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QCD deconfinement transition line up to μ_B = 400 MeV from finite volume lattice simulations

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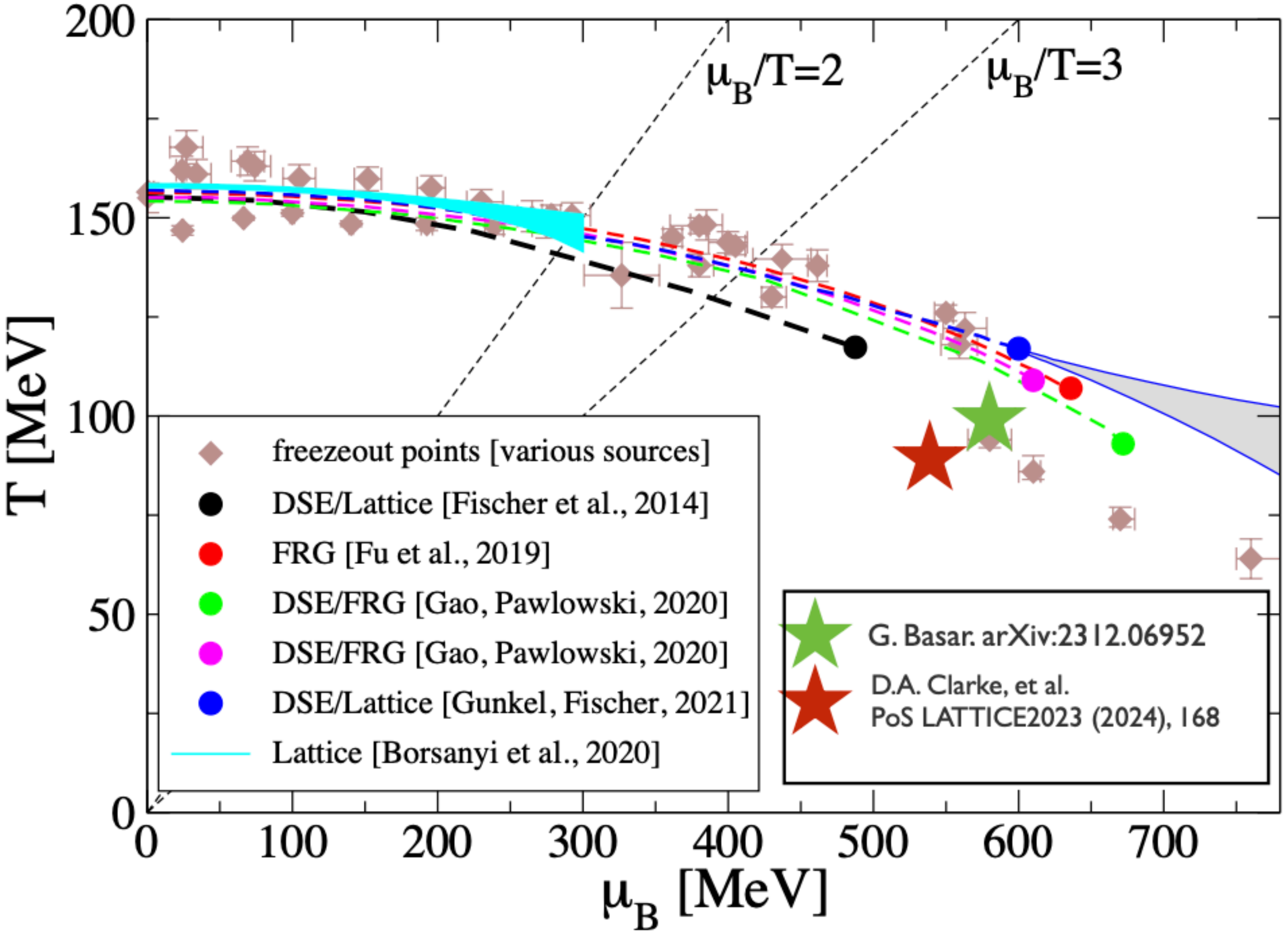
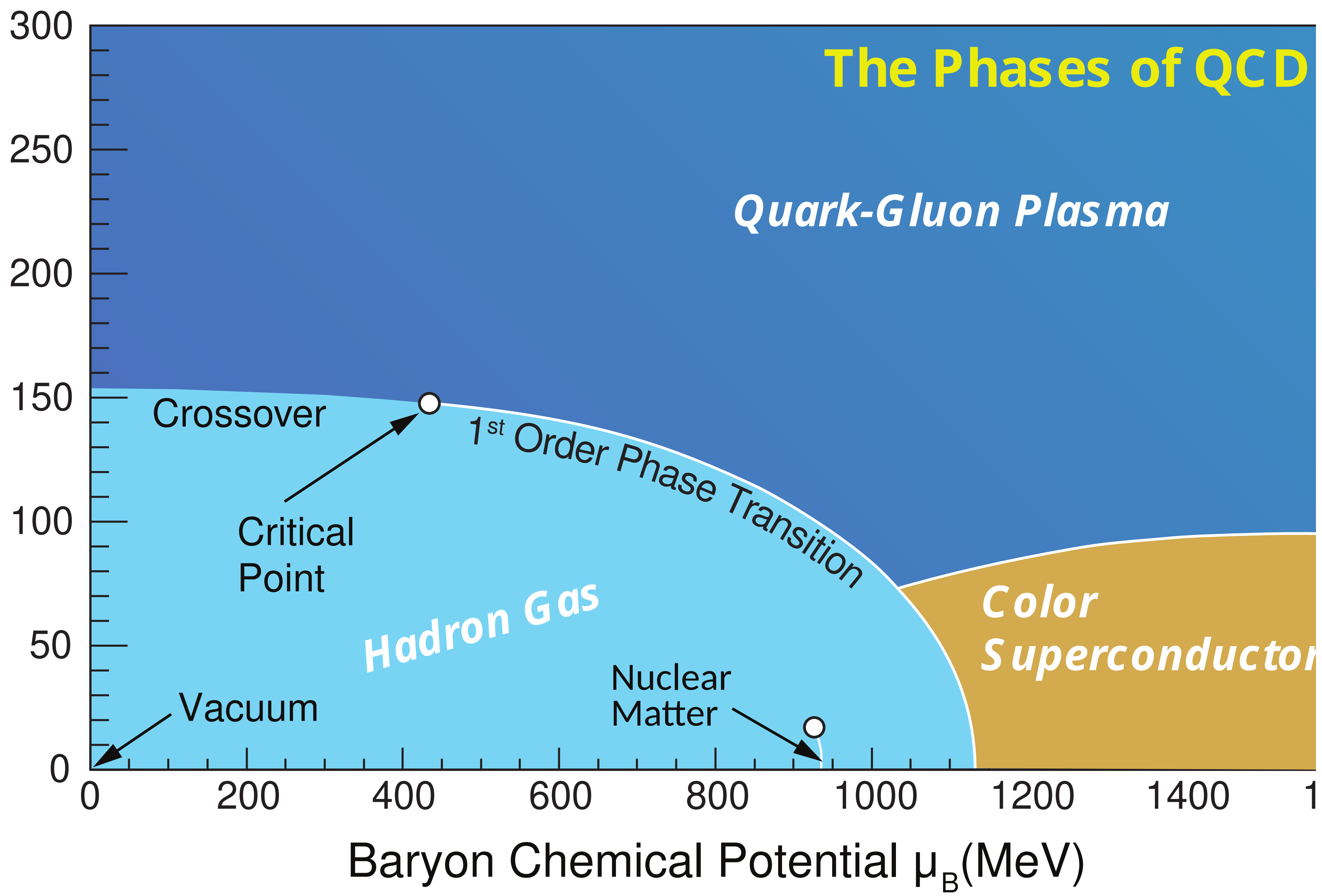
A research area that is affected by the sign problem

