Following Uwe-Jens' Journey of Major Discoveries

Biographical talk, different from rest of the workshop. First talk of this kind for me, and first SIGN workshop.

I failed to take pictures, in particular during my time at MIT, but Uwe-Jens is here!

Only covers a very small part of Uwe-Jens' work, hope for complementary information in Shailesh' talk.

Regarding this workshop, Uwe-Jens was a founding member in 2003, and probably the first who clearly pointed out the meaning of the sign problem: requirement of statistics, with stable statistical errors, grows exponentially with the volume.

For instance, C-periodic boundary conditions will be missing; (don't call it "$\mathrm{C}^{\star}$-periodic"), cf. Alessandro's talk

I will even miss Uwe-Jens most cited papers: "Computational complexity and fundamental limitations to fermionic quantum Monte Carlo simulations", with Mattias Troyer (2005), the sign problem is NP-hard!

"Monopole Condensation and Color Confinement" and "Topology and Dynamics of the Confinement Mechanism" with Andreas Kronfeld, Morten Laursen and Gerrit Schierholz (1987), lattice formulation of maximally Abelian monopoles, plausibility argument for confinement mechanism

Each of these three papers has > 500 citations in INSPIRE

- **Bern 1992**

Final stage of my Ph.D. on χPT, supervised by Heiri Leutwyler. Construction works in our office (view to the railway), moved to other office shared with Uwe-Jens: Habilitation in Jülich/Aachen, one year in Bern, arranged by Peter Hasenfratz.

Just back from Lattice92 in Amsterdam, "Blockspin scheme and cluster algorithm for quantum spin systems" with He-Ping Ying (postdoc in Bern). Uwe-Jens introduced me to lattice field theory, in particular the fermion doubling problem, topology of 2d $\mathbb{C}$P$(N-1)$ models etc.

Field theory lecture by Peter Hasenfratz: emphasis on book by Shen-Ka Ma on Critical Phenomena, Renormalization Group Transformations (RGTs) and Fixed Point Actions $\rightarrow$ inspiration for "perfect action" program.

Uwe-Jens: 2d $\mathbb{C}$P(3) model (asymp. free, $\chi_{\rm t}$ on safe grounds), later handed over to Ruedi Burkhalter, and U(1) gauge theory to Marc Blatter.

Focus on *lattice fermions*, Uwe-Jens: 1992 paper in PLB, FP fermions starting from Wilson fermion, $D_{\rm W}$, no doubling comes in.

▶ chiral Renormalization Group Transformation (RGT), δ-type : *non-local*

▶ chirality-breaking RGT (Gaussian type) : *local*

Compatible with Nielsen-Ninomiya No-Go Theorem, but RGT-breaking is non-physical: lattice Dirac operator D obeys Ginsparg-Wilson Relation, $\{D, \gamma_5\} \propto aD\gamma_5 D$, fully recognized — or rediscovered — only later.

- **1993 Rio de Janeiro; Jülich, Lattice93 Dallas, Lattice94 Bielefeld**

Point of departure for RGTs quite irrelevant (does not need to be $D_{\rm W}$). Iteration with blocking factor $n = 2$, or 3, or higher, finally $n \to \infty$: "blocking from the continuum" (in blocked lattice units).
Lattice93 (Dallas)
Final stage of SSC: 22.5 of 87.1 km tunnel built, $2 \times 10^9\,\$$ spent

Work on FPA for *staggered fermions*; applied by Erich Focht to Gross-Neveu model ("2d NJL", asympt. free); analytic perfect action at large $N_{\rm f}$. Blocking "tastes" and auxiliary scalar field separately, with tedious shifts between overlapping zones. Lattice 94 (Bielefeld)

NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS

Nuclear Physics B (Proc. Suppl.) 34 (1994) 516–524
North-Holland

Fixed Point Actions for Lattice Fermions

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The fixed point actions for Wilson and staggered lattice fermions are determined by iterating renormalization group transformations. In both cases a line of fixed points is found. Some points have very local fixed point actions. They can be used to construct perfect lattice actions for asymptotically free fermionic theories like QCD or the Gross-Neveu model. The local fixed point actions for Wilson fermions break chiral symmetry, while in the staggered case the remnant $U(1)_e \otimes U(1)_o$ symmetry is preserved. In addition, for Wilson fermions a nonlocal fixed point is found that corresponds to free chiral fermions. The vicinity of this fixed point is studied in the Gross-Neveu model using perturbation theory.

1. Wilson Fermions

The continuum limit of a lattice field theory is defined at a fixed point of the renormalization group. The lattice models on a renormalized trajectory emanating from the fixed point are free of cut-off effects and hence have perfect lattice actions. Recently, Hasenfratz and Niedermayer realized that perfect actions can be constructed explicitly for asymptotically free theories [1]. In addition, in the 2-d nonlinear σ-model the renormalization group transformation can be optimized such that the fixed point action is extremely local. This is essential for numerical simulations. The question arises if fixed point actions for fermionic theories are local as well [2]. Since the fixed point of an asymptotically free theory is close to the Gaussian fixed point this question can be studied perturbatively, to lowest order even in the free theory. The corresponding calculation for a free scalar field was done long ago by Bell and Wilson [3].

Let us consider free Wilson fermion fields $\overline{\Psi}$ and Ψ with the action $S[\overline{\Psi}, \Psi]$ on a hypercubic lattice Λ, which is then blocked to a lattice Λ' of doubled lattice spacing. Then each point $x' \in \Lambda'$ corresponds to a hypercubic block of 2^d points $x \in \Lambda$ and each point x belongs to exactly one block x' (we denote this by $x \in x'$). The block transformation is illustrated in fig.1. On the blocked lattice we define new fermion fields $\overline{\Psi}'$ and Ψ' with an effective action $S'[\overline{\Psi}', \Psi']$, and we perform a renormalization group step that leaves the partition function unchanged

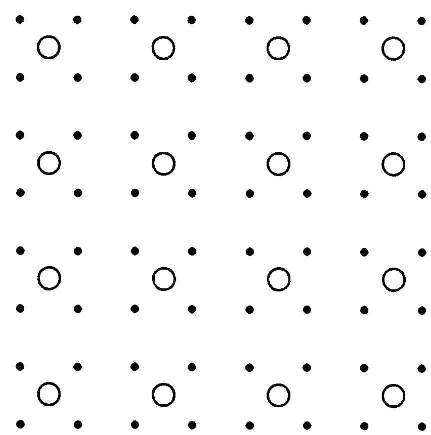

Figure 1. Blocking of a 2-d lattice.

$$\exp(-S'[\overline{\Psi}', \Psi']) = \int \mathcal{D}\overline{\Psi}\mathcal{D}\Psi \exp(-S[\overline{\Psi}, \Psi]) \times$$

*Based on two talks presented by the authors

SSDI 0920-5632(94)00326-Q

ELSEVIER

Nuclear Physics B 436 (1995) 385–413

NUCLEAR
PHYSICS B

Perfect lattice actions for the Gross–Neveu model at large $N^\star$

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Abstract

Fixed point actions for free and interacting staggered lattice fermions are constructed by iterating renormalization group transformations. At large N the fixed point action for the Gross–Neveu model is a perfect action in the sense of Hasenfratz and Niedermayer, i.e. cut-off effects are completely eliminated. In particular, the fermionic 1-particle energy spectrum of the lattice theory is identical with the one of the continuum even for arbitrarily small correlation lengths. The cut-off effects of the chiral condensate are eliminated using a perfect operator.

1. Introduction

Cut-off effects are the main source of systematic errors in numerical simulations of lattice field theories. Artifacts due to a finite lattice spacing a vanish in the continuum limit when the correlation length diverges in lattice units. In bosonic theories the lattice artifacts are usually of $O(a^2)$ and in fermionic theories they are of $O(a)$. Hence they go to zero rather slowly as the continuum limit is approached. In practice it is very difficult to work at large correlation lengths mostly because of critical slowing down. When Wilson introduced lattice field theory his idea was to use the renormalization group to map the critical continuum theory—defined at a fixed point of the renormalization group—to a noncritical theory with small correlation length which could then be

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- **1994–6, MIT, Uwe-Jens new junior faculty member**

Famous people around, including Goldstone, Jackiw, Johnson, Huang, Feshbach, Weisskopf; coffee breaks (Bjorken) and Chinese postdoc lunch

Interesting visitors: Kronfeld, Creutz, Weingarten, Sokal (before hoax in 1996), Christ (Colombia, 0.8 TeraFLOP machine), TD Lee (non-locality?)

