Conformal field theory 3 ways: integrable, probabilistic, and supersymmetric

Report of Contributions

Contribution ID: **1**   Type: **not specified**

Introduction to Liouville field theory - Part 1

Monday 22 January 2024 09:00 (1 hour)

In the the first lecture I will outline the main mathematical features of the bootstrap approach to Liouville conformal field theory, assembling the Liouville correlation function from holomorphic functions called conformal blocks. The most important properties of the conformal blocks will be described. The second lecture will review a construction of the conformal blocks using an analytic version of the free field representation. If time permits, we will outline how the free field representation can be used to derive the braid relations, which can serve as a key ingredient in the bootstrap approach.

Presenter: TESCHNER, Joerg (DESY)

Contribution ID: **2**   Type: **not specified**

On the interplay of random curves with CFT and hidden symmetries- Part 1

Monday 22 January 2024 10:30 (1 hour)

I will discuss how random curves appearing in critical models, probability theory, and complex geometry are related to evident and hidden algebraic content in CFT.

Presenter: PELTOLA, Eviliina

Contribution ID: **3**   Type: **not specified**

Bicommutant Categories from Conformal Nets, towards constructing fully extended functorial (Segal) chiral CFTs

Monday 22 January 2024 11:35 (30 minutes)

Two-dimensional chiral CFTs have three mathematical formulations, namely VOAs, conformal nets and Segal (functorial) CFTs. We are working on the construction of a fully extended 2d chiral functorial field theory given the data of a conformal net. We introduce some ingredients of the target category of Bicommutant Categories (a model for 3-Hilb) as a categorification of the Morita category of von Neumann algebras. We further give equivalent constructions of the category of solitons (solitonic representations of the net) that we assign to a point with a germ of a 1-manifold in the conformal cobordism category. This is a bicommutant category whose Drinfeld center is representations of the net. Further at the level of 1-morphisms, the composition of intervals maps to categorified Connes-Fusion.

Presenter: NIVEDITA, Nivedita

Contribution ID: **4** Type: **not specified**

AGT correspondence: a 2d CFT valued in 4d QFTs - Part 1

Monday 22 January 2024 16:30 (1 hour)

Starting from a 6d point of view, we will develop the basic dictionary associating 4d N=2 gauge theories to punctured Riemann surfaces, and explain some extensions thereof. As an application, we will explore a rich set of operators (loops, surfaces, walls) that can be added to the story, and how their use led to a conjecture for some braiding kernels in Toda CFT.

Presenter: LE FLOCH, Bruno

Contribution ID: **5**   Type: **not specified**

On the interplay of random curves with CFT and hidden symmetries- Part 2

Monday 22 January 2024 17:45 (1 hour)

I will discuss how random curves appearing in critical models, probability theory, and complex geometry are related to evident and hidden algebraic content in CFT.

Presenter: PELTOLA, Eviliina

Contribution ID: **6**   Type: **not specified**

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Presenter: TESCHNER, Joerg (DESY)

Contribution ID: **7**   Type: **not specified**

Irreducible representations in Liouville CFT

Monday 22 January 2024 18:45 (30 minutes)

In the context of the probabilistic construction of Liouville CFT, we prove the irreducibility of highest-weight representations of the Virasoro algebra on the Kac table. Joint with Baojun Wu.

Presenter: BAVEREZ, Guillaume

Contribution ID: **8**   Type: **not specified**

From Liouville to Painlevé and Heun via CFT/isomonodromy relation

Wednesday 24 January 2024 09:00 (1 hour)

I will review connections between the problem of construction of linear ordinary differential equations with prescribed monodromy and the 2D conformal field theory. This correspondence leads to a number of conjectures in the theory of Painlevé and Heun equations some of which have already been proven rigorously and some remain open. The two main applications I will focus on are the construction of the general solution of Painlevé VI equation and the computation of accessory parameter and connection formulas for Heun's equation in terms of Liouville conformal blocks.

Presenter: LISOVYI, Oleg

Contribution ID: **9**   Type: **not specified**

Probabilistic approach of Liouville theory

Tuesday 23 January 2024 10:30 (1 hour)

In these lectures, we will review the probabilistic construction of Liouville CFT. We will explain the Segal axioms for CFTs and how they can be implemented in Liouville CFT with the construction of amplitudes. Gluing amplitudes then allows us to identify the Hamiltonian of the Liouville CFT and to provide a representation of the Virasoro algebra in the Hilbert space of Liouville theory. We will explain the diagonalization of the Hamiltonian using the Virasoro representation. Finally we will explain how to combine these concepts to establish the validity of the conformal bootstrap.

Presenter: RHODES, Remi (Aix-Marseille university)

Contribution ID: **10** Type: **not specified**

(Quantum) Painleve'/ gauge theory correspondence and Heun connection problem

Wednesday 24 January 2024 10:30 (1 hour)

We discuss the relation between Painleve'equations, supersymmetric gauge theories and Liouville CFT. As an application we show how within this circle of ideas one can solve the connection problem for Heun functions in terms of explicit combinatorial formulae arising from supersymmetric localisation. We also present work in progress on the extension of the correspondence to quantum Painleve'.