- Active area of research: search for Critical End Point in (T, μ_B) plane
- High μ_B needed! But…

Plot from Christian Fischer's talk at CPOD 2024



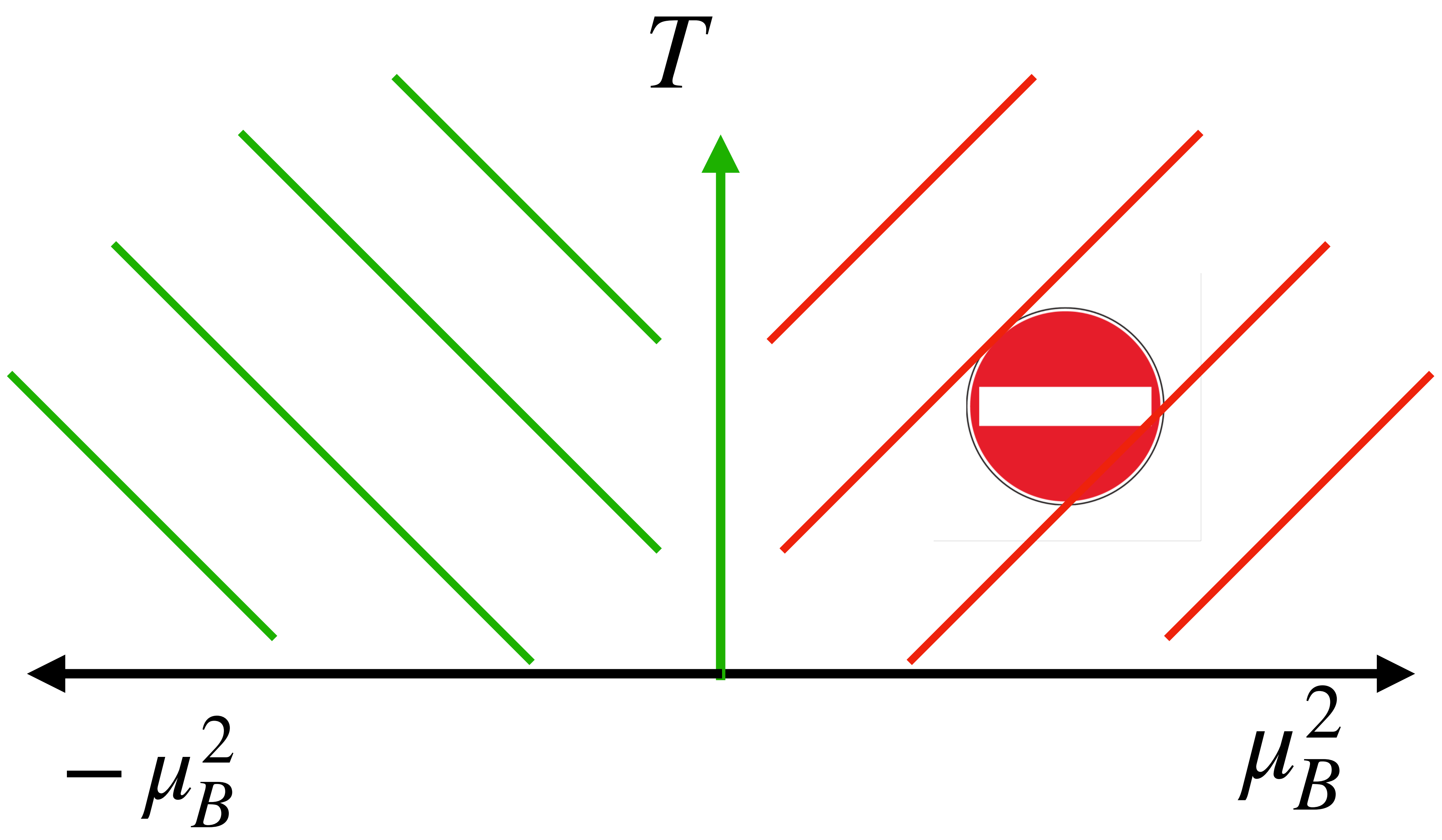
Plot from arXiv:1906.00936



…we can **not simulate there**!

