

Sign Problem Free Qubit Regularized Hamiltonian Lattice Gauge Theory

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US Department of Energy***



Collaborators



Siew



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Outline

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Qubit Regularization

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Qubit Regularization of Gauge Theories

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Classical and Quantum Dimer Models

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Conclusions

Qubit Regularization

Can we formulate quantum field theories so that we can study them using a quantum computer?

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Formulate a Hamiltonian lattice field theory
with a finite local Hilbert space
with an appropriate “continuum limit.”

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Qubit Regularization of the QFT

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Formulate a Hamiltonian lattice field theory with a finite local Hilbert space with an appropriate “continuum limit.”

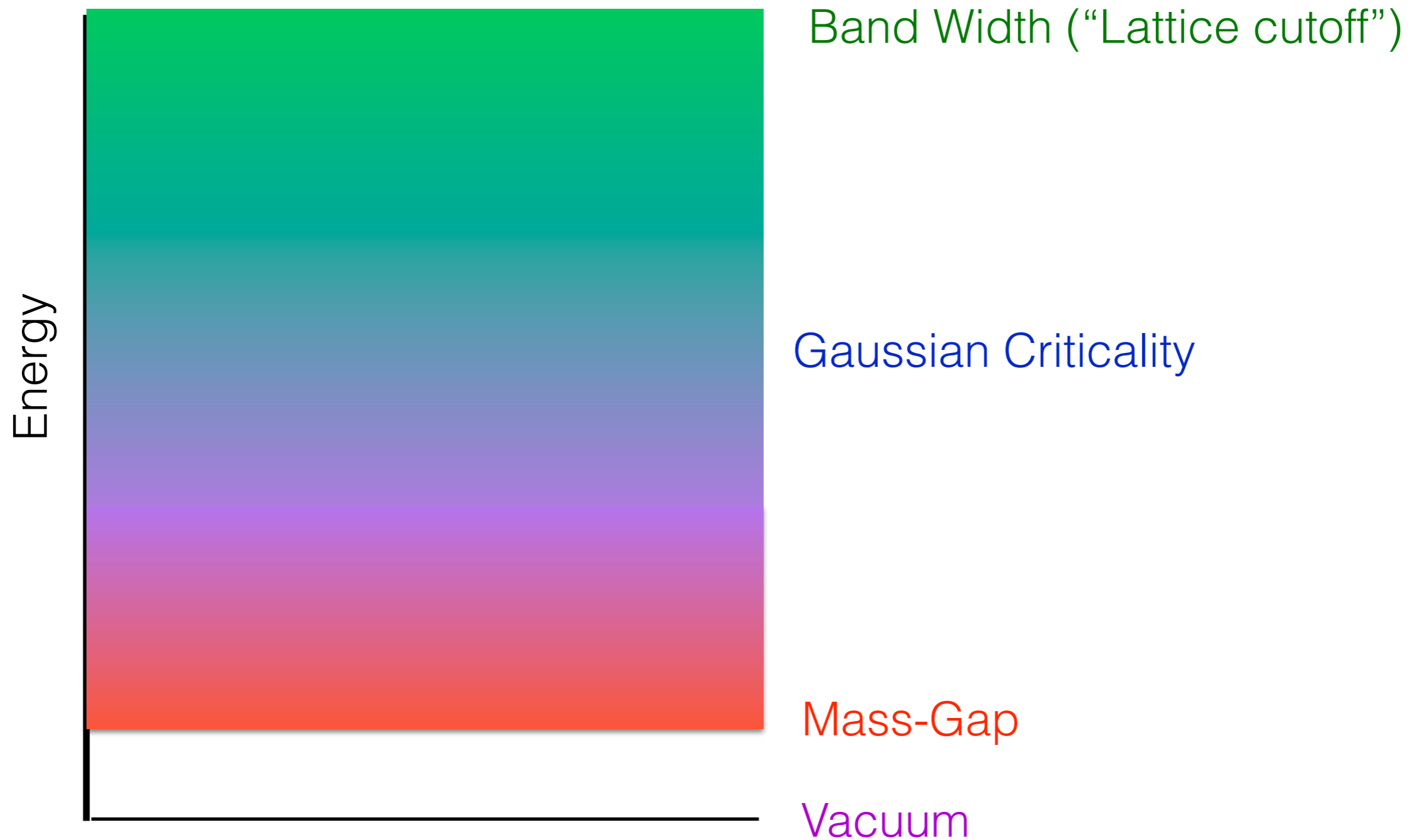


Qubit Regularization of the QFT

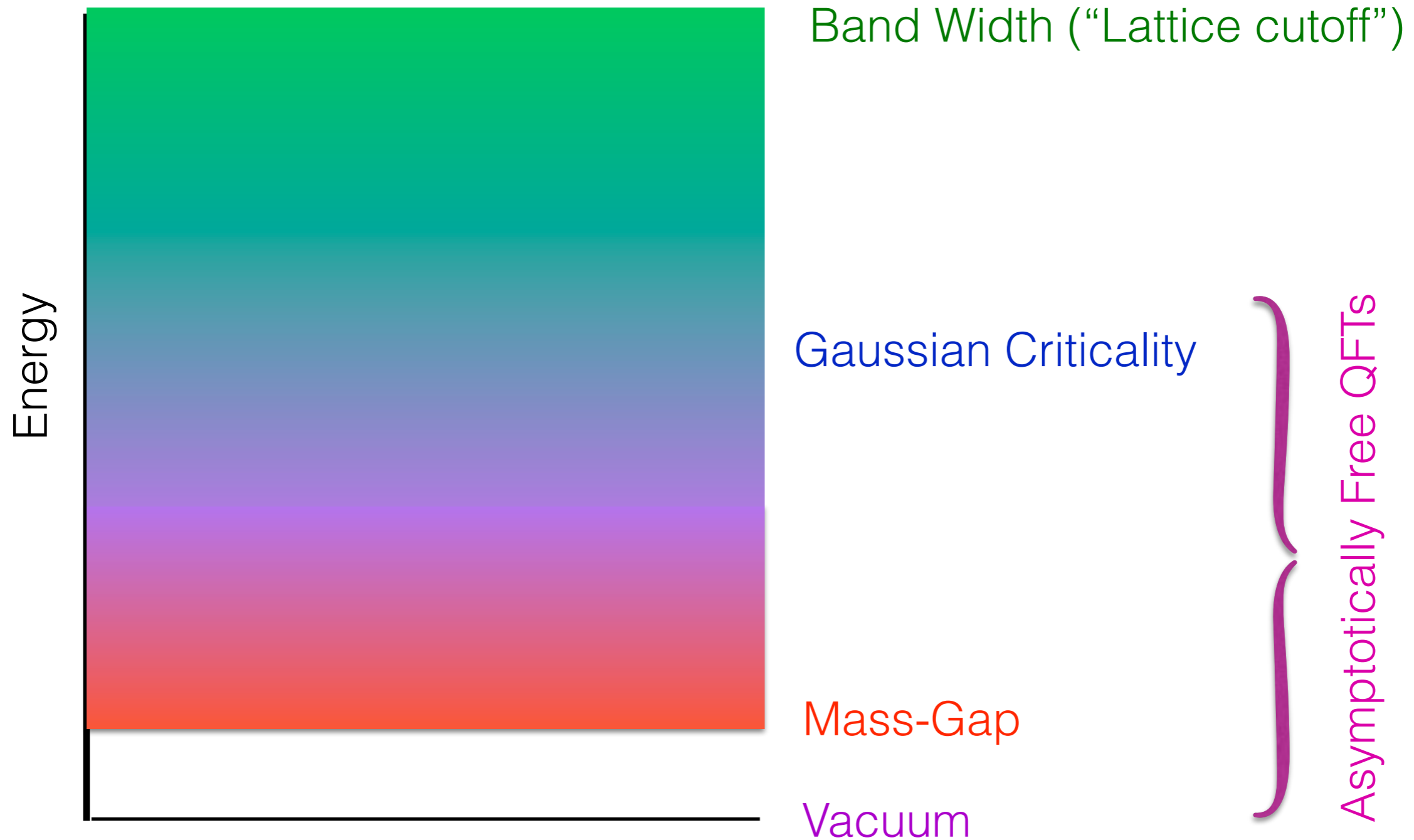
Continuum limit = UV Quantum Critical Point

Particular Interest in Asymptotically Free Massive Theories

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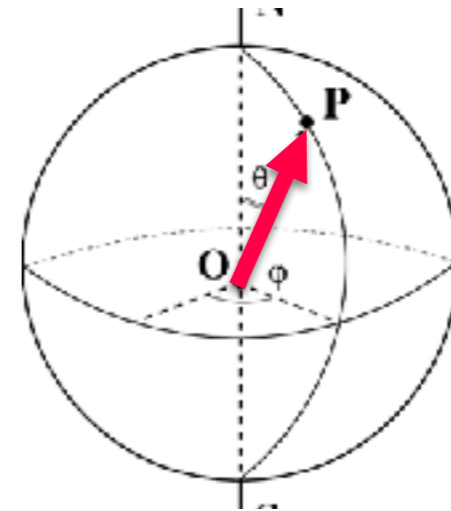
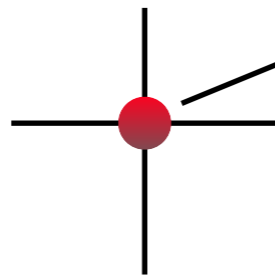


Example: $O(3)$ Non-linear sigma model

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Traditional lattice Hilbert space

At each lattice site there is a quantum particle on a surface of a unit sphere



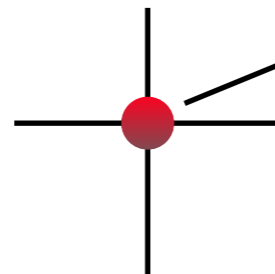
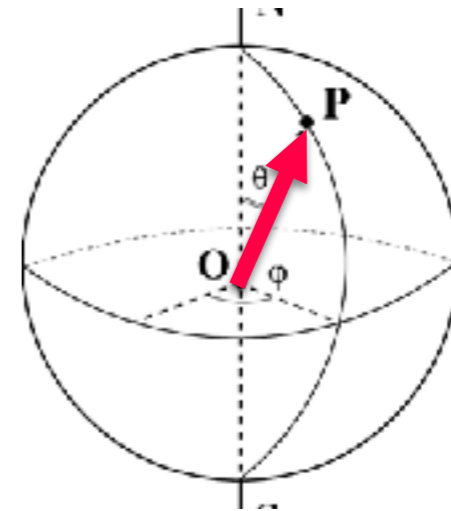
$$\vec{\phi}_x = (\phi_x^1, \phi_x^2, \phi_x^3)$$

$$\vec{\phi}_x \cdot \vec{\phi}_x = 1$$

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Basis of the traditional Hilbert space $\mathcal{H}_{\text{Trad}}$:

$$\int d\Omega |\theta, \varphi\rangle \langle \theta, \varphi| = 1$$

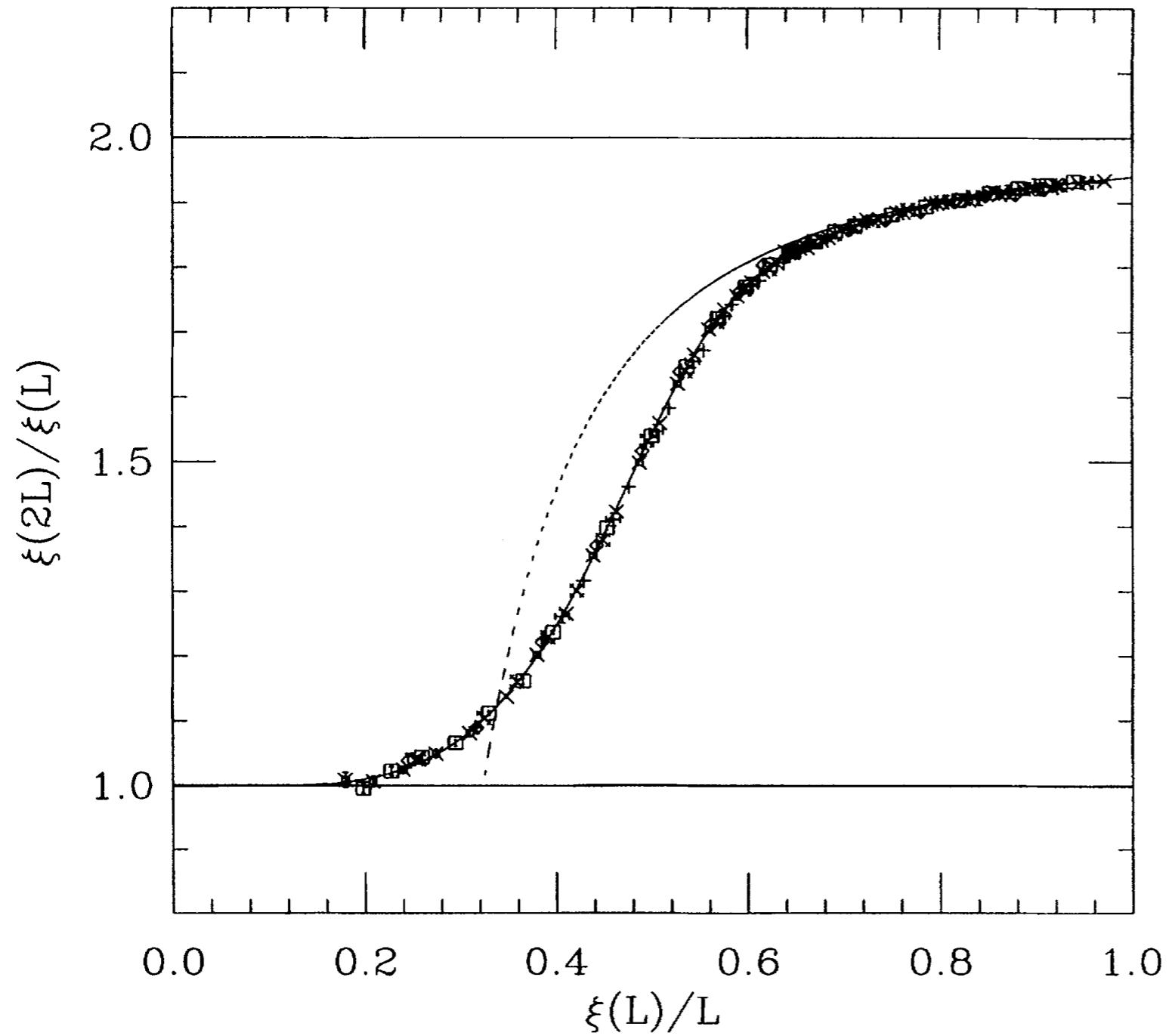
“position basis”

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l |l, m\rangle \langle l, m| = 1$$

“Representation basis”

