A new universality class describes Vicsek's flocking phase in physical dimensions

P. Jentsch, C.F. Lee, Phys. Rev. Lett. 133, 128301 (2024)

Patrick Jentsch
Chiu Fan Lee

IMPERIAL

EMBL

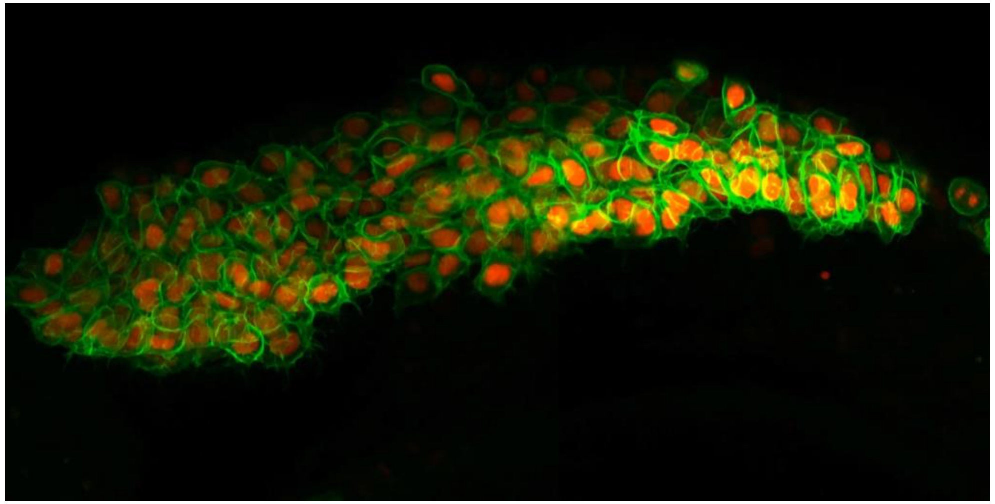
ERG2024

23/09/2024

Flocking

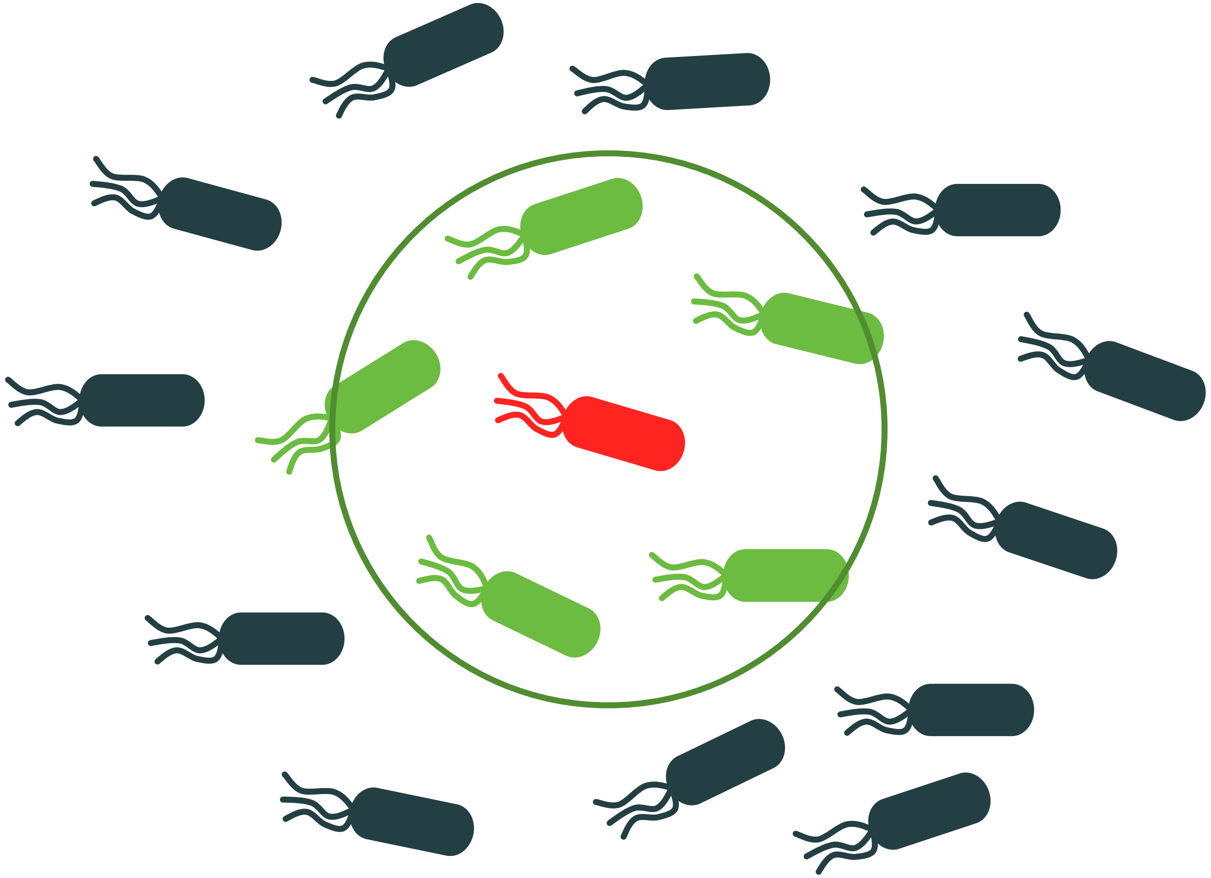
https://www.youtube.com/watch?v=q6iXT4-Oc2Q

Video by Nitesh Kamboj licensed by Pexels GmbH



https://www.quantamagazine.org/cells-blaze-their-own-trails-to-navigate-through-the-body-20220328/

The Vicsek Model



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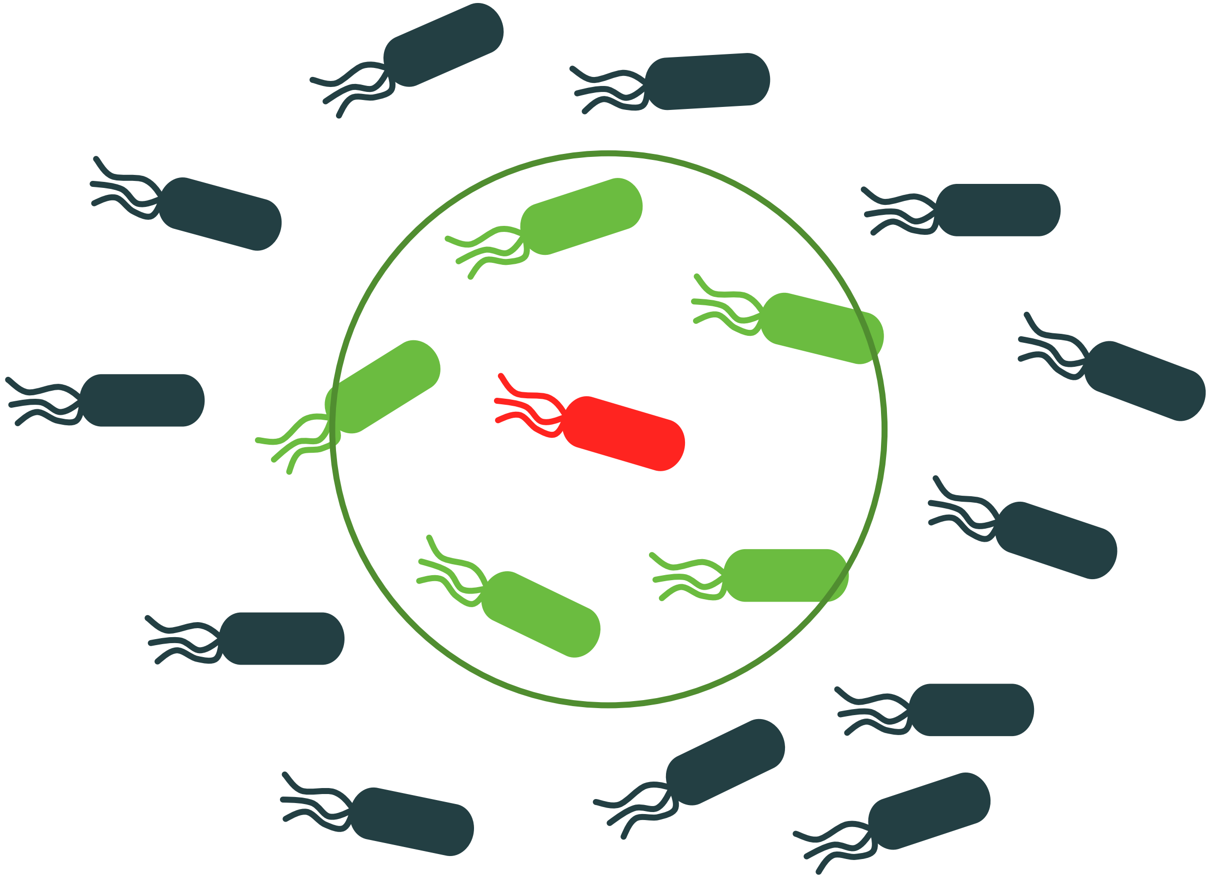