- $Z = \int DU\, e^{-S_G[U]} \det M(U, m, \mu_B)$
- $\det M(U, m, \mu_B)$ is real and positive only for $\mu_B^2 \leq 0$
- we can't measure directly any observable with Monte Carlo methods for $\mu_B^2 > 0$

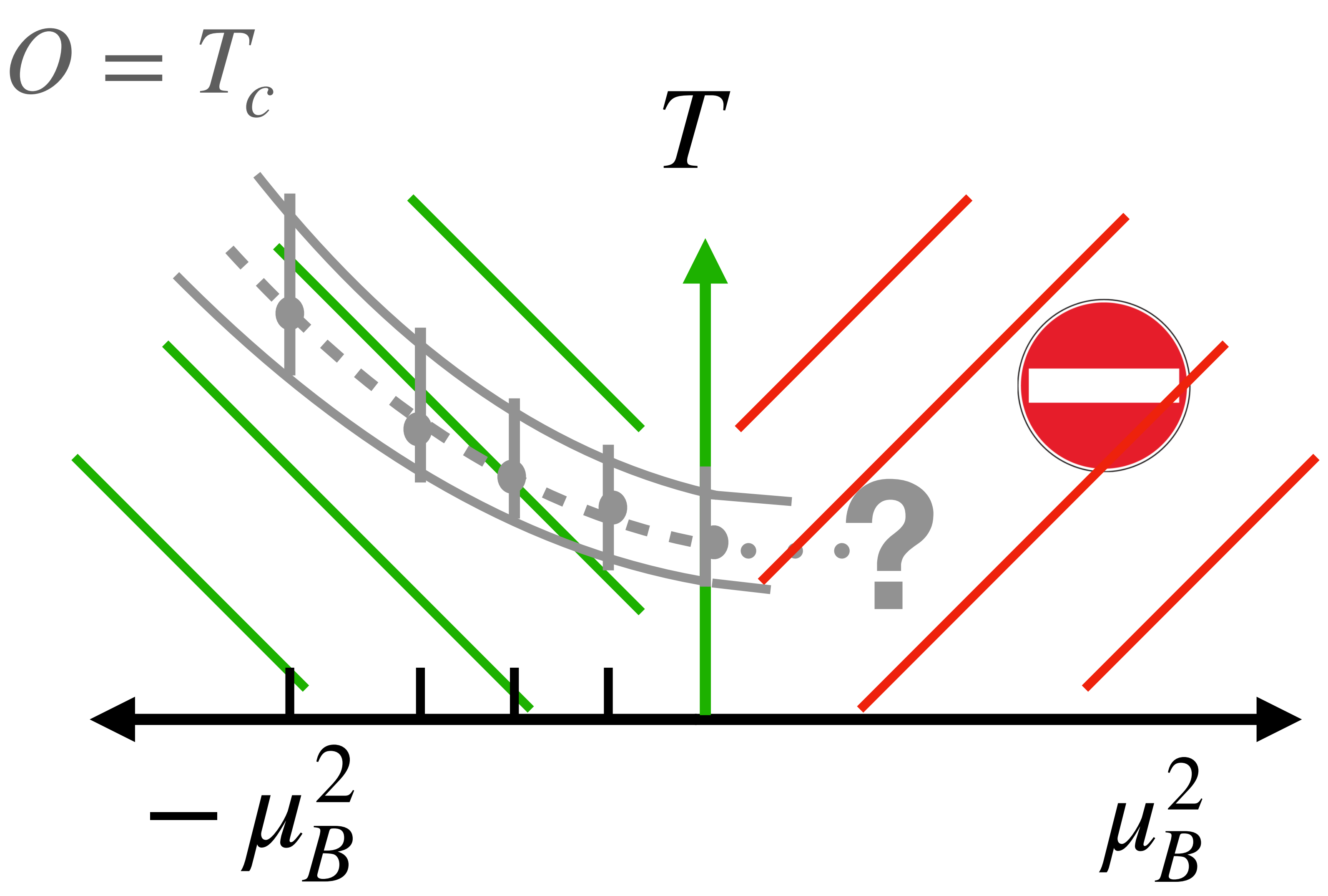
$$< O > = \frac{1}{N_{\mathrm{conf}}} \sum_{i=0}^{N_{\mathrm{conf}}} O[U_i]$$



