Hydrodynamic attractors in strongly correlated fermions

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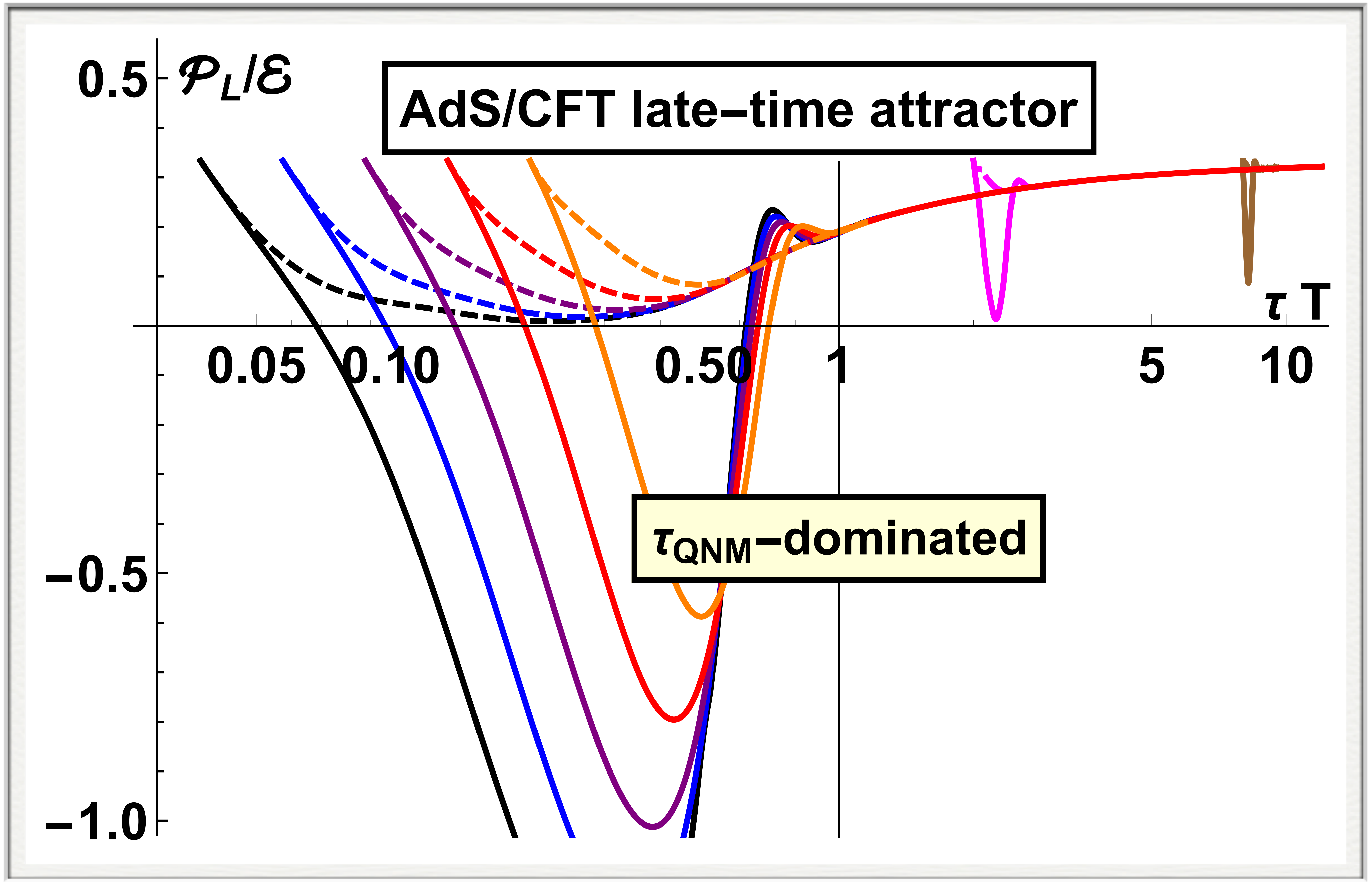
STRUCTURES CLUSTER OF EXCELLENCE

IsoQuant SFB1225

Keisuke Fujii
2404.12921
(PRL in press)

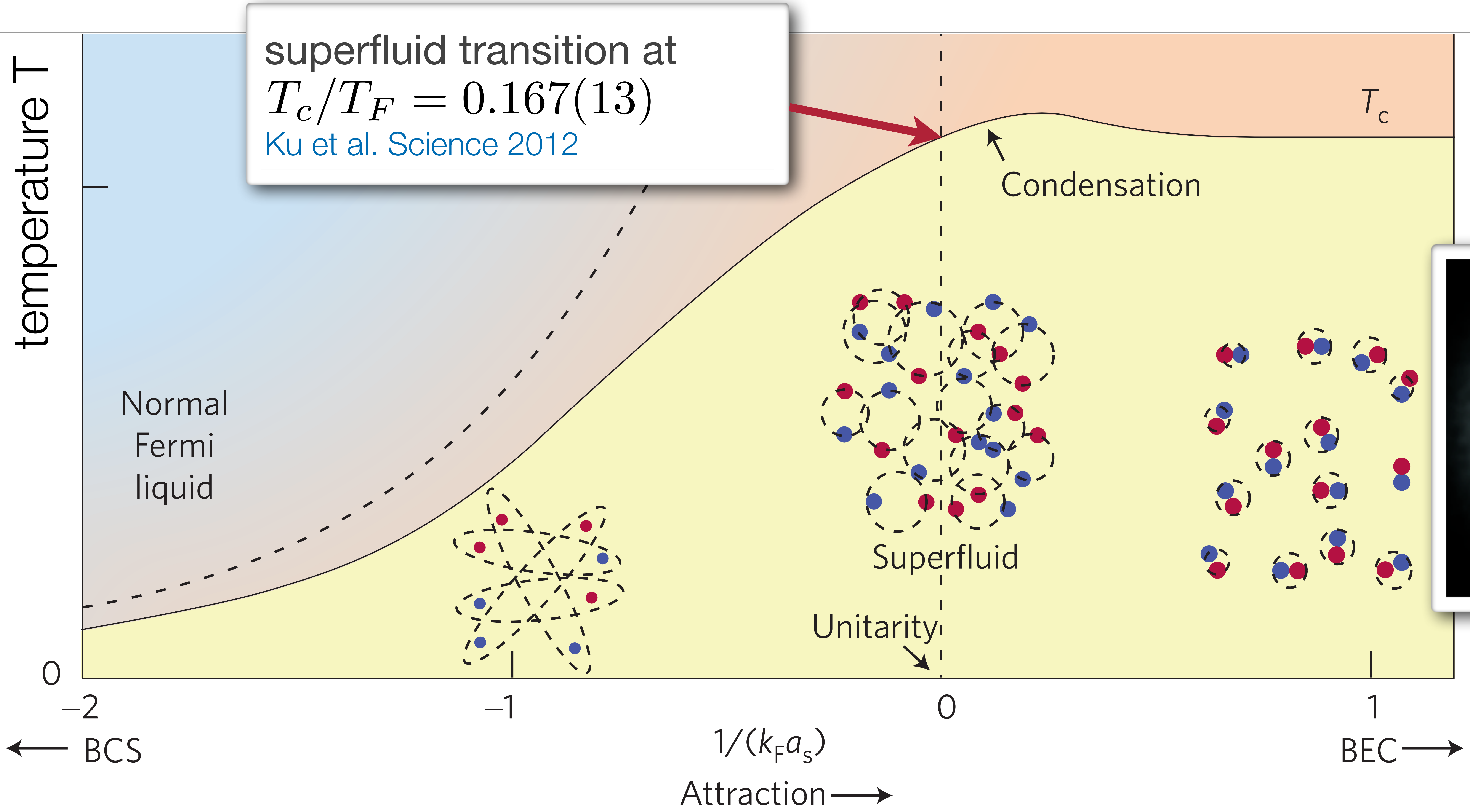
Attractor in heavy-ion collisions

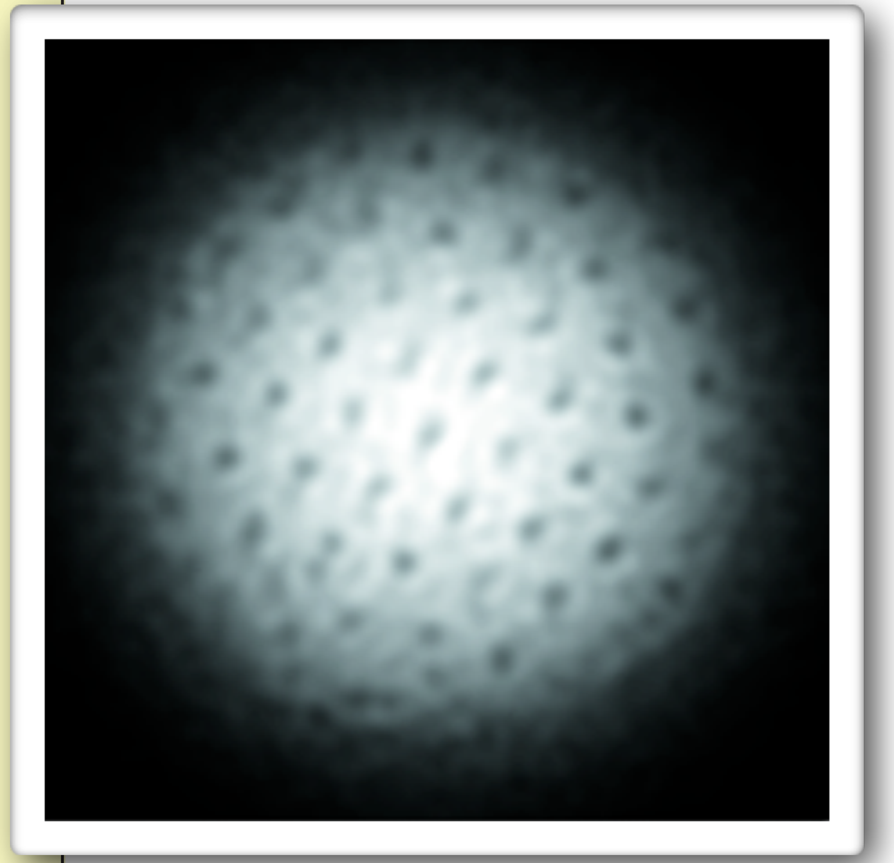
- fluid behavior observed already soon after collision, before Navier-Stokes valid
- model calculations show converge on attractor for different initial conditions
- observe real-time dynamics in experiment?