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$$\mathbf{x}_i(t+\Delta t) = \mathbf{x}_i(t) + \begin{pmatrix} \cos(\theta_i(t)) \\ \sin(\theta_i(t)) \end{pmatrix} v_m \Delta t$$

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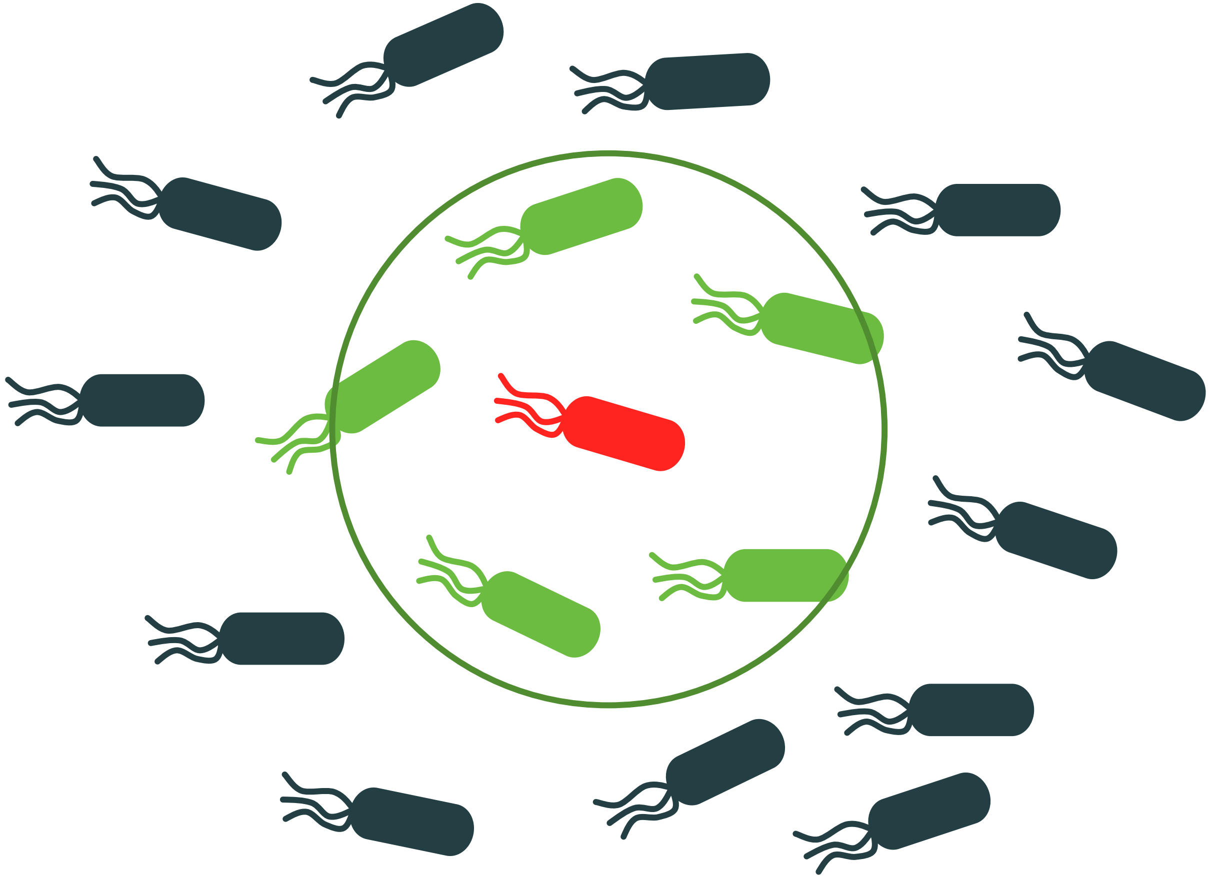

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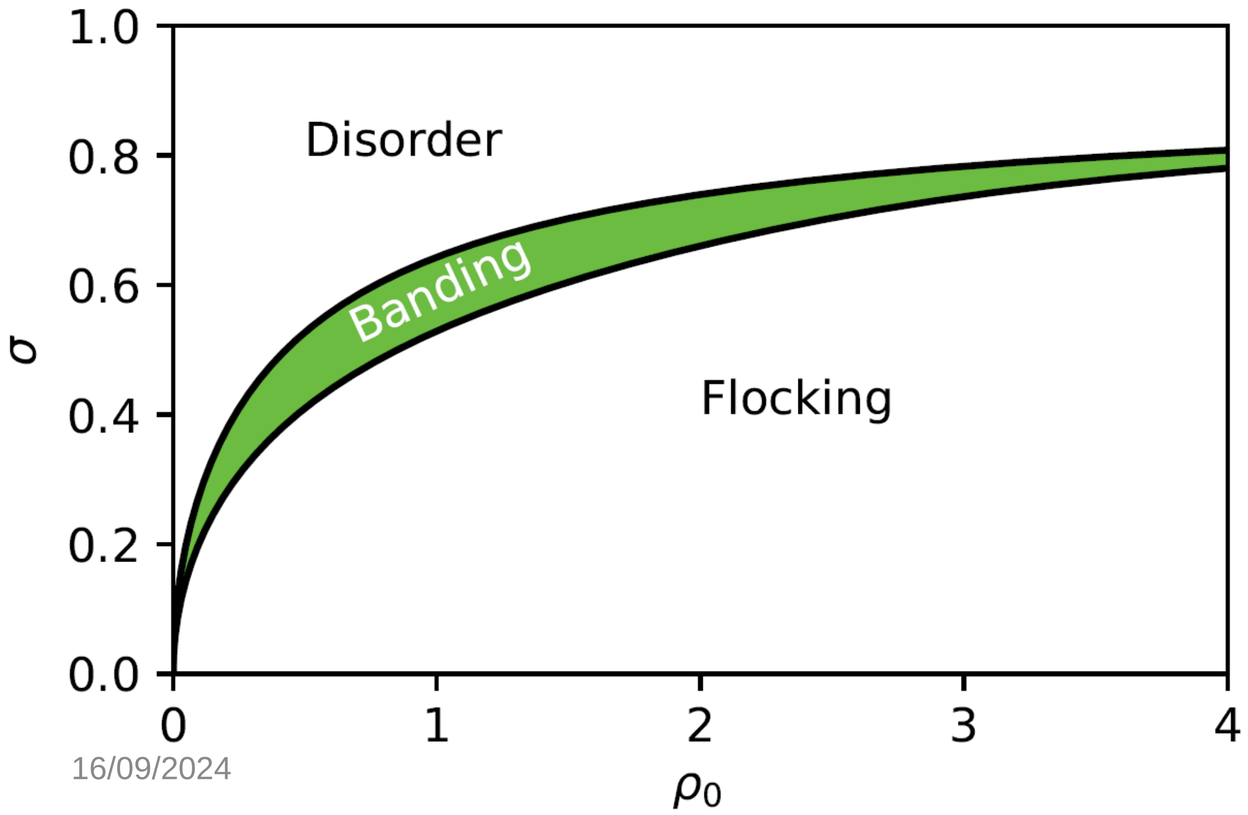
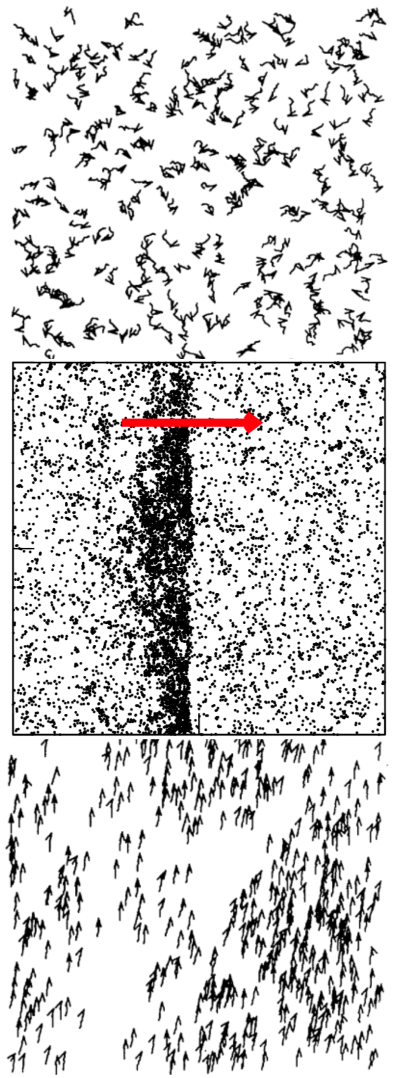
Phase diagram





