Mapping class groups: pronilpotent and cohomological approaches

Report of Contributions

Contribution ID: **1** Type: **not specified**

The Euler characteristic of the moduli space of graphs

Monday 18 September 2023 09:00 (55 minutes)

The moduli space of rank n metric graphs, the outer automorphism group of the free group of rank n and Kontsevich's Lie graph complex in degree n all have the same rational cohomology. We determine the asymptotic behavior of the associated Euler characteristic, and thereby prove that the total dimension of this cohomology grows rapidly with n. This is joint work with Michael Borinsky.

Presenter: Prof. VOGTMANN, Karen (Warwick)

Contribution ID: **2**   Type: **not specified**

The rational abelianization of the Chillingworth subgroup of the mapping class group of a surface

Monday 18 September 2023 10:20 (30 minutes)

The Chillingworth subgroup of the mapping class group of a compact oriented connected surface of genus g with one boundary component is defined as the subgroup of the mapping class group of the surface, whose elements preserve nonsingular vector fields on the surface up to homotopy. We determined the rational abelianization of the Chillingworth subgroup as a full mapping class group module. The rational abelianization is given by the first Johnson homomorphism and the Casson-Morita homomorphism for the Chillingworth subgroup. (This talk is based on arXiv:2305.11767.)

Presenter: Mr KOSUGE, Ryotaro (Tokyo)

Contribution ID: **3**   Type: **not specified**

The rings of tautological differential forms on the moduli of marked Riemann surfaces

Monday 18 September 2023 11:00 (55 minutes)

The aim of this talk is to introduce a natural lifting, to the level of smooth real differential forms, of the systems of tautological rings in the real-valued cohomology of the moduli spaces of marked compact Riemann surfaces. The system of rings of tautological forms can be described as the smallest system of forms that is closed under all tautological pullbacks and submersions, and contains all natural 2-forms obtained from the normal function sections associated to variations of Hodge structures whose monodromy representation factors through a rational representation of the symplectic group. This realizes the map from tautological forms to tautological classes as an avatar of the "primary approximation" to the cohomology of the moduli space of (one-marked) Riemann surfaces constructed by Kawazumi-Morita. We will see that the rings of tautological forms are finite dimensional vector spaces. Also we characterize a certain real-valued invariant of compact Riemann surfaces found by Kawazumi as essentially the only smooth function on moduli space whose Levi form is tautological. This talk is based on joint work with Stefan van der Lugt.

Presenter: Prof. DE JONG, Robin (Leiden)

Contribution ID: 4   Type: **not specified**

Hodge correlators, the Goldman–Turaev Lie bialgebra and Johnson homomorphisms

Monday 18 September 2023 17:00 (55 minutes)

Goncharov's Hodge correlators give a method for computing the periods of the real mixed Hodge structure on the unipotent fundamental group of a hyperbolic Riemann surface X. The Hodge correlator of X takes values in the cyclic quotient of the graded Lie bialgebra |T(H_1(X)|. The goal of the talk is to explain how Goncharov's work is related to Johnson homomorphisms and the Goldman–Turaev Lie bialgebra.

Presenter: Prof. HAIN, Richard (Duke)

Contribution ID: 5　　　　　　　　　　　　　　　　　　　Type: **not specified**

Moments of L-functions via the homology of braid groups

Monday 18 September 2023 18:00 (55 minutes)

Questions about the growth rate of zeta functions and L-functions are a central topic in analytic number theory. In 2005, Conrey, Farmer, Keating, Rubinstein, and Snaith posed a conjecture on the asymptotics of moments of quadratic L-functions. While these sorts of problems originate as questions about number fields, they have a more geometric version when posed over function fields in positive characteristic. I'll talk about how one can reinterpret the central object in this conjecture in terms of the action of the Galois group of a finite field on the cohomology of braid groups with certain coefficients coming from the braid group's interpretation as the hyperelliptic mapping class group. We will see the "arithmetic factor" in this conjecture appear in the part of this cohomology that is accessible through tools of homological stability. This is joint work with Jonas Bergström, Adrian Diaconu, and Dan Petersen.

Presenter: Prof. WESTERLAND, Craig (Minnesota)

Contribution ID: **6**   Type: **not specified**

On the stable cohomology of the IA-automorphism groups of free groups

Tuesday 19 September 2023 09:00 (55 minutes)

By combining Borel's stability and vanishing theorem for the stable cohomology of GL(n,Z) with coefficients in algebraic GL(n,Z)-representations and the Hochschild-Serre spectral sequence, we study the stable twisted cohomology of the automorphism group Aut(F_n) of the free group F_n of rank n and the stable rational cohomology of the IA-automorphism group IA_n of F_n. We propose a conjectural algebraic structure of the stable rational cohomology of IA_n, and consider some relations to known results and conjectures. If time permits, we also consider a conjectural structure of the stable rational cohomology of the Torelli groups of surfaces. This is a joint work with Mai Katada.