Kurkela, van der Schee, Wiedemann, Wu PRL 2020
Berges, Heller, Mazeliauskas, Venugopalan RMP 2021

strongly interacting Fermi gas



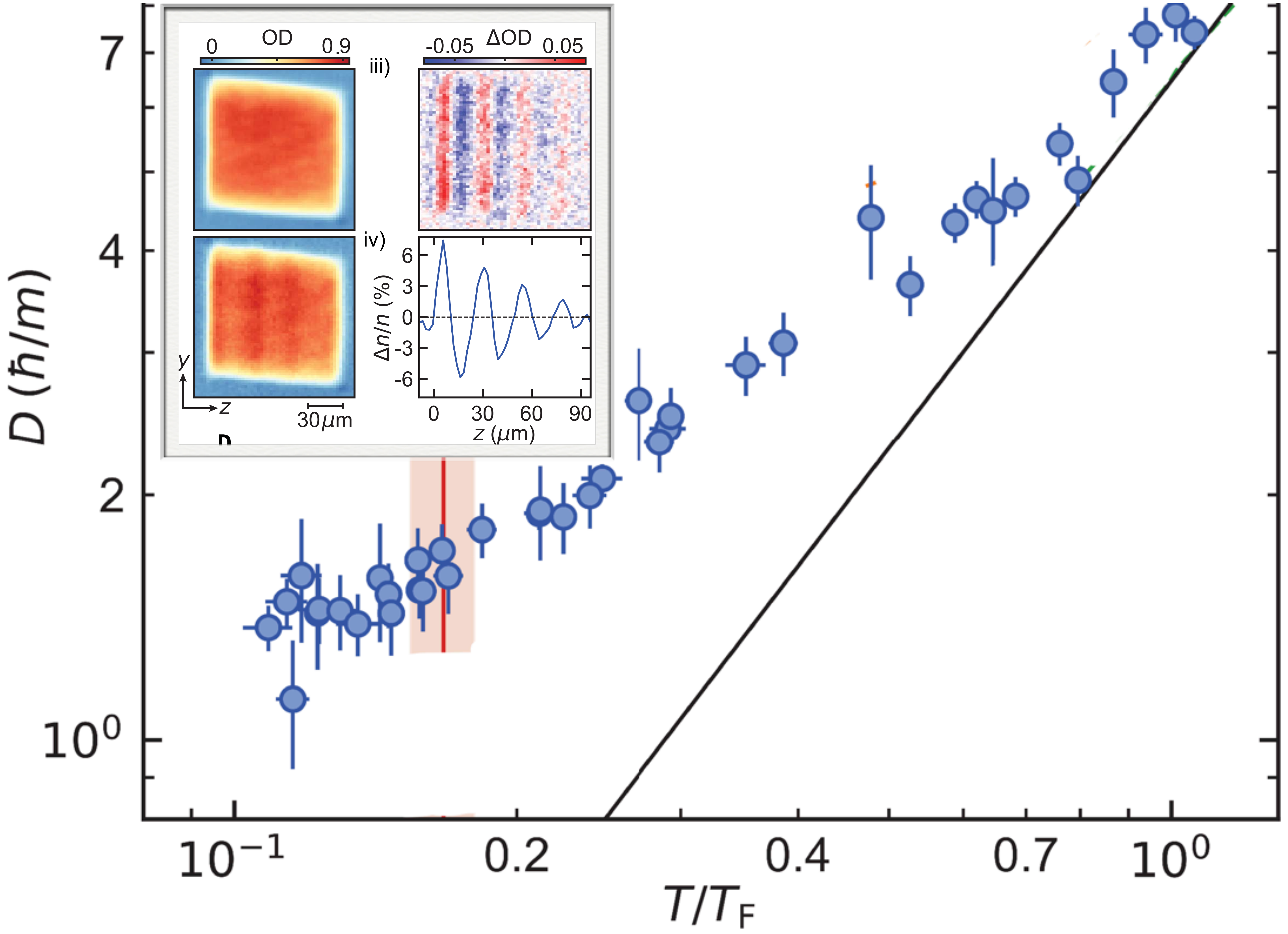


dilute gas of ↑ and ↓ fermions with contact interaction

$$\mathcal{H} = \int d\mathbf{x} \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu_\sigma \right) \psi_\sigma + g_0 \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow$$

Randeria, Zwerger & Zwierlein 2012

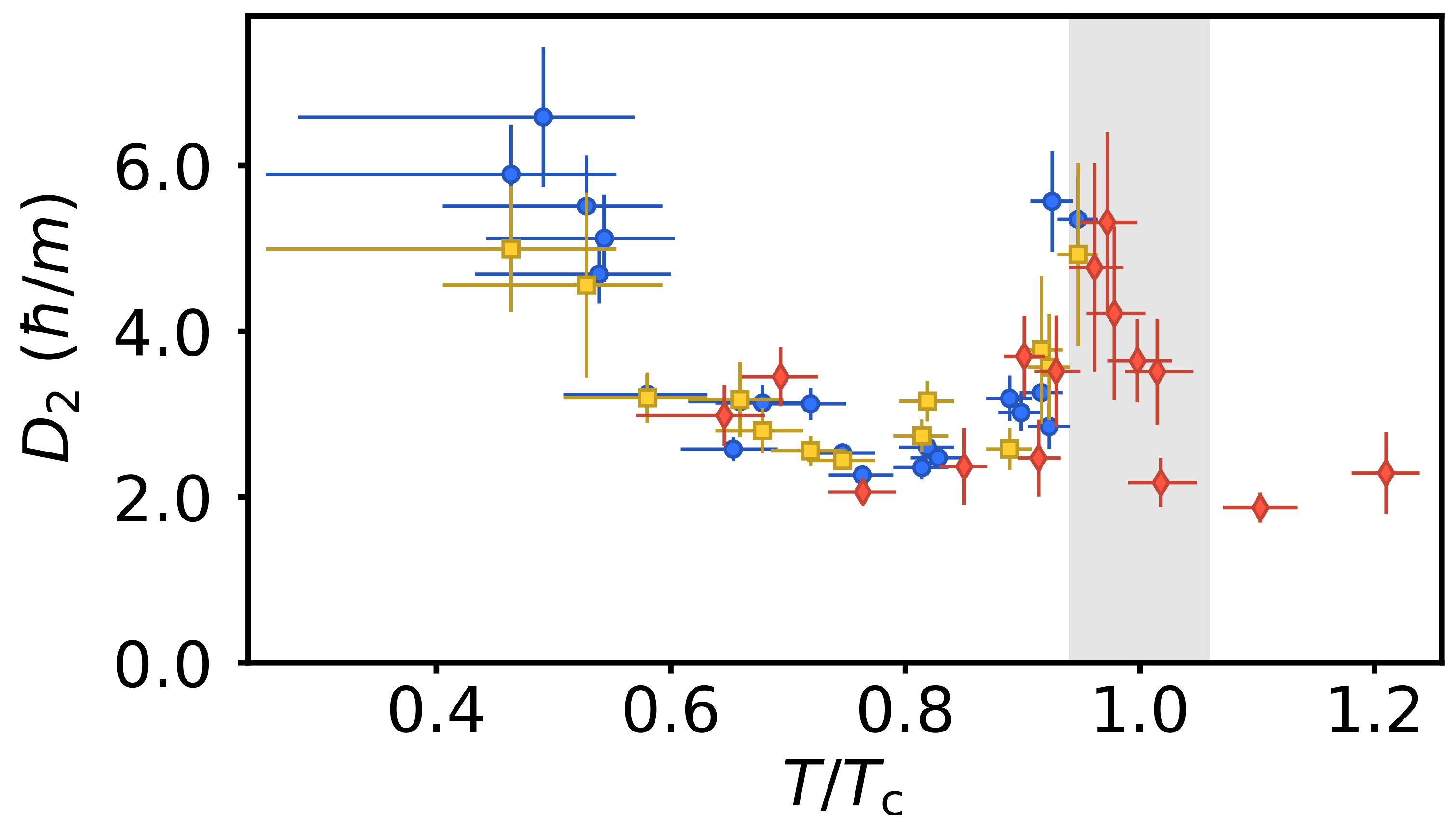
hydrodynamics in a Fermi gas: sound diffusion



$$D = \frac{4}{3}\frac{\eta}{mn} + \frac{\zeta}{mn} + \frac{c_p - c_v}{c_v}\frac{\kappa}{c_p}$$

almost perfect fluidity

$$\frac{\eta}{s} \simeq 0.5\frac{\hbar}{k_B}$$



Patel et al., Science 2020; Li et al., Science 2022; Yan et al., Science 2024

Strong fermion correlations

„Contact“ (many-body) **few-body**

pair correlation $g^{(2)}(r) = \langle \hat{n}_\uparrow(r)\hat{n}_\downarrow(0)\rangle \simeq \mathbf{C}\left(\frac{1}{r} - \frac{1}{a}\right)^2 + \dots$ $r_0 \lesssim r \lesssim \ell$

contact operator: $\hat{C}(x) = g_0^2\, \hat{n}_\uparrow(x)\, \hat{n}_\downarrow(x) = \hat{\Delta}^\dagger(x)\hat{\Delta}(x)$

local pair $\hat{\Delta}(x) = g_0 \hat{\psi}_\downarrow(x)\hat{\psi}_\uparrow(x)$

Hamiltonian $\hat{H} = \hat{H}_{\text{kin}} + \frac{\hat{C}}{g_0} = \hat{H}_{\text{resonant}} + \frac{\hat{C}}{a}$ breaks scale invariance for $\frac{1}{a} \neq 0$

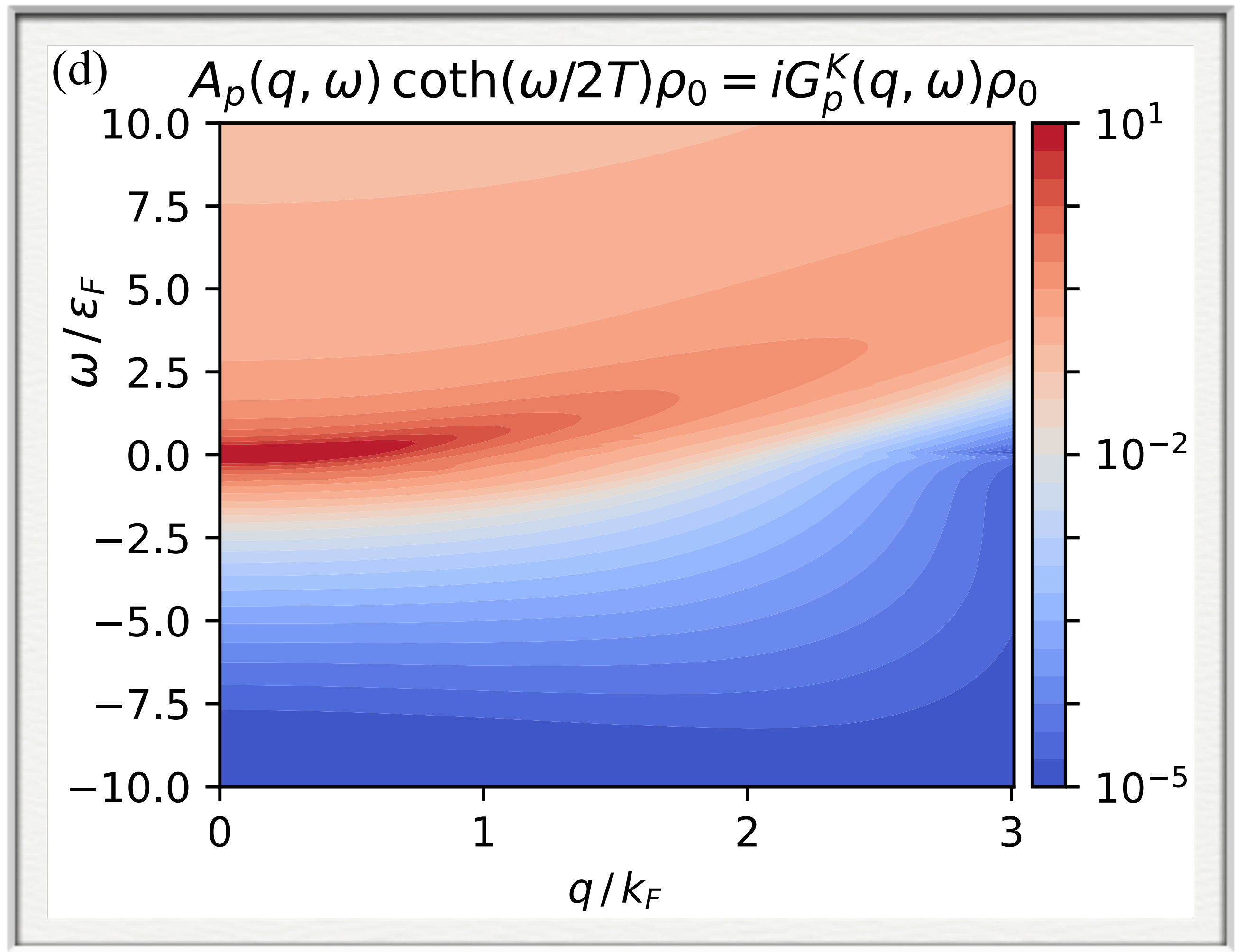
Fujii & Nishida PRA 2018
Frank, Zwerger & Enss PRR 2020

