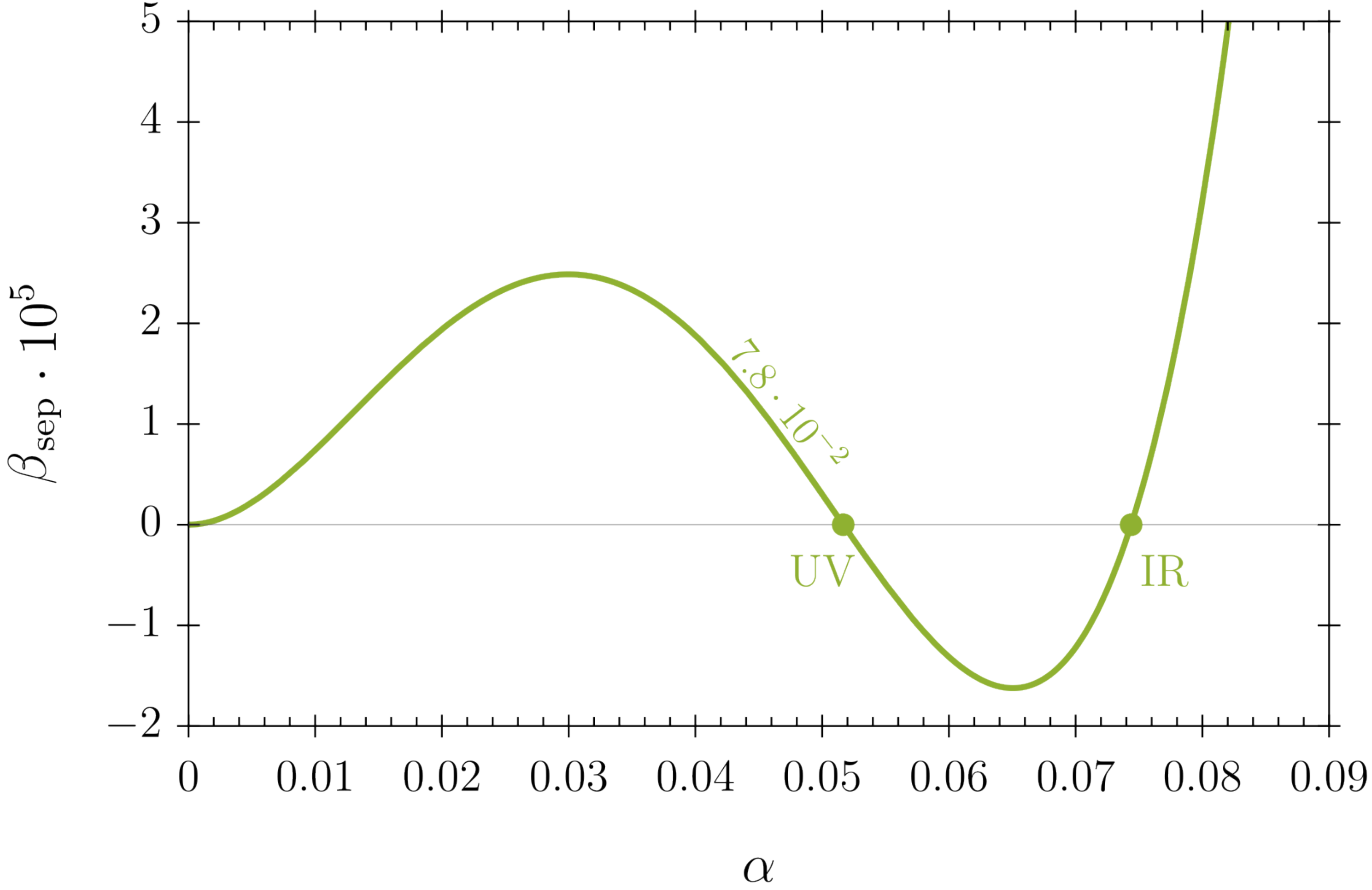
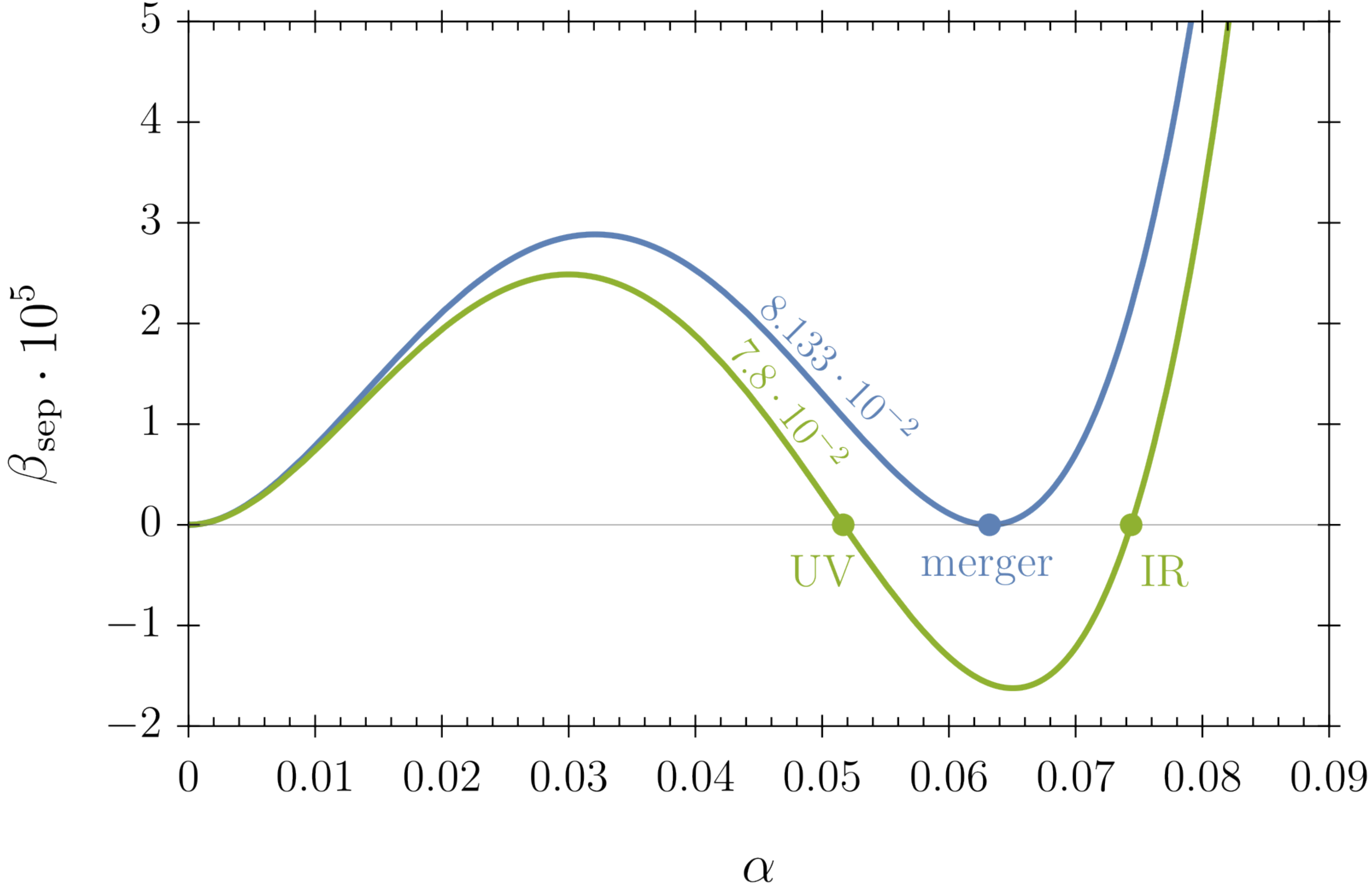
$\longrightarrow$ complete ϵ dependence up to A_4