Head of the theory group: John Negele (3 referee reports . . .); Barton Zwiebach (Ph.D. with Gell-Mann: mentioned in “The Quark and the Jaguar”), Xiangdong Ji (resurfaced at Lattice19, now U. of Maryland and Jiao Tong University, Shanghai). Paulo Bedaque (now Maryland, also active at SIGN workshops, at that time: “disordered chiral condensate”)

Blocking from the continuum for gauge fields (non-compact U(1)), perturbative construction of perfect lattice Schwinger model, axial anomaly

Extension to “Perfect lattice actions for quarks and gluons” (still non-compact gauge field), 213 citations

ELSEVIER

20 June 1996

Physics Letters B 378 (1996) 222–226

PHYSICS LETTERS B

A perturbative construction of lattice chiral fermions ⋆

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Editor: H. Georgi

Abstract

We perform a renormalization group transformation to construct perturbatively a lattice theory of chiral fermions. The field variables of the continuum theory are averaged over hypercubes to define lattice fields. Integrating out the continuum variables we derive a chirally invariant effective action for the lattice fields. This is consistent with the Nielsen-Ninomiya theorem because the effective action is nonlocal. We also construct the axial current on the lattice and we show that the axial anomaly of the continuum theory is reproduced in the Schwinger model.

It is a long-standing problem to construct a lattice regularization of chiral gauge theories such as the Standard model. Naive discretization of the continuum action leads to a multiplication of fermion species—known as the fermion doubling problem. To remove the doubler fermions Wilson has introduced a chiral symmetry breaking term [1]. In a chiral gauge theory this term is problematic because it breaks gauge invariance. Moreover, the Nielsen-Ninomiya theorem [2] excludes a chirally invariant solution of the doubling problem assuming hermiticity, locality and lattice translation invariance of the fermionic action. An early proposal to circumvent the doubling problem was made by the SLAC group using a nonlocal action [3]. That nonlocality manifests itself in a finite discontinuity of the inverse fermion propagator in momentum space. However, Karsten and Smit demonstrated that proposals of this kind fail to reproduce Lorentz invariance in the continuum limit [4]. In a refined proposal Rebbi introduced nonlocality by poles instead of finite discontinuities [5]. However, as soon as gauge interactions are switched on the doublers return as spurious ghost states [6,7]. In particular, they cancel the anomaly of the original fermion and render the theory vector-like. Currently discussed proposals to solve the doubling problem include domain wall fermions [8], overlap fermions [9], gauge fixing approaches [10], Pauli-Villars/SLAC fermions [11] as well as continuum fermions coupled to an interpolated lattice gauge field [12].

Here we construct a manifestly gauge invariant lattice theory of chiral fermions using renormalization group concepts. Iterating renormalization group transformations a recent study found a line of fixed points for free Wilson fermions [13]. The fixed point action at the line's endpoint is nonlocal and corresponds to a theory of free chiral fermions. In contrast to SLAC and Rebbi fermions here the nonlocality arises naturally by integrating out the high momentum modes of the fermion field. As for Rebbi fermions the nonlo-

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NUCLEAR PHYSICS B

Perfect lattice actions for quarks and gluons ⋆

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Abstract

We use perturbation theory to construct perfect lattice actions for quarks and gluons. The renormalized trajectory for free massive quarks is identified by blocking directly from the continuum. We tune a parameter in the renormalization group transformation such that for 1d configurations the perfect action reduces to the nearest-neighbor Wilson fermion action. The fixed point action for free gluons is also obtained by blocking from the continuum. For 2d configurations it reduces to the standard plaquette action. Classically perfect quark and gluon fields, quark–gluon composite operators and vector and axial vector currents are constructed as well. Also the quark–antiquark potential is derived from the classically perfect Polyakov loop. The quark–gluon and three-gluon perfect vertex functions are determined to leading order in the gauge coupling. This work provides a basis for the numerical construction of a lattice action for QCD, which is (approximately) perfect even beyond perturbation theory.

PACS: 11.15.Ha, 12.38.Gc

1. Introduction

Cut-off effects are the major source of systematic errors in numerical simulations of lattice QCD. For the standard Wilson action these effects are of the order of the lattice spacing. Hence they vanish rather slowly as the continuum limit is approached. Symanzik's perturbative improvement program systematically eliminates cut-off effects by introducing irrelevant higher-dimensional operators in the lattice action [1]. For QCD this program was realized by Sheikholeslami and Wohlert [2]. Their so-called "clover

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Shailesh Chandrasekharan and Richard Brower joined the project.

Non-Abelian lattice gauge fields: parametrization with large table of strange "furniture" (twisted sofa etc.). Program carried out to the end in Bern (Hasenfratz, Niedermayer, Blatter, Wenger)
Recently revitalised with machine learning by Holland, Ipp, Müller, Wenger

Perfect staggered fermions with gauge interaction

Perfect lattice topology for quantum rotor: sum up all possibles windings between the lattice sites.

Hypercube fermion (optimally local, perfect free fermion in 3^d lattice as truncation scheme, to be gauged . . .)

Fortune cookie at postdoc lunch: *"You have a yearning for perfection"*

ELSEVIER

Nuclear Physics B 495 (1997) 285-305

NUCLEAR PHYSICS B

Perfect lattice actions for staggered fermions ★

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Abstract

We construct a perfect lattice action for staggered fermions by blocking from the continuum. The locality, spectrum and pressure of such perfect staggered fermions are discussed. We also derive a consistent fixed point action for free gauge fields and discuss its locality as well as the resulting static quark–antiquark potential. This provides a basis for the construction of (classically) perfect lattice actions for QCD using staggered fermions. © 1997 Elsevier Science B.V.

PACS: 11.10.Hi; 11.15.Ha; 12.38Gc

Keywords: Improved lattice actions; Staggered fermions; QCD

1. Introduction

Recently there has been a surge of interest in lattice actions which are closer to the continuum limit than standard lattice actions, in the sense that artifacts due to the finite lattice spacing are suppressed. The hope is that such improved actions will allow Monte Carlo simulations to reach the scaling region even on rather coarse lattices.

Based on renormalization group concepts, it has been known for a long time that perfect actions, i.e. lattice actions without any cutoff artifacts, do exist [1]. However, it is very difficult to construct – or even approximate – such perfect actions. Recent progress is based on the observation that for asymptotically free theories the determination of the fixed point action (FPA) is a classical field theory problem [2]. At infinite correlation

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4 September 1997

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PHYSICS LETTERS B

Perfect lattice topology: the quantum rotor as a test case ★

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Editor: H. Georgi

Abstract

Lattice actions and topological charges that are classically and quantum mechanically perfect (i.e. free of lattice artifacts) are constructed analytically for the quantum rotor. It is demonstrated that the Manton action is classically perfect while the Villain action is quantum perfect. The geometric construction for the topological charge is only perfect at the classical level. The quantum perfect lattice topology associates a topological charge distribution, not just a single charge, with each lattice field configuration. For the quantum rotor with the classically perfect action and topological charge, the remaining cut-off effects are exponentially suppressed. © 1997 Published by Elsevier Science B.V.

PACS: 05.50.+q; 11.10.Ef; 11.10.Hi; 11.15.Ha
Keywords: Lattice; Topology; Perfect action; Renormalization group

1. Introduction

The most severe systematic errors in numerical simulations of lattice field theories are due to finite lattice spacing effects. For asymptotically free theories like QCD, Hasenfratz and Niedermayer [1] realized that in the classical limit one can numerically construct nonperturbative perfect lattice actions, which are completely free of finite lattice spacing artifacts. Using the classically perfect action of the 2-d $O(3)$ model even in the quantum theory, they found that cut-off effects are still practically eliminated. A similar behavior has been observed in the 4-d pure $SU(3)$ gauge theory [2], and it has been argued that in these cases the classically perfect action is also quantum perfect at the 1-loop level. This has been demonstrated explicitly for the 2-d $O(3)$ model [3]. In the Gross-Neveu model at large N, the classically perfect action is also perfect at the quantum level for arbitrarily short correlation lengths [4]. The same is true for the $O(N)$ model at large N. Still, one expects that in general a classically perfect action will not be quantum perfect. In this paper we investigate this question in a simple model – the quantum rotor (or 1-d XY model). In this case both, the classically and quantum perfect actions can be constructed analytically. In particular, when the classically perfect action is used at the quantum level, the size of the remaining cut-off effects can be analyzed in detail.

The topological charge of a field configuration on

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Nuclear Physics B (Proc. Suppl.) 53 (1997) 921–934

PROCEEDINGS SUPPLEMENTS

Progress on Perfect Lattice Actions for QCD

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We describe a number of aspects in our attempt to construct an approximately perfect lattice action for QCD. Free quarks are made optimally local on the whole renormalized trajectory and their couplings are then truncated by imposing 3-periodicity. The spectra of these short ranged fermions are excellent approximations to continuum spectra. The same is true for free gluons. We evaluate the corresponding perfect quark-gluon vertex function, identifying in particular the "perfect clover term". First simulations for heavy quarks show that the mass is strongly renormalized, but again the renormalized theory agrees very well with continuum physics. Furthermore we describe the multigrid formulation for the non-perturbative perfect action and we present the concept of an exactly (quantum) perfect topological charge on the lattice.