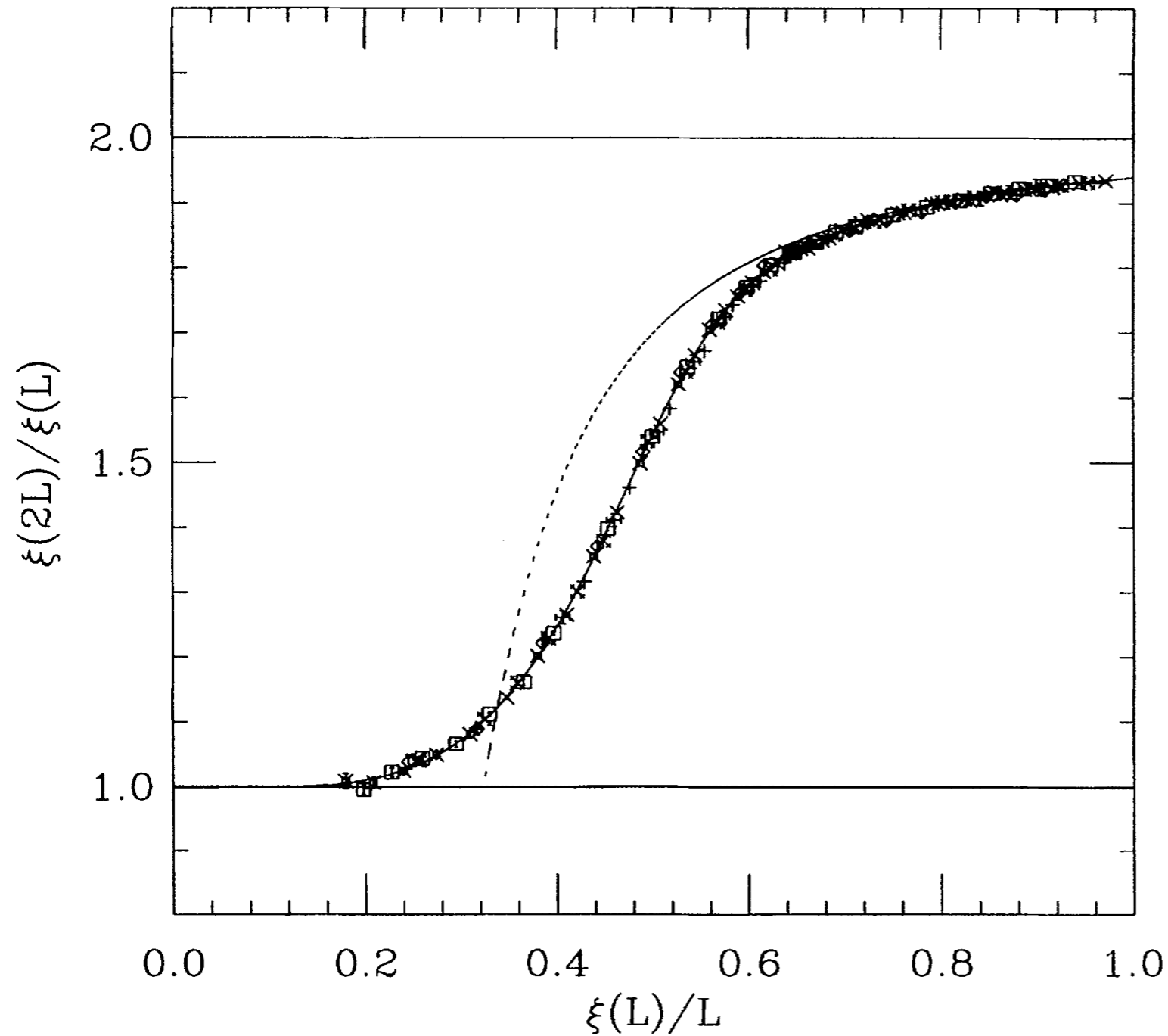
Continuum Physics: Universal step-scaling function

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Caracciolo, et.al., PRL 75, 1891 (1995)

Continuum Physics: Universal step-scaling function



Qubit Regularization



Can we reproduce this continuum physics in a lattice model with a finite lattice Hilbert space?

Caracciolo, et.al., PRL 75, 1891 (1995)

Qubit regularized model

Bhattacharya, Buser, SC, Gupta, Singh PRL 126 (2021), 172001

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Qubit Regularization: $\mathcal{H}_{\text{Trad}} \rightarrow \mathcal{H}_Q$

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$$\mathcal{H}_{\text{Trad}} = \bigoplus_{\ell=0,1,2,\dots} \mathcal{H}_{\ell}$$

$$\mathcal{H}_{\text{Q}} = \mathcal{H}_{\ell=0} \oplus \mathcal{H}_{\ell=1}$$

Qubit regularized model

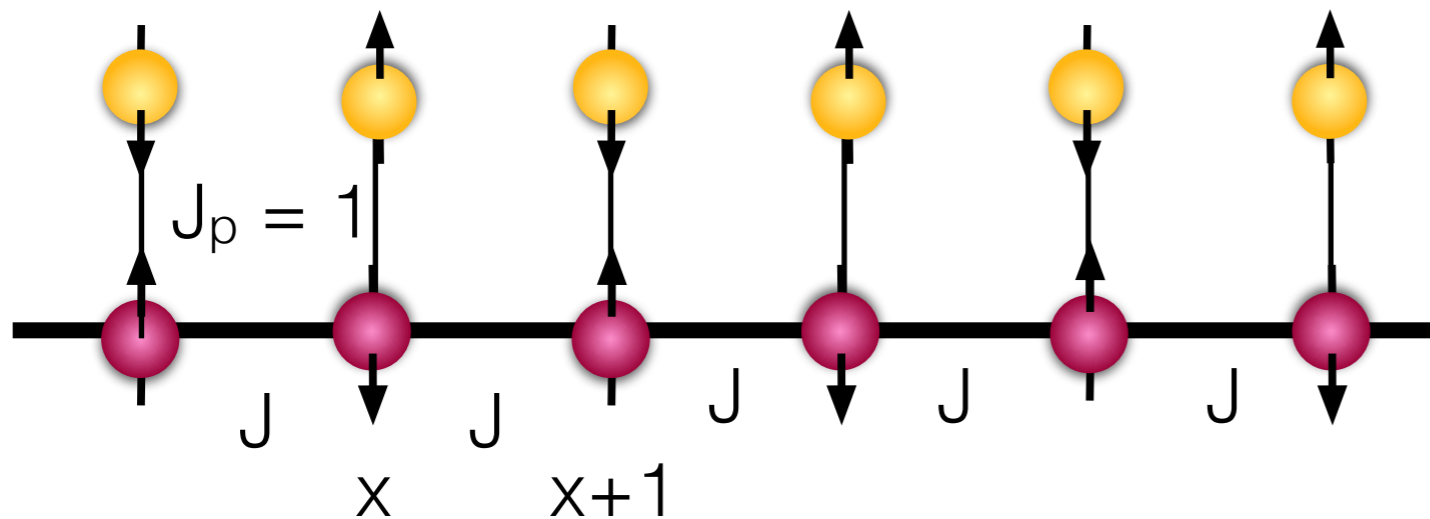
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Heisenberg-Comb



$$H = \sum_x J \mathbf{S}_{x,1} \cdot \mathbf{S}_{x+1,1} + \mathbf{S}_{x,1} \cdot \mathbf{S}_{x,2}$$

Qubit regularized model

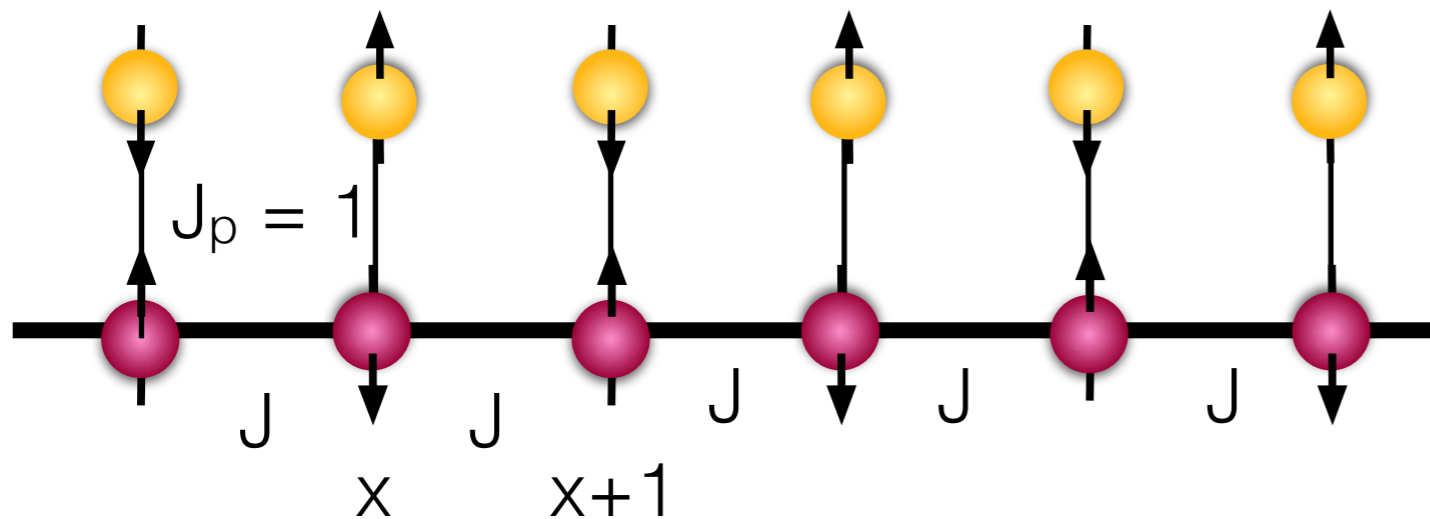
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UV Quantum Critical Point:

$$J \rightarrow \infty$$

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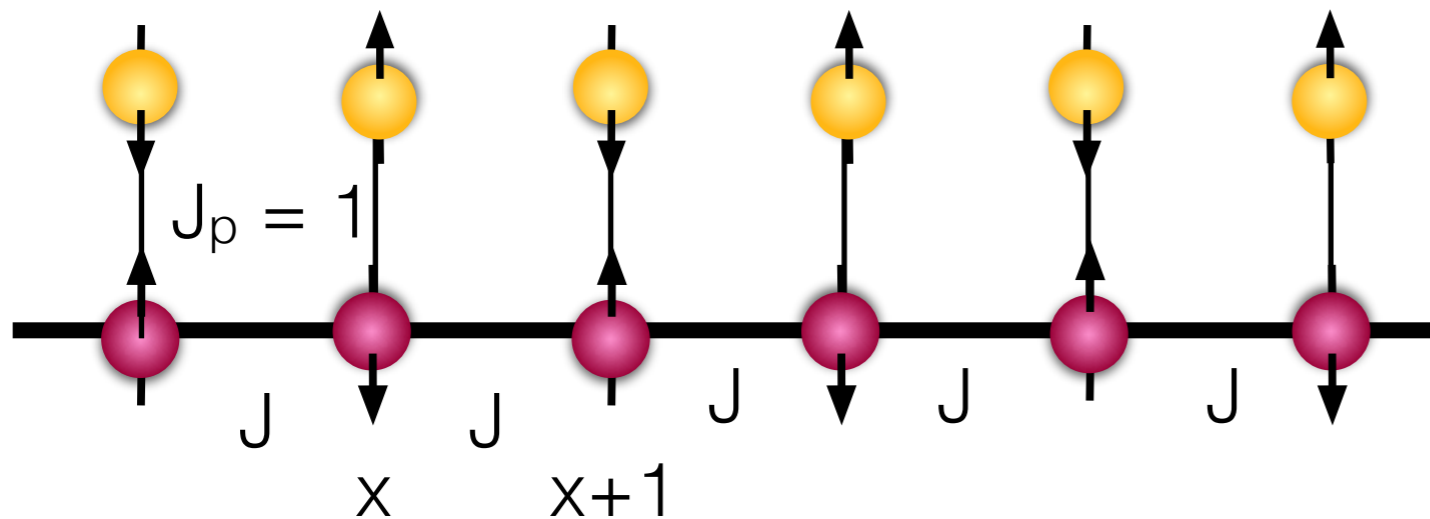
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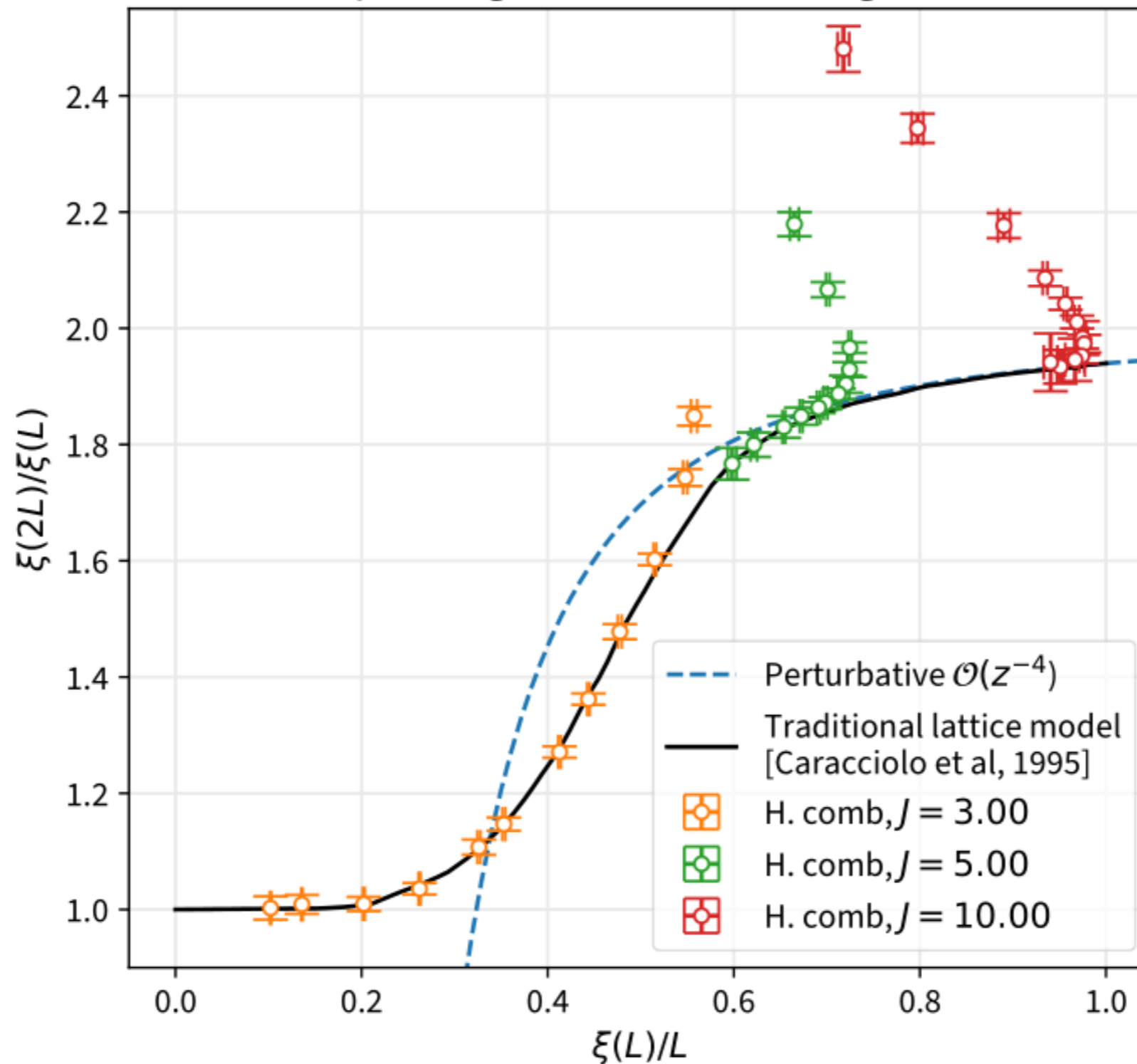
Note