$\longrightarrow$ requires 432 RGEs

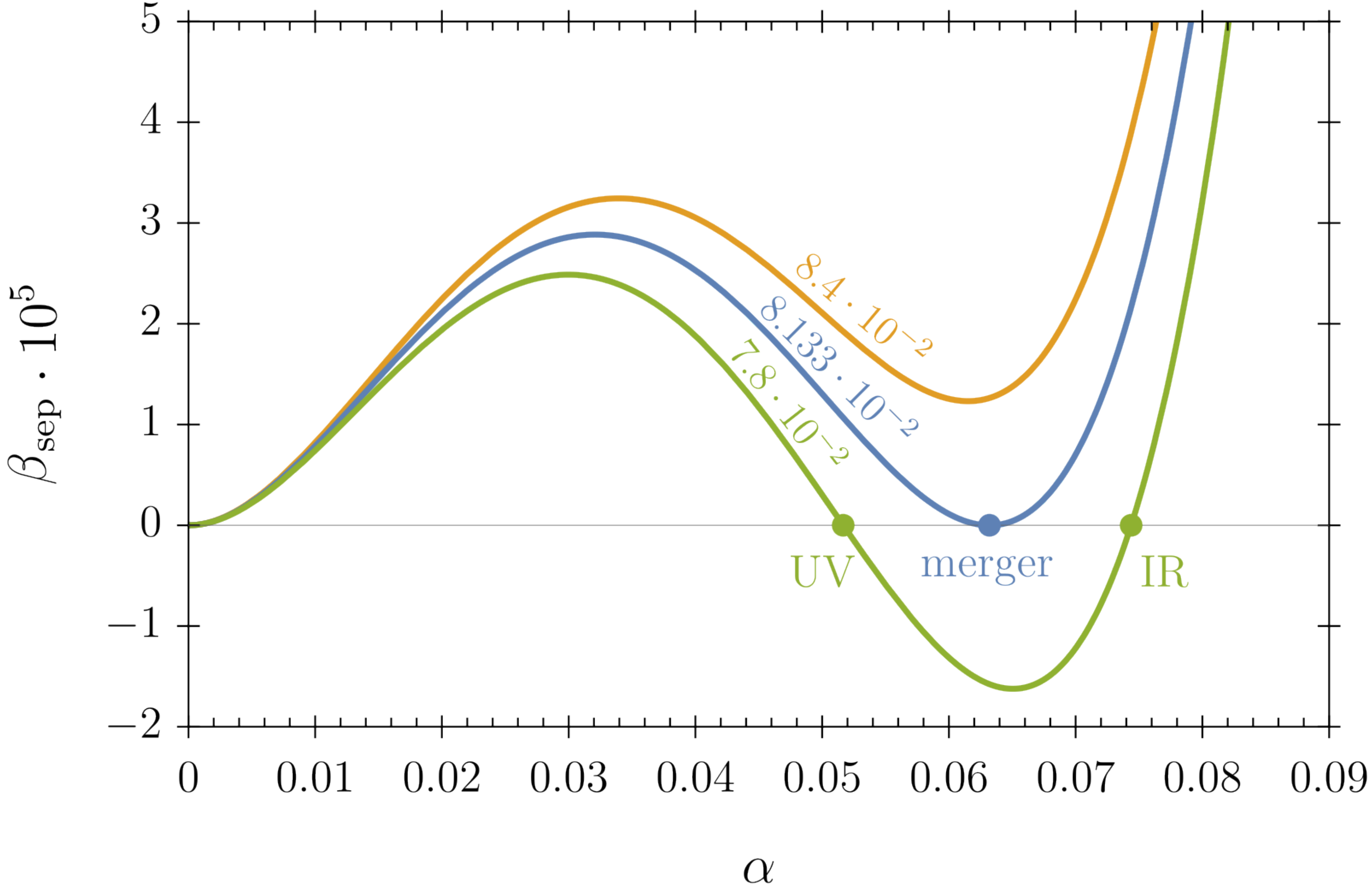
Relevant Separatrix



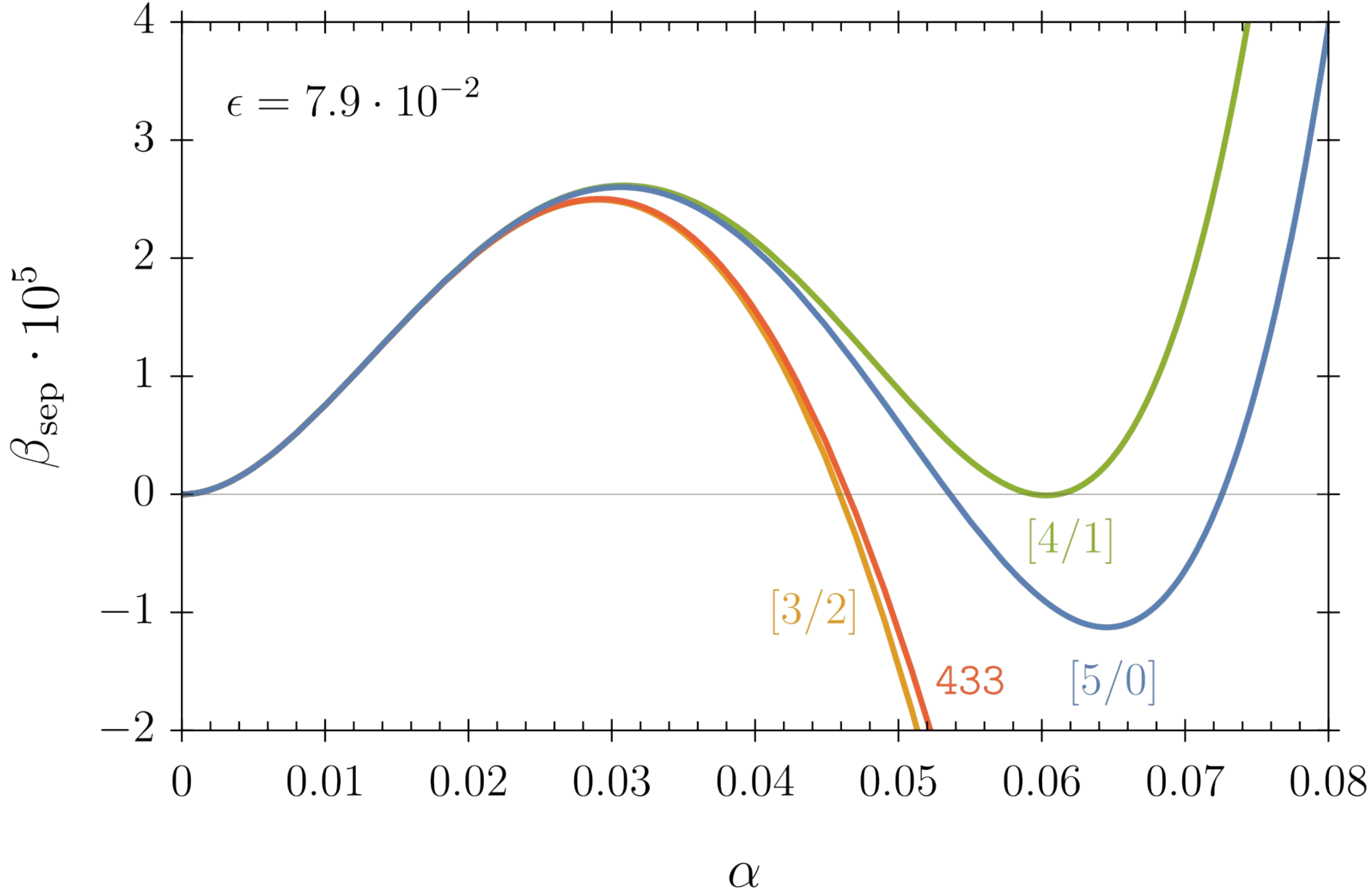
Relevant Separatrix



Relevant Separatrix



Relevant Separatrix – Padé resummation in α



Merger vs. Vacuum Instability

ϵ_{max}	$0.078 \ldots 0.081$	$0.087 \ldots 0.088$	$0.093 \ldots 0.095$
	$\beta_{\mathrm{sep}}^{[4/1]}$ β_{sep}	322 432 ϵ expansion	$\beta_{\mathrm{sep}}^{[3/2]}$ 433

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		ϵ expansion	

vacuum instability

compatible with ϵ - expansion

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single-trace merger dominated:
$\longrightarrow$ vacuum instability ?!

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» Is the problem β_{sep} vs. $\beta_{g,y,u,v}$? Higher loops?

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» Influence of quantum corrections to vacuum stability?

IV. Effective Potential

Effective Potential

» Boundedness from below of classical potential $\longrightarrow$ quantum effective potential
$\longrightarrow$ additional loop corrections for $\alpha_u^* + \alpha_v^* > 0$ [Litim, Mojaza, Sannino, 2015]

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» effective potential along close to UV FP due to RG invariance

$$\frac{V_{\text{eff}}}{(4\pi)^2} = \alpha_w^*(\epsilon,\, z_0)\, h^4 \left(\frac{z}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)} \qquad \text{with} \qquad z = \frac{(4\pi)^2}{N_f}\frac{h^2}{\mu^2}$$

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$$\frac{V_{\text{eff}}}{(4\pi)^2} = \underbrace{\alpha_w^*}_{\text{sign determines stability}}(\epsilon,\, \underbrace{z_0}_{\text{parameter}})\, \underbrace{h^4 \left(\frac{z}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)}}_{>0,\ \text{field dependence}} \qquad \text{with} \qquad z = \frac{(4\pi)^2}{N_f}\frac{h^2}{\mu^2}$$