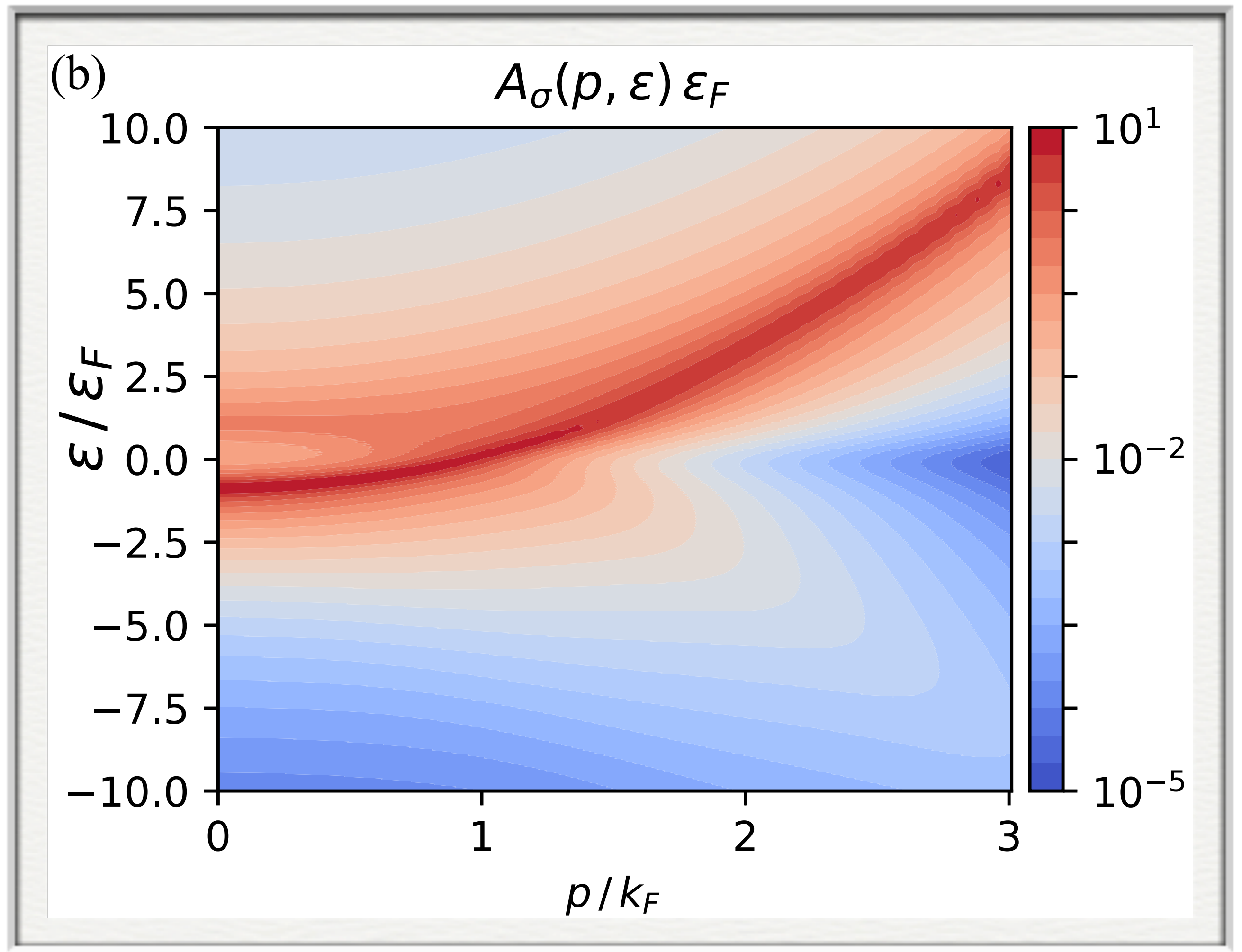
quantum many-body theory

Luttinger-Ward approach (2PI)

repeated scattering
between particles

fermion spectra
in medium





Haussmann et al. 2009; Johansen+ 2024; Enss 2024; Dizer+ 2024
Enss, Haussmann & Zwerger 2011, 2012, 2019

Enss 2024

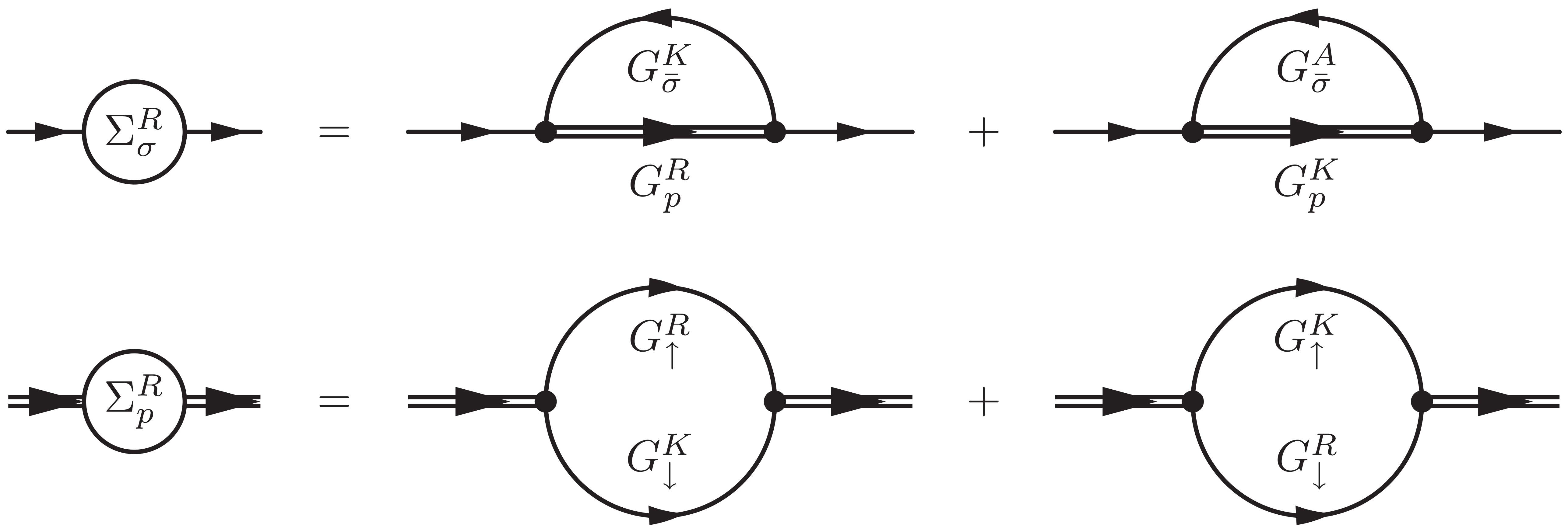
solving the Luttinger-Ward equations in real frequency

$$H = \sum_\sigma \int d\boldsymbol{r}\, \psi_\sigma^\dagger(\boldsymbol{r})\Big(-\frac{\hbar^2\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma(\boldsymbol{r}) + g_0 \int d\boldsymbol{r}\, \psi_\uparrow^\dagger(\boldsymbol{r})\psi_\downarrow^\dagger(\boldsymbol{r})\psi_\downarrow(\boldsymbol{r})\psi_\uparrow(\boldsymbol{r}).$$

$$S = \int d\boldsymbol{r} \int_0^\beta d\tau \Big[\sum_\sigma \psi_\sigma^*\Big(\partial_\tau - \frac{\nabla^2}{2m} - \mu_\sigma\Big)\psi_\sigma - \frac{1}{g_0}|\Delta|^2 - \psi_\uparrow^*\psi_\downarrow^*\Delta - \Delta^*\psi_\downarrow\psi_\uparrow\Big],$$

imaginary frequency: continue analytically (=> E. Gull, ERG 2022)

directly in real frequency (Keldysh in equilibrium):



$$\mathrm{Im}\Sigma_\sigma^R(\boldsymbol{p},\varepsilon) = -\pi \int_{\boldsymbol{p}',\varepsilon'} [f(\varepsilon') + b(\varepsilon+\varepsilon')] \times A_p(\boldsymbol{p}+\boldsymbol{p}', \varepsilon+\varepsilon')A_{\bar\sigma}(\boldsymbol{p}',\varepsilon').$$

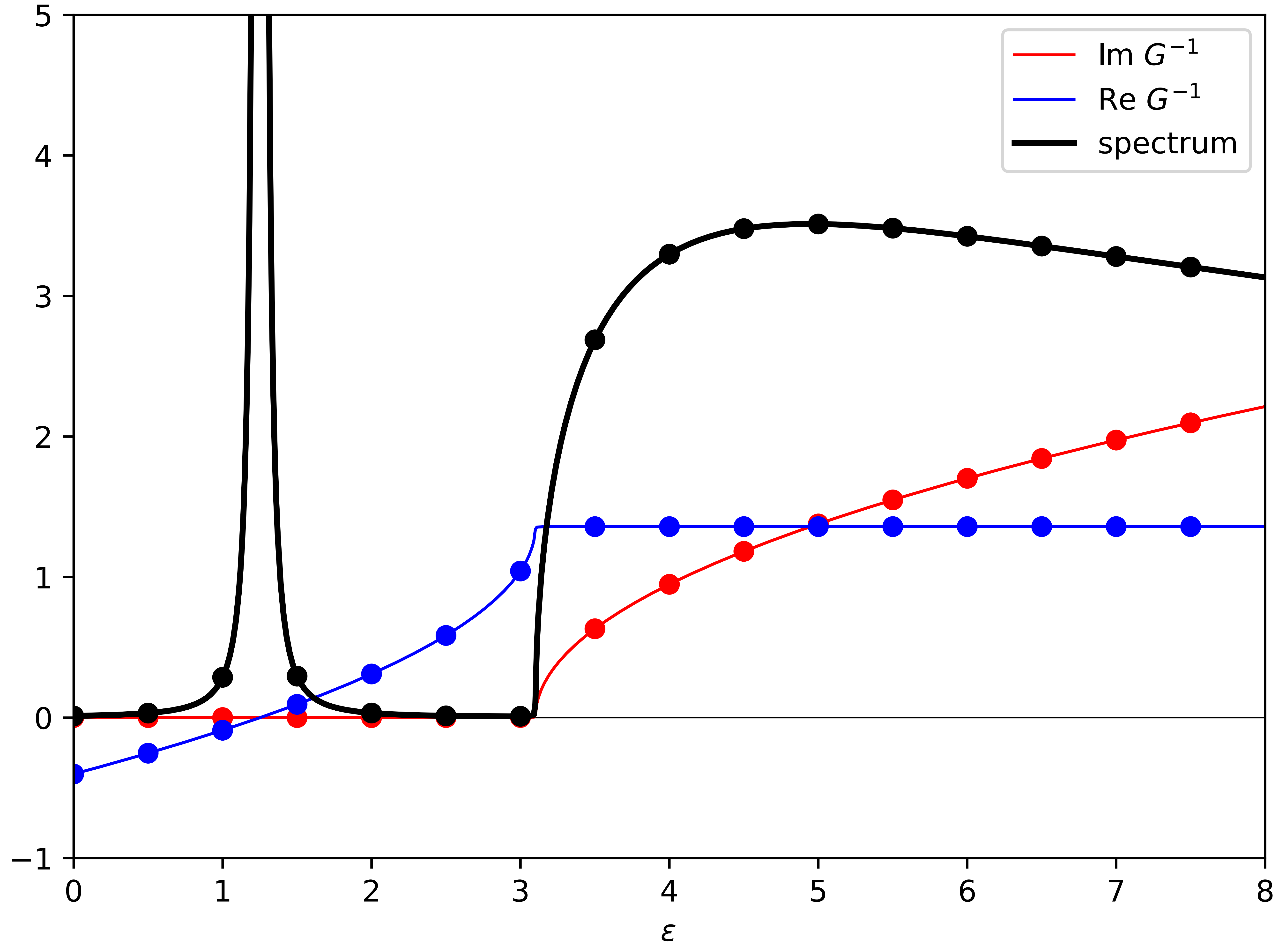
$$\mathrm{Im}\Sigma_p^R(\boldsymbol{q},\omega) = -\pi \int_{\boldsymbol{p},\varepsilon} [1-2f(\varepsilon)]A_\uparrow(\boldsymbol{p},\varepsilon) \times A_\downarrow(\boldsymbol{q}-\boldsymbol{p}, \omega-\varepsilon).$$