$$J \rightarrow \infty \neq J = \infty$$

Universal Step Scaling Function

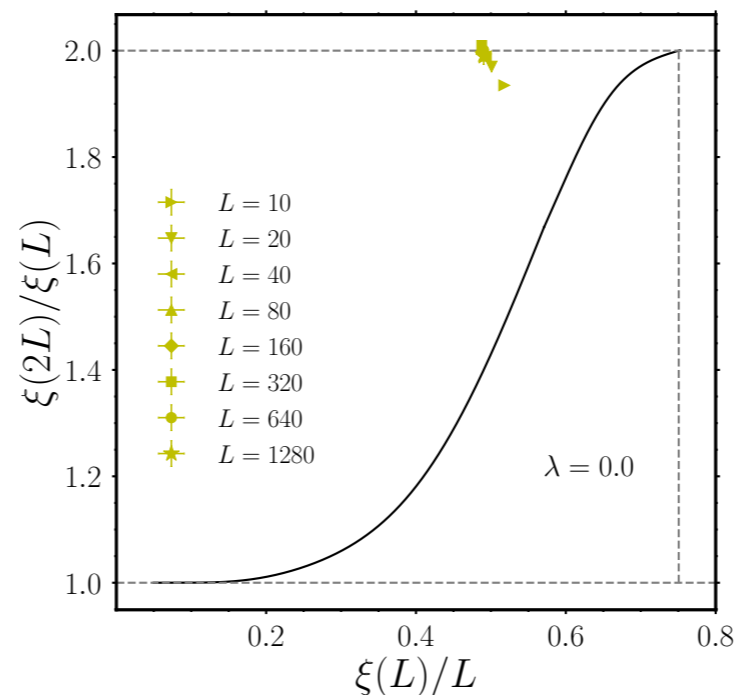
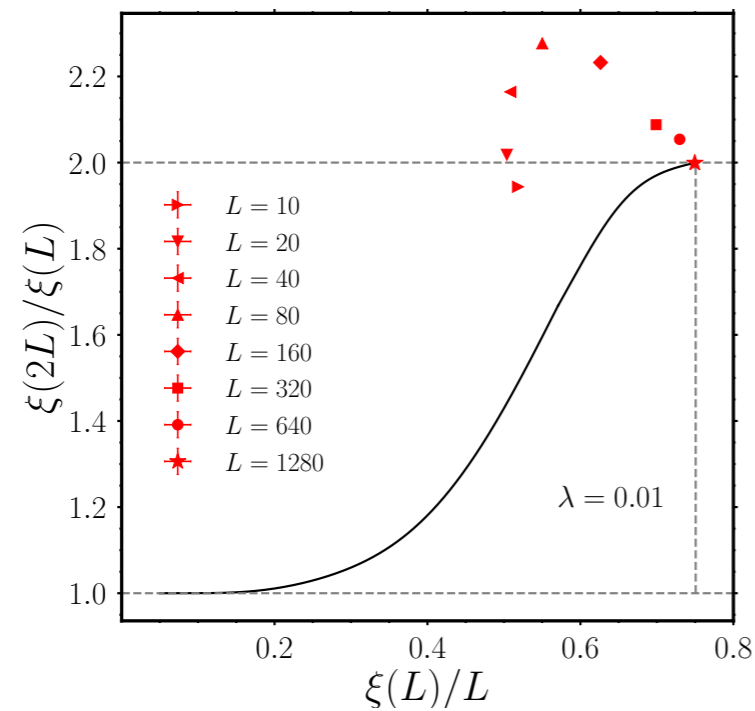
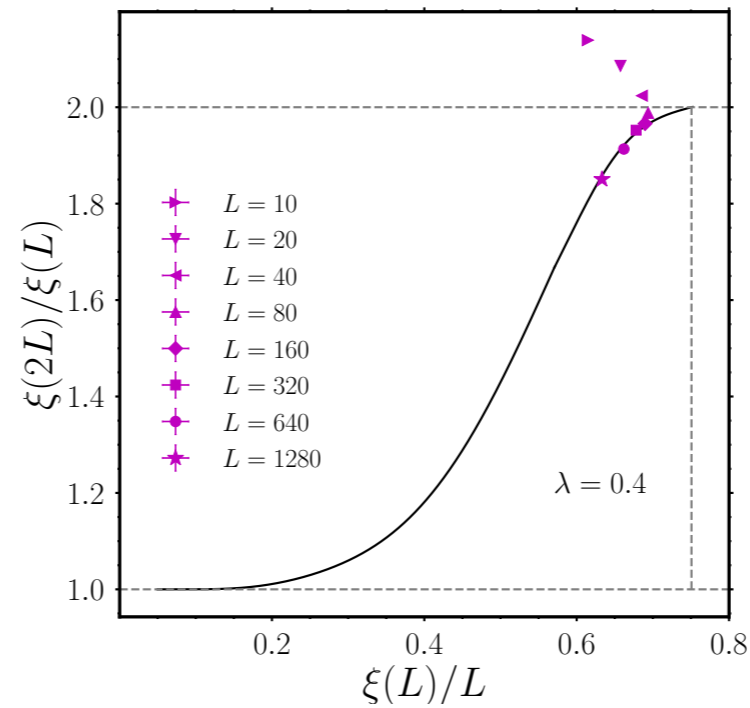
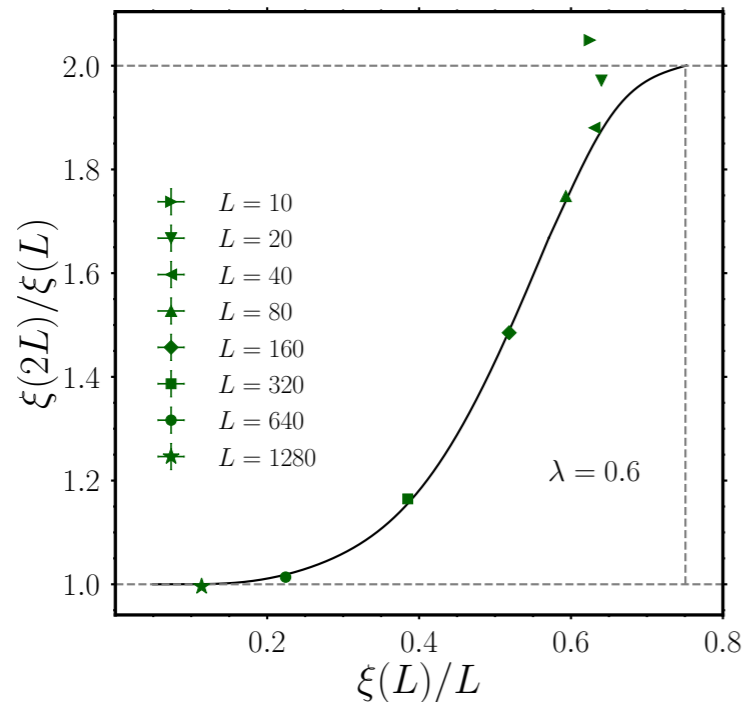
Bhattacharya, Buser, SC, Gupta, Singh PRL 126 (2021), 172001

Step scaling function: Heisenberg comb



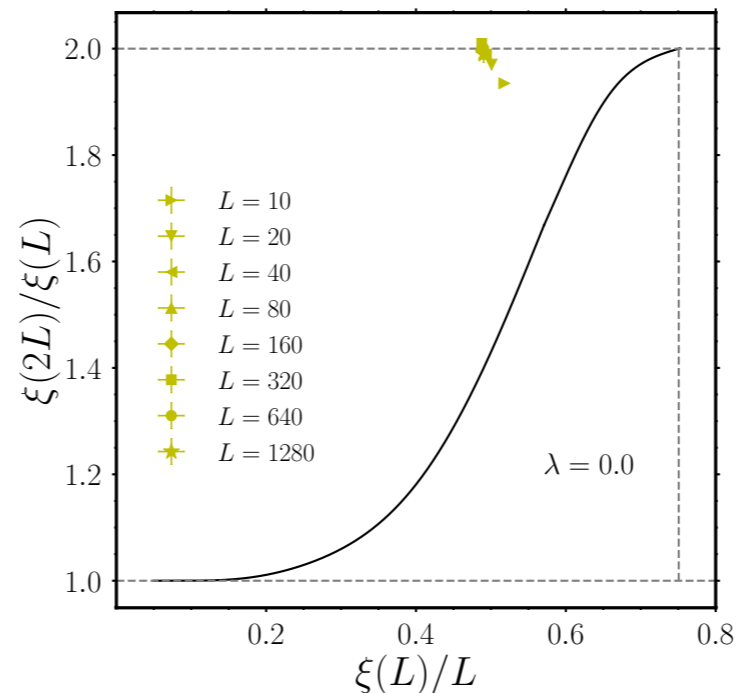
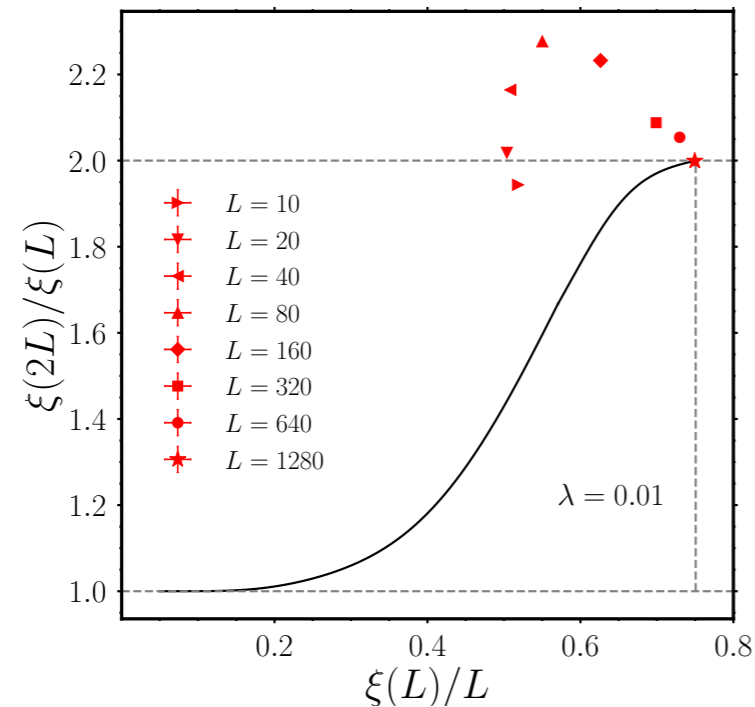
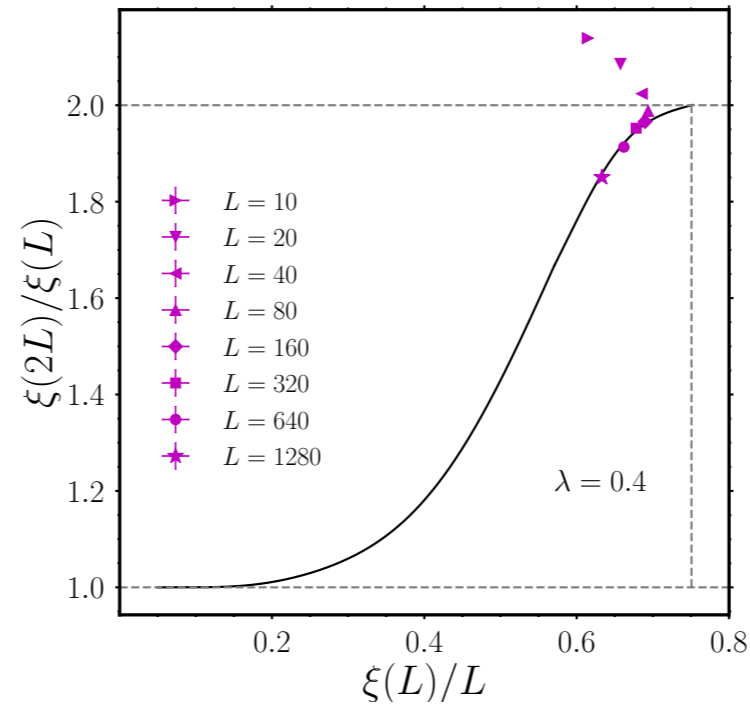
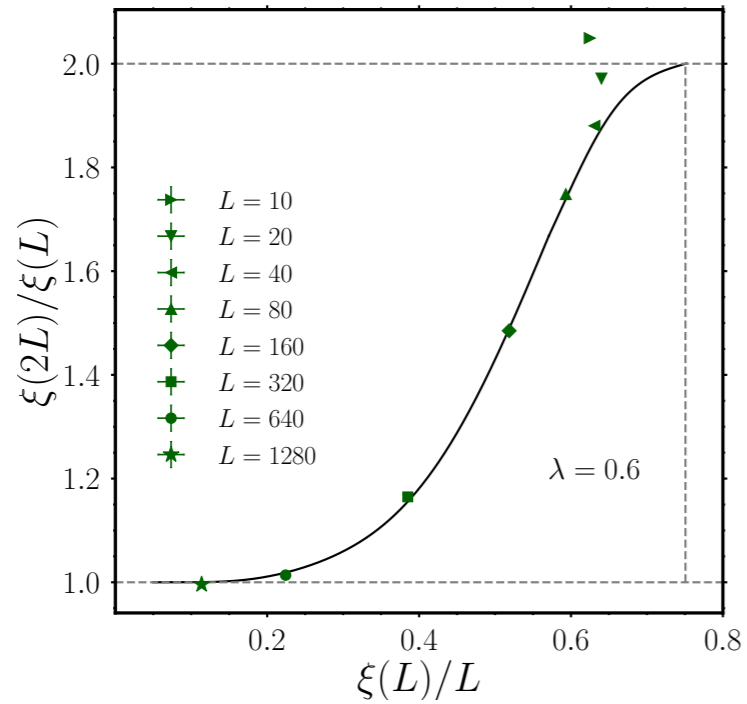
Another Example: Qubit Regularization of the BKT transition

Maiti, Banerjee, SC, Marinkovic, PRL 132 (2024), 041601



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Maiti, Banerjee, SC, Marinkovic, PRL 132 (2024), 041601



Again note:
 $\lambda \rightarrow 0 \neq \lambda = 0$

Qubit Regularization \sim D-theory

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D-theory was an idea introduced by Uwe-Jens

Lattice 1998, Plenary talk by Wiese.