A large number of contributions to this conference are devoted to improved actions; there is no doubt that they are in fashion. This indicates a consensus that they represent a ray of hope for a great leap forward in lattice QCD. Most improvement procedures follow in one way or the other Symanzik's program [1], using perturbation theory in the lattice spacing, e.g. [2]. Our work employs a different concept, utilizing renormalization group tools to construct an approximately perfect action. A fixed point action (FPA) on a critical surface is an example of a perfect action, an action without any cutoff artifacts. Using the fixed point action even at finite correlation length, a drastically improved scaling behavior has been observed for the 2d O(3) model [3] and pure 4d SU(3) gauge theory [4], and is expected also for other asymptotically free theories such as full QCD.

1. Free fermions

Fixed point actions have been constructed also for free fermions [5,6]. They can be obtained from iterating block renormalization group transformations (RGTs) with a finite blocking factor, or more efficiently by sending the blocking factor to infinity and performing only one step. This amounts to a technique that we call "blocking from the continuum". It has been used extensively in a discussion of the Schwinger model [7] and for quarks and gluons [8]. One starts from a continuum theory, divides the coordinate space into lattice cells and defines lattice fields by integrating over these cells. For free fermions the lattice action is then given by

$$e^{-S[\bar\Psi,\Psi]} = \int D\bar\psi D\psi \exp\Big\{ - s[\bar\psi,\psi] \qquad (1)$$
$$-\frac{1}{a}\sum_x \Big[\bar\Psi_x - \int_{c_x}\bar\psi(y)dy\Big]\Big[\Psi_x - \int_{c_x}\psi(y)dy\Big]\Big\}.$$

Here $\bar\psi, \psi$ are continuum fields, s is the continuum action and c_x is a unit hypercube with center x. S is the perfect action in terms of the lattice fields $\bar\Psi, \Psi$. Finally $a \geq 0$ is an arbitrary RGT parameter; for any choice of a all expectation values are invariant under this RGT. For $a \to 0$ this is a δ function RGT, and $a > 0$ "smears" the δ function to a Gaussian.

For fermions of mass m, this yields in momentum space the perfect action

$$S[\bar\Psi,\Psi] \;=\; \frac{1}{(2\pi)^d}\int_B d^dp\bar\Psi(-p)\Delta^f(p)^{-1}\Psi(p)$$

*Based on two talks presented by W.B. and S.C. and two posters presented by R.B. and U.-J.W. Work supported by U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818.

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Work on sign problem with Andrei Pochinsky: 2d O(3) model with a θ-term. Haldane-Affleck conjecture about 2nd order phase transition at $\theta \to \pi$; matches half-integer spin chain.

Sign problem at $\theta \neq 0$ tackled by multi-cluster Wolff algorithm with a powerful improved estimator: sum up topological charges for configurations obtained by all cluster flips.

Assigns to each cluster a topological charge Q with $2Q \in \mathbb{Z}$ (constraint angle $\to$ indep. of other cluster orientations), clusters with $Q = 1/2$: dynamical definition of merons; **"Meron cluster algorithm".**
Later sophisticated applications to fermionic models by Shailesh, Uwe-Jens et al. $E.g.$ talks by Thea and João yesterday.

Uwe-Jens: enhanced statistics "solves the problem half-way in a technical sense", but sufficiently powerful for precise results, conjecture confirmed beyond semi-classical arguments.

Phys. Rev. Lett. (155 citations) and Lattice 95 (Melbourne)

Meron-Cluster Simulation of the θ Vacuum in the 2D O(3) Model

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(Received 25 May 1995)

The 2D O(3) model with a θ vacuum term is formulated in terms of Wolff clusters. Each cluster carries an integer or half-integer topological charge. The clusters with charge $\pm 1/2$ are identified as merons. At $\theta = \pi$ the merons are bound in pairs inducing a second order phase transition at which the mass gap vanishes. The construction of an improved estimator for the topological charge distribution makes numerical simulations of the phase transition feasible. The measured critical exponents agree with those of the $k = 1$ Wess-Zumino-Novikov-Witten (WZNW) model. Our results are consistent with Haldane's conjecture fro 1D antiferromagnetic quantum spin chains.

PACS numbers: 75.10.Jm

Some time ago Haldane conjectured [1] that integer and half-integer 1D antiferromagnetic quantum spin chains behave qualitatively differently. While integer spin chains have a mass gap, half-integer chains should be gapless. This has been confirmed numerically for finite chains of spin 1 and spin 2 [2,3] and analytically for half-integer spins and for spin 1 [4]. The long-range physics of 1D quantum spin chains is described by an effective 2D classical O(3) model. Haldane argued that the effective action for a chain of spins S contains a topological term $i\theta Q$. Here Q is the topological charge and $\theta = 2\pi S$ is the vacuum angle. Since the physics is periodic in θ, i.e., $\theta \in]-\pi, \pi]$, integer spins have $\theta = 0$ and half-integer spins have $\theta = \pi$. The standard O(3) model at $\theta = 0$ has a mass gap in agreement with Haldane's conjecture. On the other hand, Haldane's conjecture together with the (nonrigorous) mapping of spin chains on the O(3) model imply that the mass gap disappears at $\theta = \pi$. This corresponds to a phase transition in the vacuum angle. Because of the complex action it is notoriously difficult to simulate θ vacua numerically. A previous numerical study that was limited to $|\theta| < 0.8\pi$ found no phase transition in that region [5]. In fact, Haldane's conjecture has not yet been verified in the context of the O(3) model. In this paper we use the Wolff cluster algorithm [6] combined with a reweighting technique [7] to attack this problem. The construction of an improved estimator for the topological charge distribution enables us to simulate θ vacua reliably for any value of θ.

Affleck and Haldane have suggested a dynamical mechanism that explains why the mass gap disappears at $\theta = \pi$ [3]. In this picture pseudoparticles with topological charge $\pm 1/2$—so-called merons—are the relevant degrees of freedom. At $\theta = 0$ the merons form an ideal gas. They disorder the system and thereby give nonzero mass to the physical particles. At $\theta = \pi$, on the other hand, the merons are bound in pairs and thus do not generate mass. Affleck confirmed this picture in a model where the O(3) symmetry is explicitly broken to O(2). Then the merons behave like vortices, and the phase transition in θ is analogous to the Kosterlitz-Thouless transition of the O(2) model. When the explicit O(3) breaking is switched off, it is unclear if this dynamical picture still holds. In fact, there exists no definition of merons beyond the semiclassical approximation.

Here we formulate the O(3) model in terms of Wolff clusters. Using an appropriate action on a triangular lattice each cluster has a uniquely defined integer or half-integer topological charge. It turns out that most clusters are neutral, some have charges $\pm 1/2$, and very few carry larger charges. It is natural to identify merons as Wolff clusters with charge $\pm 1/2$. Indeed, at $\theta = 0$ the Wolff clusters are completely independent, as they should be in order to resemble merons. At $\theta = \pi$, on the other hand, no clusters with half-integer charges persist. Instead they form bound pairs of integer charge. Hence the cluster formulation of the model provides a definition of merons beyond the semiclassical approximation, and it confirms Affleck's dynamical mechanism in the O(3) model even without introducing symmetry breaking terms.

Actually Affleck's picture of the phase transition is more quantitative [8]. He argues that the model at $\theta = \pi$ is a conformal field theory in the same universality class as the $k = 1$ Wess-Zumino-Novikov-Witten (WZNW) model [9]. The critical exponents of this model are known analytically. The results of our numerical simulations are consistent with them. Therefore we confirm the implication of Haldane's conjecture as well as Affleck's picture in the context of the O(3) model.

For technical reasons we work on a 2D triangular lattice with $V = 3L^2$ points. The lattice covers the volume V of a regular hexagon with periodic boundary conditions. To each lattice site x we attach a classical three-component

 0031-9007/95/75(24)/4524(4)$06.00

- **Fz Jülich (1996-98), NORDITA Copenhagen (1998-2000)**
  Aspen, Erlangen; Lattice2000 (Bangalore)
  2001: Uwe-Jens moves to Bern

Perfect action at $\mu_B > 0$; hypercube preconditioning (Schilling, Lippert)

Perturbatively perfect quark-gluon vertex function, charmonium spectrum, Kostas Orginos

Aspen (Shailesh, Richard, Chung-I Tan). Erlangen: Frieder Lenz. Bangalore: Lattice00 (Heiri's review talk; Sign Problem in 10d IIB matrix model unsolved).