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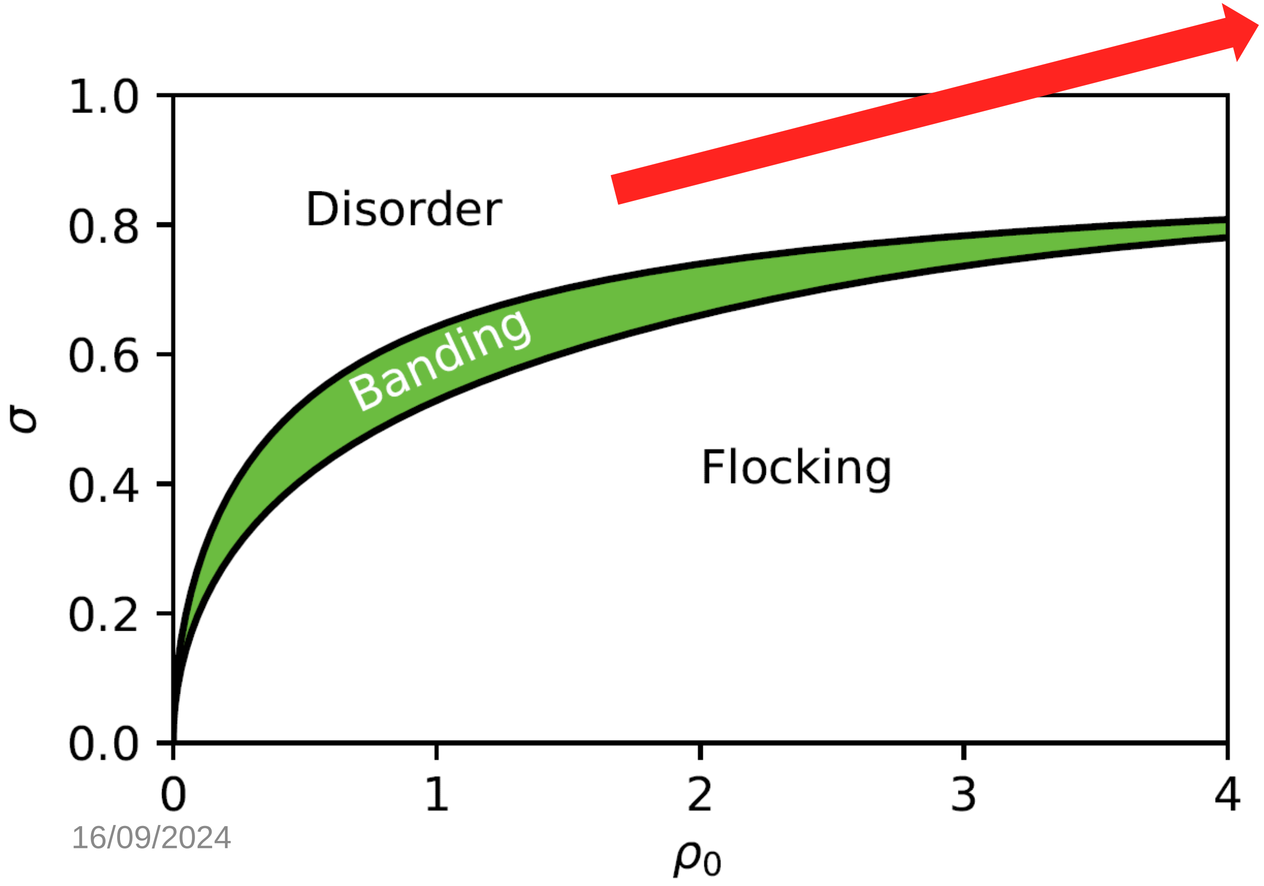