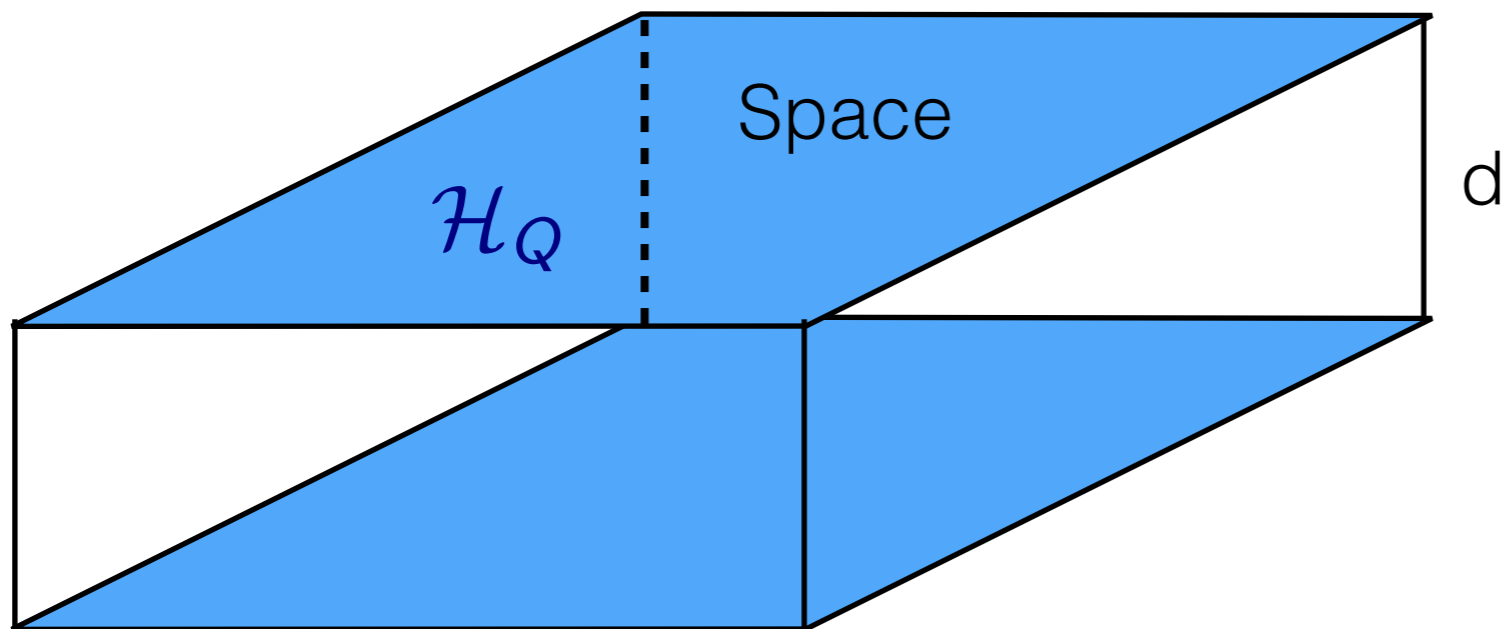
Brower, SC, Riederer, Wiese, NPB 693 (2004), 149

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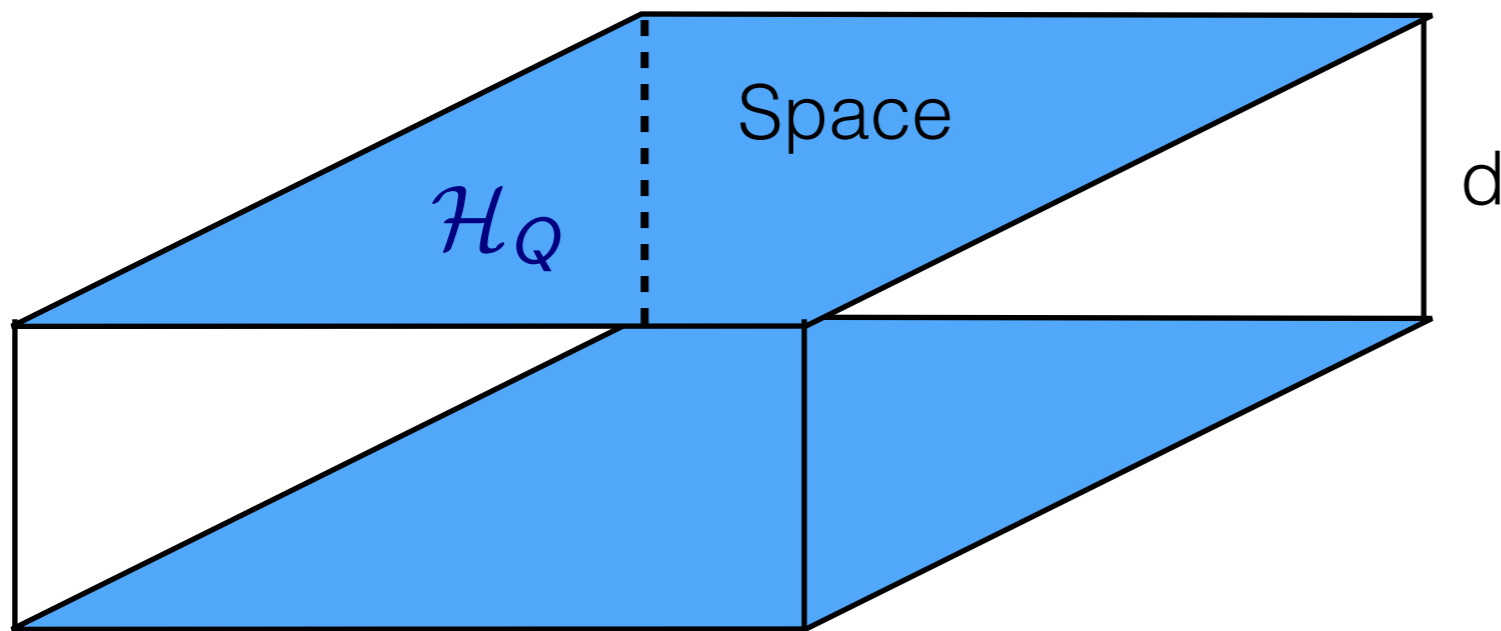
d is allowed to grow so
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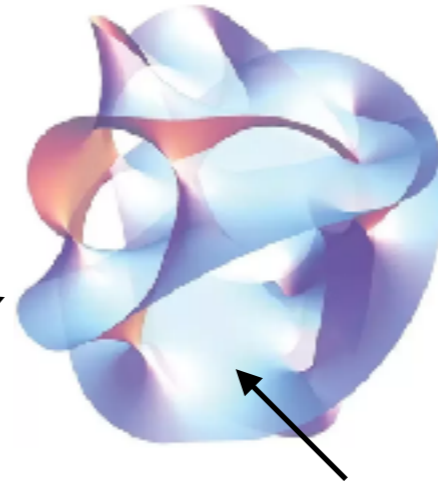
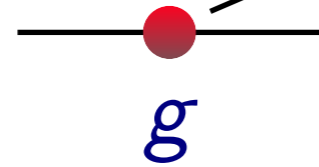
RG plays an important role!

Qubit Regularization of Gauge Theories

Traditional $SU(N)$ lattice gauge theories

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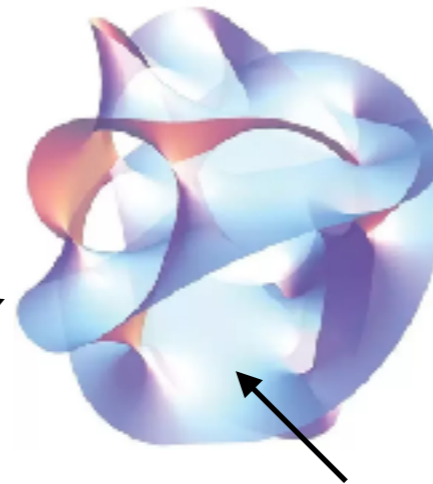
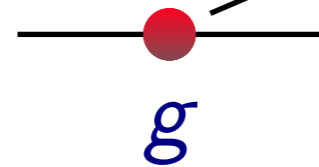
On each lattice link there is a quantum particle moving on the surface of the $SU(N)$ group manifold



$SU(N)$ manifold

Traditional SU(N) lattice gauge theories

On each lattice link there is a quantum particle moving on the surface of the SU(N) group manifold



$SU(N)$ manifold

Basis of the full Hilbert space $\mathcal{H}_{\text{Trad}}$:

$$\int [dg] |g\rangle\langle g| = I$$

“position basis”

$$\sum_{\lambda} \sum_{i,j} |D_{ij}^{\lambda}\rangle\langle D_{ij}^{\lambda}| = \mathbb{I}$$

“Representation basis”

λ labels distinct irreps of SU(N)

This means the traditional link Hilbert space is given by

$$\mathcal{H}_{\text{Trad}} = \bigoplus_{\lambda} \mathcal{H}_{\lambda} \quad \leftarrow \text{Peter-Weyl Theorem}$$

where $\mathcal{H}_{\lambda} = V_{\lambda} \otimes V_{\lambda}^*$ spanned by $\{|D_{ij}^{\lambda}\rangle\}, i, j = 1, 2, \dots, d_{\lambda}$

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Qubit Regularized Hilbert Space $\mathcal{H}_Q = \bigoplus_{\lambda \in Q} V_{\lambda} \otimes V_{\lambda}^*$

$$\dim(\mathcal{H}_Q) = \sum_{\lambda \in Q} (d_{\lambda})^2$$

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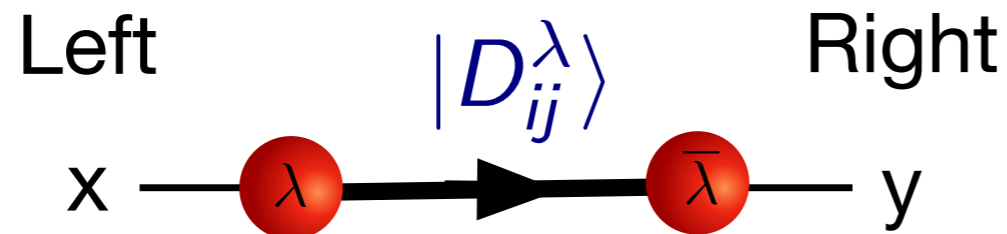
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How does the “irrep space” formulation of a lattice gauge theory look like?

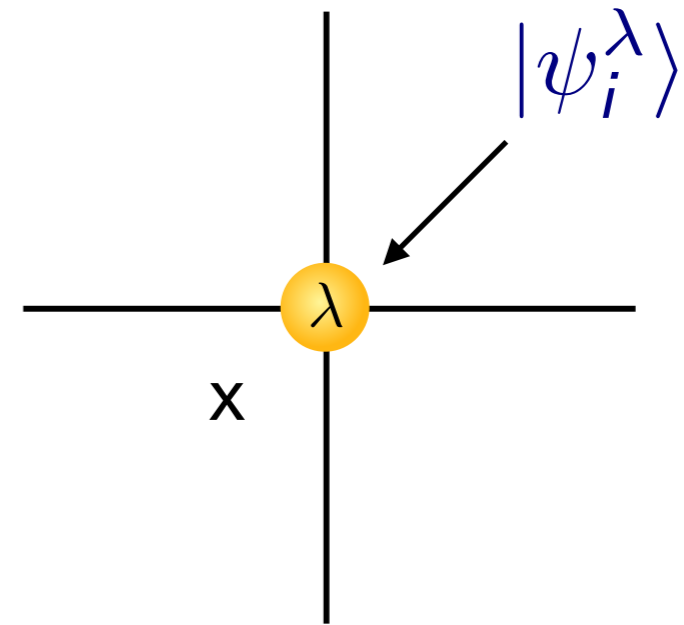
“Irrep space” formulation of
traditional lattice gauge theories

“Irrep space” formulation of traditional lattice gauge theories



Lattice Link

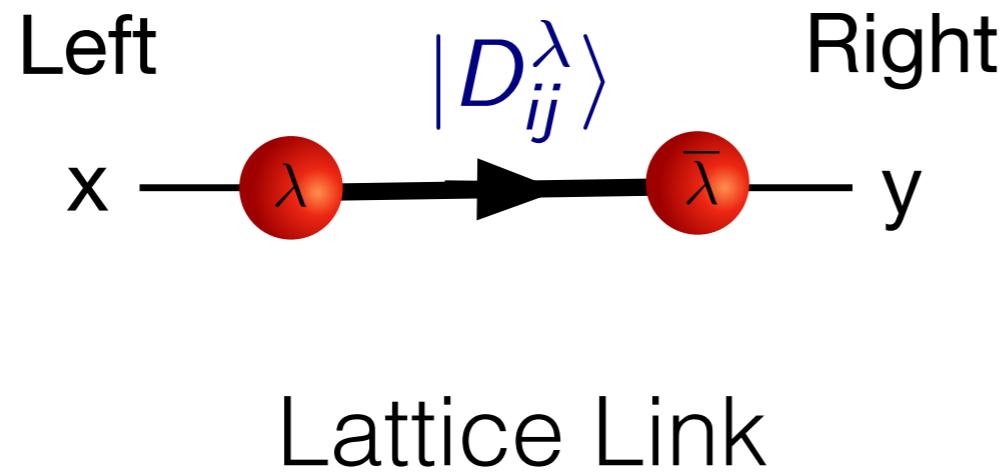
$$\dim(\mathcal{H}_\ell) = d_\lambda^2$$



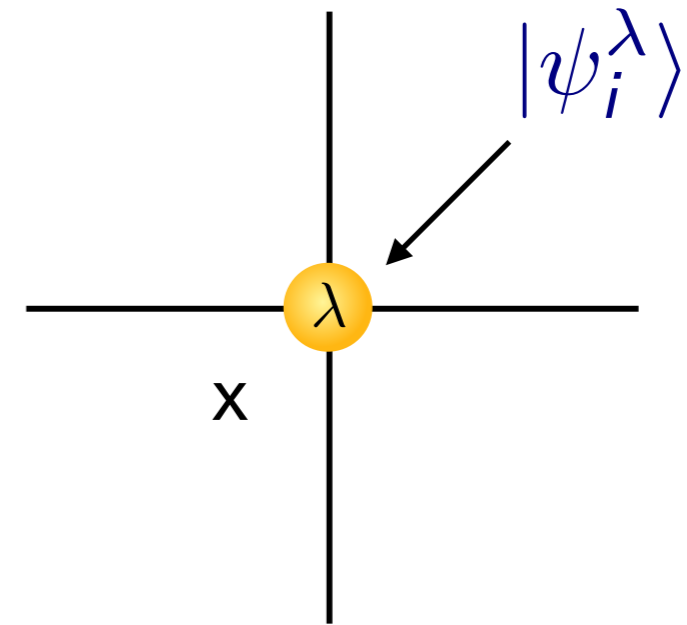
Lattice Site

$$\dim(\mathcal{H}_s) = d_\lambda$$

“Irrep space” formulation of traditional lattice gauge theories



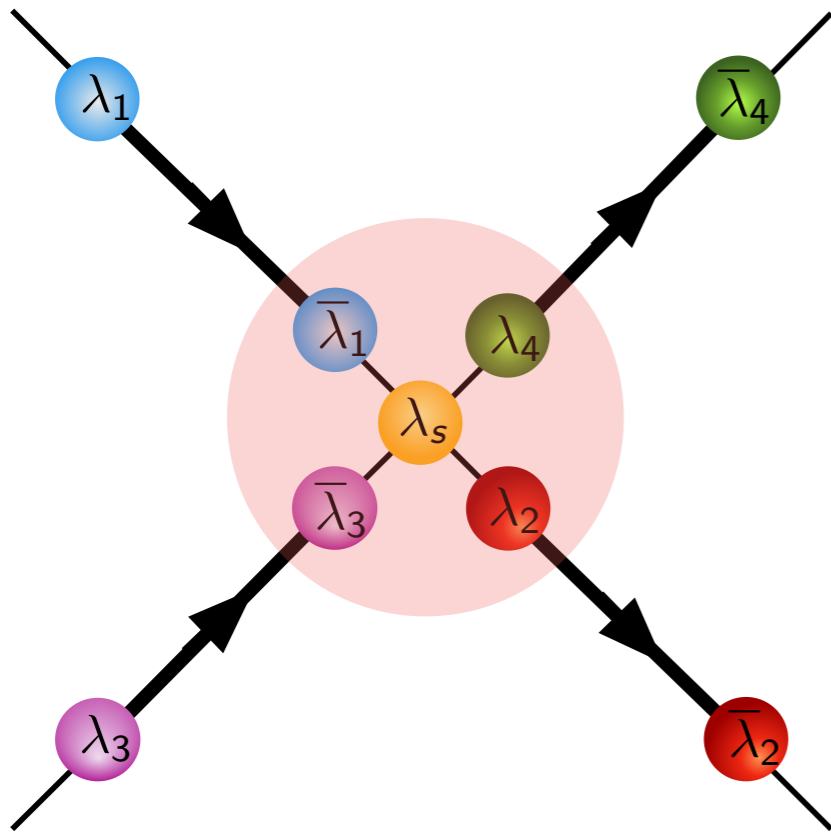
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$$\dim(\mathcal{H}_s) = d_\lambda$$