Presenter: TANZINI, Alessandro

Contribution ID: **11**   Type: **not specified**

Convergence of Nekrasov's Instanton Partition Functions

Tuesday 23 January 2024 11:35 (30 minutes)

In this talk, we will study the convergence properties of instanton partition functions in four-dimensional N=2 gauge theory with group U(N), also in the presence of matter in the adjoint or (anti)fundamental representation.

The main result is that if the considered theory is conformal its instanton function, seen as a power series in the complexified gauge coupling, has a finite radius of convergence, whereas, if the theory is asymptotically free, its instanton function is convergent over the whole complex plane.

Presenter: ARNAUDO, Paolo (SISSA)

Contribution ID: **12**   Type: **not specified**

Limiting distributions of Spherical and Spin O(N) models: Appearance of GFF

Wednesday 24 January 2024 11:35 (30 minutes)

Spherical model is a mathematical model of a ferromagnet introduced by Berlin and Kac in 1952 as a rough but analytically convenient modification of the Ising model. Since its inception it has enjoyed considerable popularity among the mathematicians and physicists as an exactly soluble model exhibiting a phase transition. In this talk we will explain its relation to the Gaussian free field in the infinite volume limit and to the spin O(N) model in the infinite spin dimensionality limit.

Presenter: KORZHENKOVA, Aleksandra

Contribution ID: **13**   Type: **not specified**

From Liouville to Painlevé and Heun via CFT/isomonodromy relation

Wednesday 24 January 2024 18:00 (1 hour)

I will review connections between the problem of construction of linear ordinary differential equations with prescribed monodromy and the 2D conformal field theory. This correspondence leads to a number of conjectures in the theory of Painlevé and Heun equations some of which have already been proven rigorously and some remain open. The two main applications I will focus on are the construction of the general solution of Painlevé VI equation and the computation of accessory parameter and connection formulas for Heun's equation in terms of Liouville conformal blocks.

Presenter: LISOVYI, Oleg

Contribution ID: **14** Type: **not specified**

AGT correspondence: a 2d CFT valued in 4d QFTs

Tuesday 23 January 2024 17:00 (1 hour)

Starting from a 6d point of view, we will develop the basic dictionary associating 4d N=2 gauge theories to punctured Riemann surfaces, and explain some extensions thereof. As an application, we will explore a rich set of operators (loops, surfaces, walls) that can be added to the story, and how their use led to a conjecture for some braiding kernels in Toda CFT.

Presenter: LE FLOCH, Bruno

Contribution ID: **15**   Type: **not specified**

Tau function, topological string and spectral theory

Wednesday 24 January 2024 19:00 (30 minutes)

Tau function not only is the essential object in the study of integrable system in mordern mathematics but also plays important role in physics; they correspond to the (chiral) conformal blocks. I will talk about our work on an application of tau function to study the physics, which is to prove a special limit of the conjectured topological string/ spectral theory (TS/ST) correspondence. I will introduce the TS/ST correspondence and the particular scaling limit we are taking. In particular, we focus on $Y^{N,0}$ geometry. Tau function is an important tool we use in the proof. And in turn, the proved duality helps us to obtain the strong coupling expression for the tau function itself.

Presenter: HAO, Qianyu (Universite de Geneve (CH))

Contribution ID: **16** Type: **not specified**

Probabilistic approach of Liouville theory

Tuesday 23 January 2024 18:15 (1 hour)

In these lectures, we will review the probabilistic construction of Liouville CFT. We will explain the Segal axioms for CFTs and how they can be implemented in Liouville CFT with the construction of amplitudes. Gluing amplitudes then allows us to identify the Hamiltonian of the Liouville CFT and to provide a representation of the Virasoro algebra in the Hilbert space of Liouville theory. We will explain the diagonalization of the Hamiltonian using the Virasoro representation. Finally we will explain how to combine these concepts to establish the validity of the conformal bootstrap.

Presenter: RHODES, Remi (Aix-Marseille university)

Contribution ID: 17   Type: **not specified**

Random surfaces and Yang-Mills gauge theory

Thursday 25 January 2024 09:00 (1 hour)

Constructing and understanding the basic properties of Euclidean Yang-Mills theory is a fundamental problem in physics. It is also one of the Clay Institute's famous Millennium Prize problems in mathematics. The basic problem is not hard to understand. You can begin by describing a simple random function from a set of lattice edges to a group of matrices. Then you ask whether you can construct/understand a continuum analog of this object in one way or another. In addition to a truly enormous physics literature, this topic has inspired research within many major areas of mathematics: representation theory, random matrix theory, probability theory, differential geometry, stochastic partial differential equations, low-dimensional topology, graph theory and planar-map combinatorics.