Ginsparg-Wilson Relation re-discovered by Peter Hasenfratz, paradigm shift. Hasenfratz/Laliena/Niedermayer: Index Theorem, Neuberger: overlap operator, Lüscher: lattice modified chiral symmetry.
"overlap-hypercube fermion" with Ivan Hip.

Uwe-Jens concentrated on *D-theory, quantum link variables* with Shailesh and Richard and *continuous-time algorithm* with Bernhard B. Beard (inspiration by Grassberger; talk by Boris Svistunov), etc.

ELSEVIER

30 April 1998

PHYSICS LETTERS B

Physics Letters B 426 (1998) 114 120

Perfect actions with chemical potential [1]

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Editor: P.V. Landshoff

Abstract

We show how to include a chemical potential μ in perfect lattice actions. It turns out that the standard procedure of multiplying the quark fields $\Psi,\overline{\Psi}$ at Euclidean time t by $\exp(\pm\mu t)$, respectively, is perfect. As an example, the case of free fermions with chemical potential is worked out explicitly. Even after truncation, cut-off effects in the pressure and the baryon density are small. Using a (quasi-)perfect action, numerical QCD simulations for non-zero chemical potential become more powerful, because coarse lattices are sufficient for extracting continuum physics. © 1998 Elsevier Science B.V. All rights reserved.

Understanding strongly interacting matter at *finite baryon density* is a long-standing and challenging problem, motivated for instance by relativistic heavy ion collision and by the physics of neutron stars. The standard procedure to formulate lattice QCD at a finite chemical potential μ includes a factor $\exp(\pm\mu)$ in the time-like link variables [1]. As a consequence, the Euclidean action is complex, the Boltzmann factor cannot be interpreted as a probability, and standard Monte Carlo techniques fail.

The usual method to handle a chemical potential is to simulate at $\mu=0$, and to include the baryon number term by some re-weighting technique in measured observables [2]. However, this method is tractable only on small physical volumes V, for a recent review see Ref. [3]. The essential numerical problem is to measure exponentially suppressed observables, like the partition function ratio $Z(\mu)/Z(0) \sim \exp(-\beta V[f(\mu)-f(0)])$. Here β is the inverse temperature and $f(\mu)$ is the free energy density. In numerical simulations the above ratio arises as an average over many positive and negative contributions. Hence its accurate determination requires tremendous statistics. An improved lattice action can not directly solve this sign problem, but it would help because it suppresses the artifacts due to the finite lattice spacing.

As a particularly troublesome effect caused by lattice artifacts, there is an upper limit for the possible fermion number density on the lattice. The value of this limit depends on the lattice action. It can be

[1] This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818.

PII S0370-2693(98)00269-

ELSEVIER

Nuclear Physics B (Proc. Suppl.) 63A-C (1998) 904–906

NUCLEAR PHYSICS B
PROCEEDINGS SUPPLEMENTS

The Perfect Quark-Gluon Vertex Function *

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We evaluate a perfect quark-gluon vertex function for QCD in coordinate space and truncate it to a short range. We present preliminary results for the charmonium spectrum using this quasi-perfect action.

Approximately perfect lattice actions have a potential to suppress lattice artifacts much more than the $O(a)$ improvement, which is very fashionable at this conference. However, it is still unclear how well this more sophisticated improvement program can be applied to QCD. Here we discuss our recent progress in the construction of a quasi-perfect action for QCD, and show preliminary results of its application to heavy quarks.

For free fermions we have derived extremely local perfect lattice actions [1] of the form,

$$S[\bar{\Psi},\Psi] = \sum_{x,y} \bar{\Psi}_x[\gamma_\mu \rho_\mu(y-x) + \lambda(y-x)]\Psi_y .$$

The couplings decay exponentially (in $d > 1$). We truncate them to a unit hypercube by means of 3-periodic boundary conditions. The resulting "hypercube fermion" has still excellent spectral and thermodynamic properties [2]. As an example, we give the couplings for $m = 1$ in Table 1 and show the dispersion relation in Fig. 1. It is very close to the continuum dispersion and approximates rotational symmetry very well.

Expansion to $O(gA_\mu)$ for full QCD yields

$$\begin{aligned} S[\bar{\Psi},\Psi,A_\mu] &= S[\bar{\Psi},\Psi] + S[A_\mu] + gV[\bar{\Psi},\Psi,A_\mu] \\ V[\bar{\Psi},\Psi,A_\mu] &= \frac{1}{(2\pi)^{2d}} \int_{B^2} dp dq \bar{\Psi}^i(-p) V_\mu(p,q) \\ &\quad A^a_\mu(p-q)\lambda^a_{ij}\Psi^j(q). \end{aligned}$$

*Based on a poster presented by K. Orginos at LAT97.

$y-x$	$\rho_1(y-x)$	$\lambda(y-x)$
(0000)	0	1.26885069540
(1000)	0.05457967484	-0.03008271460
(1100)	-0.01101007028	-0.01082956270
(1110)	-0.00325481234	-0.00471575763
(1111)	-0.00120632489	-0.00221240767

Table 1
The "hypercube fermion" couplings at $m = 1$.

The perfect action for free gluons, $S[A_\mu]$, has also been discussed in [1]. Moreover, we found an explicit but complicated expression for the *perfect quark-gluon vertex function* $V_\mu(p,q)$.

Numerically the vertex function can be evaluated and transformed to c-space, where it is represented by a set of link couplings, which depend on the fermion positions.

We truncate the $O(gA_\mu)$ perfect action and parameterize it in a gauge invariant form:

1) Take the mean value of the link couplings over short lattice paths connecting $\bar{\Psi}$ and Ψ.

2) Re-scale the path and plaquette couplings such that their sum amounts to $\lambda(r)$ for the scalar terms ($\propto \mathbb{1}$), $\rho_\mu(r)$ for the vector terms ($\propto \gamma_\mu$), $s(m) = (m/\hat{m})^2[1/m - 1/\hat{m}]$, $\hat{m} \doteq e^m - 1$ for the plaquette couplings ($\propto \sigma_{\mu\nu}$) [2] and a value $s_1(m)$ – obtained from a low $\vec{p}$ expansion – for the terms $\propto \gamma_\mu\gamma_\nu\gamma_\rho$.

For our first experiments we impose a tough truncation and arrive at the following parameterization of $V_\mu(x,y)$:

0920-5632/98/$19.00
PII S0920-5632(97)00936-5

- **Berlin (2000-2008)**

Lattice 2001 Berlin: Uwe-Jens in IAC (but rather silent)

2002 EU IHP Network Workshop on Fermion Actions and Chiral Symmetry, Bern, “mini-lattice conference”, with David Kaplan (non-local restriction; two comments . . .)

Brane world fermions: with Adrian Gfeller, construction of naturally light fermions in a 2+1 d brane world, in principle successful, but no dynamics (Pauli blocked)

Dublin: Lattice 2005 (non-commutative space, fuzzy sphere)

Cyprus (Nicosia) 2005: yet another talk about maximally Abelian monopoles . . .

Lattice 2008 (Williamsburg); plenary talk by Uwe-Jens, condensed matter oriented (Gerrit Schierholz et al., lattice OPE)

Published by Institute of Physics Publishing for SISSA/ISAS
Received: *September 24, 2003*
Accepted: *October 9, 2003*

Dimensional reduction of fermions in brane worlds of the Gross-Neveu model

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Abstract: We study the dimensional reduction of fermions, both in the symmetric and in the broken phase of the 3-d Gross-Neveu model at large N. In particular, in the broken phase we construct an exact solution for a stable brane world consisting of a domain wall and an anti-wall. A left-handed 2-d fermion localized on the domain wall and a right-handed fermion localized on the anti-wall communicate with each other through the 3-d bulk. In this way they are bound together to form a Dirac fermion of mass m. As a consequence of asymptotic freedom of the 2-d Gross-Neveu model, the 2-d correlation length $\xi = 1/m$ increases exponentially with the brane separation. Hence, from the low-energy point of view of a 2-d observer, the separation of the branes appears very small and the world becomes indistinguishable from a 2-d space-time. Our toy model provides a mechanism for brane stabilization: branes made of fermions may be stable due to their baryon asymmetry. Ironically, our brane world is stable only if it has an extreme baryon asymmetry with all states in this "world" being completely filled.

Keywords: Field Theories in Lower Dimensions, Spontaneous Symmetry Breaking, D-branes.

- **Mexico / Bern (since 2009)**

Topological lattice action (Urs Gerber, Michele Pepe, Michael Bögli), "impaired action" (hard discrete derivative), but excellent scaling behavior

Project with Peter Zoller and Innsbruck group, Wynne Evans, Urs Gerber and Héctor Mejía: $\mathbb{C}\mathrm{P}(N-1)$ quantum computing with cold alkaline-earth atoms on optical lattice. Feasible, but not implemented so far.