The physical Hilbert space is obtained by projecting to a gauge-invariant sector

Gauss Law

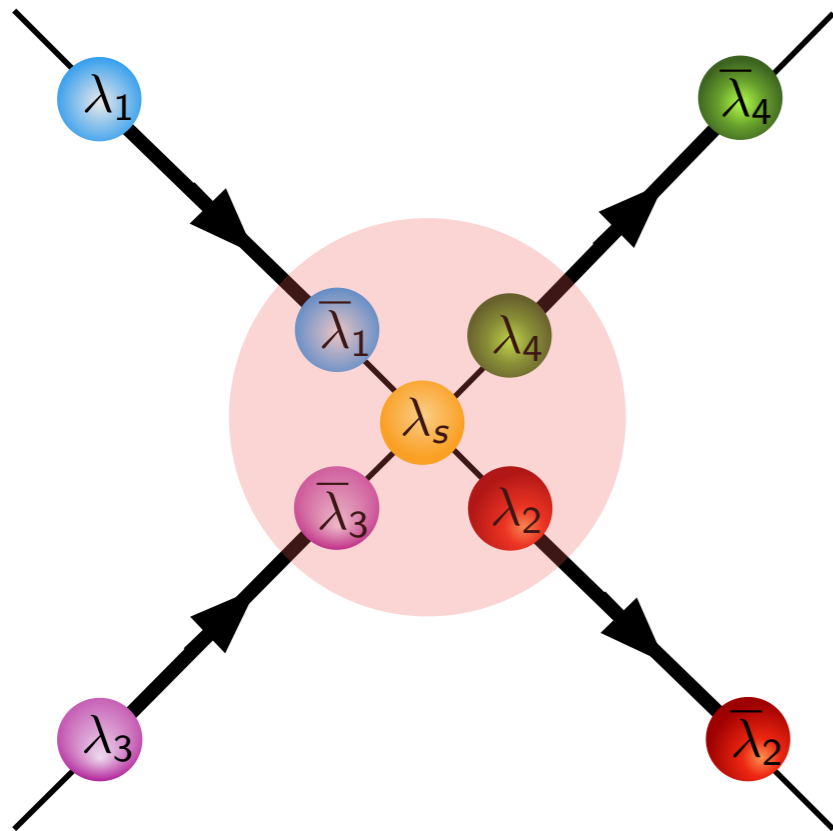


$$\mathcal{H}_s^g = \bar{\lambda}_1 \otimes \lambda_2 \otimes \bar{\lambda}_3 \otimes \lambda_4 \otimes \lambda_s.$$

α_s labels the basis states of the singlet space of \mathcal{H}_s^g

$$\alpha_s = 1, 2, \dots, \mathcal{D}(\mathcal{H}_s^g)$$

Gauss Law



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$$\alpha_s = 1, 2, \dots, \mathcal{D}(\mathcal{H}_s^g)$$

A basis of the physical Hilbert space

$$|\{\lambda_s\}, \{\lambda_\ell\}, \{\alpha_s\}\rangle$$

All Clebsch-Gordan Coefficients have disappeared!

All irreps $\{\lambda_\ell\}$ are allowed in the traditional theory

All irreps $\{\lambda_e\}$ are allowed in the traditional theory

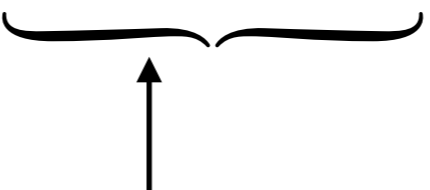
Qubit regularization works with a subset of these irreps

All irreps $\{\lambda_\ell\}$ are allowed in the traditional theory

Qubit regularization works with a subset of these irreps

Antisymmetric qubit regularization scheme

Hanqing Liu, SC Symmetry 14 (2022) 2 305,

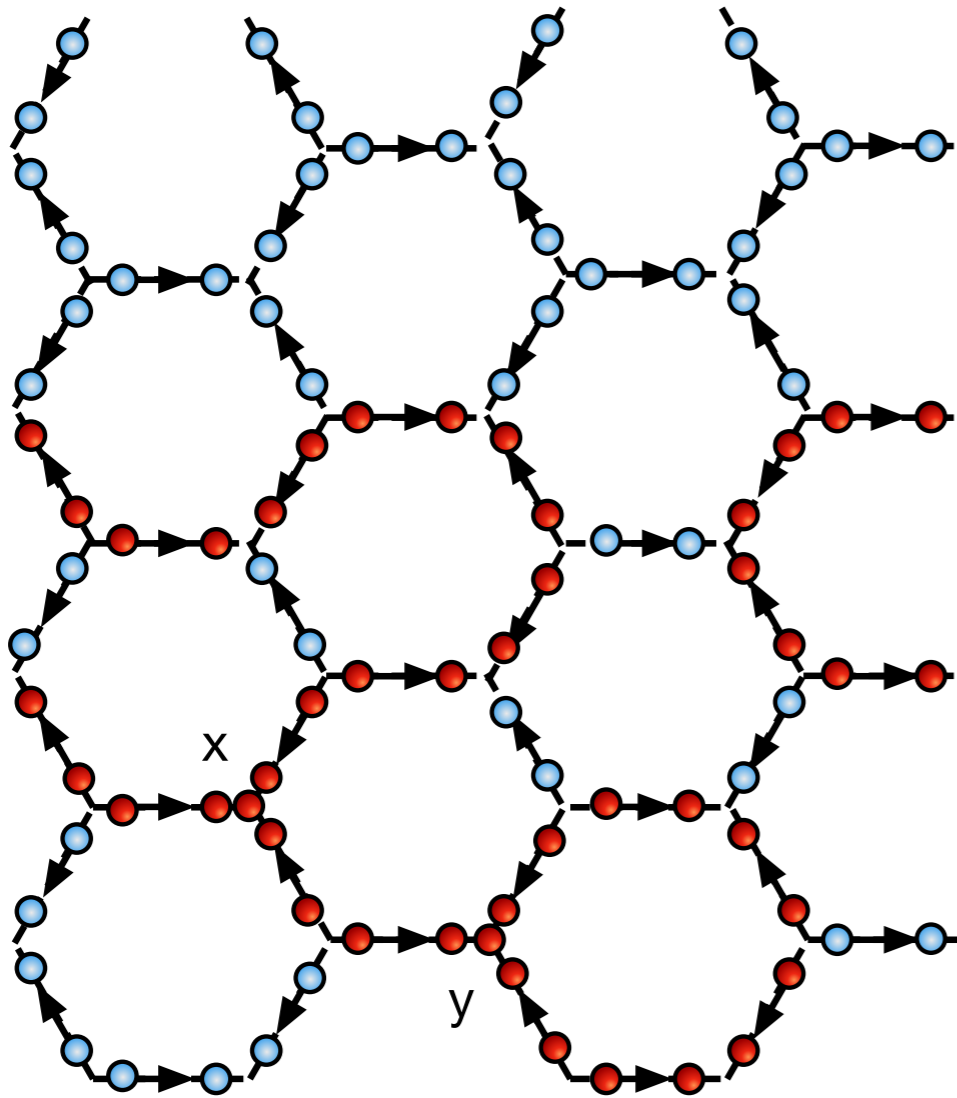
$$Q = \{1, \square, \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array}, \dots\}$$


All anti-symmetric irreps

Classical and Quantum Dimer Models

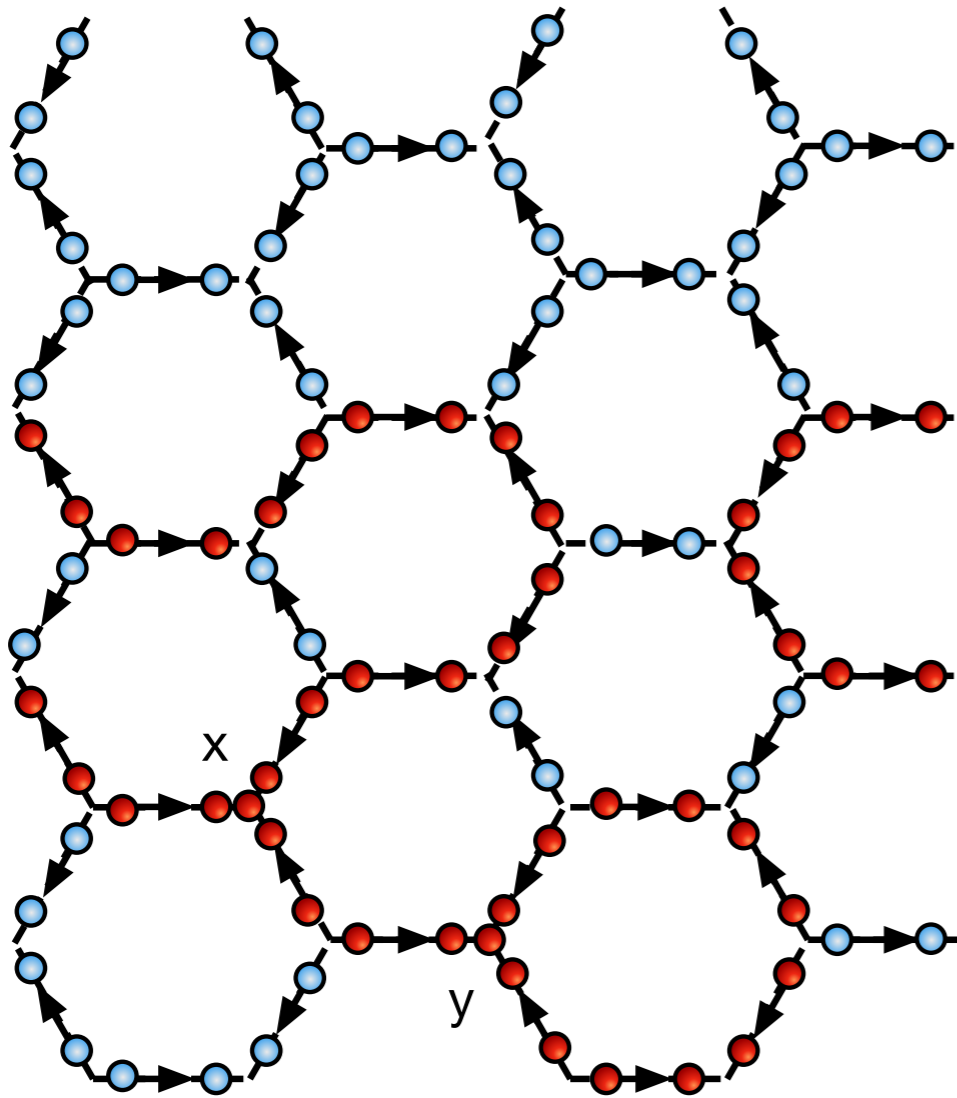
Every physical basis state of a lattice gauge theory in irrep formulation can be viewed as a configuration of monomers and dimers

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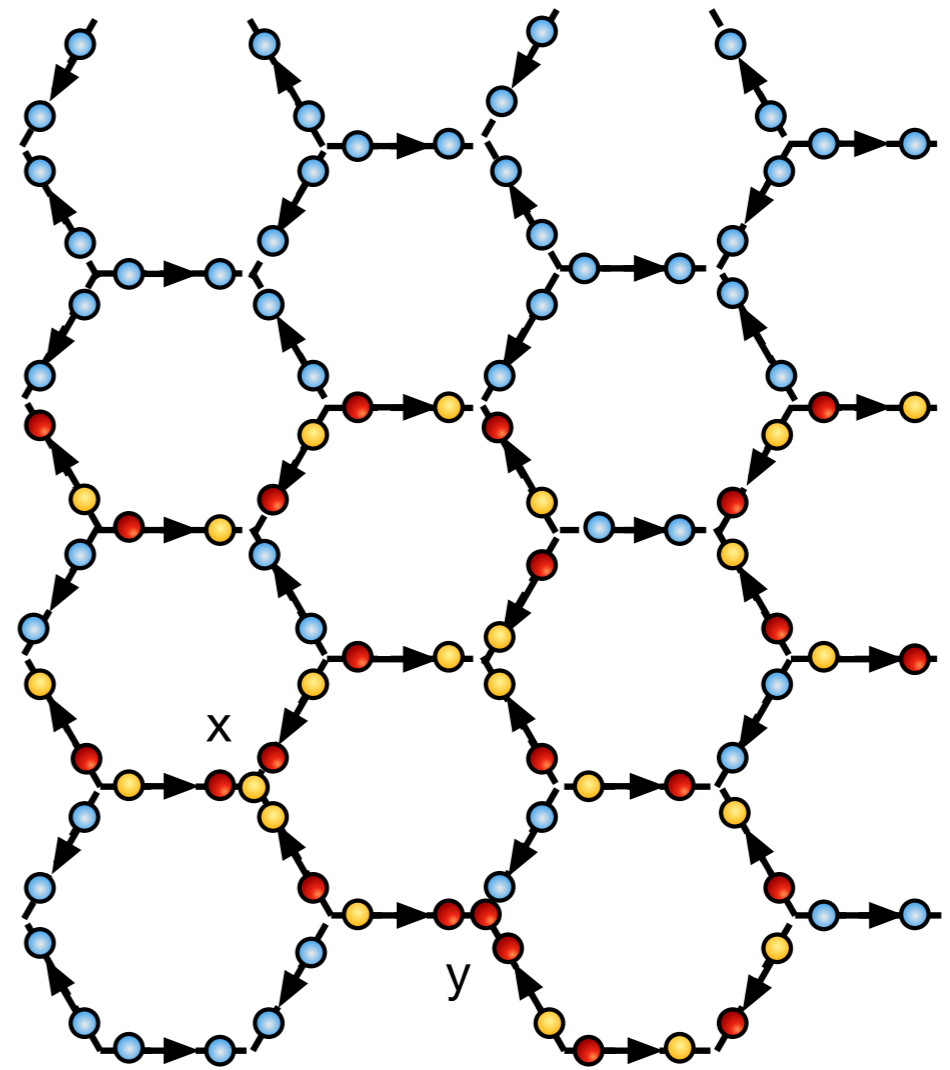


SU(2)

Every physical basis state of a lattice gauge theory in irrep formulation can be viewed as a configuration of monomers and dimers



SU(2)



SU(3)