- Let's see how to circumvent the problem

Analytical continuation

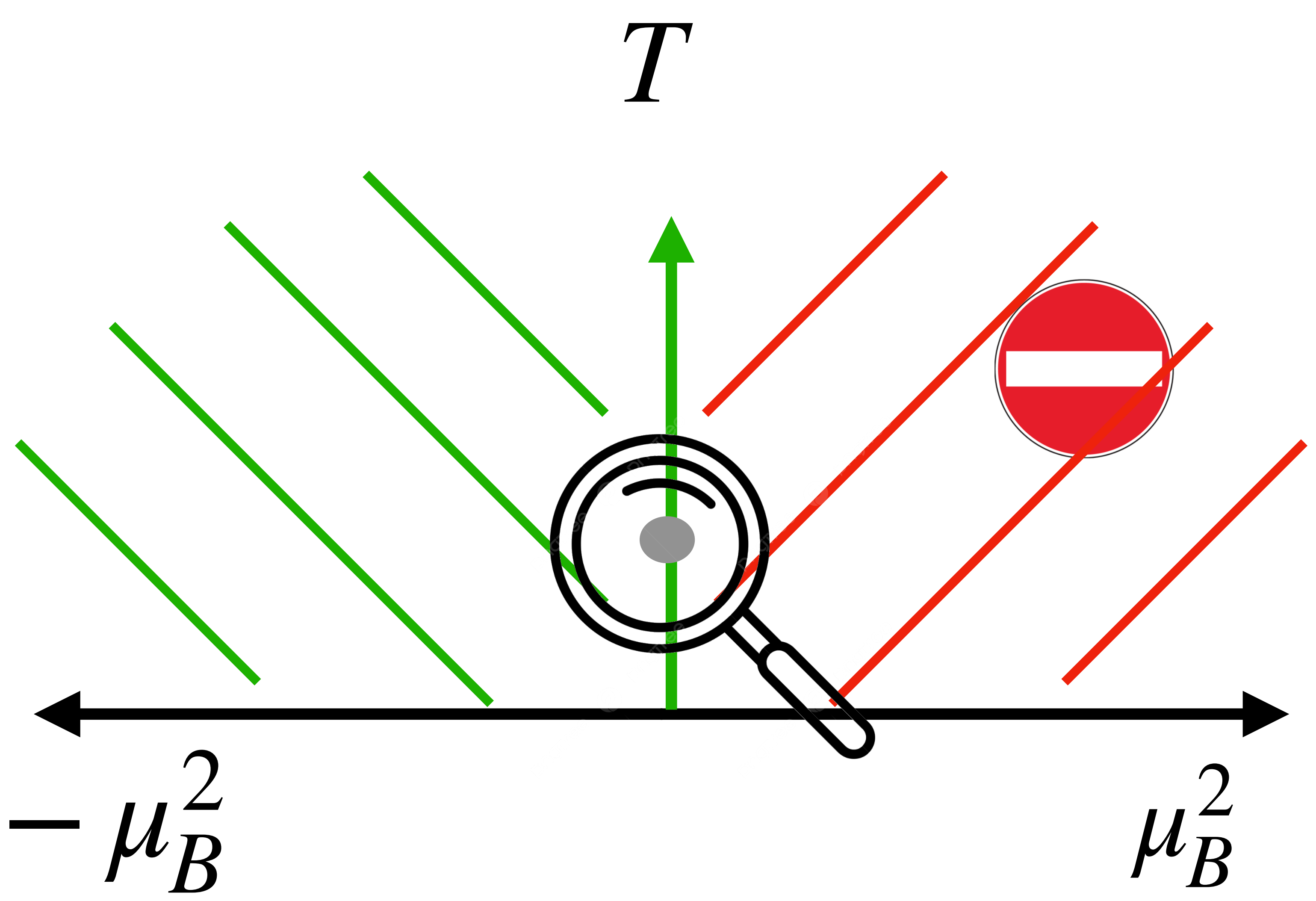
- simulate at $\mu_B^2 < 0$ and extrapolate to $\mu_B^2 > 0$



Taylor/T' expansion

- compute the derivatives at $\mu_B = 0$

$$O(\mu_B) = O(0) + \frac{1}{2!}\mu_B^2 \frac{\partial^2 O}{\partial \mu_B^2}(0) + \frac{1}{4!}\mu_B^4 \frac{\partial^4 O}{\partial \mu_B^4}(0) + \dots$$



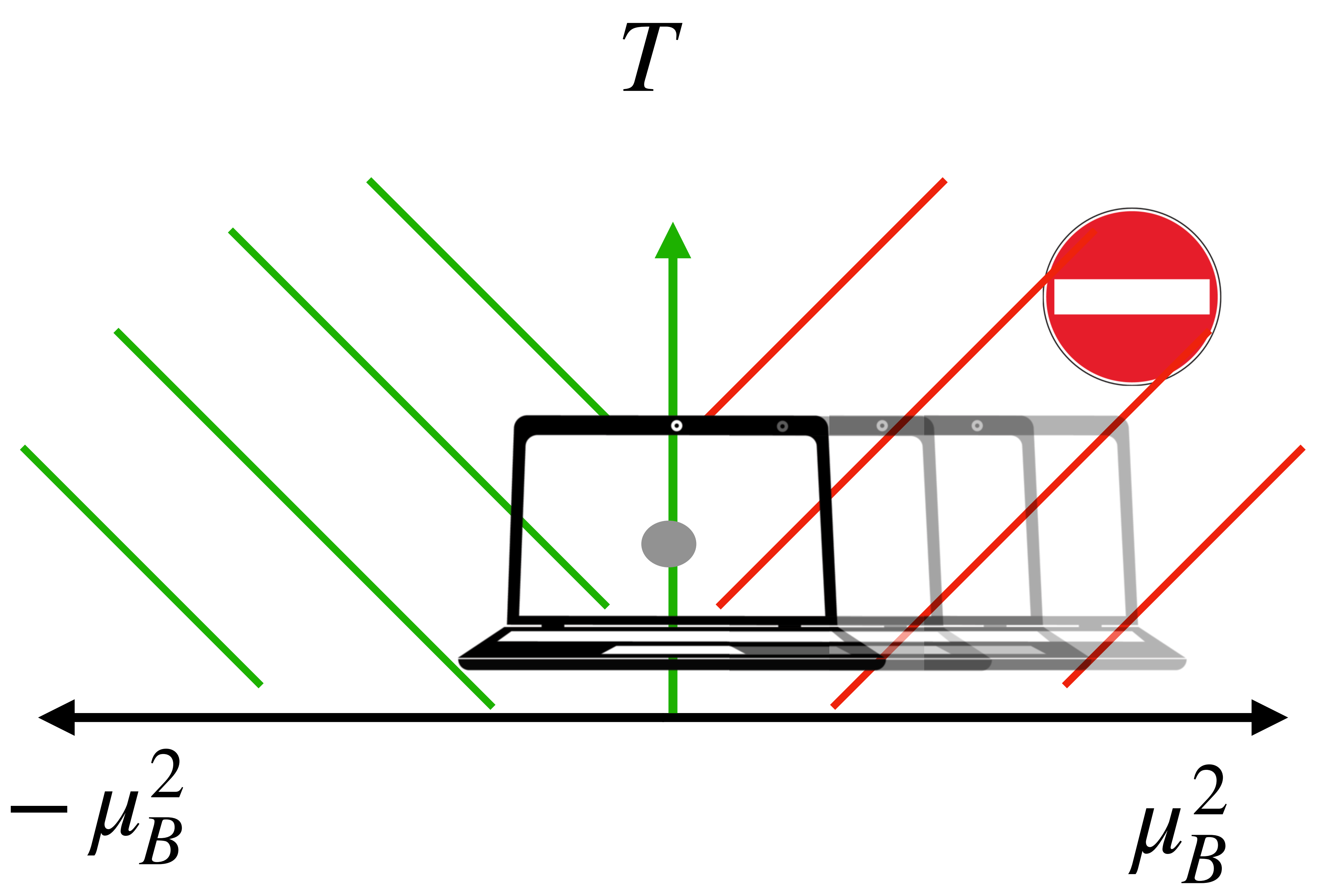
Reweighting

- make the simulations at $\mu_B = 0$ and correct the weights in the observable measure