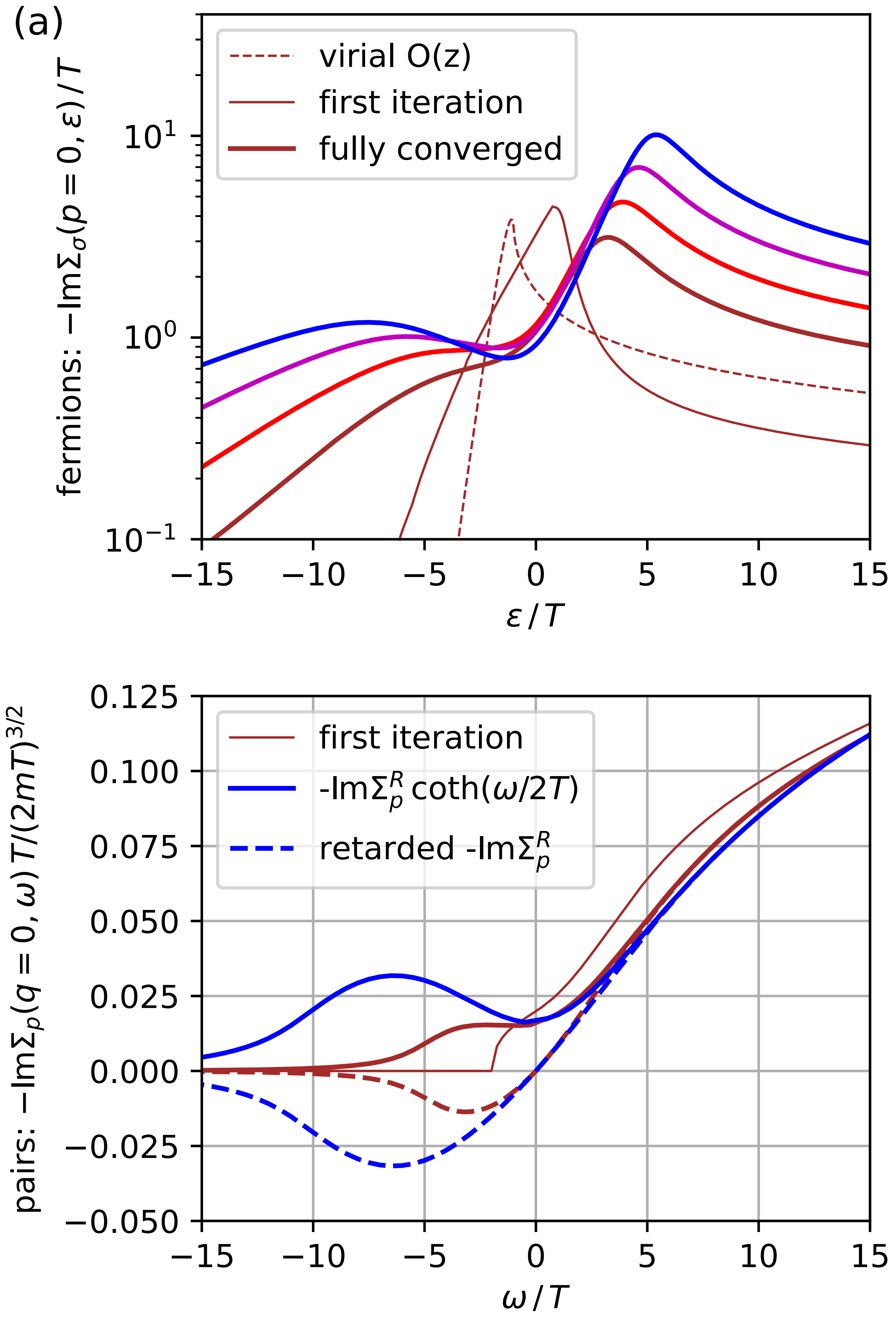
Johansen+ 2024; Enss 2024; Dizer+ 2024

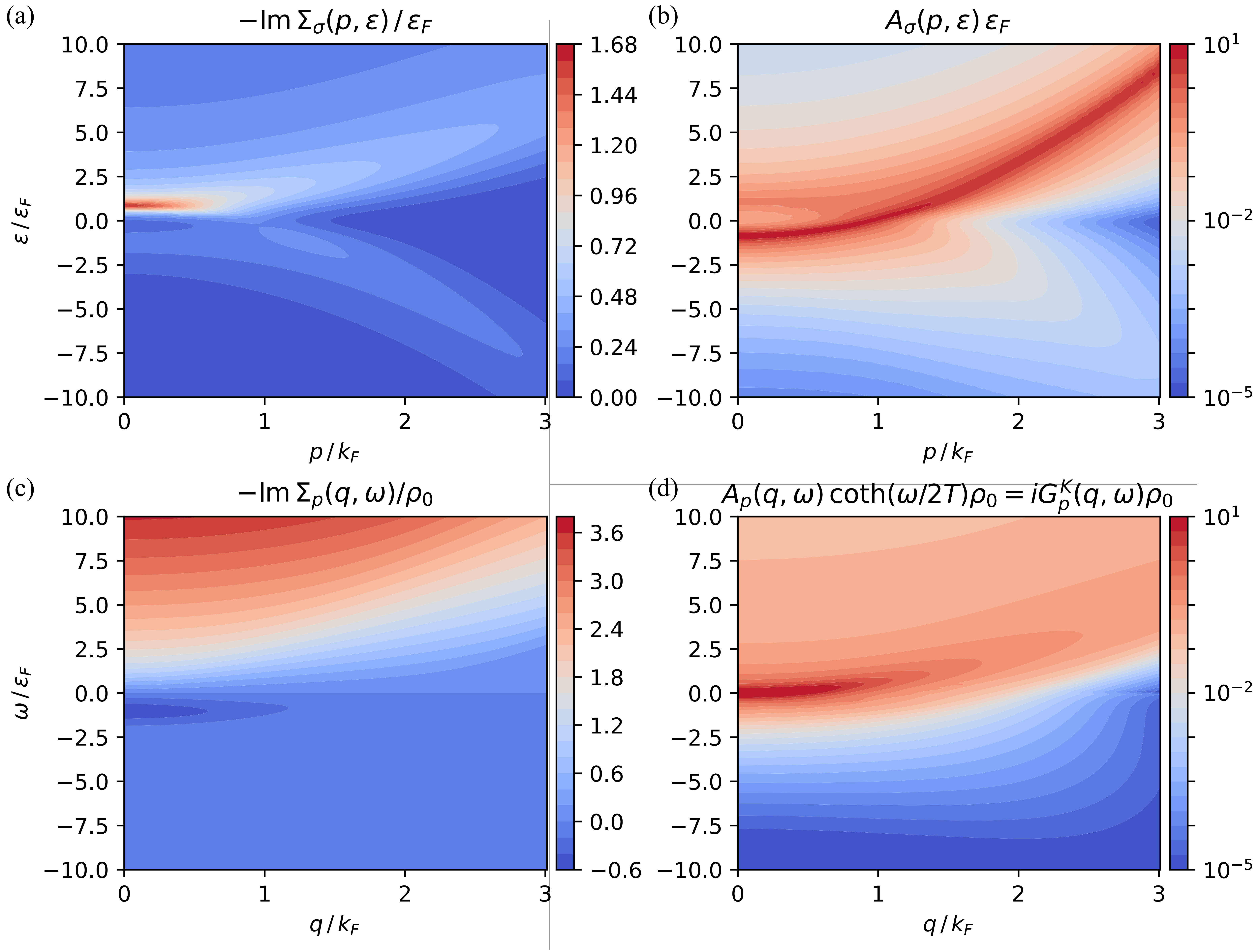
pair spectrum

sharp peaks in real frequency:

- convolution by Fourier transform
  Johansen, Frank, Lang 2024
- adaptive mesh to resolve peak
  Dizer, Horak, Pawlowski 2024
- linearize inverse propagator between grid points
  Enss 2024

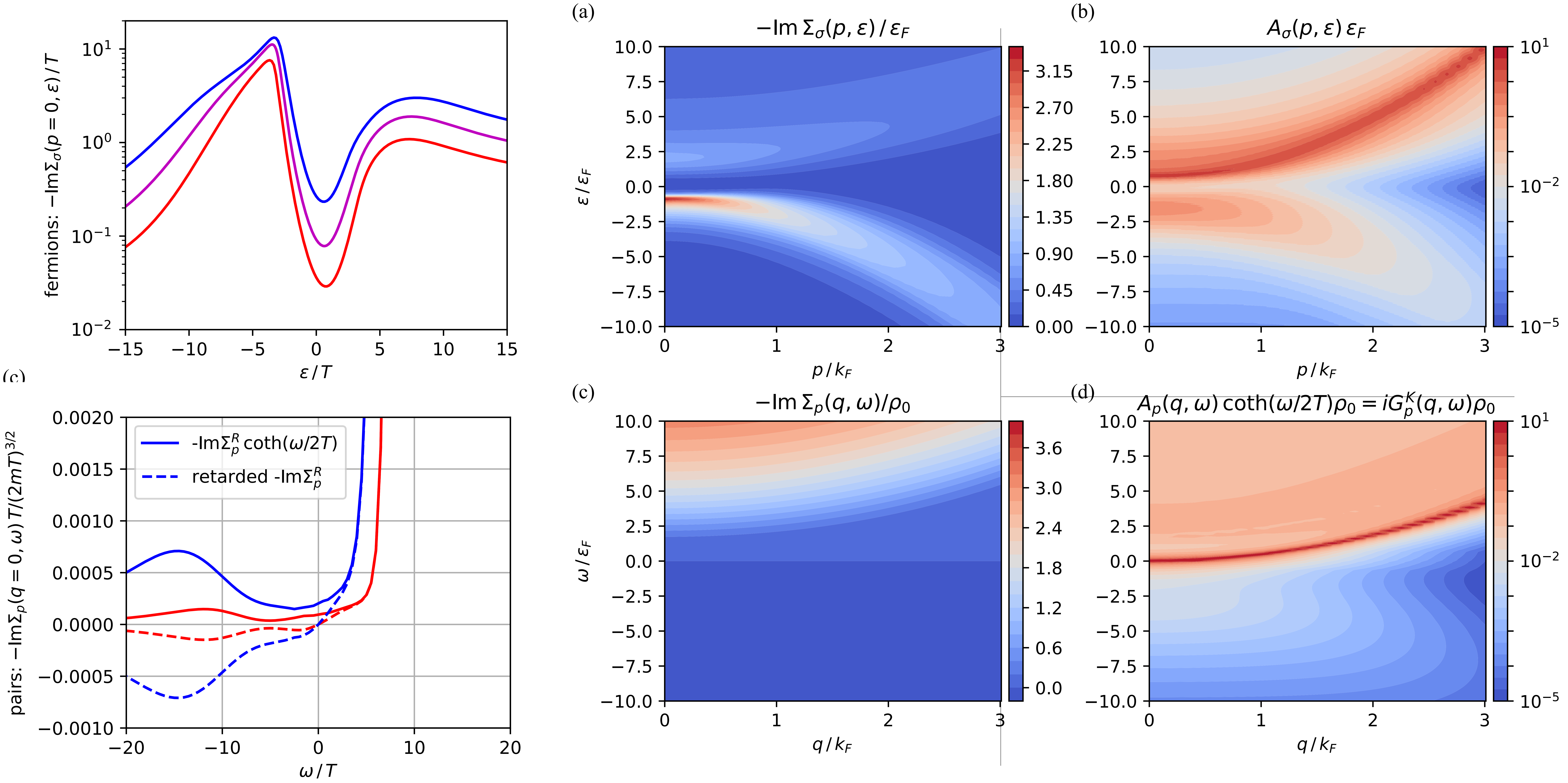






Fermions and pairs at unitarity

Enss PRA 2024

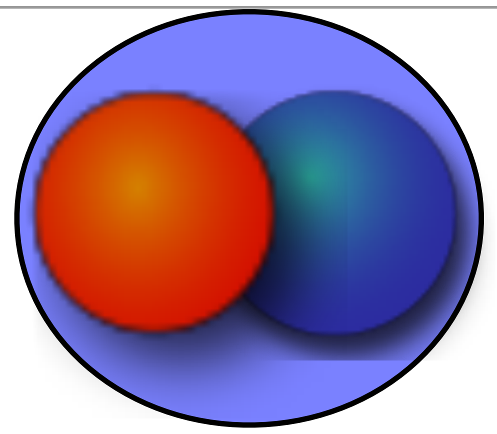


Fermions and pairs at strong binding (BEC side)