With Urs Gerber (now at Swiss school in Mx) and Fernando Rejón: BKT mechanism at constant energy, entropy effect, only convincing continuum extrapolation of helicity modulus Υ.
Work along these lines by Philippe de Forcrand et al.

Visits in Bern: Debasish Banerjee, Guillermo Palma, Fu-Jiun Jiang etc.

Collaboration with Christoph Hofmann, João Pinto Barros, Stephan Caspar, Manes Hornung

Published for SISSA by Springer

Received: *September 20, 2010*
Accepted: *November 9, 2010*
Published: *December 3, 2010*

Topological lattice actions

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Abstract: We consider lattice field theories with topological actions, which are invariant against small deformations of the fields. Some of these actions have infinite barriers separating different topological sectors. Topological actions do not have the correct classical continuum limit and they cannot be treated using perturbation theory, but they still yield the correct quantum continuum limit. To show this, we present analytic studies of the 1-d $O(2)$ and $O(3)$ model, as well as Monte Carlo simulations of the 2-d $O(3)$ model using topological lattice actions. Some topological actions obey and others violate a lattice Schwarz inequality between the action and the topological charge Q. Irrespective of this, in the 2-d $O(3)$ model the topological susceptibility $\chi_t = \langle Q^2 \rangle / V$ is logarithmically divergent in the continuum limit. Still, at non-zero distance the correlator of the topological charge density has a finite continuum limit which is consistent with analytic predictions. Our study shows explicitly that some classically important features of an action are irrelevant for reaching the correct quantum continuum limit.

Keywords: Field Theories in Lower Dimensions, Nonperturbative Effects, Lattice Quantum Field Theory, Sigma Models

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JHEP12(2010)020

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Topological lattice actions for the 2d XY model

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Abstract: We consider the 2d XY Model with topological lattice actions, which are invariant against small deformations of the field configuration. These actions constrain the angle between neighbouring spins by an upper bound, or they explicitly suppress vortices (and anti-vortices). Although topological actions do not have a classical limit, they still lead to the universal behaviour of the Berezinskii-Kosterlitz-Thouless (BKT) phase transition — at least up to moderate vortex suppression. In the massive phase, the analytically known Step Scaling Function (SSF) is reproduced in numerical simulations. However, deviations from the expected universal behaviour of the lattice artifacts are observed. In the massless phase, the BKT value of the critical exponent η_c is confirmed. Hence, even though for some topological actions vortices cost zero energy, they still drive the standard BKT transition. In addition we identify a vortex-free transition point, which deviates from the BKT behaviour.

JHEP03(2013)141

Annals of Physics 370 (2016) 117–127

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journal homepage: www.elsevier.com/locate/aop

$\mathbb{C}$P($N-1$) quantum field theories with alkaline-earth atoms in optical lattices

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SU(N) spin models
Continuum limit
CP($N-1$) theory
Quantum simulation

ABSTRACT

We propose a cold atom implementation to attain the continuum limit of $(1+1)$-d $\mathbb{C}$P($N-1$) quantum field theories. These theories share important features with $(3+1)$-d QCD, such as asymptotic freedom and θ-vacua. Moreover, their continuum limit can be accessed via the mechanism of dimensional reduction. In our scheme, the $\mathbb{C}$P($N-1$) degrees of freedom emerge at low energies from a ladder system of SU(N) quantum spins, where the N spin states are embodied by the nuclear Zeeman states of alkaline-earth atoms, trapped in an optical lattice. Based on Monte Carlo results, we establish that the continuum limit can be demonstrated by an atomic quantum simulation by employing the feature of asymptotic freedom. We discuss a protocol for the adiabatic preparation of the ground state of the system, the real-time evolution of a false θ-vacuum state after a quench, and we propose experiments to unravel the phase diagram at non-zero density.

PoS PROCEEDINGS OF SCIENCE

Meron- and Semi-Vortex-Clusters as Physical Carriers of Topological Charge and Vorticity*

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In O(N) non-linear σ-models on the lattice, the Wolff cluster algorithm is based on rewriting the functional integral in terms of mutually independent clusters. Through improved estimators, the clusters are directly related to physical observables. In the $(N-1)$-d O(N) model (with an appropriately constrained action) the clusters carry an integer or half-integer topological charge. Clusters with topological charge $\pm 1/2$ are denoted as merons. Similarly, in the 2-d O(2) model the clusters carry pairs of semi-vortices and semi-anti-vortices (with vorticity $\pm 1/2$) at their boundary. Using improved estimators, meron- and semi-vortex-clusters provide analytic insight into the topological features of the dynamics. We show that the histograms of the cluster-size distributions scale in the continuum limit, with a fractal dimension D, which suggests that the clusters are physical objects. We demonstrate this property analytically for merons and non-merons in the 1-d O(2) model (where $D = 1$), and numerically for the 2-d O(2), 2-d O(3), and 3-d O(4) model, for which we observe fractal dimensions $D < d$. In the vicinity of a critical point, a scaling law relates D to a combination of critical exponents. In the 2-d O(3) model, meron- and multi-meron-clusters are responsible for a logarithmic ultraviolet divergence of the topological susceptibility.

37th International Symposium on Lattice Field Theory - Lattice2019
16-22 June 2019
Wuhan, China

*This work was supported by UNAM-DGAPA-PAPIIT, grant number IG100219, by the Albert Einstein Center for

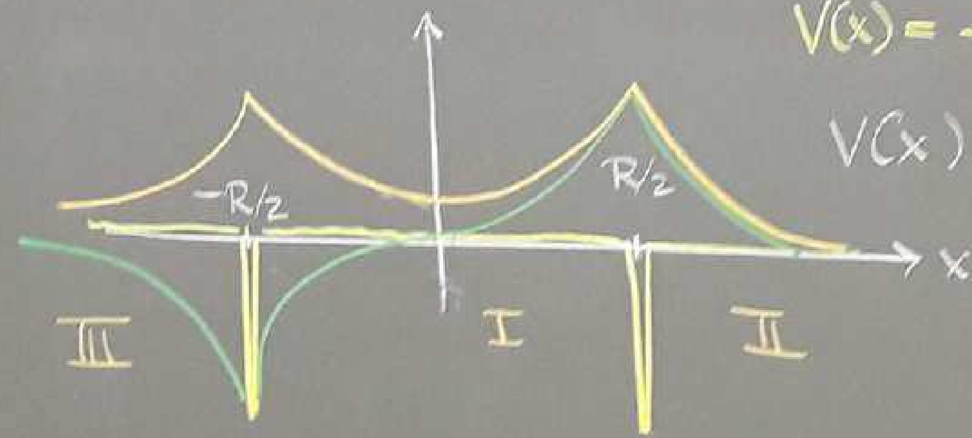
• **Textbook** **“Uncovering Quantum Field Theory and the Standard Model: from Fundamental Concepts to Dynamical Mechanisms”** (sorry for the commercial and for repetitions)

Origin: Uwe-Jens’ habilitation lecture notes from 1992 in Aachen (Early Universe), later updated and extended. Begin for me in Berlin, during Uwe-Jens’ visit in September 2003, walk in the evening near Brandenburger Tor, where he generously offered to include me in this book project.

Progress somewhat discontinuous over more than 10 years.

Sabbatical in Bern, winter 2016/7: unfortunate start: stolen laptop, SIGN Workshop (March 2017, U. of Washington); QM lecture, black board.

Book proposal submitted with 5 chapters plus one appendix, Table of Contents for 462 pages, to Cambridge University Press. Accepted by 6 referees. Contract: maximum of 600 pages, deadline July 1, 2018.



$$V(x) = -aV_0\left(\delta\left(x-\frac{R}{2}\right) + \delta\left(x+\frac{R}{2}\right)\right)$$

$$V(x) = V(-x) \Rightarrow$$

$$\psi(x) = \pm\,\psi(-x)$$

(un)gerade Lösungen der zeitunabhängigen SG: $\psi(x) = \pm\,\psi(-x)$

$$-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}\psi(x) = E\,\psi(x)\,, \quad x \neq \pm\frac{R}{2}$$

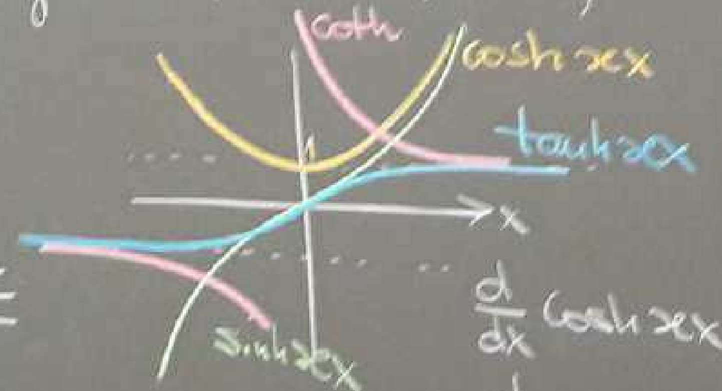
(b)