- $$< O >_{\mu_B} = \frac{\int DU\, e^{-S_G[U]}\, O(U) \frac{\det M(U,m,\mu_B)}{\det M(U,m,0)} \det M(U,m,0)}{\int DU\, e^{-S_G[U]} \frac{\det M(U,m,\mu_B)}{\det M(U,m,0)} \det M(U,m,0)} =$$

$$= \frac{\left\langle O \frac{\det M(\mu_B)}{\det M(0)} \right\rangle_{\mu_B=0}}{\left\langle \frac{\det M(\mu_B)}{\det M(0)} \right\rangle_{\mu_B=0}}$$

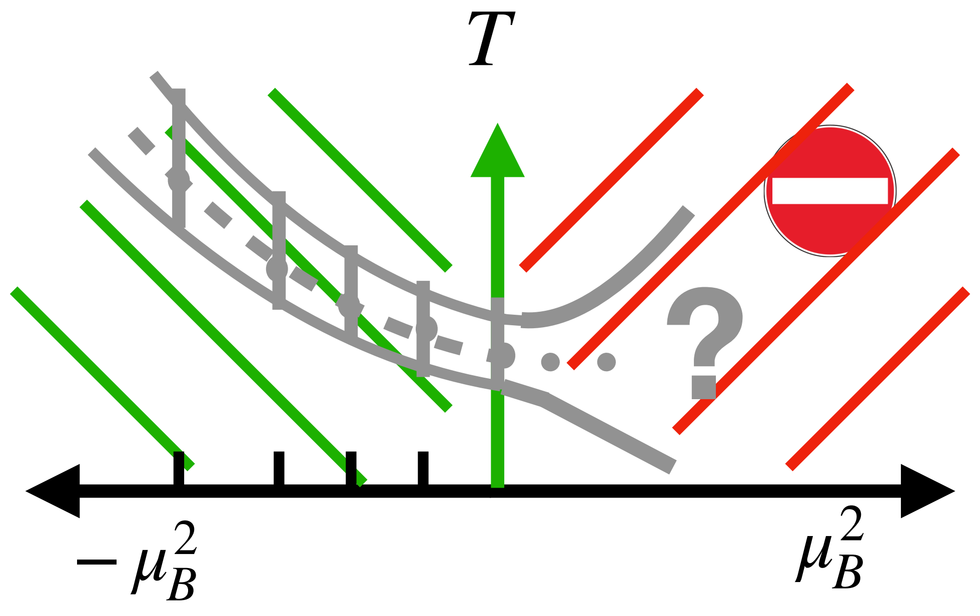
$$\frac{Z(\mu_B)}{Z(0)} = \left\langle \frac{\det M(\mu_B)}{\det M(0)} \right\rangle_{\mu_B=0}$$



- Computation of the complex determinant can be circumvented with these methods

- Is that enough?

Watch out!

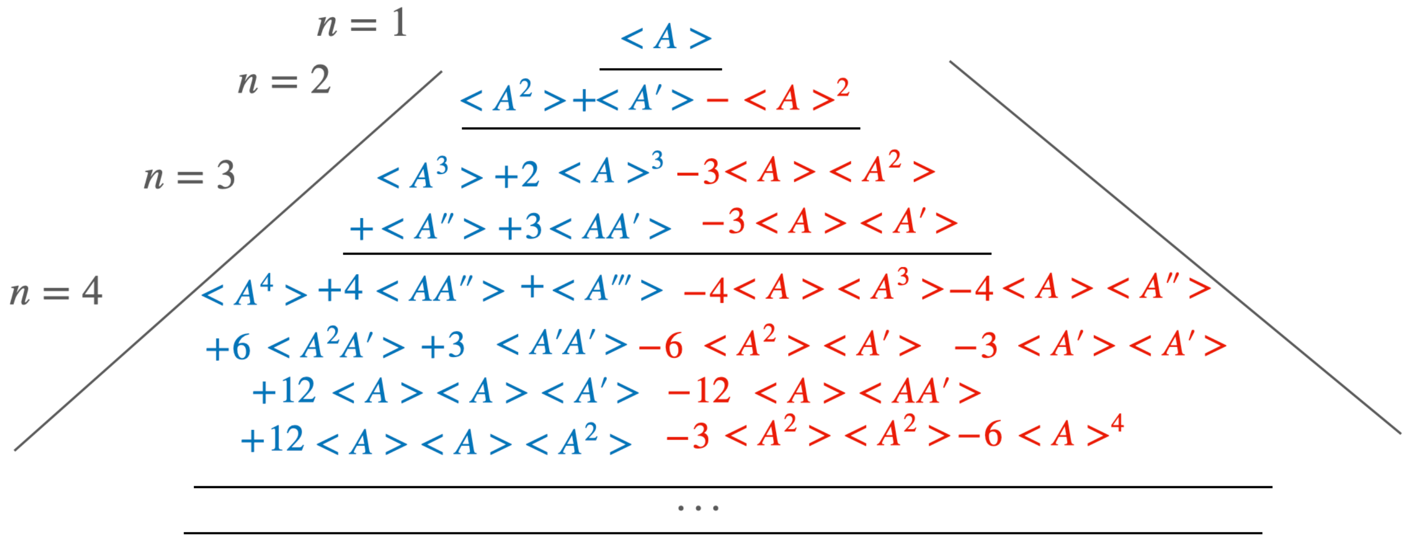


- Analytical continuation: lots of systematics

- Taylor / T' methods:

  - a lot of cancellations inside the terms

  - uncontrolled truncation in μ_B



- Reweighting:

  - need some care if you use staggered fermions

arXiv:2308.06105

A small trip in

Reweighting with staggered fermions

arXiv:2308.06105

- Staggered fermions: rooting

- We look at 2+1 flavours theory with $\mu_u = \mu_d = \mu_q$, $\mu_s = 0$

- $Z_{2+1}(T, \mu_q) = \int DU\, e^{-S_G[U]} \det M(U, m_q, \mu_q)^{\frac{1}{2}} \det M(U, m_s, 0)^{\frac{1}{4}}$

- $\frac{Z_{2+1}(\mu_q)}{Z_{2+1}(0)} = \left\langle \frac{\det M(U, m_q, \mu_q)^{\frac{1}{2}}}{\det M(U, m_q, 0)^{\frac{1}{2}}} \right\rangle_{\mu_B=0}$ can be computed with reduced matrix formalism

- the eigenvalues $\{\lambda_k\}$ of the reduced matrix do not depend on μ_q
- solve the rooting ambiguity: take the square root for each eigenvalue
- $$\frac{\det M(U, m_q, \mu_q)^{\frac{1}{2}}}{\det M(U, m_q, 0)^{\frac{1}{2}}} = e^{-3N_s^3 \mu_q/T} \prod_{k=1}^{6N_s^3} \sqrt{\frac{\lambda_k[m, U] - e^{\mu_q/T}}{\lambda_k[m, U] - 1}}$$
- it could have an overlap problem (long tails in the weights) but we crosscheck with other 2 reweighting methods
  - phase reweighting $\frac{Z_{2+1}}{Z_{2+1}^{PQ}} = \left\langle \frac{\det M(U, m_q, \mu_q)^{\frac{1}{2}}}{|\det M(U, m_q, \mu_q)^{\frac{1}{2}}|} \right\rangle_{PQ}$
  - sign reweighting $\frac{Z_{2+1}}{Z_{2+1}^{SQ}} = \left\langle \frac{\mathrm{Re}\det \mathrm{M}(\mathrm{U}, \mathrm{m_q}, \mu_\mathrm{q})^{\frac{1}{2}}}{|\mathrm{Re}\det \mathrm{M}(\mathrm{U}, \mathrm{m_q}, \mu_\mathrm{q})^{\frac{1}{2}}|} \right\rangle_{SQ}$

- Now, let's look at the light quark density (e.g. for equation of state studies)

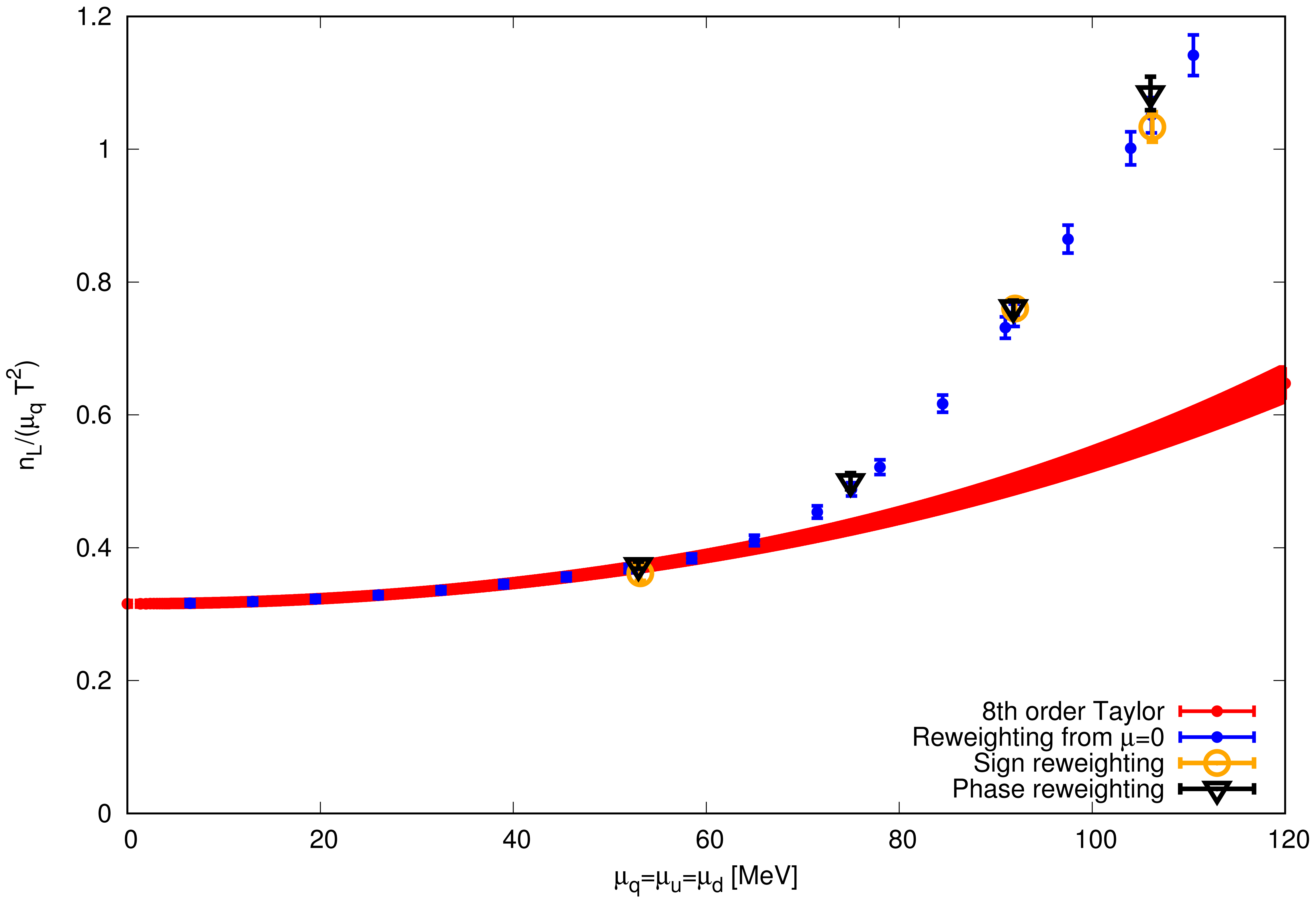
- Plot: $\hat{n}_L/\hat{\mu}_q$, $16^3 \times 8$ lattice, $m_\pi = 135$ MeV