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field dependence

$$\frac{V_{\text{eff}}}{(4\pi)^2} = \alpha_w^*(\epsilon,\, z_0)\, h^4 \underbrace{\left(\frac{z}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)}}_{>\,0} \qquad \text{with} \qquad z = \frac{(4\pi)^2}{N_f}\frac{h^2}{\mu^2}$$

parameter

sign determines stability

» determine α_w^* by matching fixed order computation at $z = z_0$
⟶ two loops for α_w^* up to ϵ^3

Fixed-order Effective Potential

» Expansion of scalar field around classical background field $U(N_f)_L \times U(N_f)_R \mapsto U(N_f)_V$

$$\phi_{ij} = h\,\delta_{ij} + \frac{\delta_{ij}}{\sqrt{2N_f}}\left(S + i\tilde{S}\right) + t^a_{ij}\,(R^a + iI^a)$$

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subleading in large-N

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» two loops matching:

$\alpha_w^{(0)} = \alpha_u + \alpha_v$ $\qquad\qquad$ $\alpha_w^{(\ell)} \propto \alpha^{\ell+1}$ $\qquad\qquad$ also logs: $\quad \propto \alpha^{\ell+1} \log^{\ell} \dfrac{\alpha_{F,R,I}}{z_0}$

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classical field subleading in large-N adjoint quantum fields

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only here

$$\longrightarrow \quad \alpha_F = \left(\tfrac{11}{2} + \epsilon\right)\alpha_y \qquad \alpha_R = 6\,\alpha_u + 2\,\alpha_v \qquad \alpha_I = 2\,\alpha_u + 2\,\alpha_v$$

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“Goldstone catastrophe”

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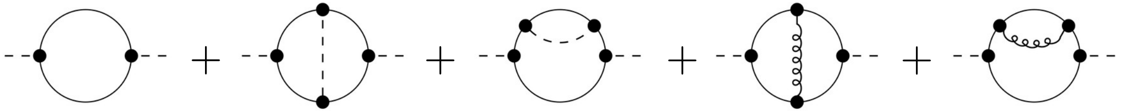
$\tilde{\alpha}_I = \alpha_I + \Delta_I$

$\Delta_I =$  $\sim \alpha_{g,y}$

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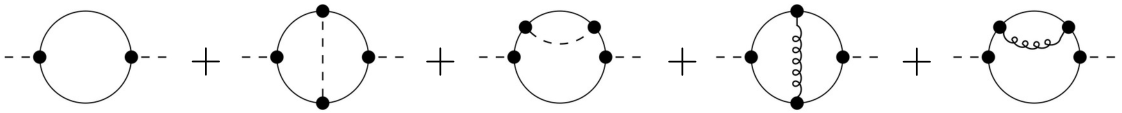
$\Delta_I =$  $\sim \alpha_{g,y}$

$$\alpha_I \log \frac{\alpha_I}{z_0} \mapsto \alpha_I \log \frac{\tilde{\alpha}_I}{z_0} - \Delta_I + \mathcal{O}\left(\Delta_I^2\right)$$

Logs

» Logarithms $\log\dfrac{\alpha_{F,R}}{z_0}$ and $\log\dfrac{\tilde{\alpha}_I}{z_0}$ can be partially removed by choosing z_0

$$\alpha_w^*(\epsilon,\, z_0') = \left(\frac{z_0'}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)} \alpha_w^*(\epsilon,\, z_0)$$

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» hierarchy at UV FP

$$0 < \underset{0.13\epsilon}{\tilde{\alpha}_I} \ll \underset{0.92\epsilon}{\alpha_R} \approx \epsilon < \underset{1.16\epsilon}{\alpha_F} \ll 1$$

Logs

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Logs

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» hierarchy at UV FP $\quad 0 < \tilde{\alpha}_I \ll \alpha_R \approx \epsilon < \alpha_F \ll 1$

$0.13\epsilon \quad 0.92\epsilon \quad 1.16\epsilon$

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best choice: $\quad z_0 \neq \tilde{\alpha}_I \quad \boxed{\alpha_R \leq z_0 \leq \alpha_F}$

Logs

» Logarithms $\log \frac{\alpha_{F,R}}{z_0}$ and $\log \frac{\tilde{\alpha}_I}{z_0}$ can be partially removed by choosing z_0

$$\alpha_w^*(\epsilon,\, z_0') = \left(\frac{z_0'}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)} \alpha_w^*(\epsilon,\, z_0)$$

» hierarchy at UV FP $\quad 0 < \tilde{\alpha}_I \ll \alpha_R \approx \epsilon < \alpha_F \ll 1$

$0.13\epsilon \quad 0.92\epsilon \quad 1.16\epsilon$

» soft logs in potential : $\quad \alpha_I^2 \log \frac{\tilde{\alpha}_I}{z_0} + \alpha_R^2 \log \frac{\alpha_R}{z_0} \quad \longrightarrow \quad \alpha_I^2 \log \frac{\tilde{\alpha}_I}{\alpha_R}$ better than $\alpha_R^2 \log \frac{\alpha_R}{\tilde{\alpha}_I}$

best choice: $\quad z_0 \neq \tilde{\alpha}_I \quad \boxed{\alpha_R \leq z_0 \leq \alpha_F}$

$\longrightarrow$ uncertainty estimate for higer loop corrections

Logs

» Logarithms $\log \frac{\alpha_{F,R}}{z_0}$ and $\log \frac{\tilde{\alpha}_I}{z_0}$ can be partially removed by choosing z_0

$$\alpha_w^*(\epsilon,\, z_0') = \left(\frac{z_0'}{z_0}\right)^{2\gamma_h^*/(1-\gamma_h^*)} \alpha_w^*(\epsilon,\, z_0)$$