Presenter: Prof. HABIRO, Kazuo (Tokyo)

Contribution ID: **7**   Type: **not specified**

About torsion in the cokernels of the Johnson homomorphisms

Tuesday 19 September 2023 10:20 (30 minutes)

The Johnson homomorphisms encode the action of the mapping class group on the nilpotent quotients of the fundamental group of the surface, embedding the graded space associated with the Johnson filtration in some Lie algebra of derivations. In this talk, we will use the infinitesimal Dehn-Nielsen representation of the mapping class group to study torsion in the cokernel of the Johnson homomorphisms.

Presenter: Dr FAES, Quentin (Tokyo)

Contribution ID: **8**   Type: **not specified**

A non-commutative Reidemeister-Turaev torsion of homology cylinders

Tuesday 19 September 2023 11:00 (55 minutes)

A homology cylinder is a 3-manifold that is homologically the product of a surface and an interval. In this talk, we introduce the Reidemeister-Turaev torsion of homology cylinders which takes values in the K_1-group of the I-adic completion of the group ring of the fundamental group of a surface over the rationals, and prove that its reduction by the ideal \hat{I}^{d+1} is a finite-type invariant of degree d.

We also show that the 1-loop part of the LMO homomorphism and the Enomoto-Satoh trace can be recovered from the leading term of our torsion.

This is joint work with Masatoshi Sato and Masaaki Suzuki.

Presenter: Prof. NOZAKI, Yuta (Yokohama)

Contribution ID: **9**   Type: **not specified**

Bordification of the moduli space of tropical abelian varieties, and unstable cohomology of the general linear group GL_g(Z)

Tuesday 19 September 2023 17:00 (55 minutes)

I will explain a geometric argument to construct infinitely many non-zero unstable cohomology classes for the group GL_g(Z), some of which were known or conjectural, and others which are new.

Presenter: Prof. BROWN, Francis (Oxford)

Contribution ID: **10**   Type: **not specified**

Stable Koszulness of mapping class groups

Tuesday 19 September 2023 18:00 (55 minutes)

By a deep result of Hain, we know generators and relations of the (relative) Malcev completion of mapping class groups. In the limit where the genus goes to infinity, there is a description of that Lie algebra as the cohomology of a certain graph complex (closely related to higher genus Grothendieck-Teichmüller Lie algebras). By computing the cohomology of the Koszul dual graph complex, one can deduce stable Koszulness of Hain's Lie algebras.

This is joint work with M. Felder and T. Willwacher.

Presenter: Prof. NAEF, Florian (Dublin)

Contribution ID: **11**　　　　　　　　　　　　　　　　　　　Type: **not specified**

Automorphism groups of free groups, subgroups and functor categories

Wednesday 20 September 2023 09:00 (55 minutes)

Automorphism groups of free groups are related to mapping class groups and gives rise to nice subgroups IA, analogous to Torelli groups, with two fundamental filtrations: the Johnson-Andreadakis one, and its lower central series. All these objects carry a deep structure and are very hard to approach. In this talk, we will give a survey of what the use of functor categories can bring to their study, in particular: some stable homological computations with twisted coefficients or nice and precise ways to express stable properties (most of them being still open), in the spirit of representation stability.

Presenter:　Prof. DJAMENT, Aurélien (Paris)

Contribution ID: **12**   Type: **not specified**

Stable cohomology of mapping class groups with some particular twisted coefficients

Wednesday 20 September 2023 10:20 (30 minutes)

The twisted cohomology of mapping class groups of compact orientable surfaces (with one boundary) is difficult to compute generally speaking. In this talk, I will describe the computation of the stable cohomology groups of these mapping class groups with twisted coefficients given by the first homology of the unit tangent bundles of the surfaces. This type of computation is out of the scope of the traditional framework for cohomological stability. Indeed, these twisted coefficients define a contravariant functor over the classical category associated to mapping class groups to study homological stability, rather than a covariant one. I will also explain the computations of the stable cohomology algebras with with twisted coefficients given by the exterior powers of these representations. This represents a joint work with Nariya Kawazumi. I will finally present some recent progresses on the computations of the stable cohomology groups of mapping class groups with twisted coefficients given by the Moriyama representations.

Presenter: Dr SOULIÉ, Arthur (Pohang)

Contribution ID: **13**   Type: **not specified**

Stable cohomology of Aut(F_n) with bivariant twisted coefficients

Wednesday 20 September 2023 11:00 (55 minutes)

The stable cohomology of Aut(F_n) has been studied by several authors. Galatius proved that the stable cohomology groups with coefficients in Q are trivial. With coefficients in tensor powers of H=H_1(F_n,Q), or of its dual H, *the stable cohomology groups were independently computed by Djament and Vespa (using functor homology methods) and by Randal-Williams (by extending the methods of Galatius). For mixed tensor powers of H and H* ("bivariant" twisted coefficients), a conjectural description was given by Djament. Furthermore, Kawazumi and Vespa proved that the collection of stable cohomology groups with all different bivariant twisted coefficients has the structure of a so-called "wheeled PROP" and that the wheeled sub-PROP generated by a specific cohomology class, which had been previously introduced by Kawazumi, made the conjectural description of Djament a lower bound for the stable cohomology groups. In this talk, I will review these results and explain how these cohomology groups can be computed, confirming the conjecture of Djament, by extending the methods of Galatius and Randal-Williams a bit further.