A steep rise at $\mu_q \sim \frac{m_\pi}{2}$!! **Why?**

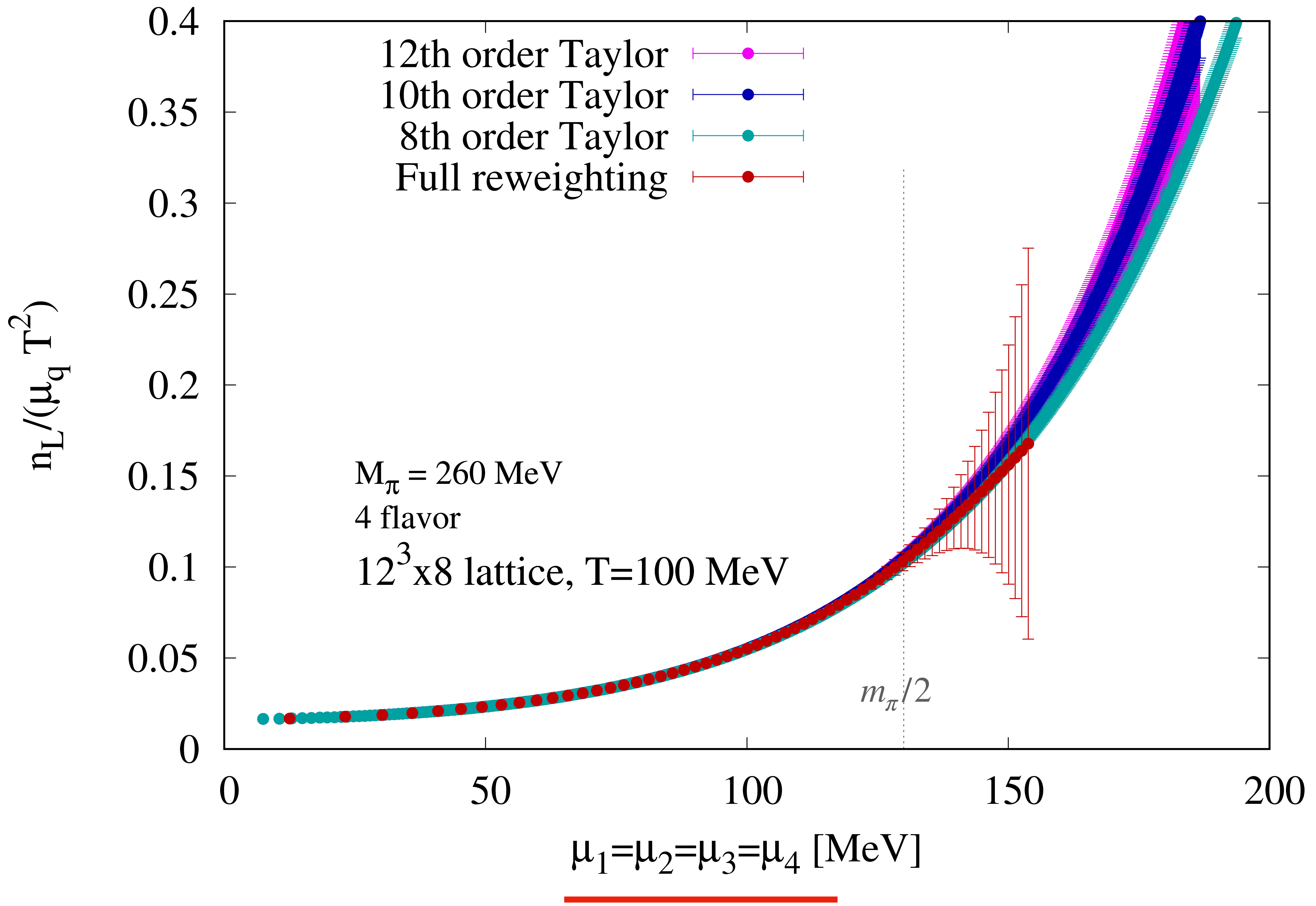
- Is it an overlap problem?