I: $\psi_{I}(x) = A\,\underset{\sinh}{\cosh}\,\varkappa x\,, \quad -\frac{\hbar^2}{2M}\varkappa^2 = E$

II: $\psi_{II}(x) = B\,e^{-\varkappa x}$

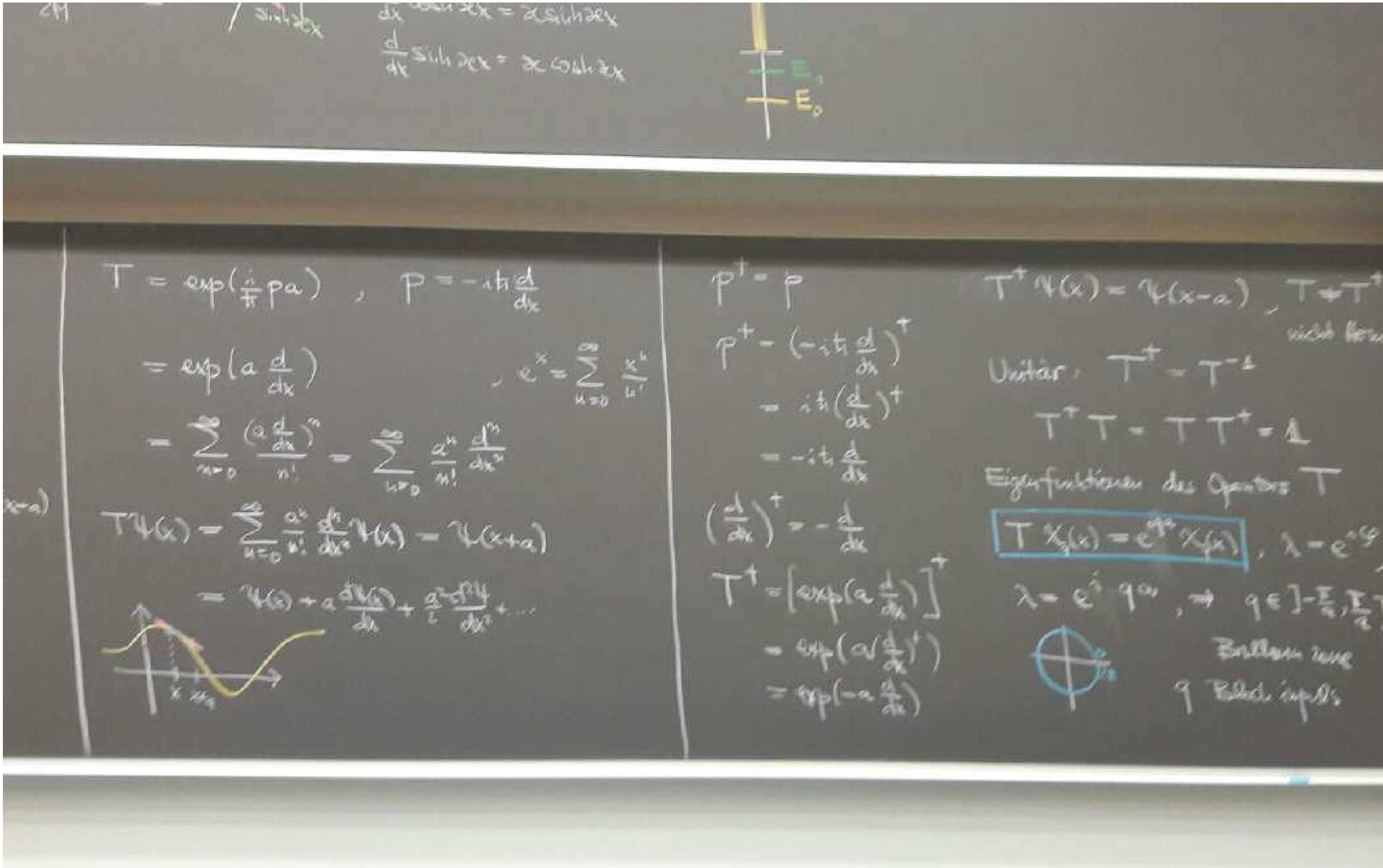
(III: $\psi(x) = -B\,e^{\varkappa x}$)

$\frac{V_0 M}{\hbar^2}$



$\frac{d}{dx}\cosh\varkappa x$

$\frac{d}{dx}\sinh\varkappa x$


d/dx sinh ϰx = ϰ cosh ϰx
E1
E0
T = exp(i/ħ p a) , p = -iħ d/dx
= exp(a d/dx) , e^x = Σ_{n=0}^∞ x^n/n!
= Σ_{n=0}^∞ (a d/dx)^n/n! = Σ_{n=0}^∞ a^n/n! d^n/dx^n
Tψ(x) = Σ_{n=0}^∞ a^n/n! d^n/dx^n ψ(x) = ψ(x+a)
= ψ(x) + a dψ(x)/dx + a²/2 d²ψ/dx² + ...
p† = p
p† = (-iħ d/dx)†
= iħ (d/dx)†
= -iħ d/dx
(d/dx)† = -d/dx
T† = [exp(a d/dx)]†
= exp(a (d/dx)†)
= exp(-a d/dx)
T† ψ(x) = ψ(x-a) , T ≠ T†
Unitär: T† = T^-1
T† T = T T† = 1
Eigenfunktionen des Operators T
T χq(x) = e^{iqa} χq(x) , λ = e^{iφ}
λ = e^{iqa} , ⇒ q ∈]-π/a, π/a]
Brillouin zone
q Blochimpuls

SECTION I SPECIFIC TERMS AND CONDITIONS

1 THE WORK

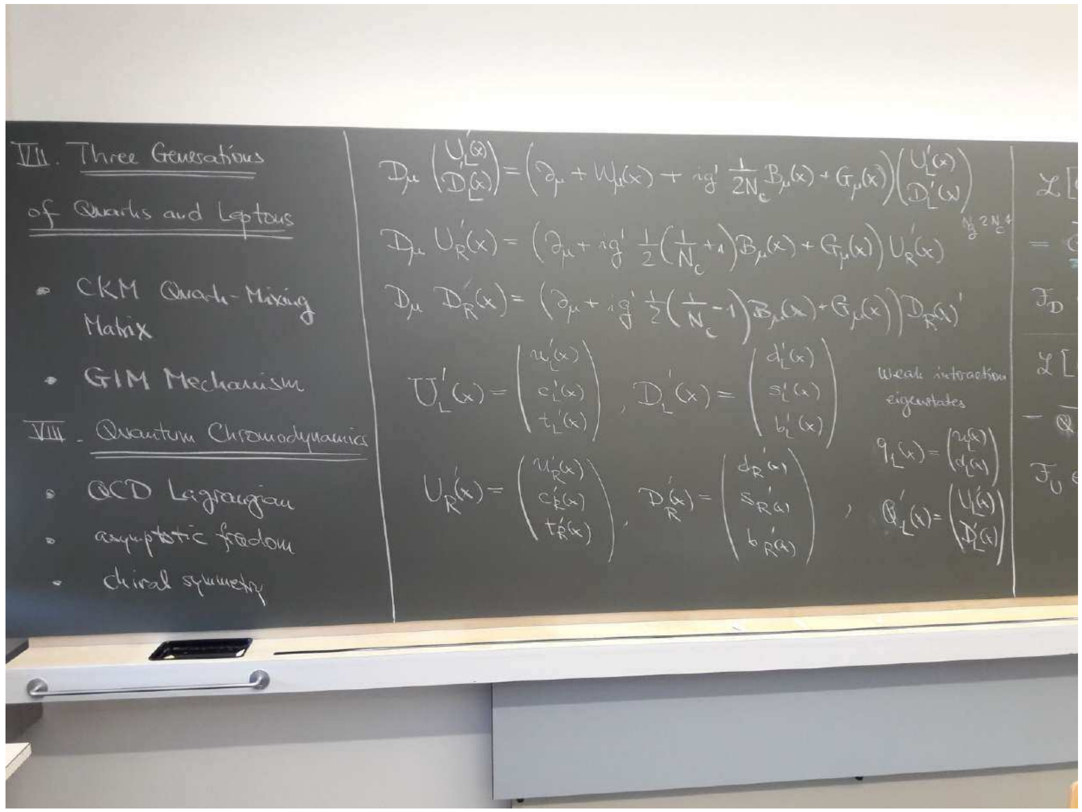
1.1 The Author shall create and deliver the Work in accordance with the terms and conditions of this Agreement. The Work shall, comprise of the following Components to be delivered by the Author to Cambridge in accordance with the Technical Specifications listed below and in accordance with the provisions of Clause 4, Section II:

Components:	Responsibility of:	Description: (eg length / quantity / quantity)	Technical Specifications: (eg file types / delivery requirements)	Delivery Date/s
Final Typescript:	Author	A maximum of 600pp	To be delivered as electronic files with an identical hardcopy printed therefrom	01 July 2018
Illustrative Materials:	Author	A maximum of 500 black and white line illustrations A maximum of 50 black and white line photographs	To be delivered as electronic files with a minimum resolution of 300 dpi	With Final Typescript
Permissions Clearance:	Cleared by: Author At the expense of: Author	Written evidence of Permissions Clearance for all relevant Third-party Materials	To be delivered as: executed permissions licence(s)	With Final Typescript
Index:	Supplied by: Author At the expense of: Author	List of entries	Further instructions to be provided by Cambridge	With Final Typescript
Ancillary Materials:	Author	To include: N/A	To be delivered as: N/A	N/A

1.2 The Author shall deliver the Work, and each relevant Component listed in Clause 1.1 above, on or before the date(s) specified above.