» hierarchy at UV FP $\quad 0 < \tilde{\alpha}_I \ll \alpha_R \approx \epsilon < \alpha_F \ll 1$

$\tilde{\alpha}_I$: 0.13ϵ, α_R: 0.92ϵ, α_F: 1.16ϵ

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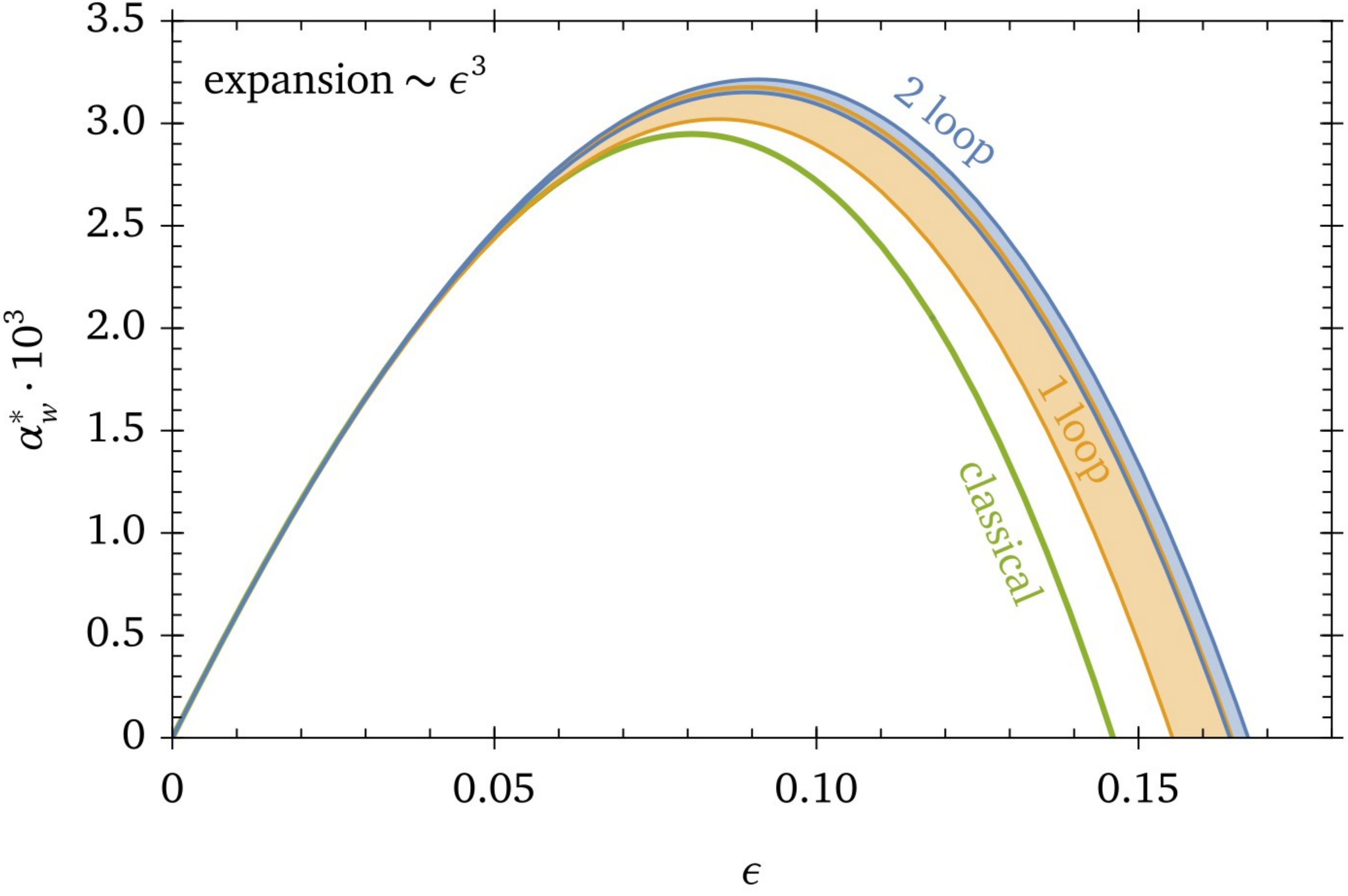
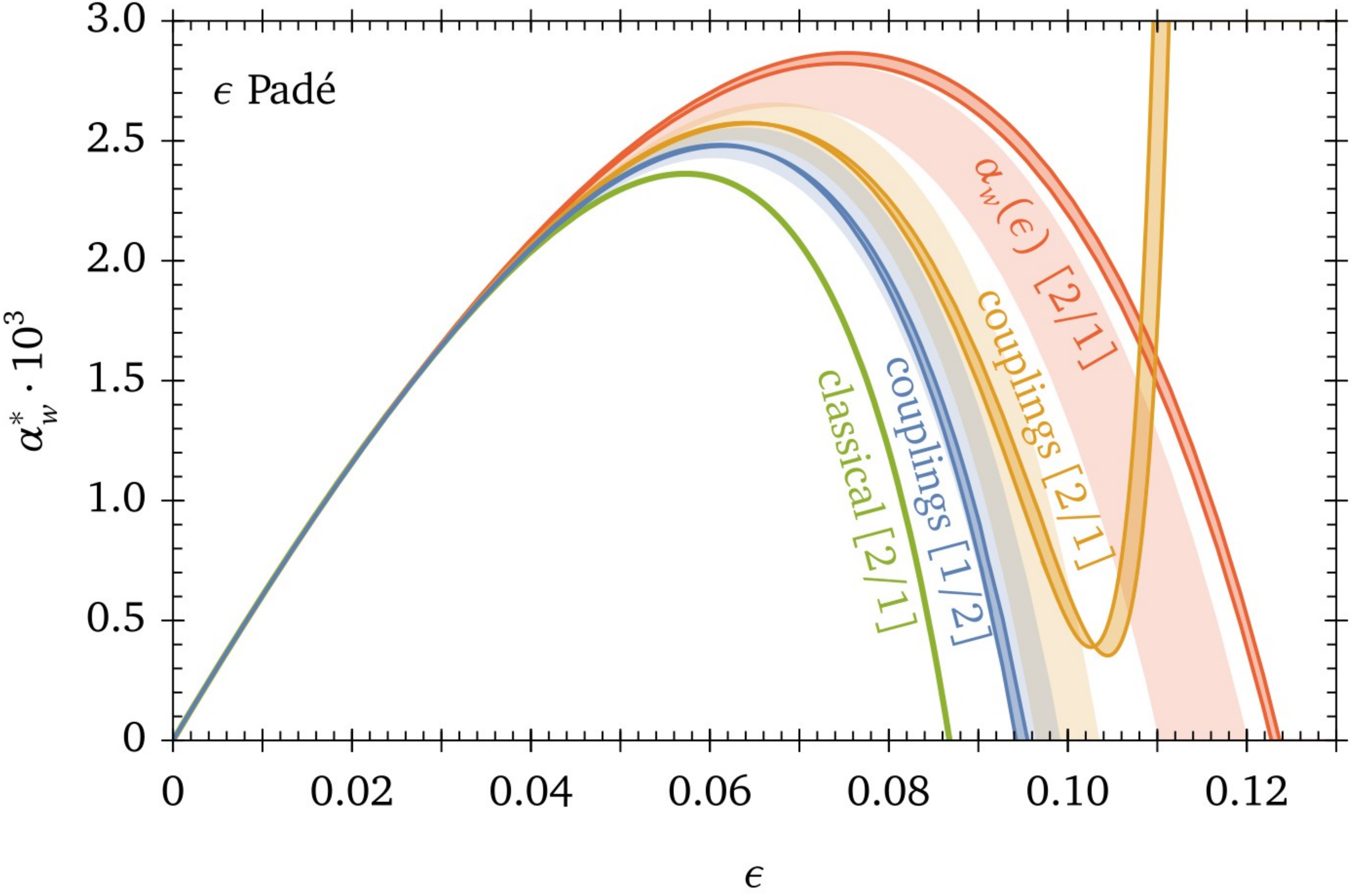
$\longrightarrow$ uncertainty estimate for higer loop corrections