Enss PRA 2024

Transport in linear response

- no assumption of „molecular chaos"
  inelastic scattering

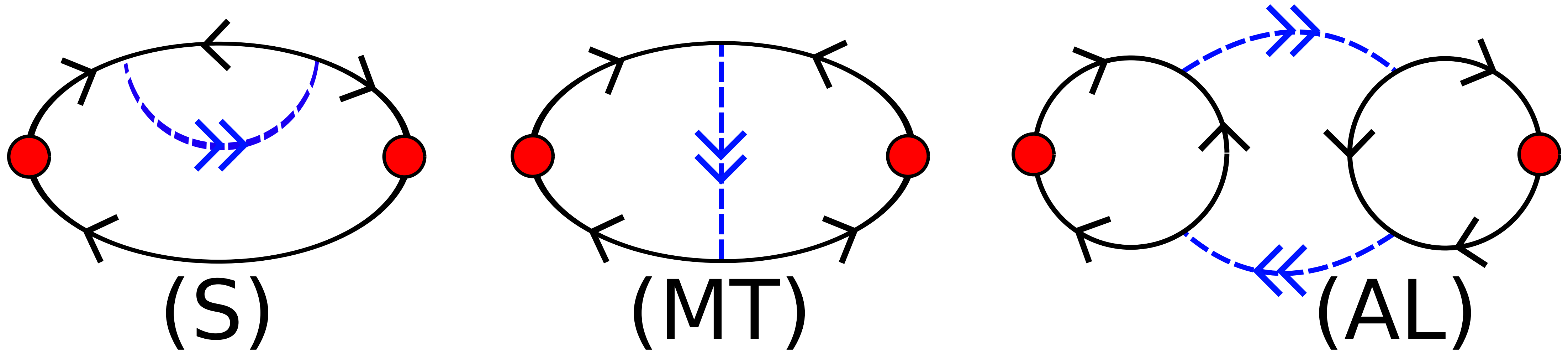


- shear viscosity from stress correlations (Kubo formula)

$$\eta(\omega) = \frac{1}{\omega} \mathrm{Re} \int_0^\infty dt\, e^{i\omega t} \int d^3x \left\langle \left[\hat{\Pi}_{xy}(\boldsymbol{x}, t), \hat{\Pi}_{xy}(0,0)\right] \right\rangle$$

- physical ingredients:

η(ω) =

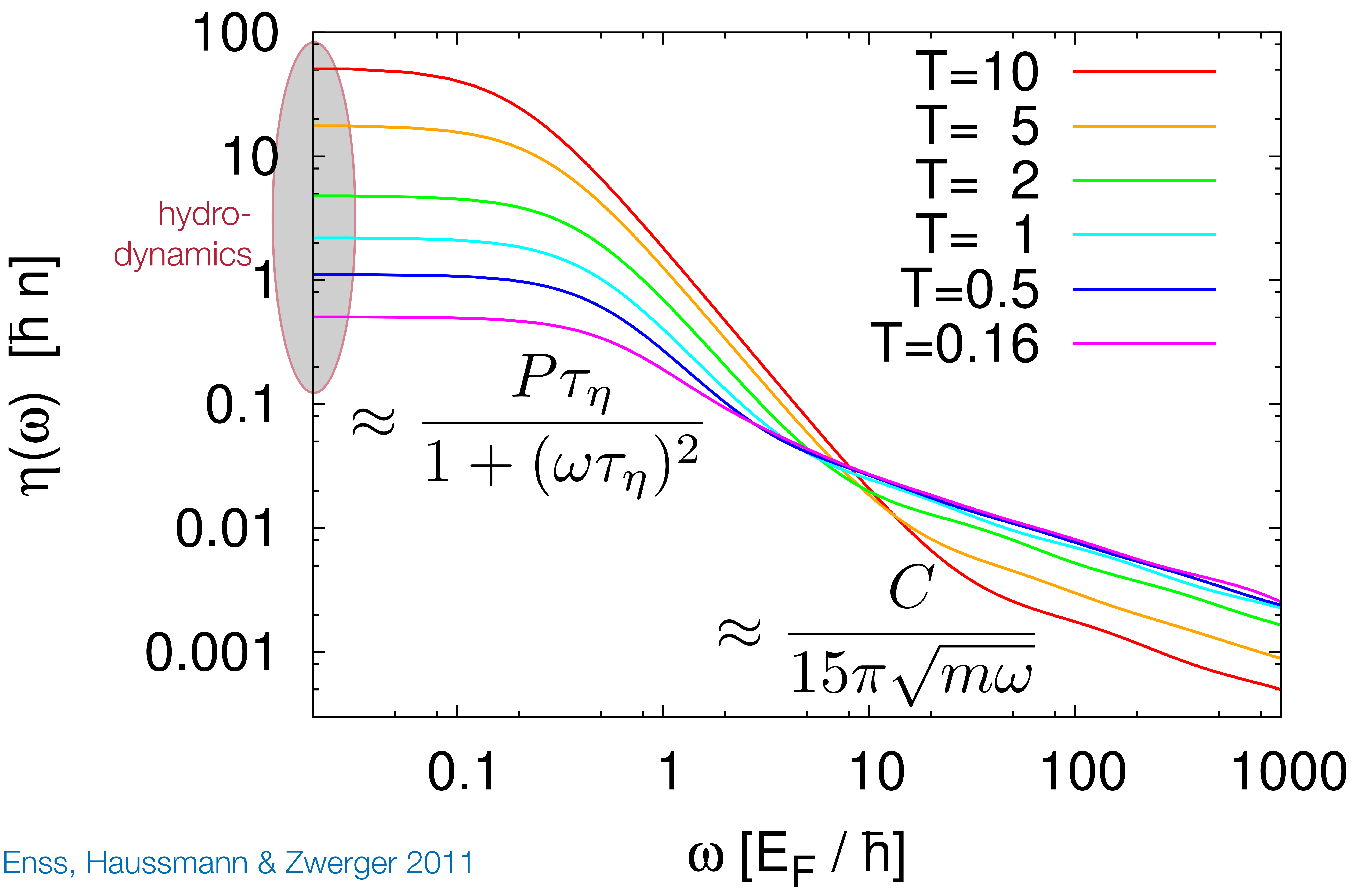


(resummed to infinite order)

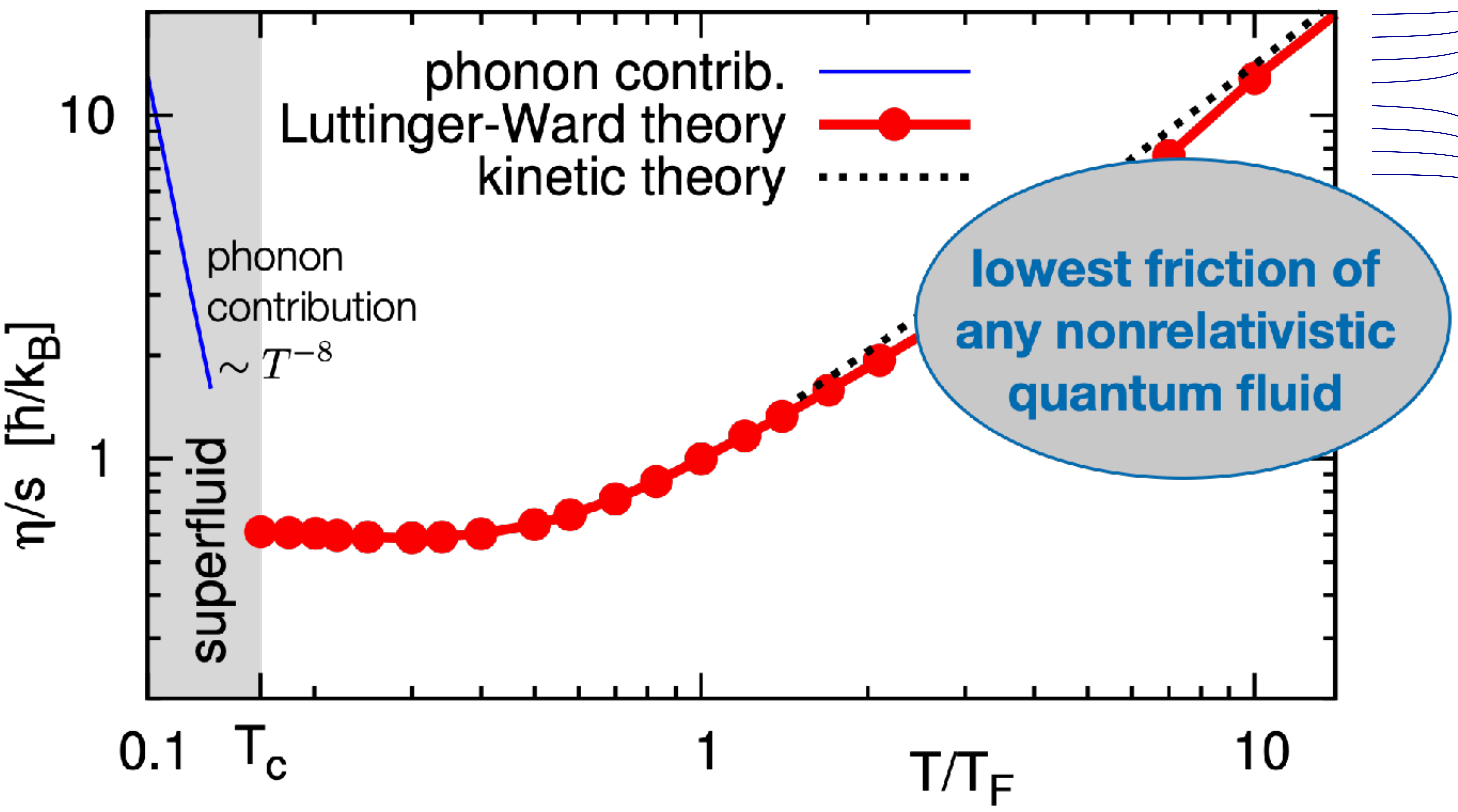
- transport via fermions and **pairs (superfluid fluctuations)**

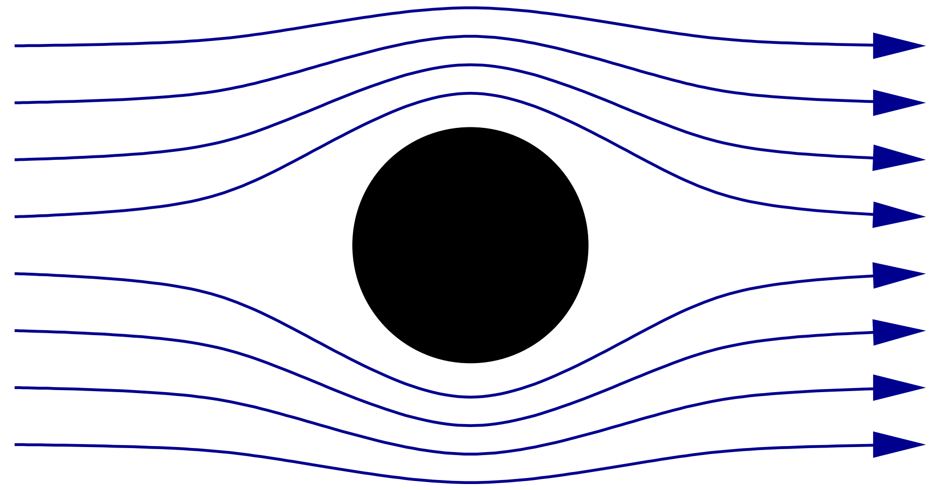
Enss, Haussmann & Zwerger, Annals Physics 2011

