Phase mixing, kinetic theory and fluid mechanics - SwissMAP Research Station

Titles and abstracts

David Gérard-Varet, *Boundary layer theory*

We shall review in these lectures some mathematical aspects of boundary layer theory, which aims at describing the dynamics of high Reynolds number flows near a rigid wall. The focus will be put on well-posedness issues for reduced boundary layer models, and on stability of boundary layer flows. After an informal description of the theory, we will discuss:

- Boundary layers for steady flows
- The unsteady Prandtl equation
- Stability of boundary layer solutions of the Navier-Stokes equation

Isabelle Gallagher, *On the derivation of the Boltzmann equation from a particle system*

The aim of this course is to study the derivation of the Boltzmann equation from a system of hard spheres interacting through binary elastic collisions. We will first present the proof of Lanford's theorem and then more recent advances concerning fluctuations and large deviations around this limit equation.

Thierry Gallay, *Dynamics of vortex pairs in two-dimensional viscous fluids*

The goal of these lectures is to present a rigorous mathematical analysis of the interaction of planar vortices in two-dimensional viscous flows at high Reynolds numbers. To keep the discussion as simple as possible, we focus on the particular case of a pair of vortices with equal or opposite circulations, in the well-prepared situation where these structures originate from point vortices at initial time. In the large Reynolds number regime, we construct an approximate solution of the 2D Navier-Stokes equations as a power series in the (time-dependent) aspect ratio of the viscous vortex pair. In particular, we compute the deformation of the stream lines due to vortex interactions, and we determine the leading order correction to the translation or rotation speed of the vortex centers due to finite size effects. We then show that the exact solution remains close to our approximation over a time interval that increases boundlessly as the viscosity parameter goes to zero. The proof relies on stability estimates derived from Arnold's variational characterization of the steady states of the 2D Euler equation. These lectures rely on ongoing work with M. Dolce (EPFL).

Pierre Germain, *The kinetic wave equation and turbulent spectra*

The kinetic wave equation aims at describing weakly nonlinear dispersive equations in a chaotic (turbulent) regime. I will present recent advances on its rigorous derivation and analysis. I will explain how this equation might be the key to rigorously prove the emergence of turbulent spectra (power laws for the distribution of energy across scales) from nonlinear dispersive or wave equations - which includes numerous examples from Fluid Mechanics.

Mahir Hadzic, *Damping vs. oscillations for the gravitational Vlasov-Poisson system*

We explore the mechanism of gravitational Landau damping around the space-inhomogeneous compactly supported equilibria of the gravitational Vlasov-Poisson system. We show that the damping occurs if and only if the steady galaxy is sufficiently regular at its vacuum boundary.

Michele Dolce, *Taylor dispersion and enhanced dissipation in the non-cutoff Boltzmann equation*

We investigate stability properties of a global Maxwellian background in the non-cutoff Boltzmann equation with soft potentials. Our focus is on the large Knudsen number (Kn) regime, which applies to gases in the upper atmosphere. We prove that, for initial data sufficiently small (independent of Kn), the solution exhibits several dynamics resulting from the interplay between the singular collision and the transport operators. In the periodic box, we quantify the enhanced dissipation mechanism, that is an exponential convergence towards $x-$averages on a time-scale $O(Kn^{1/3})$. In the whole space, we obtain the Taylor dispersion, showing that the perturbation decay polynomially fast on a time-scale $O(1)$. Both are faster relaxation time-scales compared to the $O(Kn)$ expected when neglecting the transport. Furthermore, for macroscopic quantities as the density, the bounds we obtain imply almost-uniform in Kn decay of the $x-$derivative of the density in L^∞, due to phase mixing and dispersive decay. This is a joint work with M. Coti Zelati and J. Bedrossian

Daniel Han-Kwan, *Stability of non-localized equilibria for Hartree and Vlasov: a review*

The goal of the talk is to review and compare asymptotic stability results for the Hartree and Vlasov equations set in the whole space. Along the way, I will try to discuss some open questions.

Klaus Widmayer, *Landau damping near the Poisson equilibrium in* R^3

While "Landau damping" is regarded as an important effect in the dynamics of hot, collisionless plasmas, its mathematical understanding is still in its infancy. This talk begins with a discussion of stabilizing mechanisms in the linearized Vlasov-Poisson equations near a class of homogeneous equilibria on R^3, showing how both oscillatory and damping effects arise. We then explain how these mechanisms imply a nonlinear stability result in the specific setting of the Poisson equilibrium. This is based on joint work with A. Ionescu, B. Pausader and X. Wang.