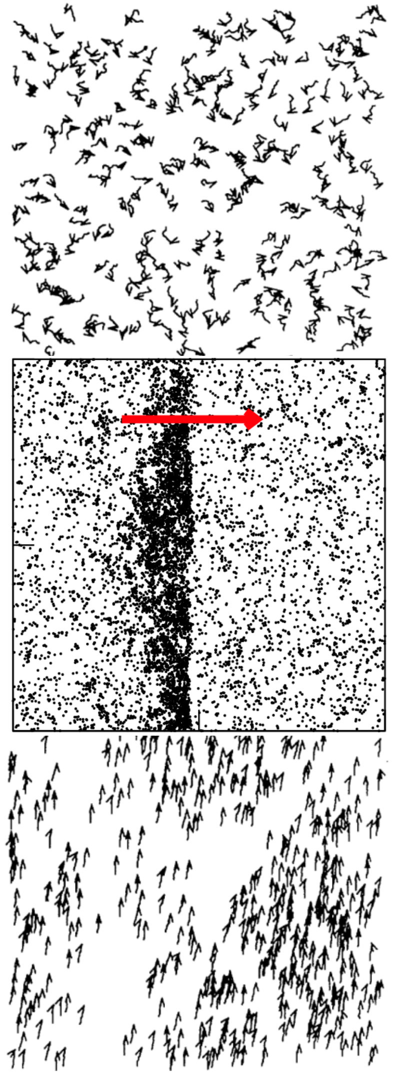

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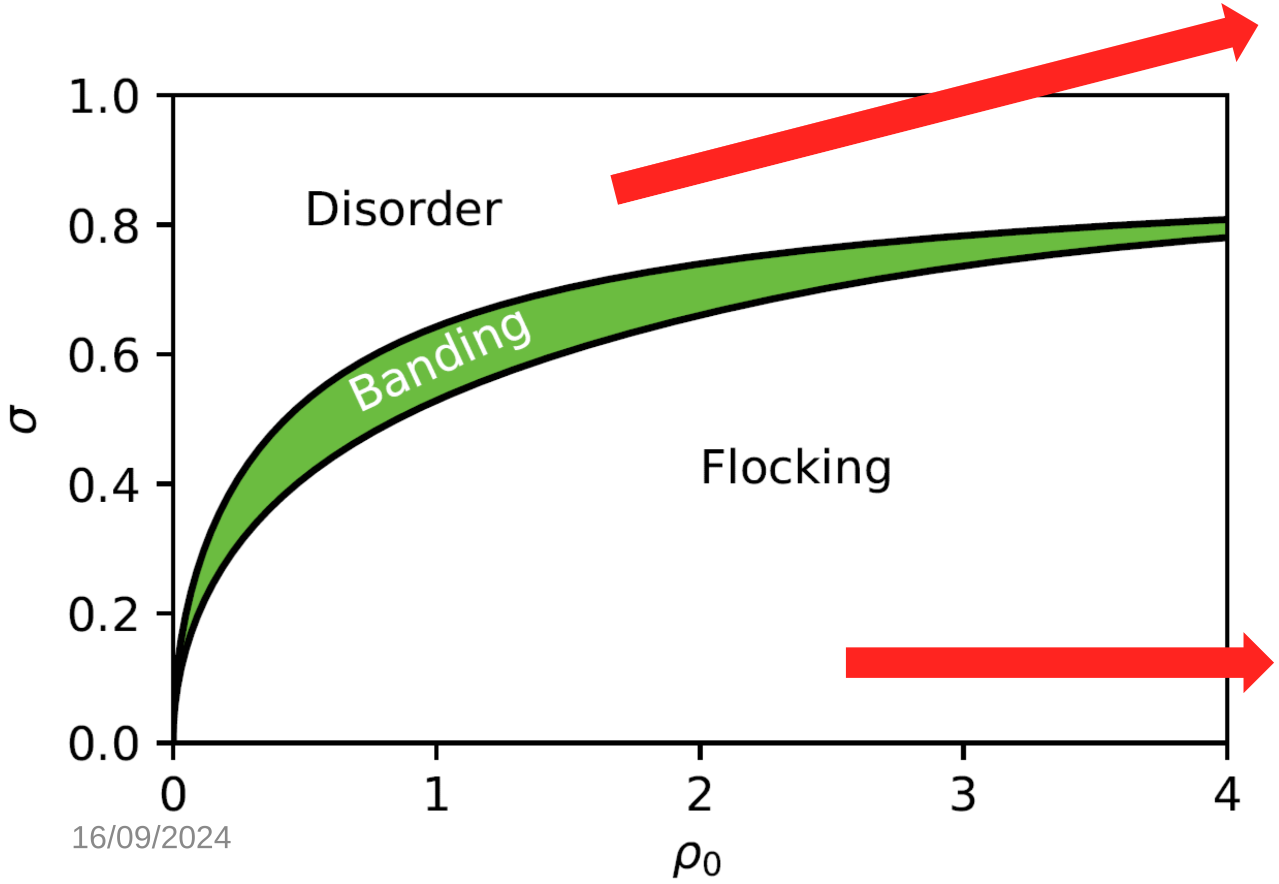

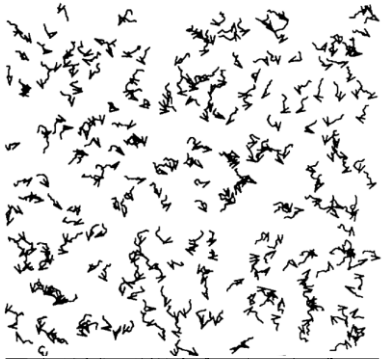

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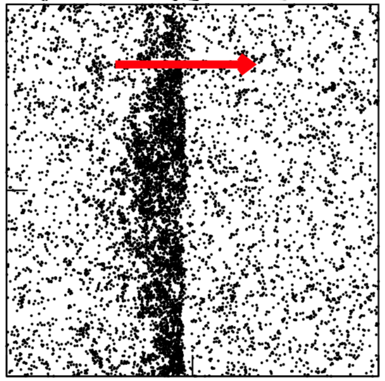

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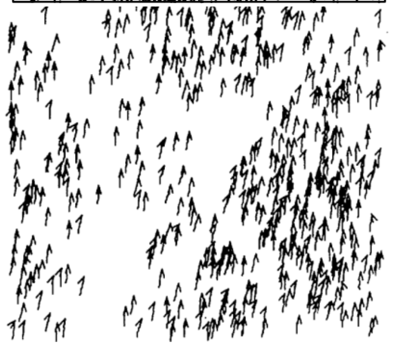

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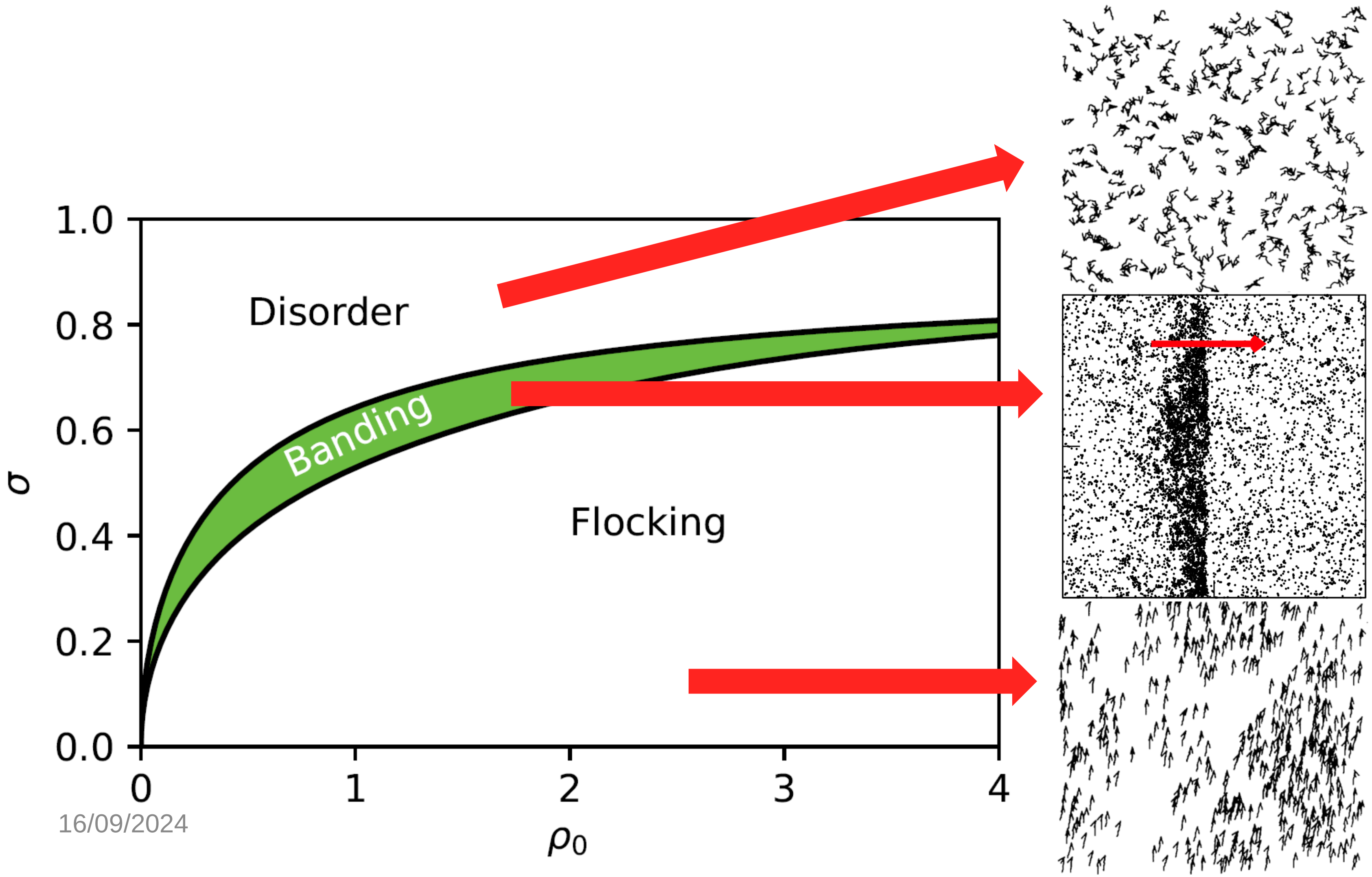

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Mahault, Ginelli, Chaté, PRL (2019)

Target: Scaling Exponents of Flocking Phase

$$\langle \delta\mathbf{g}(t, x, \mathbf{r}_\perp)\delta\mathbf{g}(0,0,\mathbf{0})\rangle = {r_\perp}^{2\chi} S_g\left(\frac{t}{{r_\perp}^{z}}, \frac{x - v_0 t}{{r_\perp}^{\zeta}}\right)$$

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$\eta > 0$ → Stable Ordered Phase in $d = 2$

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Under assumptions of:

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- Rotation sym.
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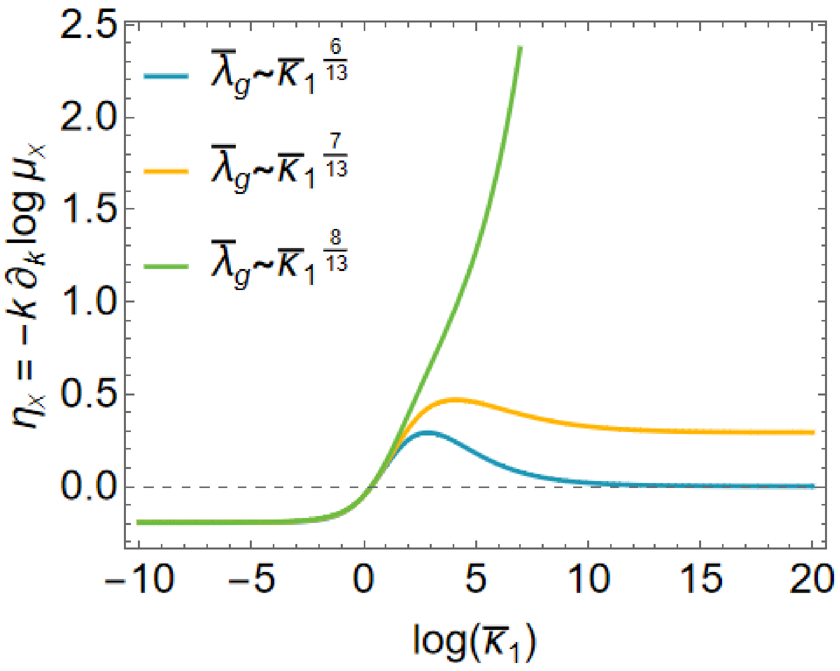
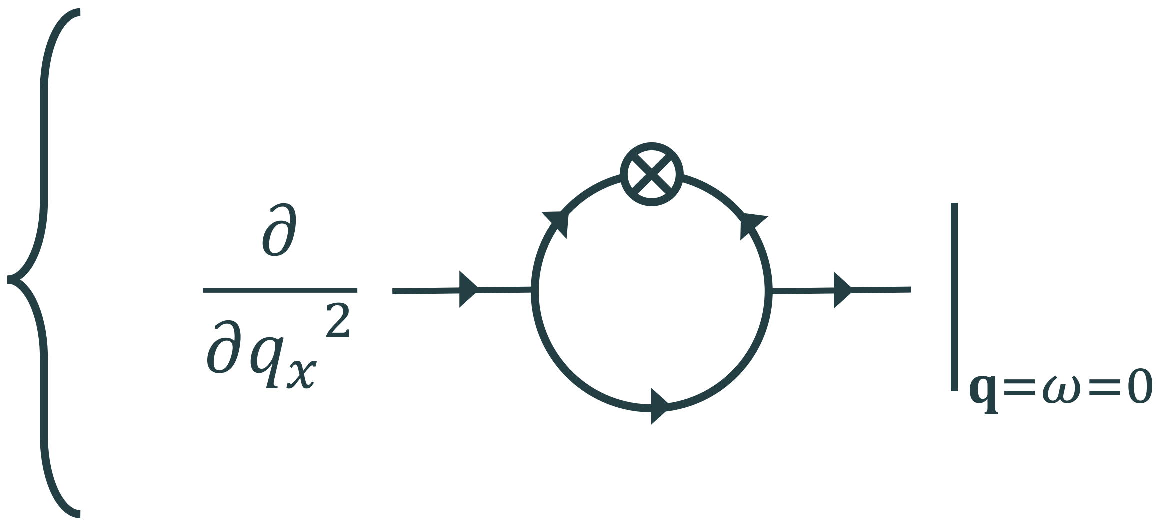
- q_x and ω remain unregulated

→ ω and $\boldsymbol{q}_\perp$ Integrals can be performed analytically

Unprecedented scaling relation





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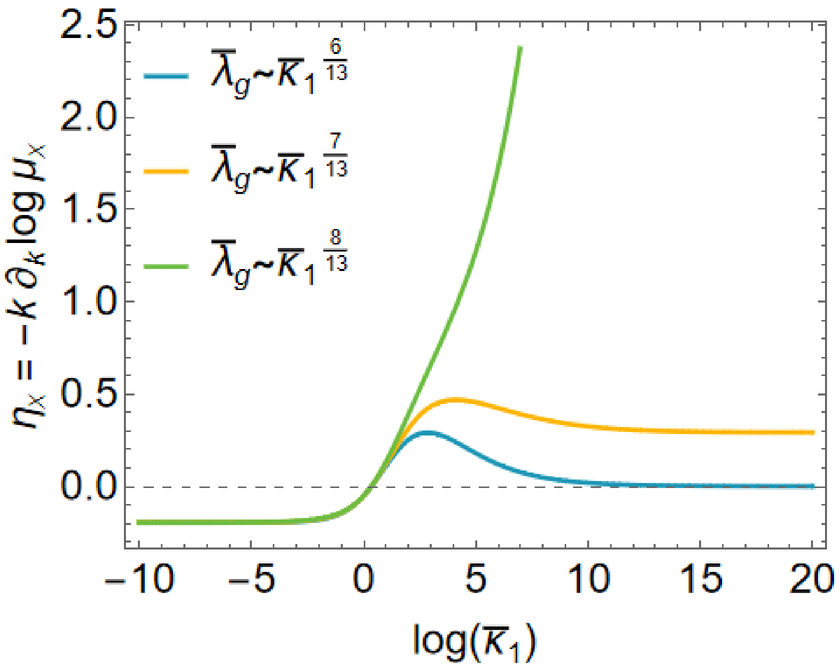

Two possible fixed points

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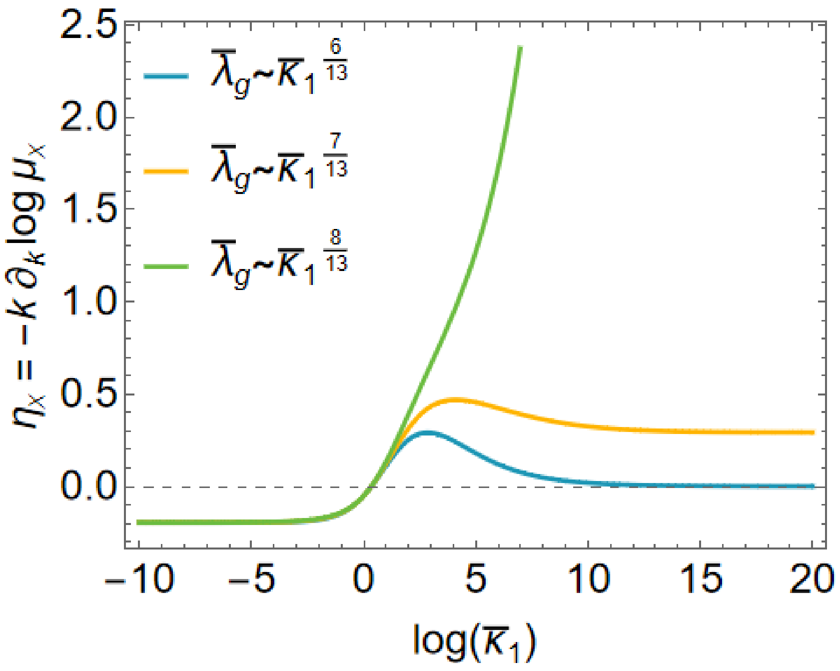

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- $\frac{\bar{\kappa}_1^7}{\bar{\lambda}_g^{13}} = 0, \eta_x = 0$ → TT95