Attempts to understand this problem in the 1970's and 1980's helped inspire the study of "random surfaces" including Liouville quantum gravity surfaces. Various relationships between Yang-Mills theory and random surface theory have been obtained over the years, but many of the most basic questions have remained out of reach. I will discuss our own recent work in this direction, as contained in two long recent papers relating "Wilson loop expectations" (the fundamental objects in Yang-Mills gauge theory) to "sums over spanning surfaces."

1. Wilson loop expectations as sums over surfaces on the plane (joint with Minjae Park, Joshua Pfeffer, Pu Yu)

2. Random surfaces and lattice Yang-Mills (joint with Sky Cao, Minjae Park)

The first paper explains how in 2D (where Yang-Mills theory is more tractable) one can interpret continuum Wilson loop expectations purely in terms of flat surfaces. The second explains a general-dimensional interpretation of the Wilson loop expectations in lattice Yang-Mills theory in terms of discrete-and not-necessarily-flat surfaces, a.k.a. embedded planar maps.

Presenter: SHEFFIELD, Scott

Contribution ID: **19** Type: **not specified**

AGT in algebraic geometry

Thursday 25 January 2024 10:30 (1 hour)

The Alday-Gaiotto-Tachikawa correspondence between conformal field theory and 4D gauge theory has a very interesting incarnation in geometric representation theory. Here the objects on the two sides of the correspondence are W-algebras of type gl_r and moduli spaces of rank r sheaves on algebraic surfaces. We prove a q-deformed version of the correspondence, which reveals some higher (more precisely, categorified) structures at play.

Presenter: NEGUT, Andrei (MIT)

Contribution ID: **20**   Type: **not specified**

Affine Yangian approach to integrability in 2D CFT

Thursday 25 January 2024 11:35 (1 hour)

I will review a recently developed approach to studying integrability in
CFT based on affine Yangian symmetry. I will focus on the derivation of
Bethe's ansatz equations for the spectrum of integrals of motion.

Presenter: LITVINOV, Alexey (Skoltech)

Contribution ID: **21**   Type: **not specified**

On the anlytical continuation of Liouville field theory path integral

Thursday 25 January 2024 17:15 (1 hour)

Presenter: SANTACHIARA, Raoul (Paris-Saclay)

Contribution ID: **22**   Type: **not specified**

A distant descendant of the six-vertex model

Thursday 25 January 2024 18:15 (1 hour)

In this talk I present a new solution of the star-triangle relation having positive Boltzmann weights. The solution defines an exactly solvable two-dimensional Ising-type (edge interaction) model of statistical mechanics where the local spin variables" can take arbitrary integer values, i.e., the number of possible spin states at each site of the lattice is infinite. There is also an equivalentdual" formulation of the model, where the spins take continuous real values on the circle. From algebraic point of view this model is closely related to the to the 6-vertex model. It is connected with the construction of an intertwiner for two infinite-dimensional representations of the quantum affine algebra $U_q(\widehat{sl}(2))$ without the highest and lowest weights. The partition function of the model in the large lattice limit is calculated by the inversion relation method. Amazingly, it coincides with the partition function of the off-critical 8-vertex free-fermion model. A connection of the model to a surface roughening transition as well as its possible interpretation as a Berezinskii-Kosterlitz-Thouless transition is briefly discussed. This talk is based on recent paper with Sergey Sergeev arXiv:2310.08427 .

Presenter: BAZHANOV, Vladimir

Contribution ID: **23**   Type: **not specified**

Quantum algebras, wild curves and Painlevé equations

Friday 26 January 2024 09:00 (1 hour)

Presenter: MAZZOCCO, Marta (Birmingham)

Contribution ID: 24   Type: **not specified**

A probabilistic approach to Toda Conformal Field Theories

Friday 26 January 2024 10:30 (30 minutes)

Toda Conformal Field Theories form a family of two-dimensional quantum field theories generalizing Liouville theory. One of their features is that they enjoy, in addition to conformal invariance, an enhanced level of symmetry encoded by W-algebras.

In this talk we describe their mathematical definition and study some of their properties. Namely we will explain how the understanding of its symmetries allows to provide a probabilistic derivation of the Fateev-Litvinov formula for a family of structure constants of the theory. Along with the proof we will shed light on some unexpected connections between a reflection principle in Toda CFTs and its probabilistic counterpart.

Presenter: CERCLE, Baptiste (EPFL)

Contribution ID: **25**   Type: **not specified**

From Segal amplitudes to conformal block

Friday 26 January 2024 11:15 (1 hour)

We shall explain how to construct conformal blocks on Riemann surfaces for Liouville CFT from the probabilistic representation of Segal amplitudes in Liouville theory and the diagonalisation of the Hamiltonian done using analytic methods.
This is based on several joint works with Baverez, Kupiainen, Rhodes, Vargas.

Presenter: GUILLARMOU, Colin (Paris Saclay and CNRS)