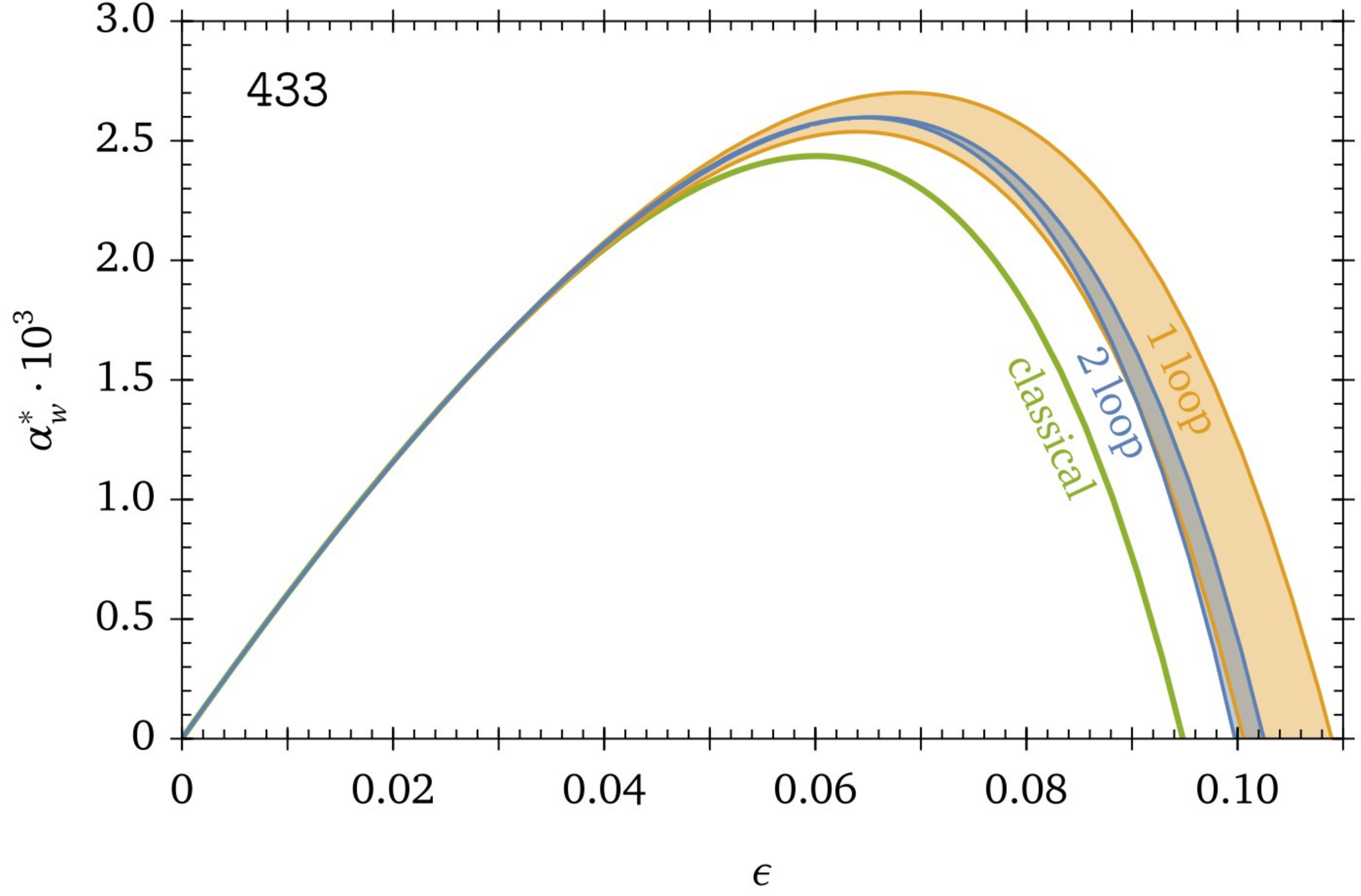
$\longrightarrow$ consecutive loop orders: nested ranges $\leftrightarrow$ relible loop expansion