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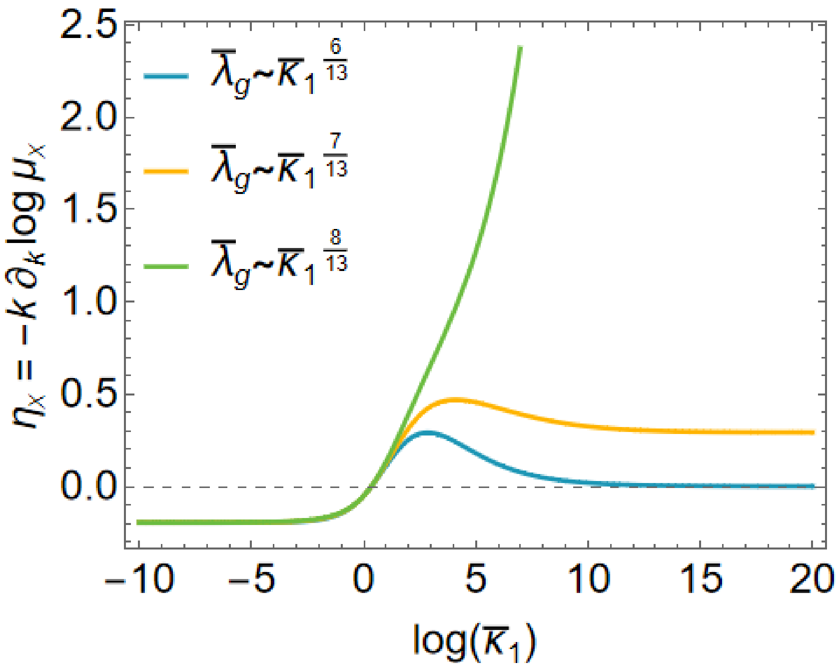

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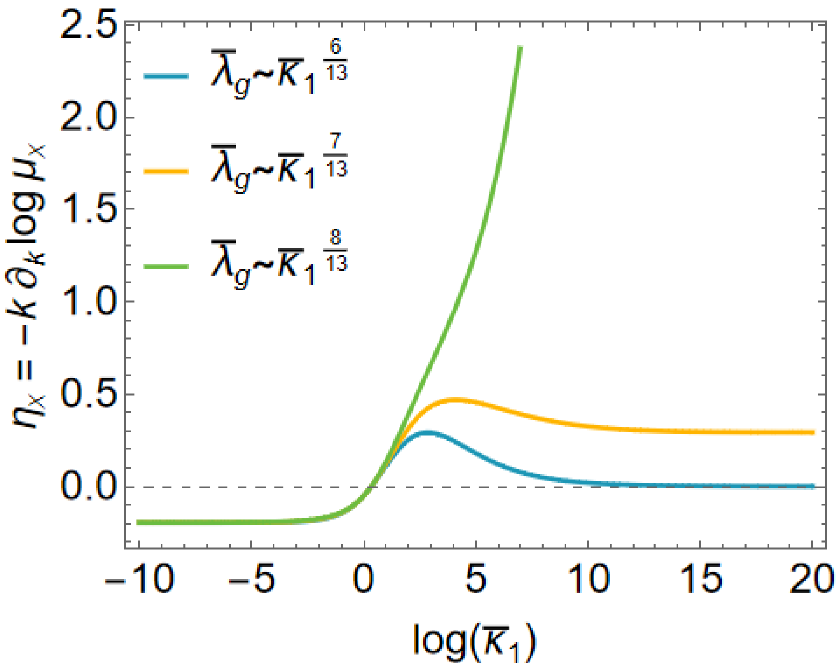

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Nontrivial Scaling Relation

$7(2z-2) = 13(z-\zeta)$

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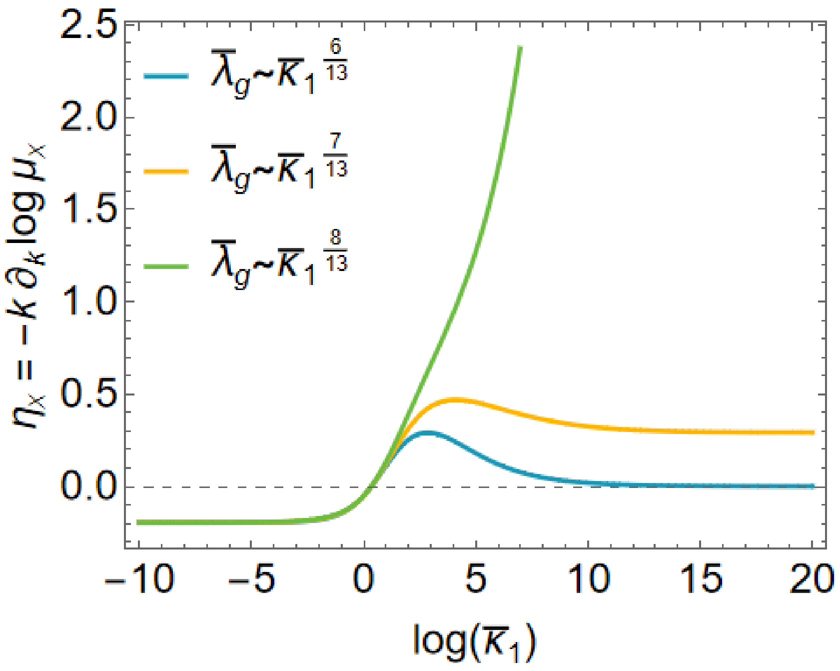

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2 other vanishing loop corrections
→ 3 scaling relations fix exponents

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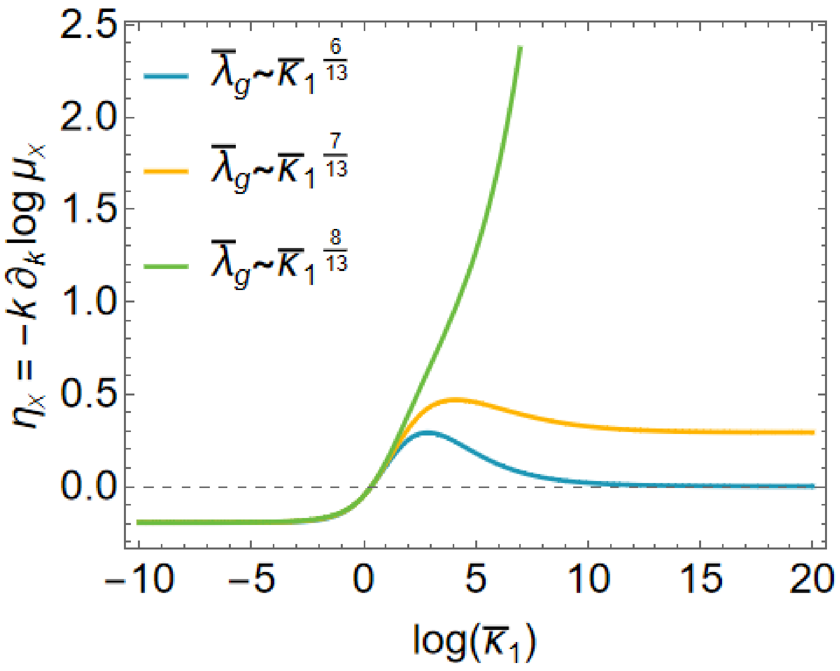

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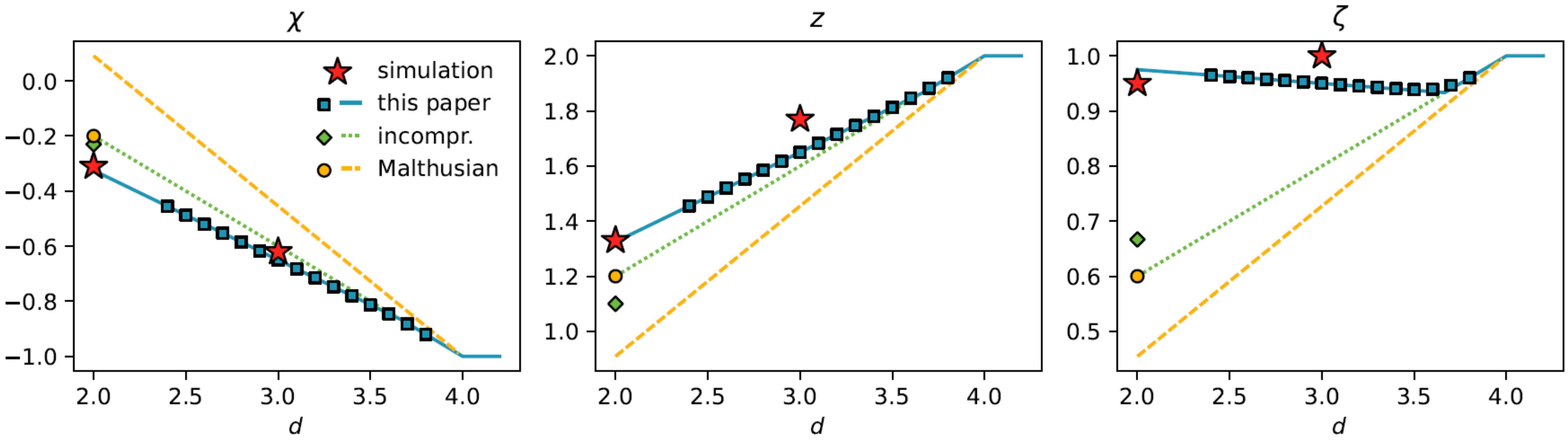
$$\chi = \frac{13(1-d)}{40}, \qquad z = \frac{27+13d}{40}, \qquad \zeta = \frac{41-d}{40}$$