Presenter: Dr LINDELL, Erik (Stockholm)

Contribution ID: **14**   Type: **not specified**

Graph complexes and their interrelations

Thursday 21 September 2023 09:00 (55 minutes)

There are many different graph complexes which are often used in applications, e.g. in the deformation quantization theories, in the algebraic topology, in the theory of moduli spaces of algebraic curves, in the Lie theory, etc. Examples include the Kontsevich graph complex, its directed version, the complex of oriented graphs, the one of sourced graphs, the one of sourced-and-targeted graphs, several variants based on the notion of ribbon graph.

We shall attempt to give an overview of the current status of our knowledge about these complexes, about their (sometimes surprising) inter-relations and their applications. We shall not assume prior knowledge of the theories of graph complexes.

Presenter: Prof. MERKULOV, Sergei (Luxembourg)

Contribution ID: **15**   Type: **not specified**

Globalising Jones and Alexander Polynomials (and their quantum generalisations) via configurations in the punctured disc

Thursday 21 September 2023 10:20 (30 minutes)

Presenter: Dr PALMER-ANGHEL, Cristina (University of Geneva)

Contribution ID: **16**   Type: **not specified**

Quantum representations of mapping class groups: factorization homology techniques

Thursday 21 September 2023 11:00 (55 minutes)

Suitable representation categories of Hopf algebras or vertex operator algebras give rise to systems of representations of mapping class groups. These are also referred to as modular functors. In my talk, I will give an approach to these representations via factorization homology, a replacement for the classical skein-theoretic methods. Particular emphasis will lie on the insights that we can gain from this new description in the situation where the modular functors are built from non-semisimple representation categories. This is based on different joint works with Adrien Brochier and Lukas Müller.

Presenter: Dr WOIKE, Lukas (Dijon)

Contribution ID: **17**
Type: **not specified**

Associators in mould theory

Thursday 21 September 2023 17:00 (55 minutes)

In my talk I will explain several basic notions and properties developed in Ecalle's mould theory and Sauzin's dimould theory. By introducing the balance map for dimoulds and extending the notion of Zag, I will explain how associators are reformulated in terms of mould theory. My talk is based on my joint work with M.Hirose and N.Komiyama.

Presenter: Prof. FURUSHO, Hidekazu (Nagoya)

Contribution ID: **18**   Type: **not specified**

The injection from the grt Lie algebra into the double shuffle Lie algebra: an elementary proof

Thursday 21 September 2023 18:00 (55 minutes)

By interpreting the pentagon equation defining the Grothendieck-Teichm¥"uller Lie algebra as a condition on the normal form of certain elements in the 5-strand Lie braid algebra, we show how the double shuffle defining equations arise naturally in grt. This gives an intrinsic and elementary proof of the injection of grt into double shuffle first shown by H. Furusho using double polylogarithms.

Presenter: Prof. SCHNEPS, Leila (Paris)

Contribution ID: **19**   Type: **not specified**

Stable rational homology of the IA-automorphism groups of free groups

Friday 22 September 2023 09:00 (55 minutes)

We study the quotient of the rational homology of the IA-automorphism group IA_n of the free group F_n that is obtained as the image of the map induced by the abelianization map of IA_n on homology. We call it the Albanese homology of IA_n. In this talk, we determine the third Albanese homology of IA_n in a stable range. Moreover, in higher degree, we obtain a subquotient of the Albanese homology of IA_n in a stable range, which is conjecturally equal to the entire Albanese homology of IA_n. We also consider the Albanese homology of the Torelli groups of surfaces.

Presenter: Dr KATADA, Mai (Tokyo)

Contribution ID: **20**   Type: **not specified**

Homological instability for moduli spaces of 4-manifolds

Friday 22 September 2023 10:20 (55 minutes)

We prove that homological stability with respect to connected sums of Sˆ2×Sˆ2 fails for moduli spaces BDiff(X) of simply-connected closed 4-manifolds X. This makes a striking contrast with all other even dimensions, where analogous stability has been established by Harer in dimension 2 and by Galatius and Randal-Williams in dimension higher than 4. The proof of the above result is based on a new characteristic class constructed by using 4-dimensional gauge theory. This is joint work with Jianfeng Lin.

Presenter: Prof. KONNO, Hokuto (Tokyo)