Another half-year sabbatical in Bern, summer 2018

Progress, book keeps growing, but again side-tracked by other subjects, termination still behind the horizon



Blackboard in Uwe-Jens' office, ExWi 122: fermion generations, now Chapter 17. YM gauge fixing included, but no SUSY.

Pandemics

Regular video calls, always a pleasure to say hello to Marija.

Uwe-Jens: intensive work, $e.g.$ on canonical formulation of fermion fields.

Instructive comments on specific chapters by Oliver Bär, Debasish Banerjee, Detlev Buchholz, Wilfried Buchmüller, Klaus Fredenhagen, Urs Gerber, Carlo Giunti, Kieran Holland, Gurtej Kanwar, Martin Lüscher, Alessandro Mariani, Colin Morningstar, Mike Peardon, Michele Pepe, João Pinto Barros, Lilian Prado, Simona Procacci, Christopher Smith, Rainer Sommer, Youssef Tammam, Christiane Tretter, Christof Wetterich, Edward Witten.

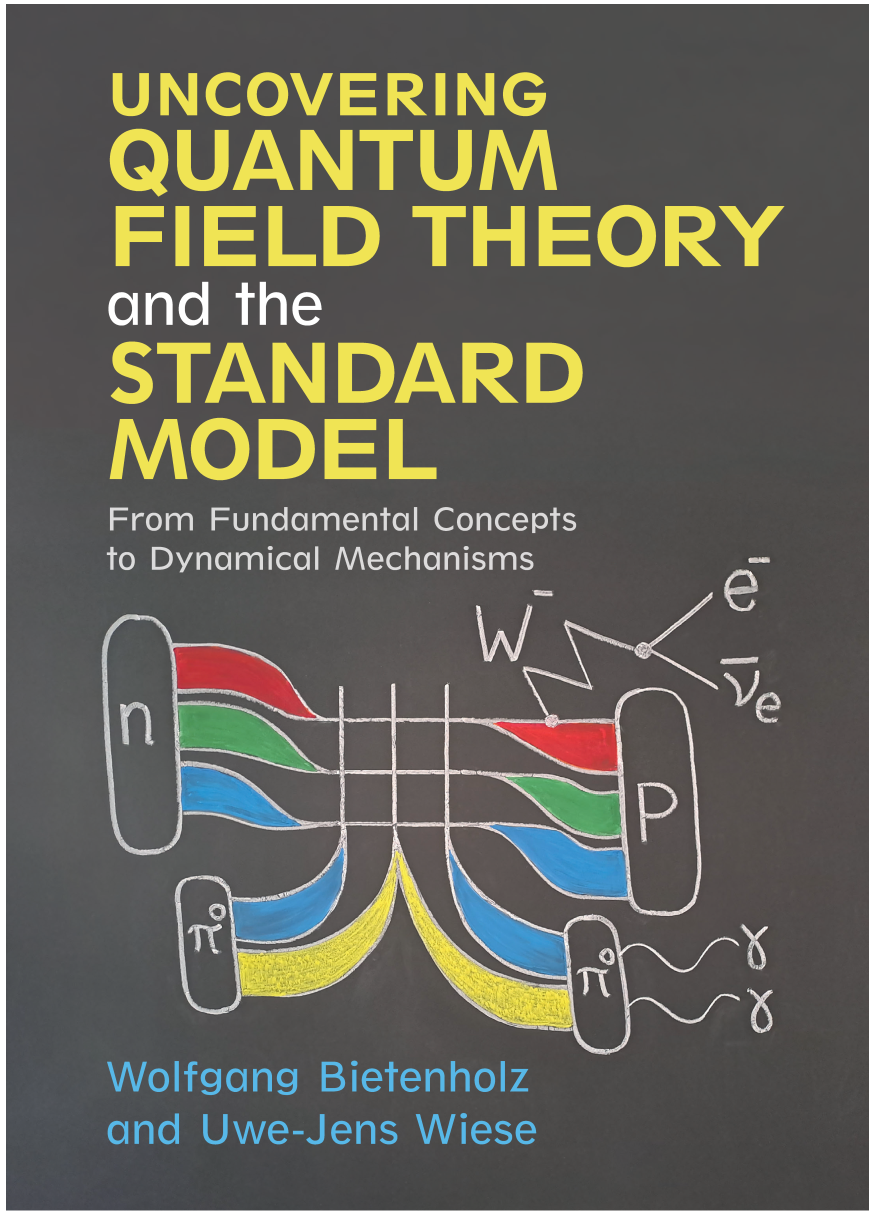
Initially heavy introduction of 14 pages, overview of the concepts. Martin Lüscher pointed out: reader who does not know *e.g.* gauge theory gets lost already after ≤ 4 pages. New structure: **Ouverture, Intermezzo, Finale**

Finally submitted in December 2022, supposed to take "on average 9 months", hope for faster procedure because we already used the Cambridge style file (?).

No way: summer 2023: edited pdf file with O(10^4) modifications (lots of commas, but also a few serious points), convergence after some cycles of revision. [Note: Klein-Gordon, sine-Gordon, but Clebsch-Gord**a**n]

Credit to Sunantha Ramamoorthy, competent, helpful, cooperative and friendly, but could not avoid a weird last-minute change of the fonts, (exponents, indices etc. displaced). Reduction from 772 to 732 pages.

Title page with chalk drawing of β- and π^0-decay by Nadiia Vlasii


UNCOVERING
QUANTUM
FIELD THEORY
and the
STANDARD
MODEL
From Fundamental Concepts
to Dynamical Mechanisms
Wolfgang Bietenholz
and Uwe-Jens Wiese

Most frequent names in the author index:

10 entries: Callen, Lüscher, 't Hooft, Wilczek

9 entries: Weinberg

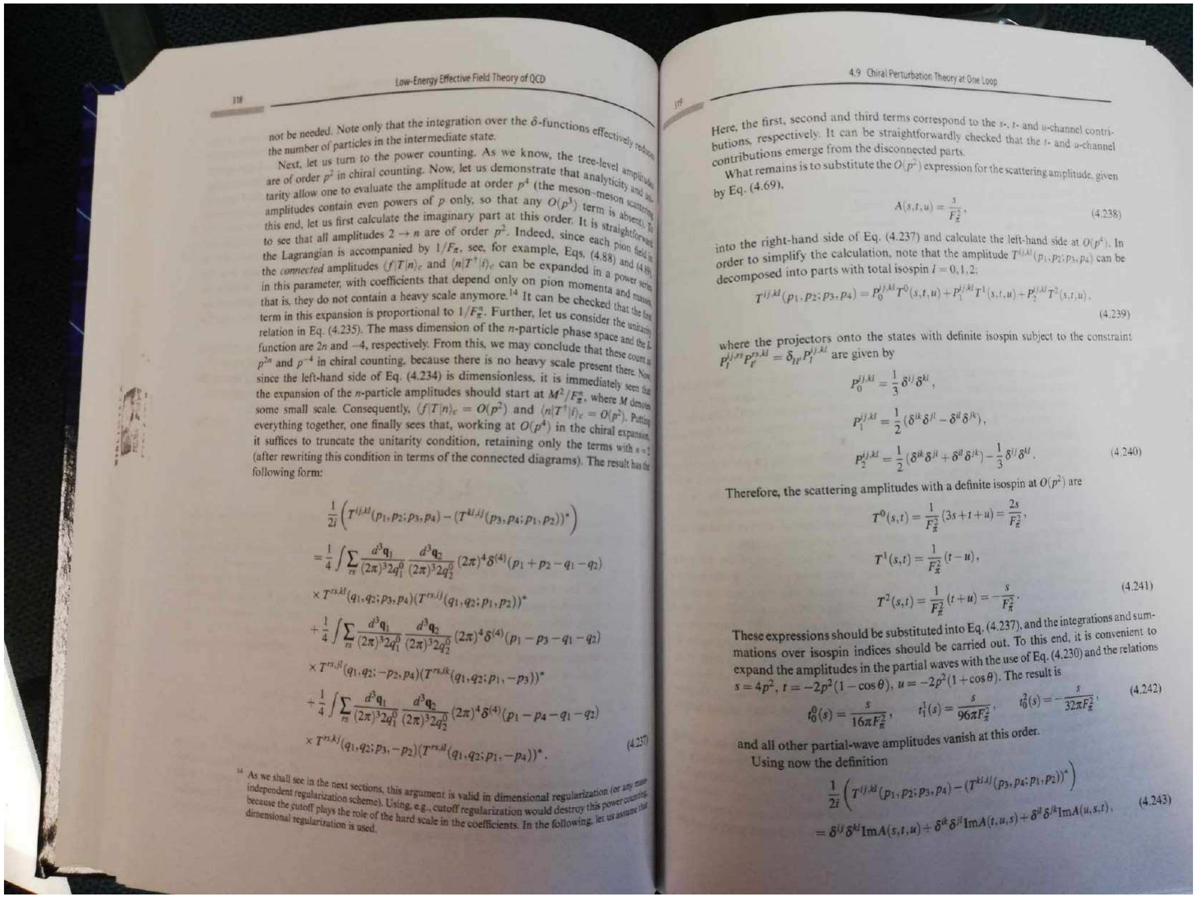
7 entries: Einstein, Feynman, Wilson
(Referee: "Wilson could have written such a book, but . . . ")