Scaling Exponents



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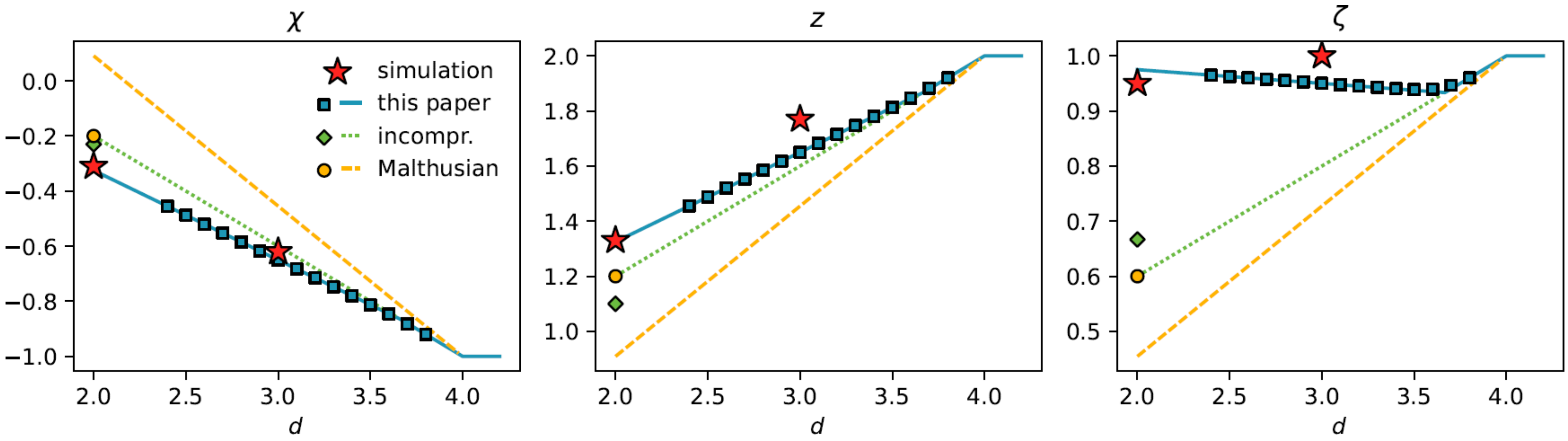
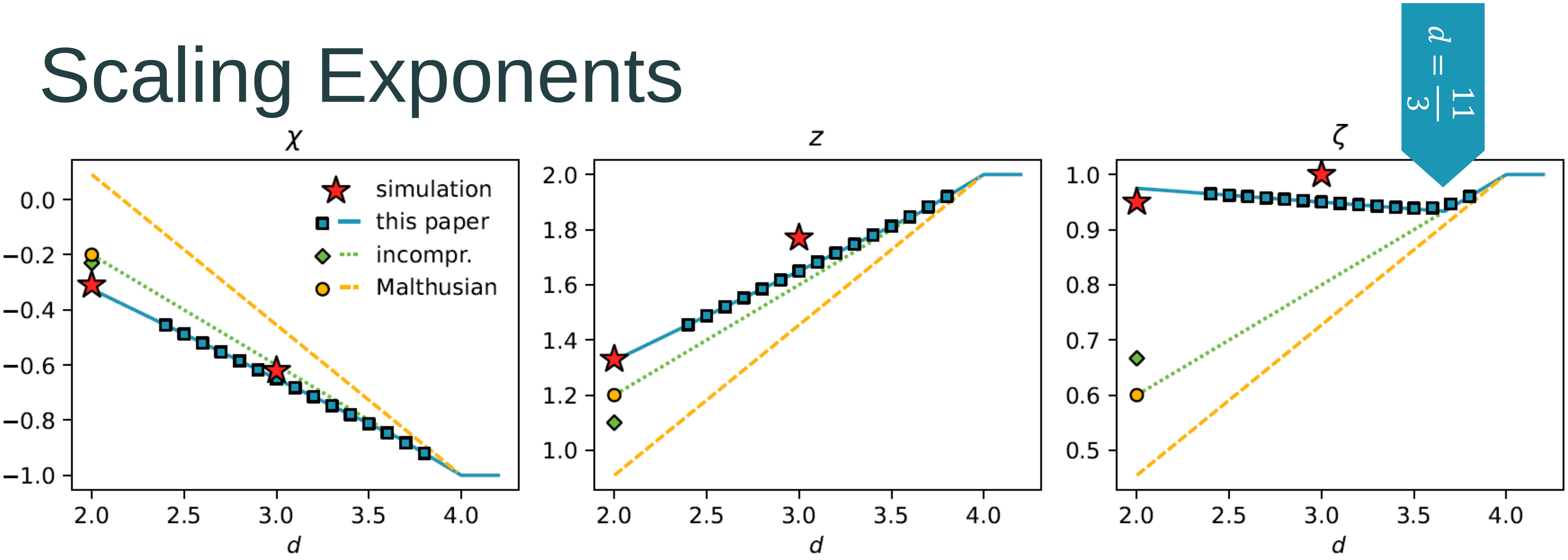


Spatial dimension (d)	χ	z	ζ
$d=2:$			
TT 95	−0.20	1.20	0.6
simulation	−0.31(2)	1.33(2)	0.95(2)
new UC	−0.325	1.325	0.975
$d=3:$			
TT 95	−0.60	1.60	0.8
simulation	−0.62	1.77	1
new UC	−0.65	1.65	0.95