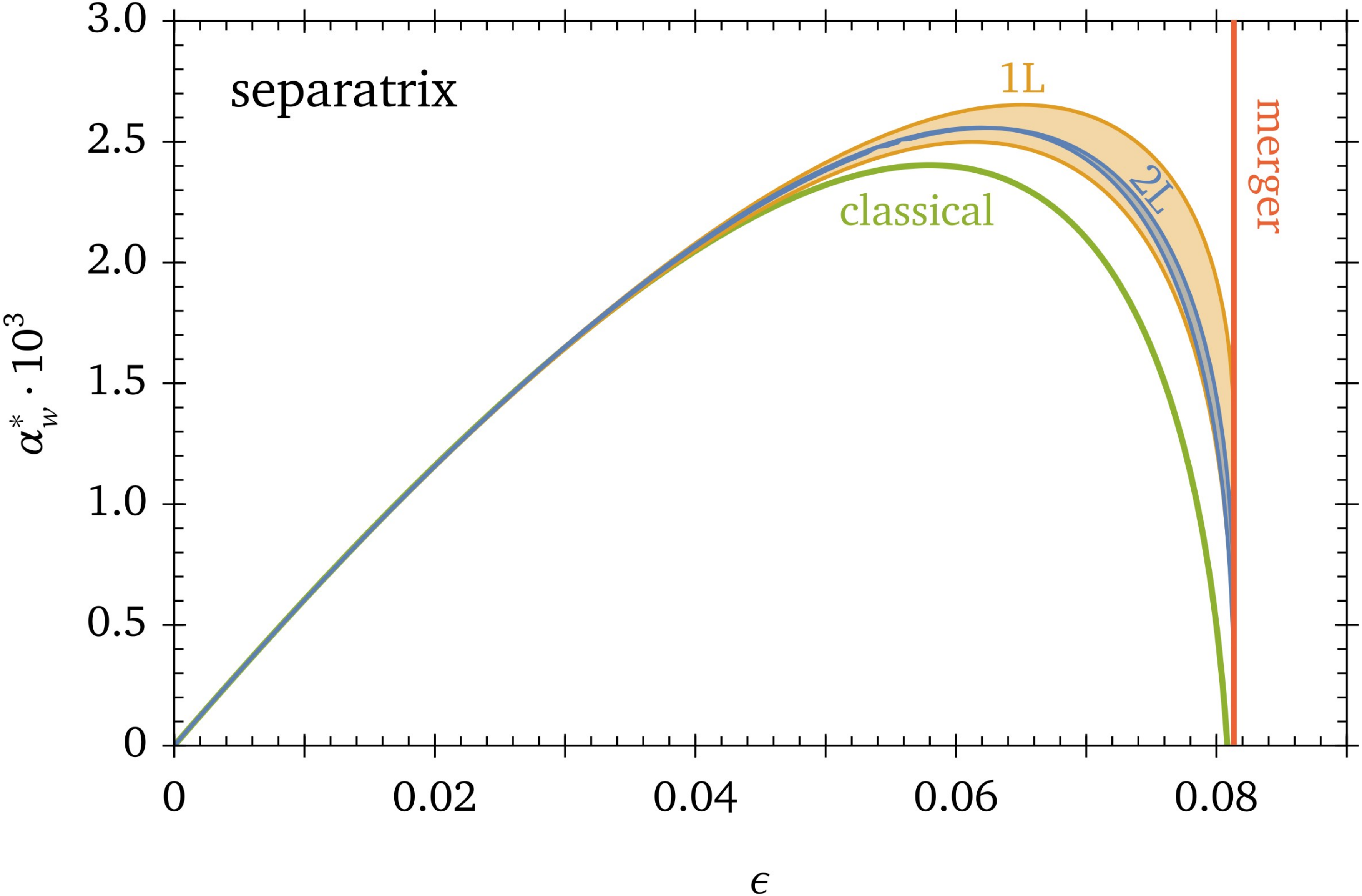


expansion $\sim \epsilon^3$
2 loop
1 loop
classical
$\alpha_w^* \cdot 10^3$
ϵ
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.05
0.10
0.15



ϵ Padé
$\alpha_w(\epsilon)$ [2/1]
couplings [2/1]
couplings [1/2]
classical [2/1]
$\alpha_w^* \cdot 10^3$
ϵ
3.0
2.5
2.0
1.5
1.0
0.5
0
0.02
0.04
0.06
0.08
0.10
0.12


433
1 loop
2 loop
classical
$\alpha_w^* \cdot 10^3$
ϵ
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.02
0.04
0.06
0.08
0.10


separatrix
1L
2L
classical
merger
$\alpha_w^* \cdot 10^3$
ϵ
3.0
2.5
2.0
1.5
1.0
0.5
0
0
0.02
0.04
0.06
0.08