6 entries: Gross, Leutwyler, Witten

5 entries: Hasenfratz, Pauli, Schwinger

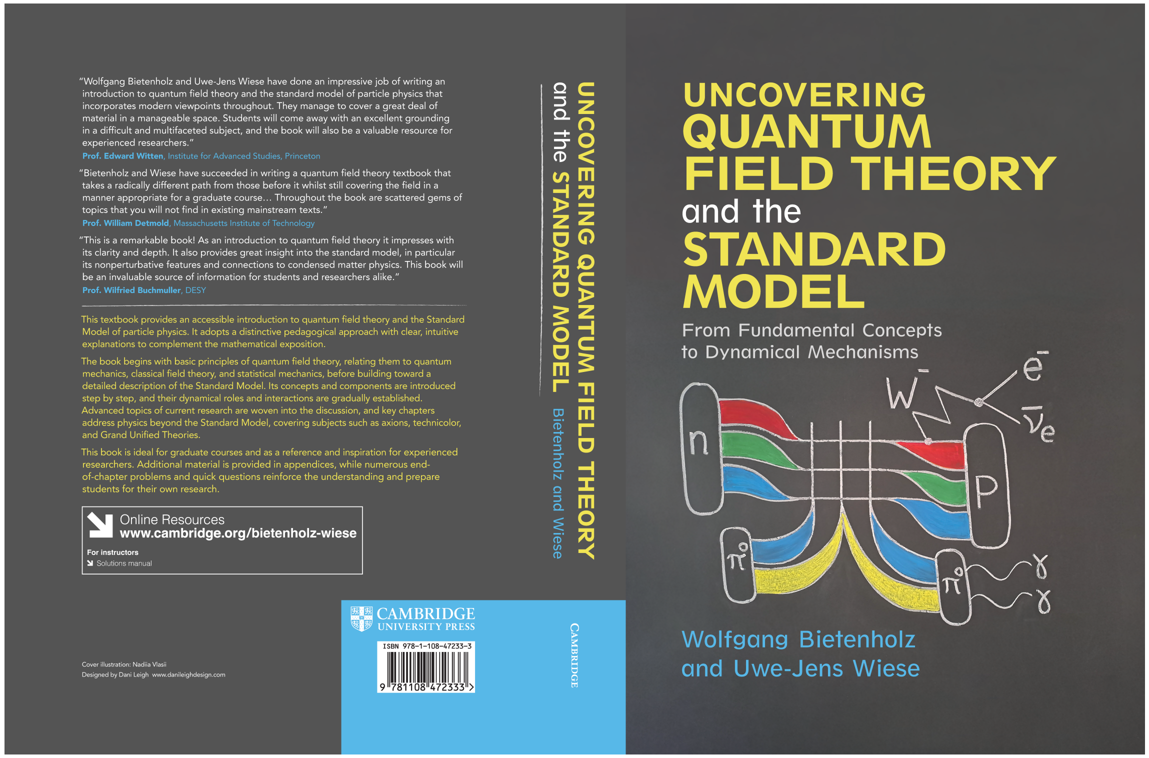
4 entries: Bogoliubov, Coleman, Dashen, Dirac, Gasser, Gell-Mann, Glashow, Niedermayer, Politzer, Polyakov, Symanzik, Wess, Yang, Zimmermann, Zinn-Justin, Zumino

The Meißner-Rusetsky problem:



Ulf-G Meißner and Akaki Rusetsky, Effective Field Theories, Cambridge University Press, 2022 (613 pages). Christian Schubert: must be related to Fermat's Last Theorem.

Confusion about endorsers, but finally 5 positive statements



Endorsements by Edward Witten, William Detmold, Wilfried Buchmüller, Poul Damgaard, Tereza Mendes

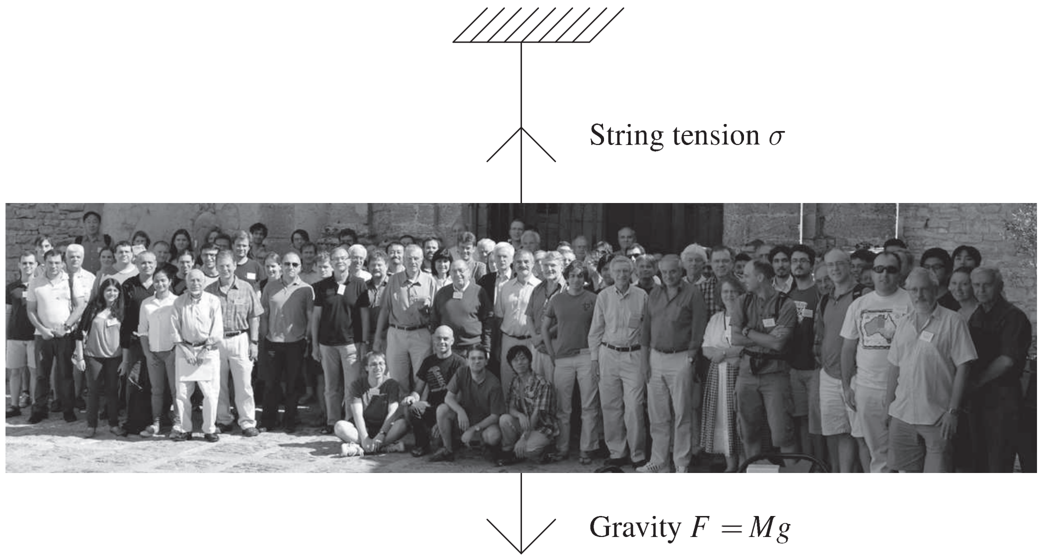


Fig. 14.4 A single Yang–Mills string is strong enough to support more than 100 people. A "Yang–Mills elevator" was installed by one of the authors as a Gedankenexperiment at the Erice workshop "From Quarks and Gluons to Hadrons and Nuclei" in 2011, in order to "elevate" our intuitive understanding for the strength of the strong force (Wiese, 2012).

Quick Question 14.6.1 Overloaded Yang–Mills elevator

What happens when about 1000 (adult) people enter the Yang–Mills elevator?

14.7 Roughening Transition

2024: announced by Cambridge University Press for January, shifted to February . . . inductive process until June.

Summer 2024: cyber attack, Cambridge editorial paralyzed, sad news: Sunantha passed away, Shanthy Jaganathan takes over

Promised for November, “in production”, shifted to December, finally published on January 2, 2025.

Happy ending !

Nomination for Anatoly Larkin Award in Theoretical Physics (U. Minnesota) declined.

However, Uwe-Jens was a recipient of an outstanding and well-deserved European Research Council (ERC) Advanced Grant for the period from 2014 to 2019, and of a Humboldt Research Award in 2022.

From my personal side: thanks to Uwe-Jens for his role as my tutor in lattice field theory, 11 joint papers and 10 lattice proceeding contributions with O(1000) citations, and countless explanations of all kind of physical questions.

Generally, one of the few trendsetters in the lattice community and beyond, as we see at this workshop.