Local Hamiltonians implement local changes to
the monomer-dimer configurations

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Classical Hamiltonian

$$H_Q = \sum_{\ell} \hat{\mathcal{E}}_{\ell} \qquad \hat{\mathcal{E}}_{\ell} |D_{ij}^{\lambda}\rangle = (1 - \delta_{\lambda,1}) |D_{ij}^{\lambda}\rangle$$

Local Hamiltonians implement local changes to the monomer-dimer configurations

Classical Hamiltonian

$$H_Q = \sum_{\ell} \hat{\mathcal{E}}_{\ell} \quad \hat{\mathcal{E}}_{\ell} |D_{ij}^{\lambda}\rangle = (1 - \delta_{\lambda,1}) |D_{ij}^{\lambda}\rangle$$

Quantum Hamiltonian

$$H_Q = \sum_{\ell} \hat{\mathcal{E}}_{\ell} - \delta \sum_P (\hat{U}_P + \hat{U}_P^{\dagger})$$

Non-traditional plaquette operators

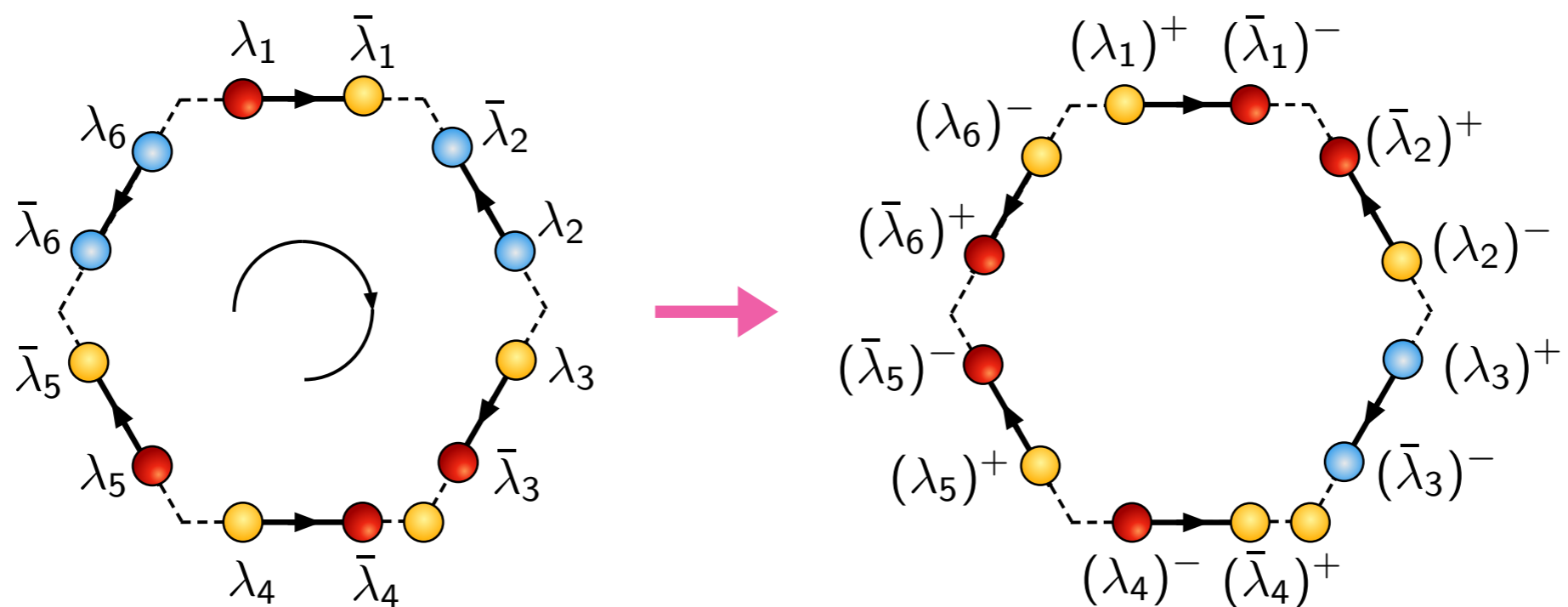
$$\hat{U}_P = \sum_{\lambda, \alpha, \lambda', \alpha'} c(\lambda, \alpha, \lambda', \alpha') |\lambda', \alpha'\rangle \langle \lambda, \alpha|$$

$$\hat{U}_P^\dagger = \sum_{\lambda, \alpha, \lambda', \alpha'} c^*(\lambda, \alpha, \lambda', \alpha') |\lambda, \alpha\rangle \langle \lambda', \alpha'|$$

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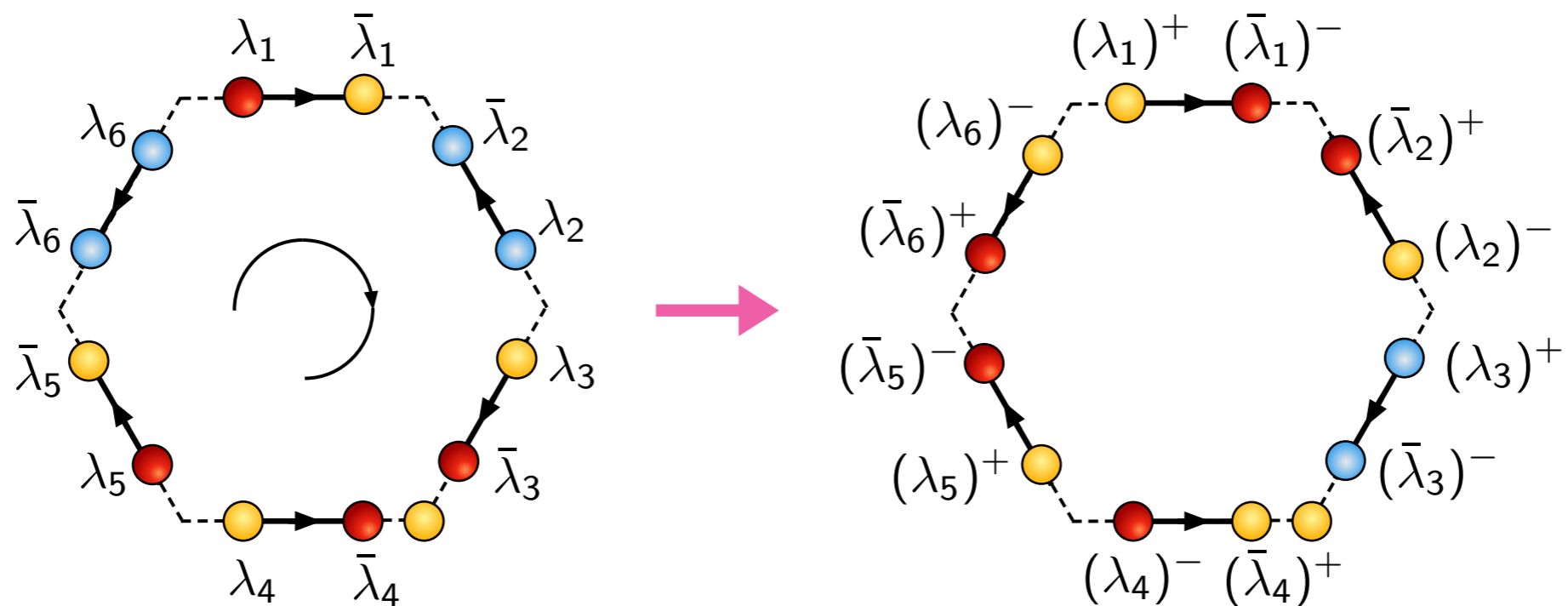
A simple example in the AS scheme:



$$\hat{U}_P = \sum_{\lambda, \alpha, \lambda', \alpha'} c(\lambda, \alpha, \lambda', \alpha') |\lambda', \alpha'\rangle \langle \lambda, \alpha|$$

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A simple example in the AS scheme:



Models are sign problem free
if the coefficients are positive and real!

Classical Lattice Gauge Theories: Results

Asymptotic Freedom of Yang Mills theory



Deconfined
phase at high
temperatures

Confined massive
phase at zero
temperatures

Asymptotic Freedom of Yang Mills theory



Deconfined
phase at high
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Confined massive
phase at zero
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Classical lattice gauge theories may already show this
finite temperature phase transitions

Asymptotic Freedom of Yang Mills theory



Deconfined
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Classical lattice gauge theories may already show this
finite temperature phase transitions

Think about the analogy with the Ising model!

Classical Hamiltonian

$$H_Q = \sum_l \hat{\mathcal{E}}_l$$

$$\hat{\mathcal{E}}_l |D_{ij}^\lambda\rangle = (1 - \delta_{\lambda,1}) |D_{ij}^\lambda\rangle$$

Classical Hamiltonian

$$H_Q = \sum_{\ell} \hat{\mathcal{E}}_{\ell} \quad \hat{\mathcal{E}}_{\ell} |D_{ij}^{\lambda}\rangle = (1 - \delta_{\lambda,1}) |D_{ij}^{\lambda}\rangle$$

A “confinement” observable at
finite temperatures

$$\chi = \frac{1}{V} \sum_{x,y} \frac{Z_{x,y}}{Z}$$

Confined phase: $\chi \sim \text{Const}$

Deconfined phase: $\chi \sim L^3$

Classical Hamiltonian

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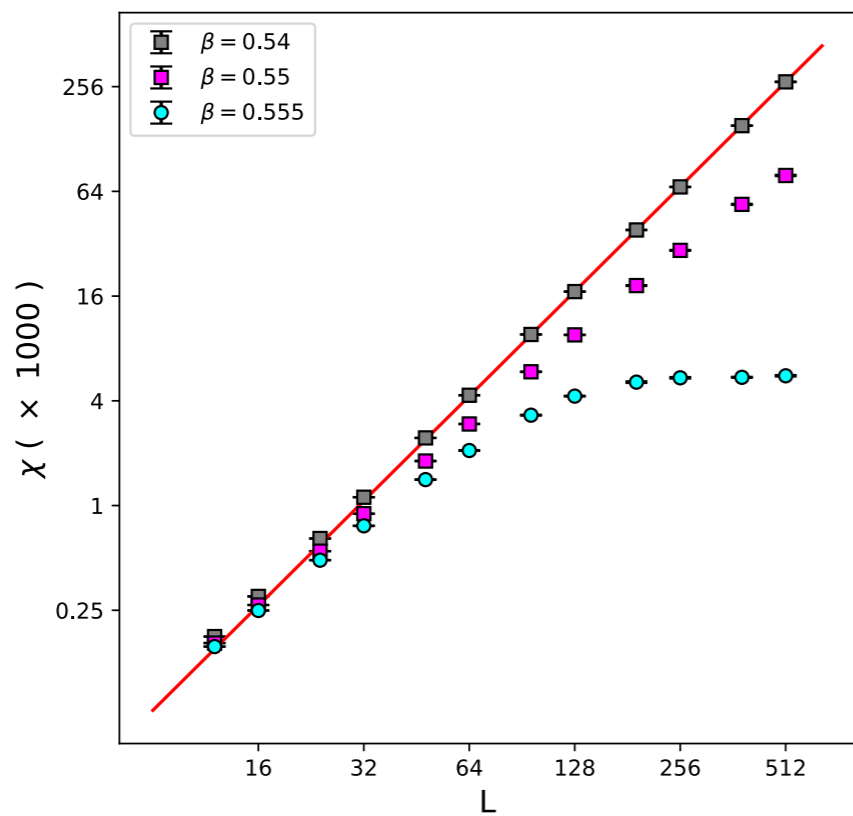
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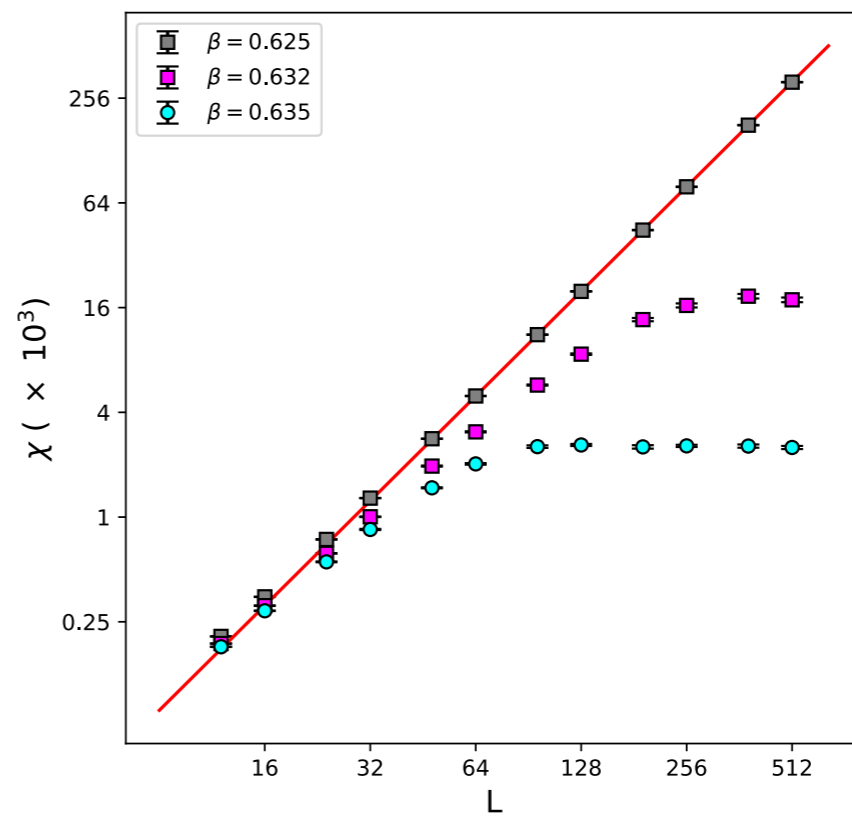
Expectation from traditional lattice gauge theory

SU(2): Ising transition SU(3): Z3 transition

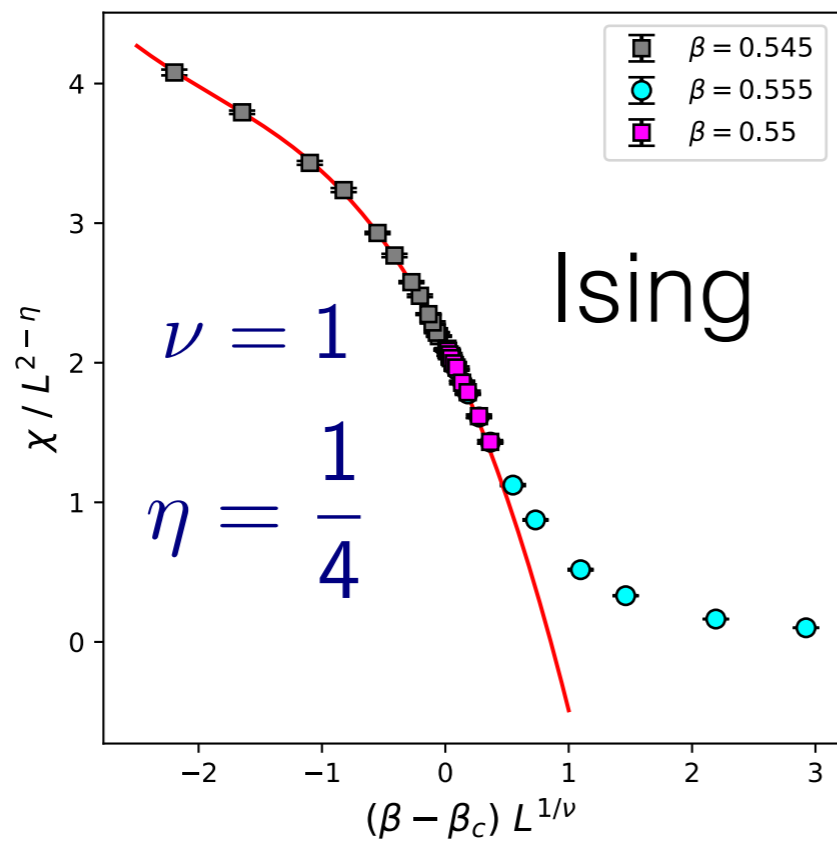
d=2



SU(2)