Dynamical stress correlations (shear viscosity)



Enss, Haussmann & Zwerger 2011





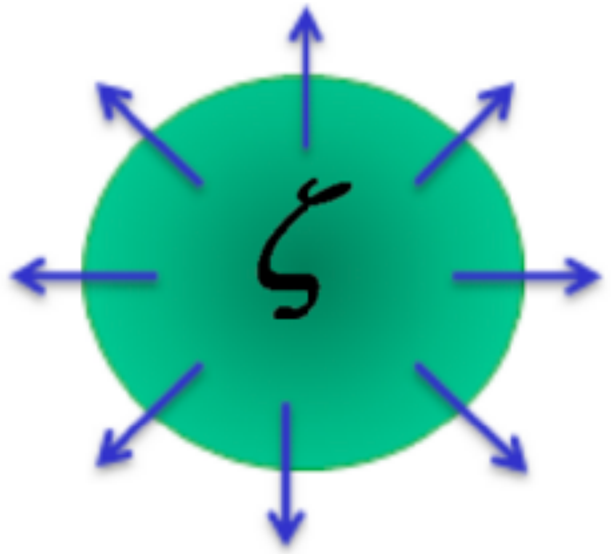
Shear viscosity/entropy
of the unitary Fermi gas

Enss, Haussmann & Zwerger 2011

bulk viscosity probes scaling violation

Kubo formula: pressure correlation function cf. Fujii & Nishida PRA 2020

$$\zeta(\omega) = \frac{1}{\omega}\int_0^\infty dt\, e^{i\omega t}\int d\boldsymbol{x}\,\langle[\delta\hat{p}(\boldsymbol{x},t),\delta\hat{p}(0,0)]\rangle$$



dilute quantum gas: pressure fluctuations

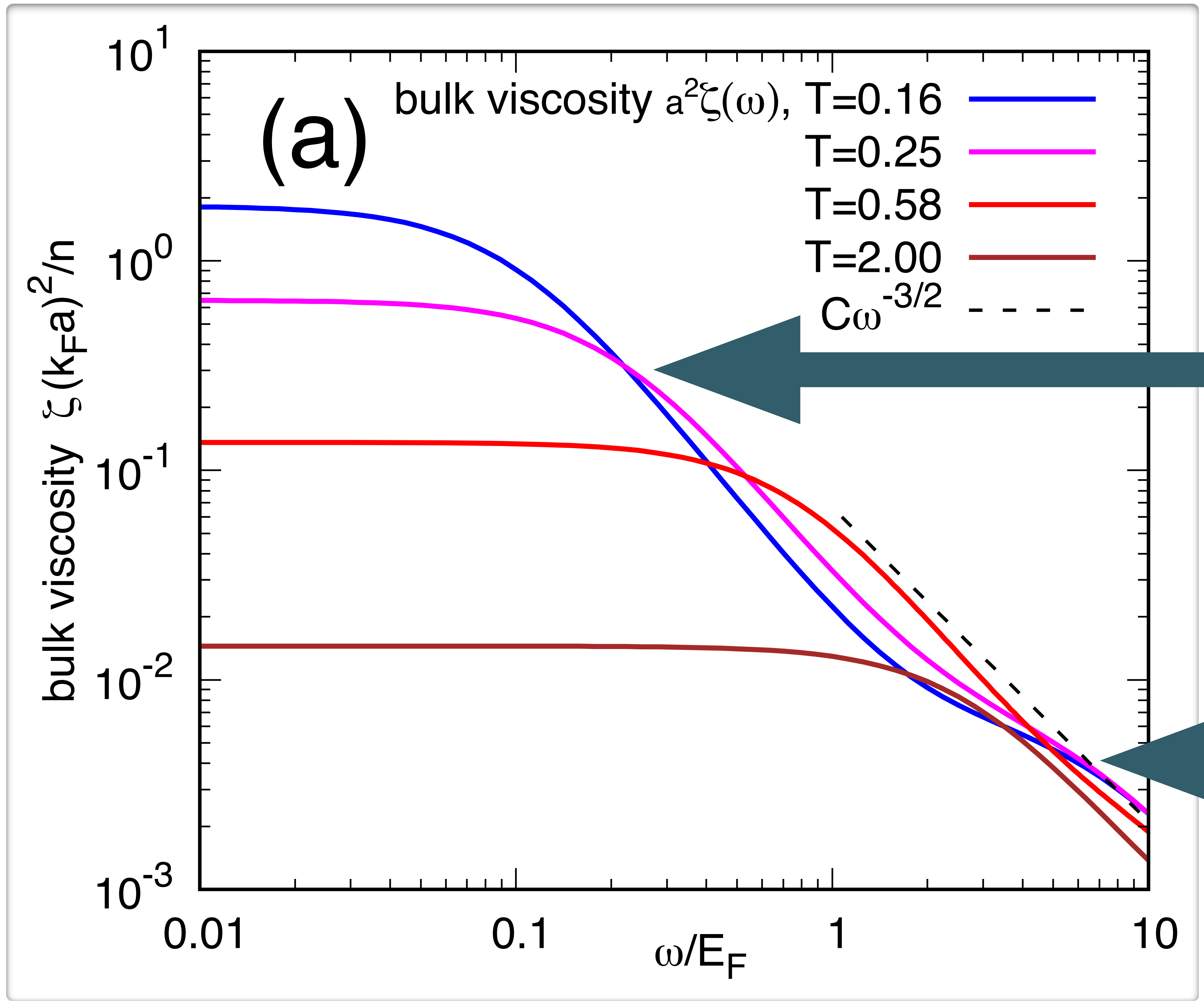
$$\delta\hat{p} = \frac{2}{3}\hat{H} + \frac{\hat{C}}{12\pi m a} - \left(\frac{\partial p}{\partial E}\right)_n \hat{H} - \left(\frac{\partial p}{\partial n}\right)_E \hat{n} \qquad (\beta \text{ function } \frac{\partial H_{\text{int}}}{\partial \ln|a|})$$

bulk viscosity probes **contact correlation** **(local pair fluctuations):**

$$\zeta(\omega>0) = \frac{1}{\omega}\int_0^\infty dt\, e^{i\omega t}\int d\boldsymbol{x}\,\langle[\frac{\hat{C}(\boldsymbol{x},t)}{12\pi m a},\frac{\hat{C}(0,0)}{12\pi m a}]\rangle \sim \langle[\Delta^\dagger\Delta(\boldsymbol{x},t),\Delta^\dagger\Delta(0,0)]\rangle$$

Enss PRL 2019; cf. Martinez & Schäfer PRA 2017, Fujii & Nishida PRA 2018

dynamical bulk viscosity (Luttinger-Ward theory)



transport peak (Drude form)

$$\zeta(\omega) = \frac{\chi\tau_\zeta}{1+(\omega\tau_\zeta)^2}$$

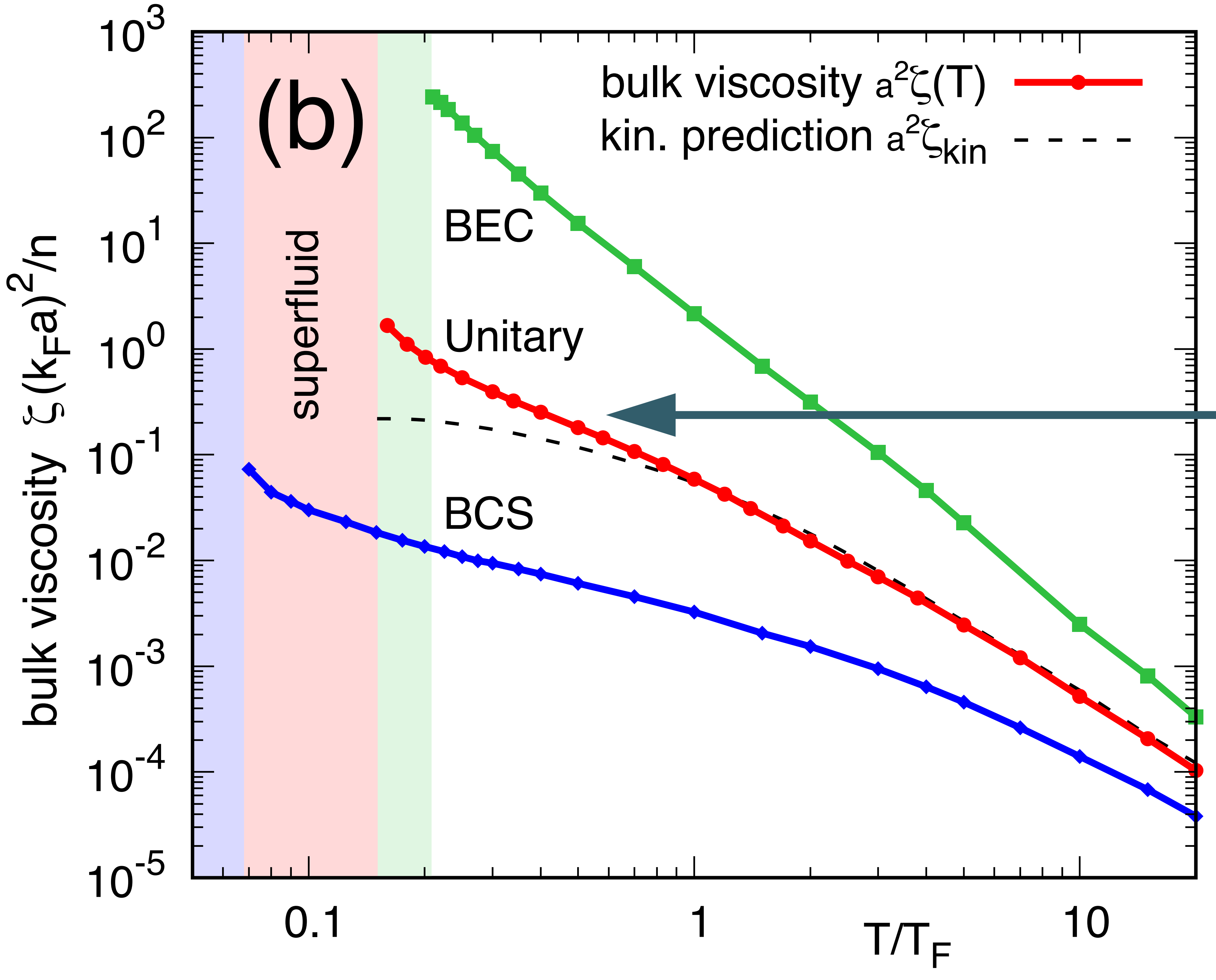
width $\tau^{-1} \simeq 0.6\, k_B T/\hbar$:
T-linear scaling of scattering rate
independent of density!

contact tail $C/\omega^{3/2}$
at high frequency

Enss PRL 2019

quantum degenerate regime (Luttinger-Ward theory)