Evelyne Miot, *Dynamics of point vortices for the lake equations*

We study the asymptotic dynamics of point vortices for the lake equations with positive depth, when the vorticity is initially sharply concentrated around N points. More precisely, we show that the vorticity remains concentrated in some strong sense around N points for all times, and that the trajectories follow the level lines of the depth function. This is joint work with Lars Eric Hientzsch (Universität Bielefeld) and Christophe Lacave (Université Savoie Mont-Blanc).

Maria Colombo, *Anomalous dissipation and nonselection via vanishing viscosity in scalar turbulence*

Kolmogorov's K41 theory of turbulence advances quantitative predictions on anomalous dissipation in incompressible fluids. This phenomenon can be described as follows: although smooth solutions of the Euler equations conserve the kinetic energy, in turbulent fluids the energy can be transferred to high frequencies and anomalously dissipated. Hence turbulent solutions of the Navier-Stokes equations are expected to converge, in the vanishing viscosity limit, to irregular solutions of the Euler equations, with decreasing kinetic energy.

In rigorous analytical terms, however, this phenomenon is little understood. In this talk, I will present the recent developments on this topic and focus on a joint work with G. Crippa and M. Sorella which considers the case of passive-scalar advection, where anomalous dissipation is predicted by the Obukhov-Corrsin theory of scalar turbulence. I will discuss the construction of a velocity field and a passive scalar exhibiting anomalous dissipation in the supercritical Obukhov-Corrsin regularity regime. The techniques developed in this context allow also to answer the question of (lack of) selection for passive-scalar advection under vanishing diffusivity. Finally, I will present a joint work with E. Brue?, G. Crippa, C. De Lellis, and M. Sorella, where we use the previous construction to give example of anomalous dissipation for the forced Navier-Stokes equations in the supercritical Onsager regularity regime.

Hyunju Kwon, *Strong Onsager conjecture*

Smooth solutions to the incompressible 3D Euler equations conserve kinetic energy in every local region of a periodic spatial domain. In particular, the total kinetic energy remains conserved. When the regularity of an Euler flow falls below a certain threshold, a violation of total kinetic energy conservation has been predicted due to anomalous dissipation in turbulence, leading to

Onsager's theorem. Subsequently, the L^3-based strong Onsager conjecture has been proposed to reflect the intermittent nature of turbulence and the local evolution of kinetic energy. This conjecture states the existence of Euler flows with regularity below the threshold of $B^{1/3}_{3,\infty}$ which not only dissipate total kinetic energy but also exhibit intermittency and satisfy the local energy inequality. In this talk, I will discuss the resolution of this conjecture based on recent collaboration with Matthew Novack and Vikram Giri.

Megan Griffin-Pickering, *The quasi-neutral limit for the ionic Vlasov-Poisson system with rough data*

Vlasov-Poisson type systems are well known as kinetic models for plasma. The version of the equation describing ions includes an additional exponential nonlinearity in the equation for the electrostatic potential compared to the electron case, which creates several new mathematical difficulties.

The quasineutral limit refers to the limit of vanishing Debye length, a length scale governing electrostatic interactions and typically very small in physical plasmas. In the case of the ionic model, the formal limit is the kinetic isothermal Euler system; however, this limit is highly non-trivial to justify rigorously and known to be false in general without very strong regularity conditions and/or structural conditions.

I will present a recent work, joint with Mikaela Iacobelli, in which we prove the quasi-neutral limit for the ionic Vlasov-Poisson system for a class of rough (L^∞) data: that is, data that may be expressed as perturbations of an analytic function, vanishing like a single exponential of a power of the inverse Debye length, in the sense of Wasserstein/Monge-Kantorovich distances from optimal transport. The condition we obtain is much less restrictive than previous results, and the single exponential form is essentially optimal.

Claudia Garcia, *Periodic solutions around localized radial profiles for the 2D Euler equations*

In this talk, we address for the 2D Euler equations the existence of rigid time periodic solutions close to stationary radial vortices of type $f_0(|x|)$ supported on the unit disk, with f_0 being a strictly monotonic profile with constant sign. We distinguish two scenarios according to the sign of the profile: defocusing and focusing. In the first regime, we have scarcity of the bifurcating curves associated with lower symmetry. However, in the focusing case we get a countable family of bifurcating solutions associated with large symmetry. The approach developed in this work is new and flexible, and the explicit expression of the radial profile f_0 is no longer required as in previous works. The alternative for that is a refined study of the associated spectral problem based on Sturm-Liouville differential equation with a variable potential that changes the sign depending on the shape of the profile and the location of the time period. This is a joint work with Taoufik Hmidi and Joan Mateu.

Frederic Rousset, *TBA*