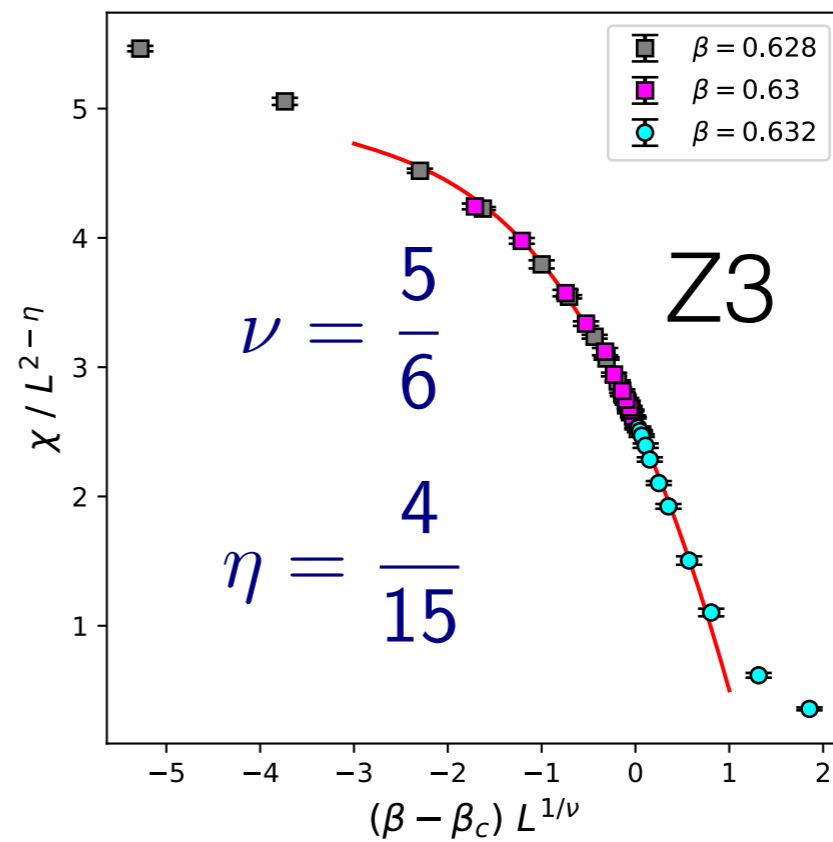
SU(3)



Ising

$$\nu = 1$$

$$\eta = \frac{1}{4}$$

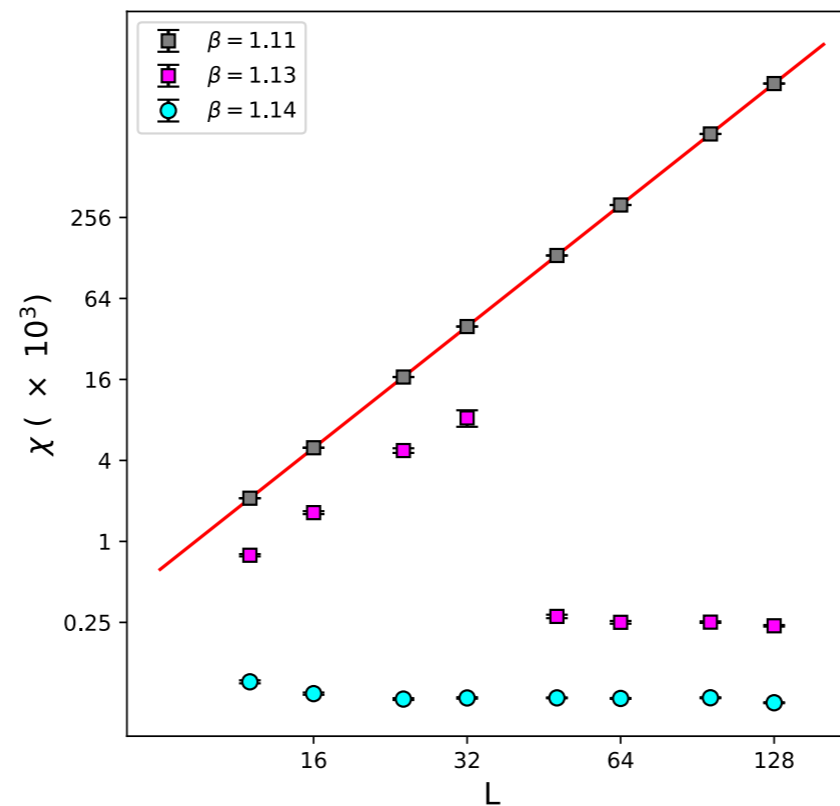
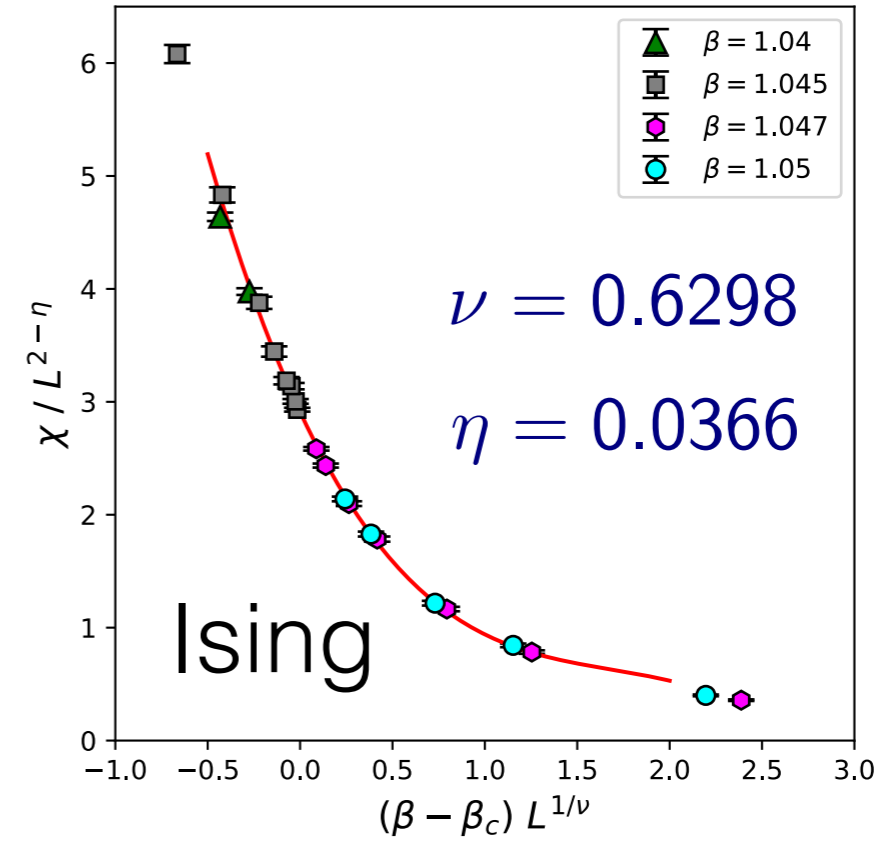
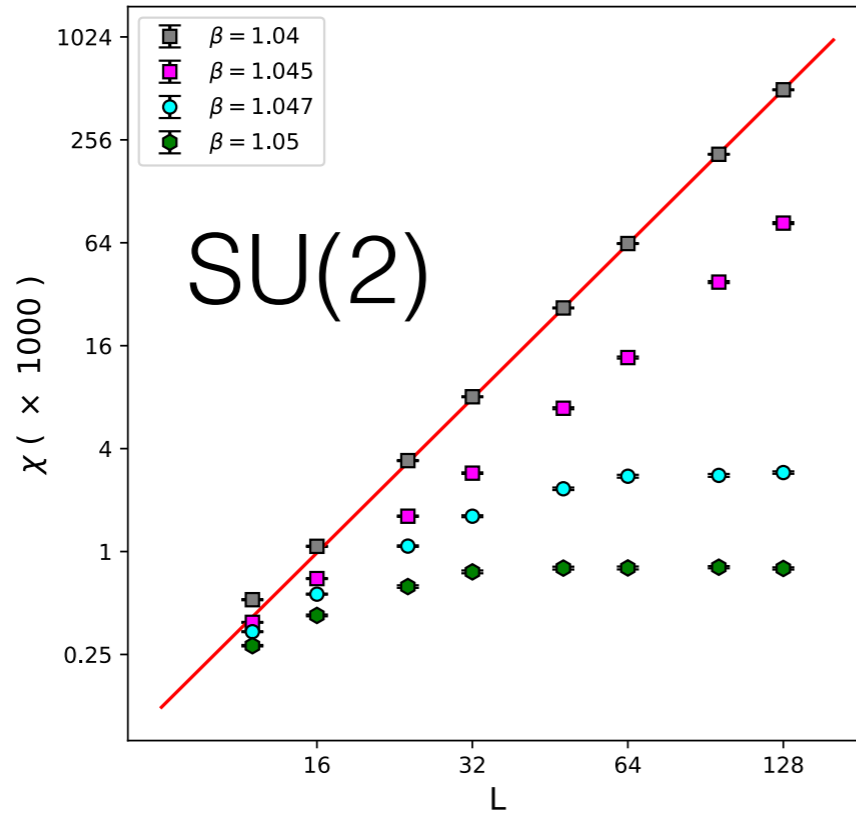


Z3

$$\nu = \frac{5}{6}$$

$$\eta = \frac{4}{15}$$

d=3



SU(3)
First order

Quantum Lattice Gauge Theories: Results

Quantum Hamiltonian

$$H_Q = \sum_{\ell} \hat{\mathcal{E}}_{\ell} - \delta \sum_P (\hat{U}_P + \hat{U}_P^{\dagger})$$

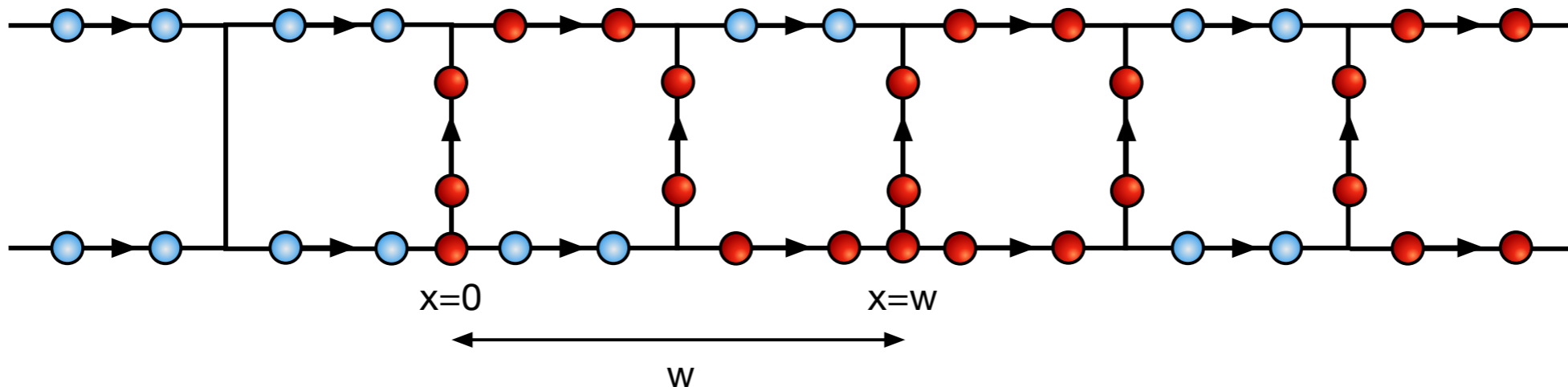
Non-traditional plaquette operators

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Non-traditional plaquette operators