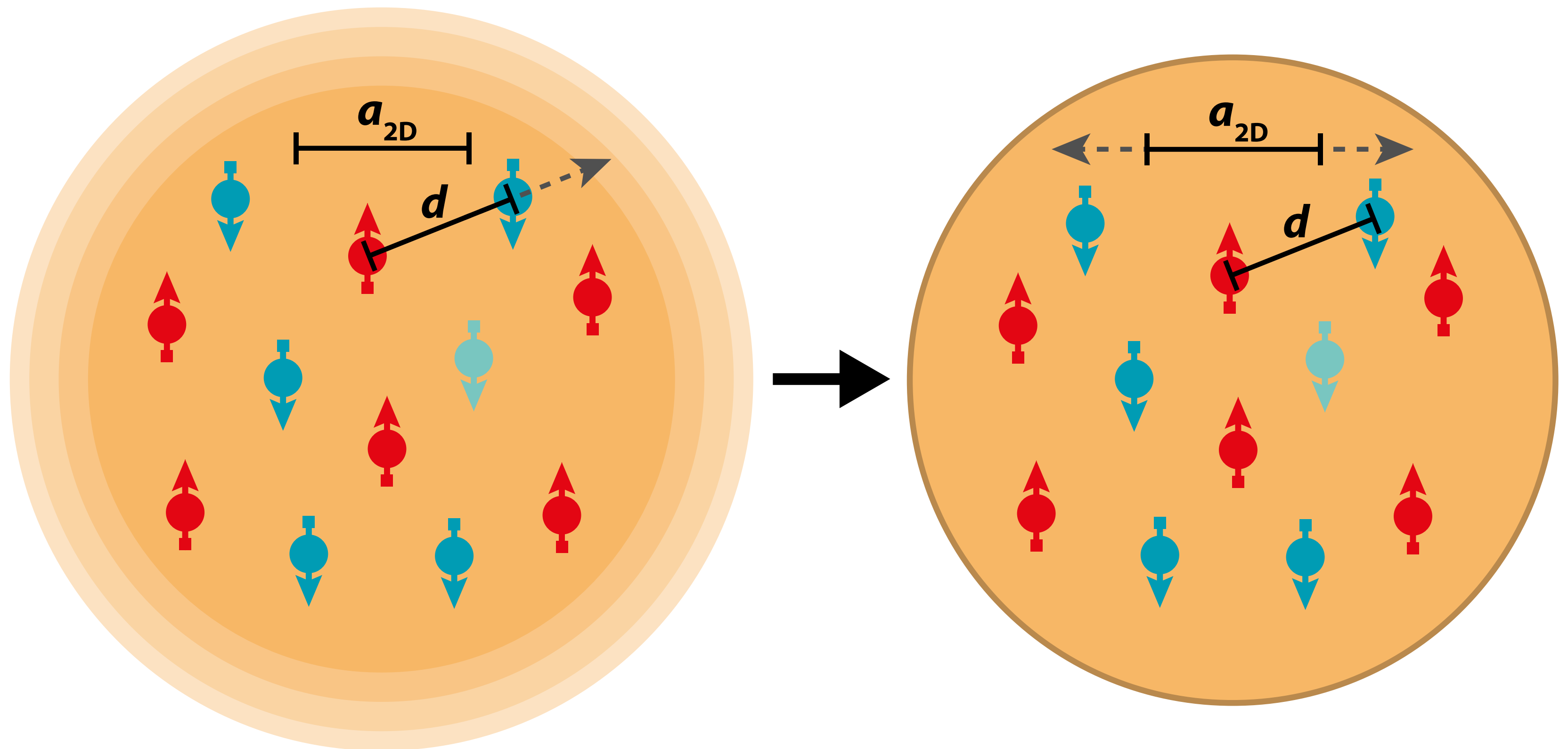
strong enhancement in quantum degenerate regime ($\zeta > \eta$)



Enss PRL 2019

larger than kinetic theory prediction for $T < T_F$

$$\frac{\zeta}{\eta} \simeq \left(\frac{P - 2E/3}{P}\right)^2 \simeq \left(\frac{C/a}{P}\right)^2$$



cf. Fujii & Nishida PRA 2018

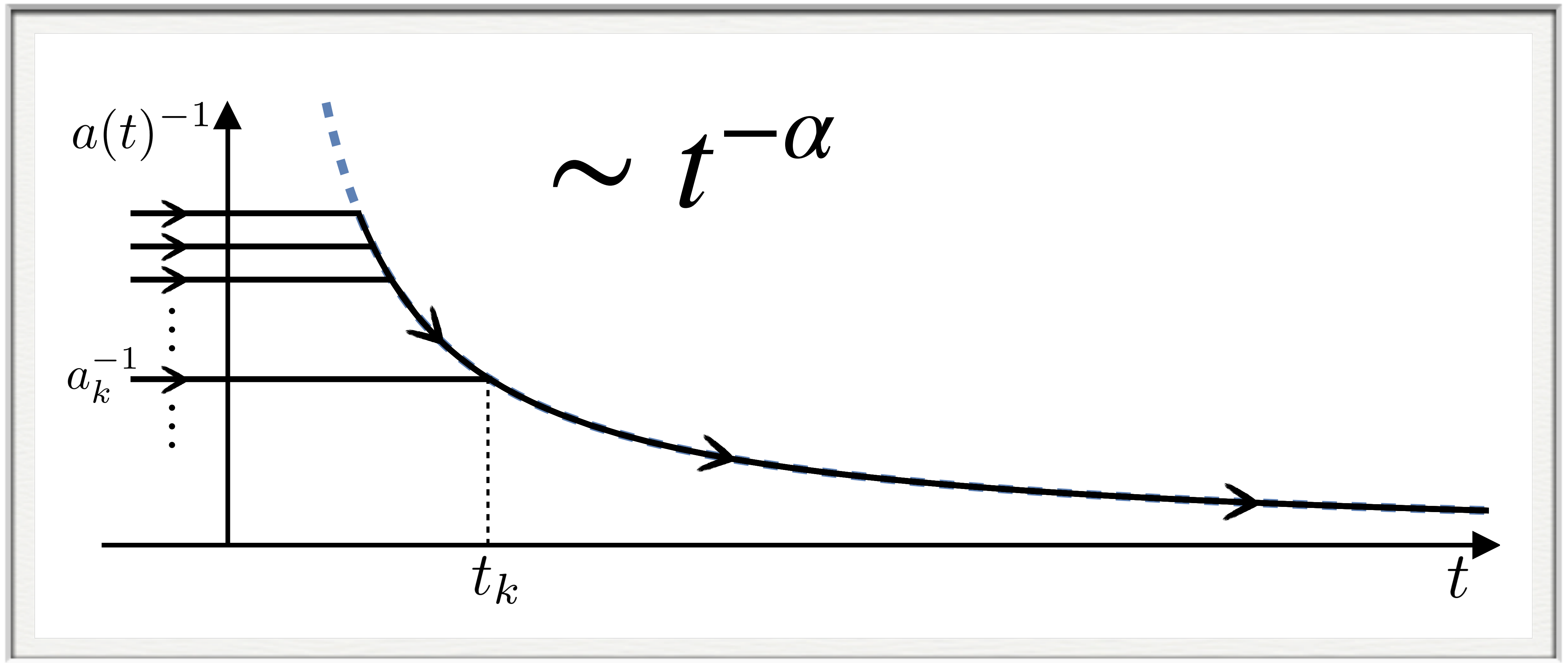
attractor

Fujii & Enss, 2404.12921

drive scattering length => resonance

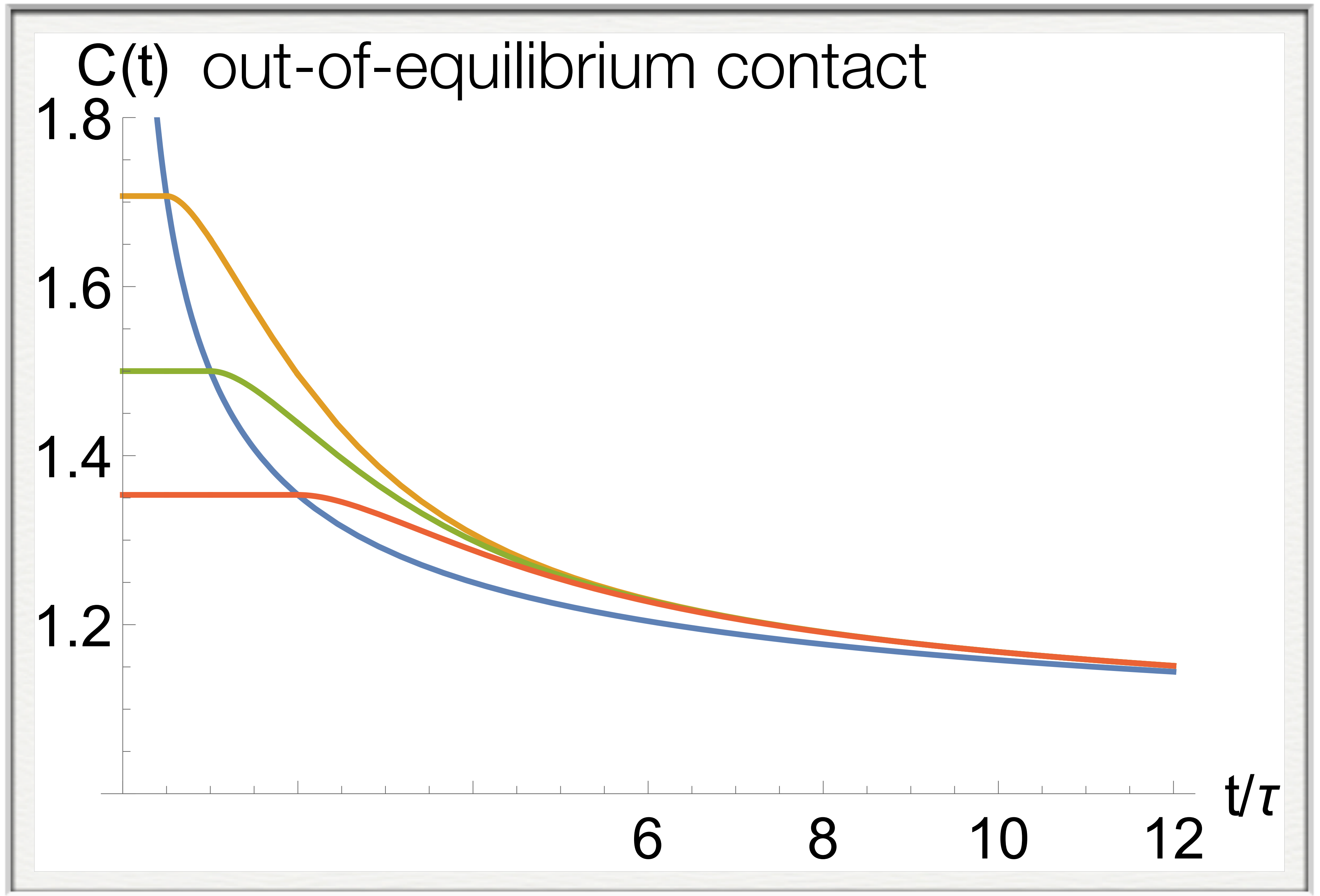
(fast, then slow)

$$\delta C(t) = \int_{-\infty}^{t} dt' \, \frac{\partial C(t)}{\partial a^{-1}(t')} \, \delta a^{-1}(t')$$



parametrization of transport peak:

$$t \sim \tau: \quad \left(\frac{\partial C}{\partial a^{-1}}\right)_{\text{eq}} \frac{e^{-(t-t')/\tau}}{\tau}$$



hydrodynamic perspective: effective field theory

Fujii & Nishida PRA 2018

- effective Hamiltonian

$$\hat{H} = \hat{H}_{\text{reso}} + \frac{\hat{C}}{4\pi m a(t)}$$

a(t) acts as „external field“
conjugate „magnetization“ C

- dissipative bulk pressure

$$\pi(t) = \frac{C(t) - C_{\text{eq}}(a(t))}{12\pi m a(t)} = -\zeta V_a$$

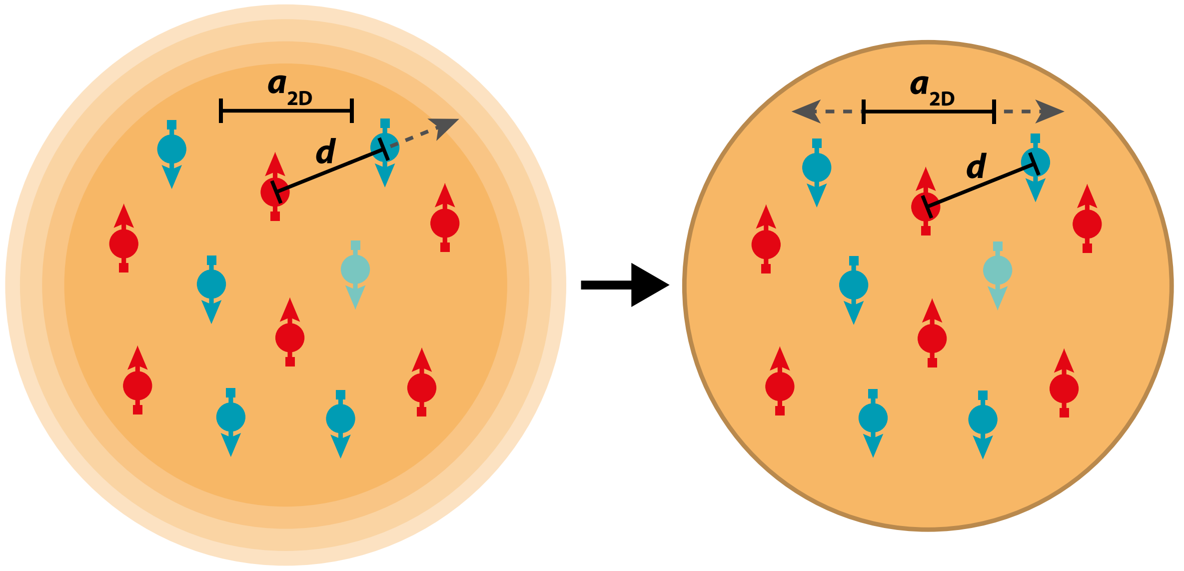
with drive $V_a = \nabla \cdot v - d\frac{\dot{a}}{a}$