Scaling Exponents



Spatial dimension (d)	χ	z	ζ
$d = 2:$			
TT 95	−0.20	1.20	0.6
simulation	−0.31(2)	1.33(2)	0.95(2)
new UC	−0.325	1.325	0.975
$d = 3:$			
TT 95	−0.60	1.60	0.8
simulation	−0.62	1.77	1
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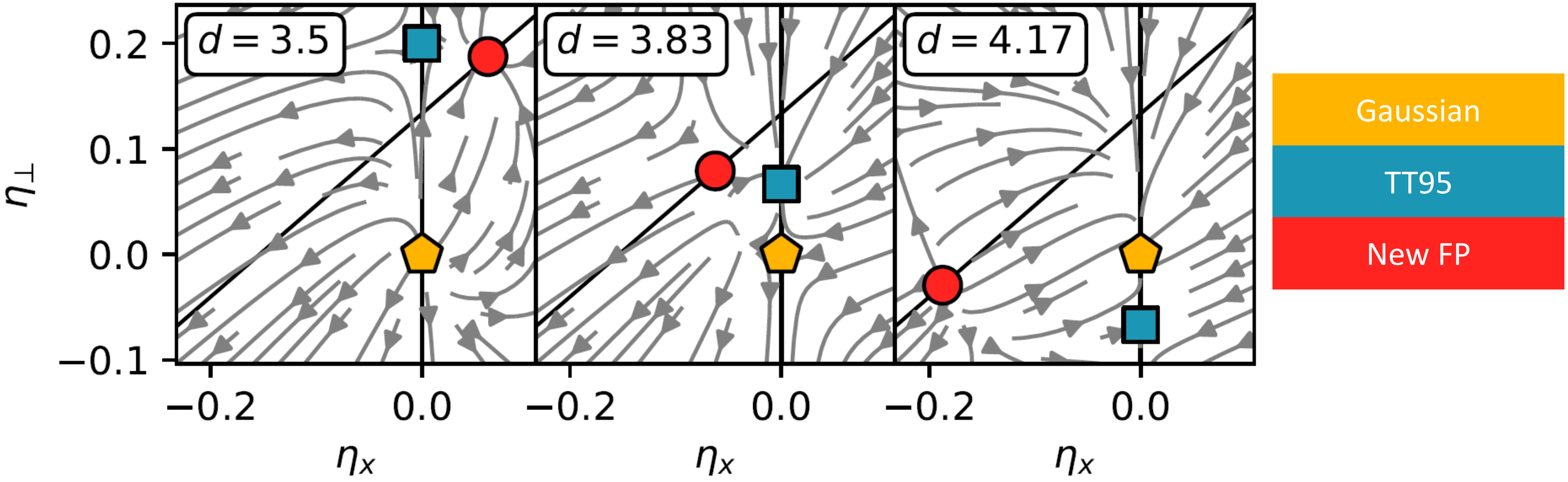
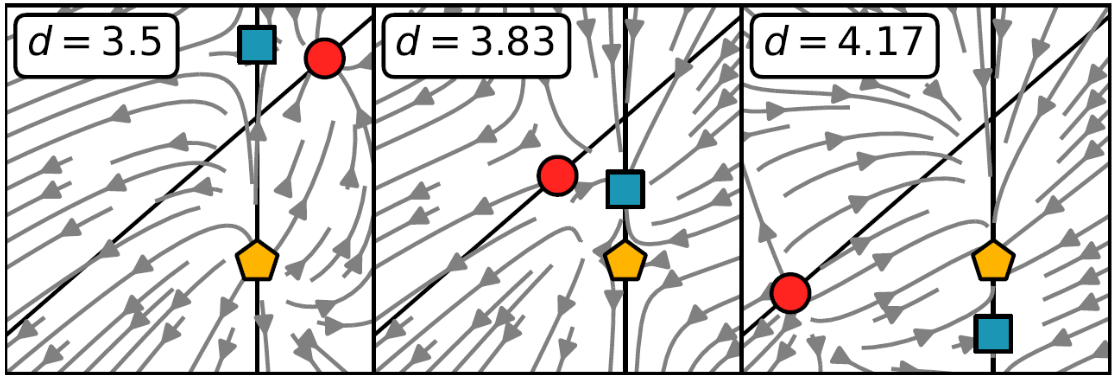
Fixed points*



*Qualitative Flow: Flow lines are fictitious

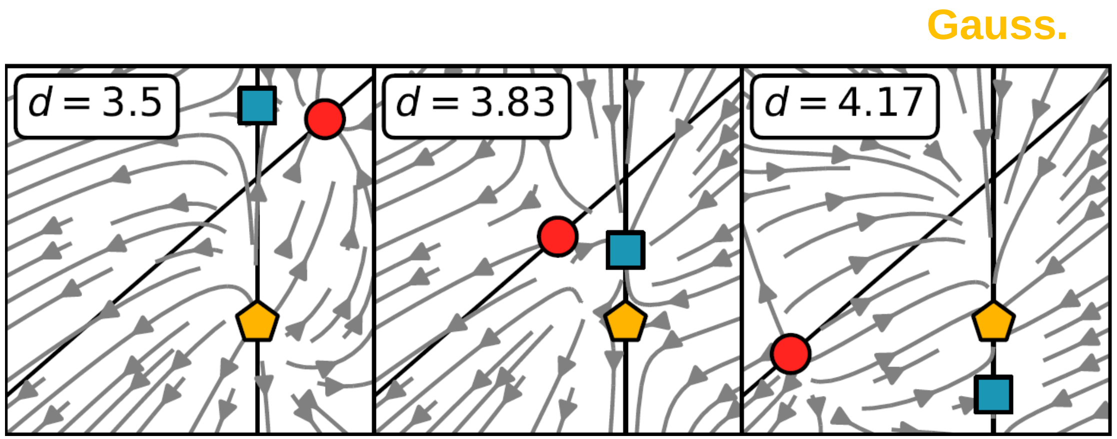
Summary

Gauss.



Summary

- We used nonperturbative, functional RG to study a simplified TT model

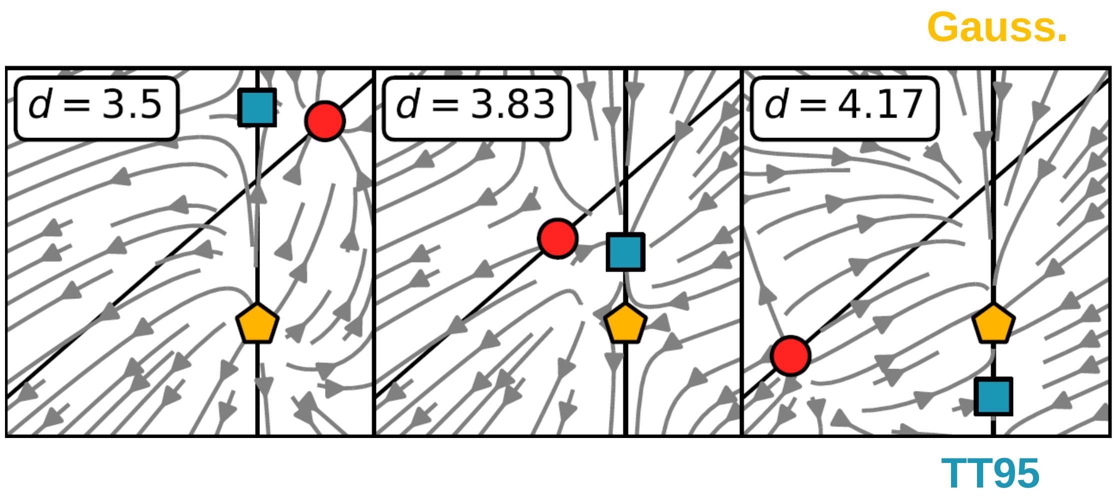


Summary

- We used nonperturbative, functional RG to study a simplified TT model

- TT UC applies for

$$\frac{11}{3}(\approx 3.67) < d < 4$$



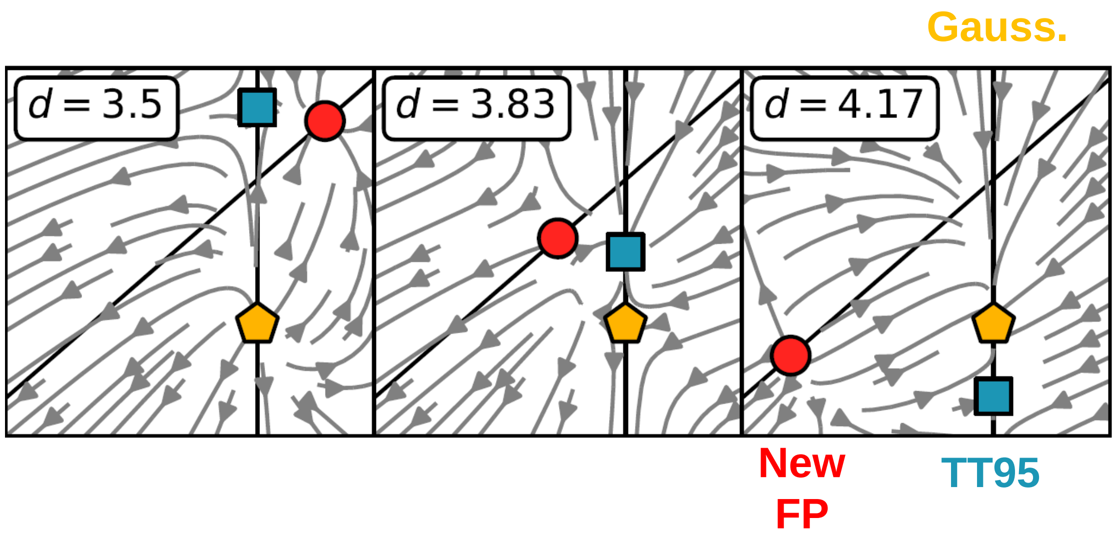
Summary

- We used nonperturbative, functional RG to study a simplified TT model

- TT UC applies for

$$\frac{11}{3}(\approx 3.67) < d < 4$$

- Below $d = 11/3$, a new UC emerges, whose scaling exponents agree remarkably well with simulation in 2D & 3D



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Thank you!

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