SU(2) gauge theory

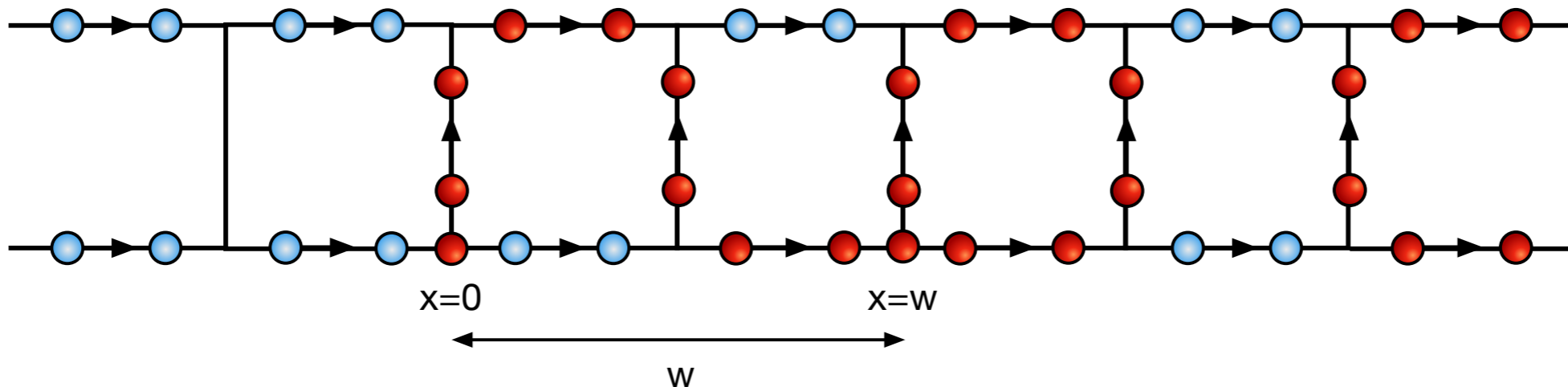


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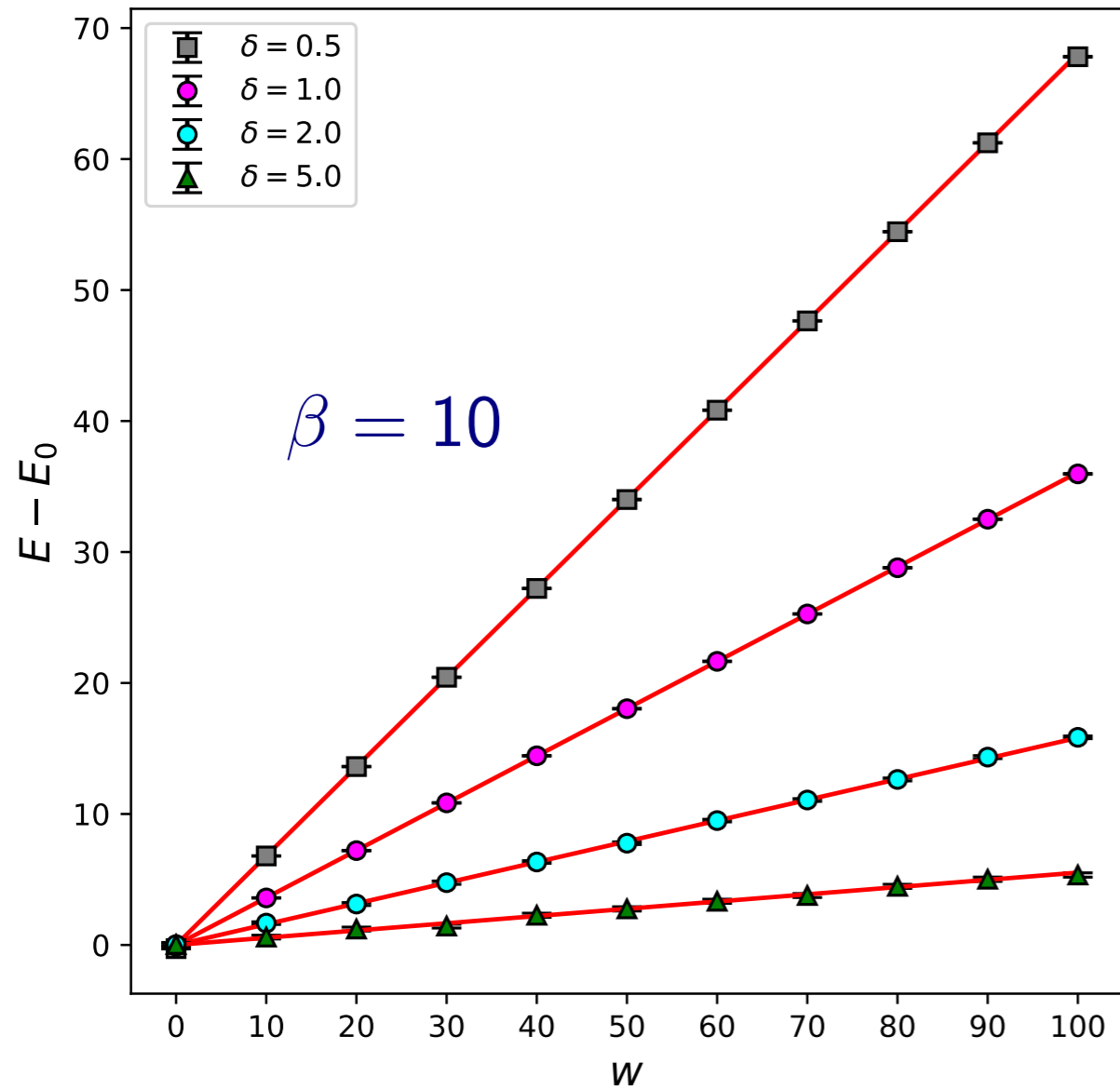
SU(2) gauge theory



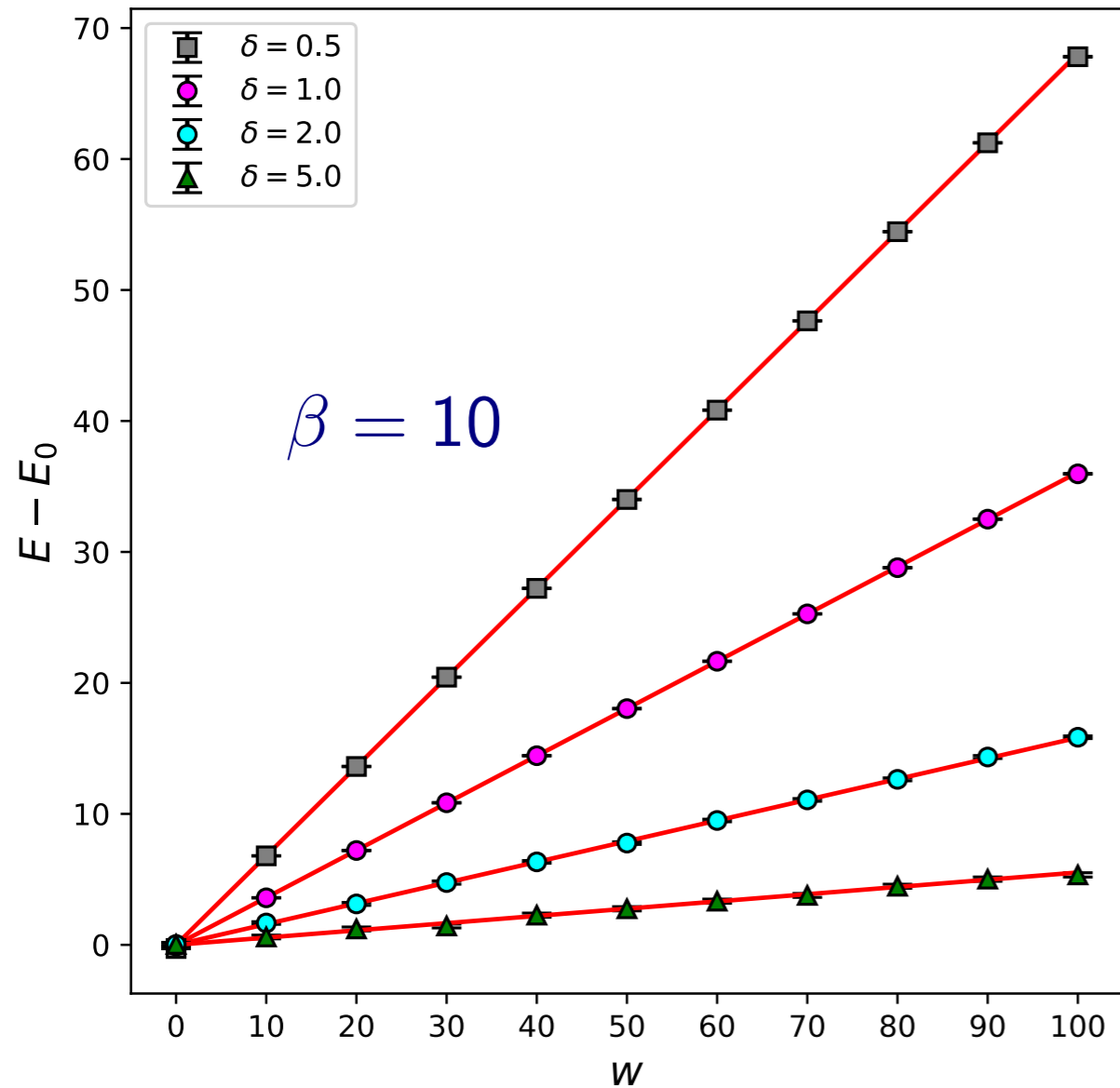
Confinement Observable: $E_{\beta}(w) = \text{Tr}(H_Q e^{-\beta H_Q}) / \text{Tr}(e^{-\beta H_Q})$

For low temperatures: $E_{\beta}(w) = E_0 + \sigma w$

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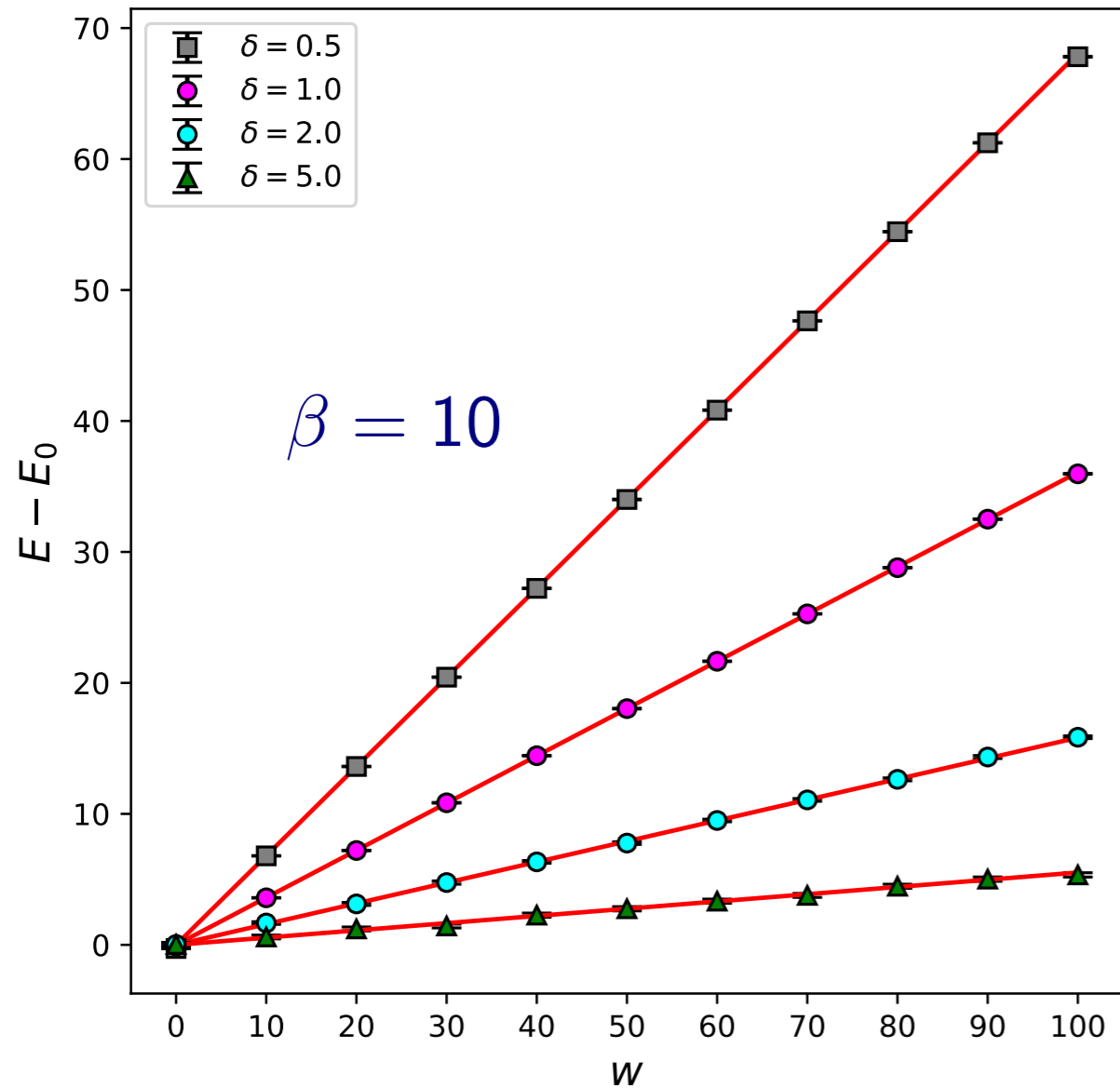
For low temperatures: $E_\beta(w) = E_0 + \sigma w$



$$E_0 \approx -2L\delta + \frac{3L}{2} - \frac{9L}{32}\delta^{-1} - \frac{L-1}{8}\delta^{-2},$$

$$\sigma \approx \frac{1}{4}\delta^{-1} + \frac{1}{8}\delta^{-2}.$$

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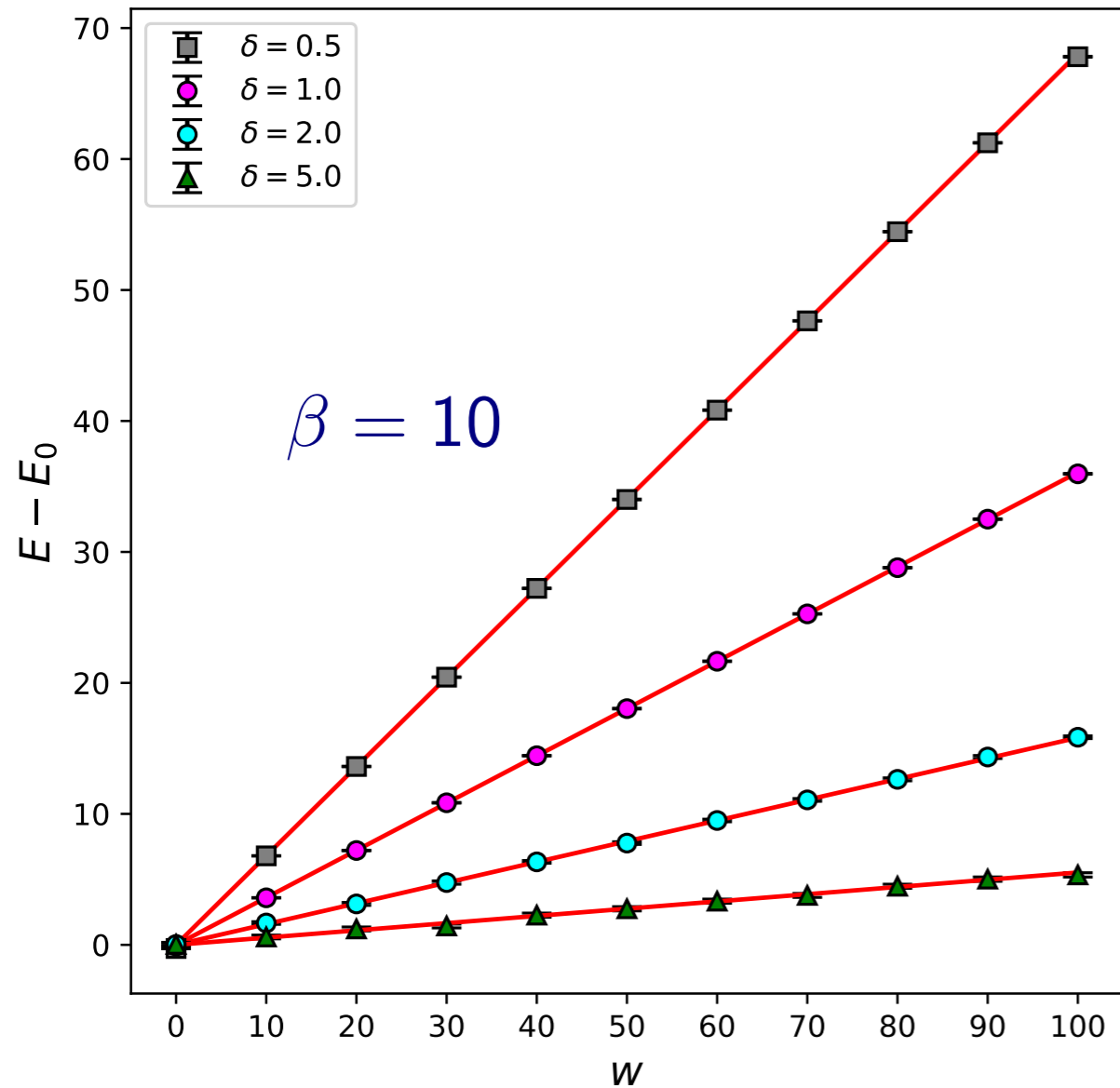


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δ	Monte Carlo			PT Eq. (43)	
	E_0	σ	χ^2/DOF	E_0	σ
0.5	-24.17(2)	0.6805(3)	0.34	-55.75	1.000
1.0	-89.04(4)	0.3608(6)	0.40	-90.50	-0.375
2.0	-267.36(8)	0.158(2)	0.40	-267.16	0.156
5.0	-856.1(2)	0.055(2)	0.51	-856.12	0.055

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Increasing δ reduces the string tension as expected

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But at least there are no sign problems!

Conclusions

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Can Yang-Mills theory arise at a quantum critical point of some quantum dimer model?