- continuity equation

$$\partial_t \hat{H} + \partial_i \hat{Q}_i = \frac{\hat{C}}{4\pi m} \partial_t a^{-1}$$

- constitutive relation

$$\hat{p} = \frac{2}{3}\hat{H} + \frac{\hat{C}}{12\pi m a}$$



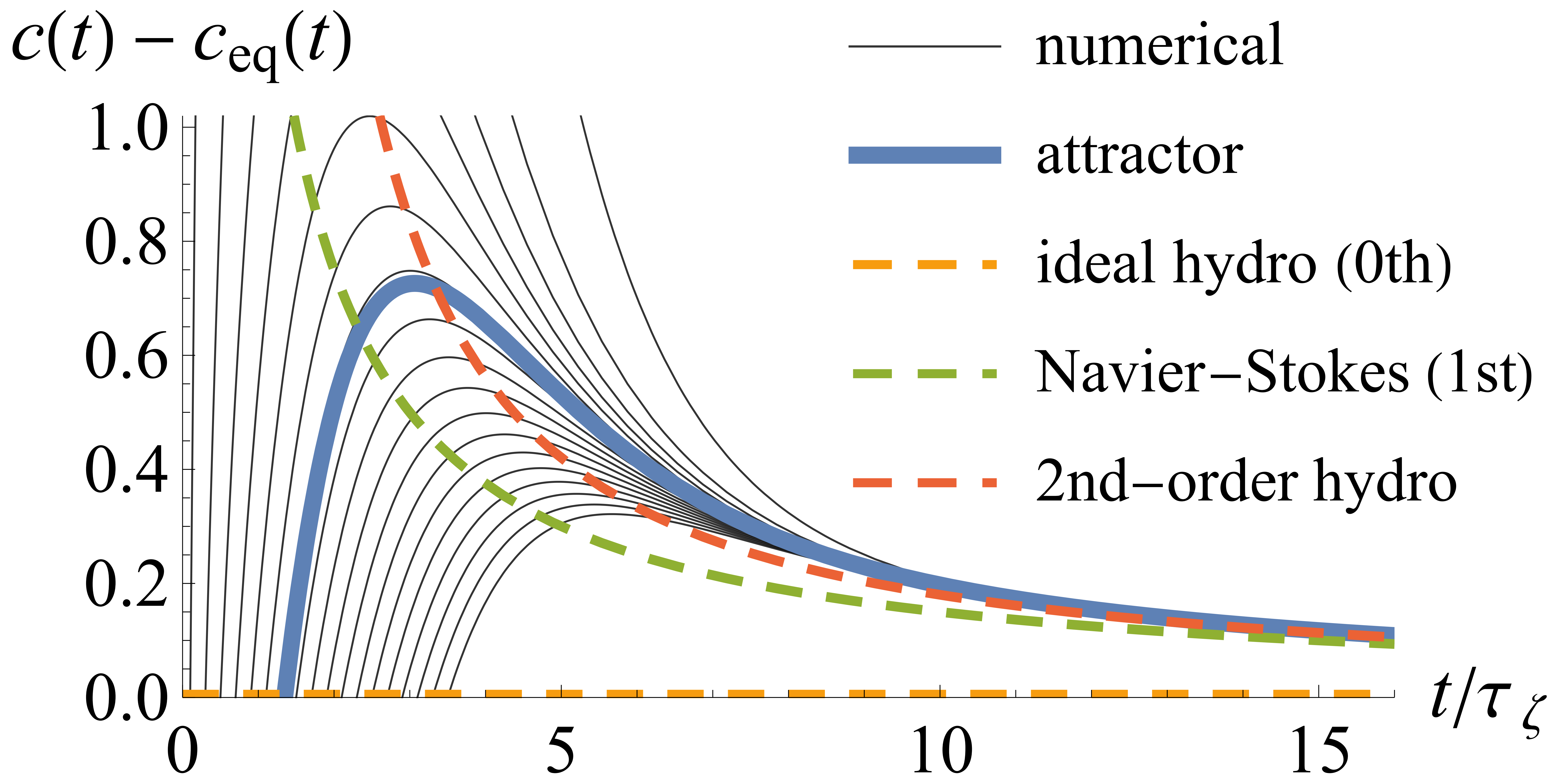
attractor solution

Fujii & Enss, 2404.12921

- Navier-Stokes: $\pi(t) = -\zeta V_a$
- from microscopic computation: $\tau\dot{\pi} + \pi = -\zeta V_a \sim t^{-1-2\alpha}$
- analytical solution
  $\pi(t) = \pi_{ini}\, e^{-(t-t_{ini})/\tau} + \pi_{att}(t)$, attractor $\pi_{att}(t) \simeq \chi e^{-t/\tau}\,\Gamma(-2\alpha, -t/\tau)$
- long times $\tau/t \ll 1$: π_{ini} term nonperturb. (nonhydro mode)
  $\pi_{att} \sim (\tau/t)^{1+2\alpha}\,[1 + (2\alpha+1)\tau/t + \ldots]$
  2nd order
- asympt. series $a_n \sim (n+2\alpha)!$ (Borel resummation)

attractor solution

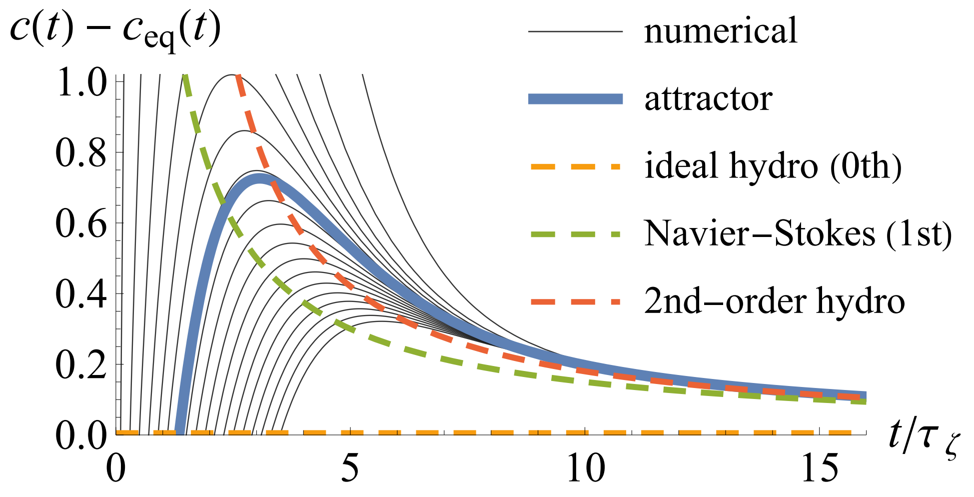
Fujii & Enss, 2404.12921



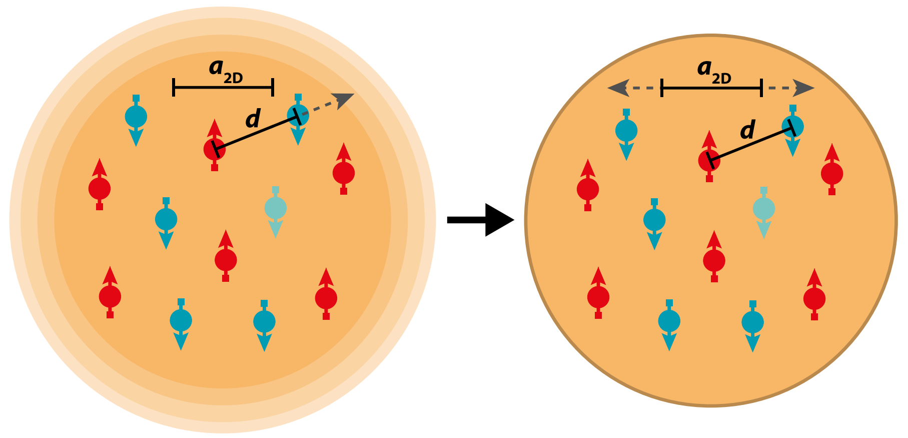
conclusions

arXiv:2404.12921

- cold atom experiment can probe hydrodynamics beyond Navier-Stokes in **real time**

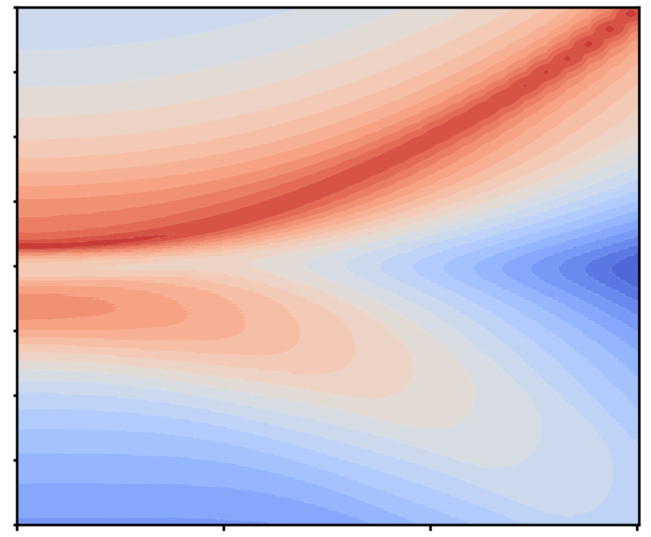


- probe isotropic expansion & **local dissipation** by external drive (no moving parts!)

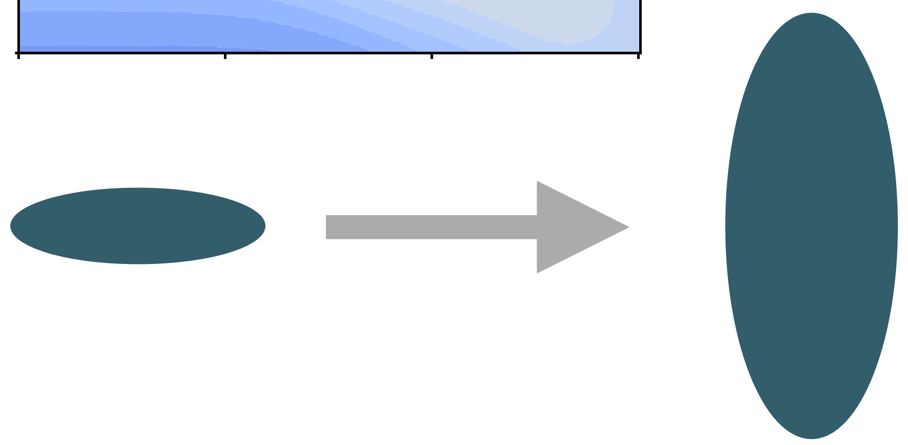


- quantum transport theory $\tau^{-1} \sim T$ (density independent)

- dynamical response functions in real frequency



- **far-from-equilibrium** response: Keldysh (ongoing)



- open PhD & postdoc position