Summary

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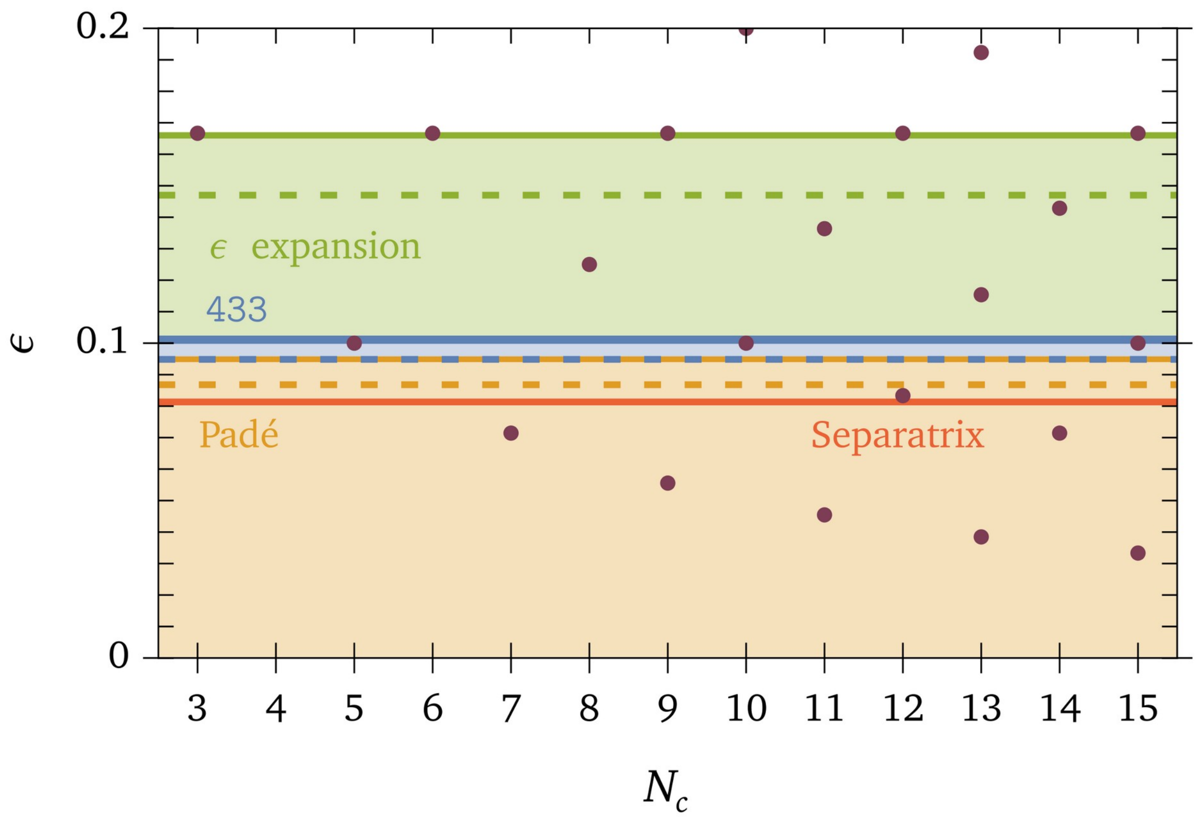

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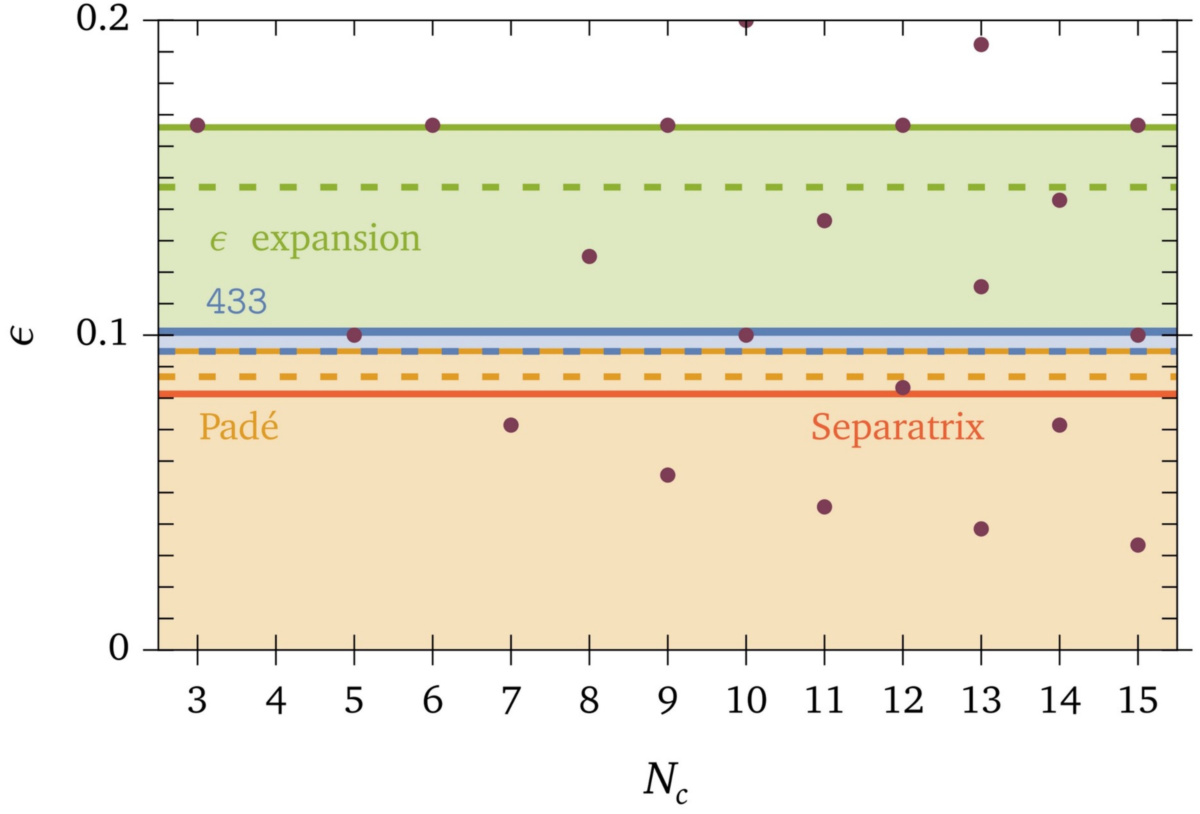

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- higher loops? γ_5-problem

  different approaches?