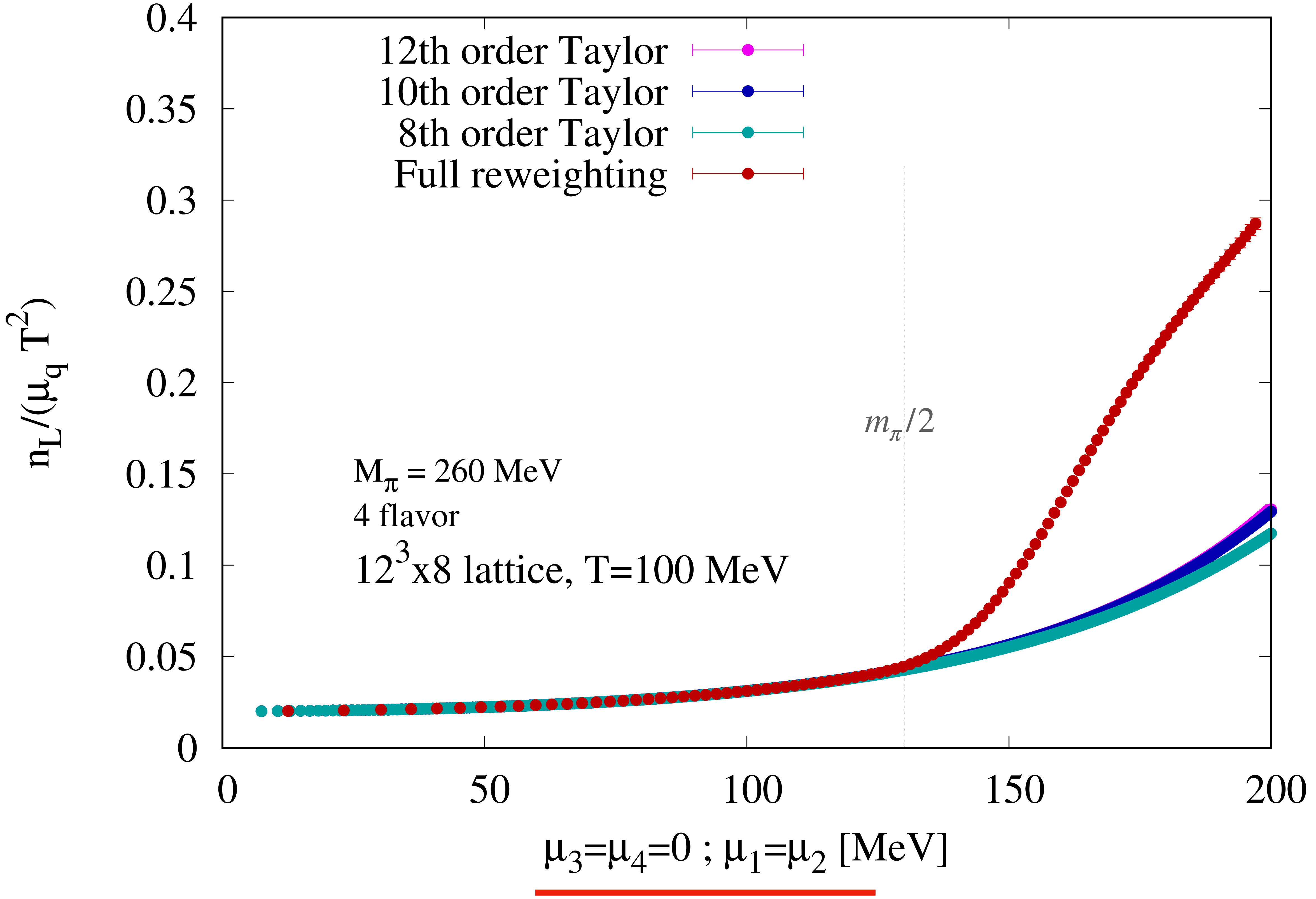
- No, the 3 reweighting techniques do the same

- It also a discretisation effect



T=130 MeV. 2 stout smearing,





- → no rooting
- reweighting and Taylor (up to 12th order) agree

- rise at $m_\pi/2$ for the reweighted case

- the culprit is actually the rooting!

- it seems that at the moment our favourite technique is Taylor method then
- But also there we need to be careful

Cancellations problem in Taylor method

- Goal: derivatives of an observable O w.r.t. μ_i (i: flavour) $\partial_i O$
- Given $A_i = \dfrac{\partial \log(\det M)^{\frac{1}{4}}}{\partial \mu_i} = \dfrac{1}{4}\mathrm{Tr}(\mathrm{M}_i^{-1}\mathrm{M}_i') \rightarrow \; < A_i > = \partial_i \log Z$
- for O we have the generic chain rule formula
- $\partial_i < O > = \; < O A_i > + < \partial_i O > - < O > < A_i >$

- the bigger is the order n of derivative, the more terms we have…
- and the bigger is the cancellation between them!

- Below: $\partial^n \log Z / \partial\mu^n$ (ignoring the flavour)

$n = 1$

$$< A >$$

$n = 2$

$$< A^2 > + < A' > - < A >^2$$

$n = 3$

$$< A^3 > +2 < A >^3 -3 < A > < A^2 >$$
$$+ < A'' > +3 < AA' > -3 < A > < A' >$$

$n = 4$

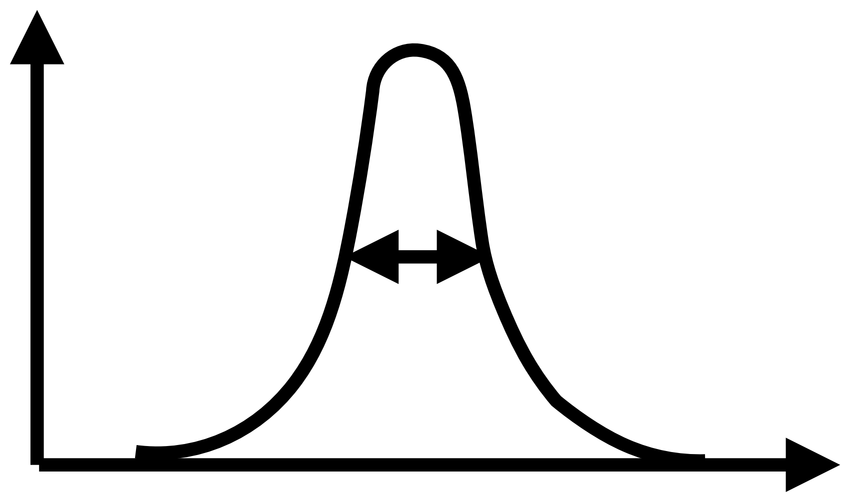
$$< A^4 > +4 < AA'' > + < A''' > -4 < A > < A^3 > -4 < A > < A'' >$$
$$+6 < A^2A' > +3 < A'A' > -6 < A^2 > < A' > -3 < A' > < A' >$$
$$+12 < A > < A > < A' > -12 < A > < AA' >$$
$$+12 < A > < A > < A^2 > -3 < A^2 > < A^2 > -6 < A >^4$$

$$\cdots$$

- This cancellation actually scales with the volume → how?
- Derivatives of $\log Z$ are related to quark number susceptibilities

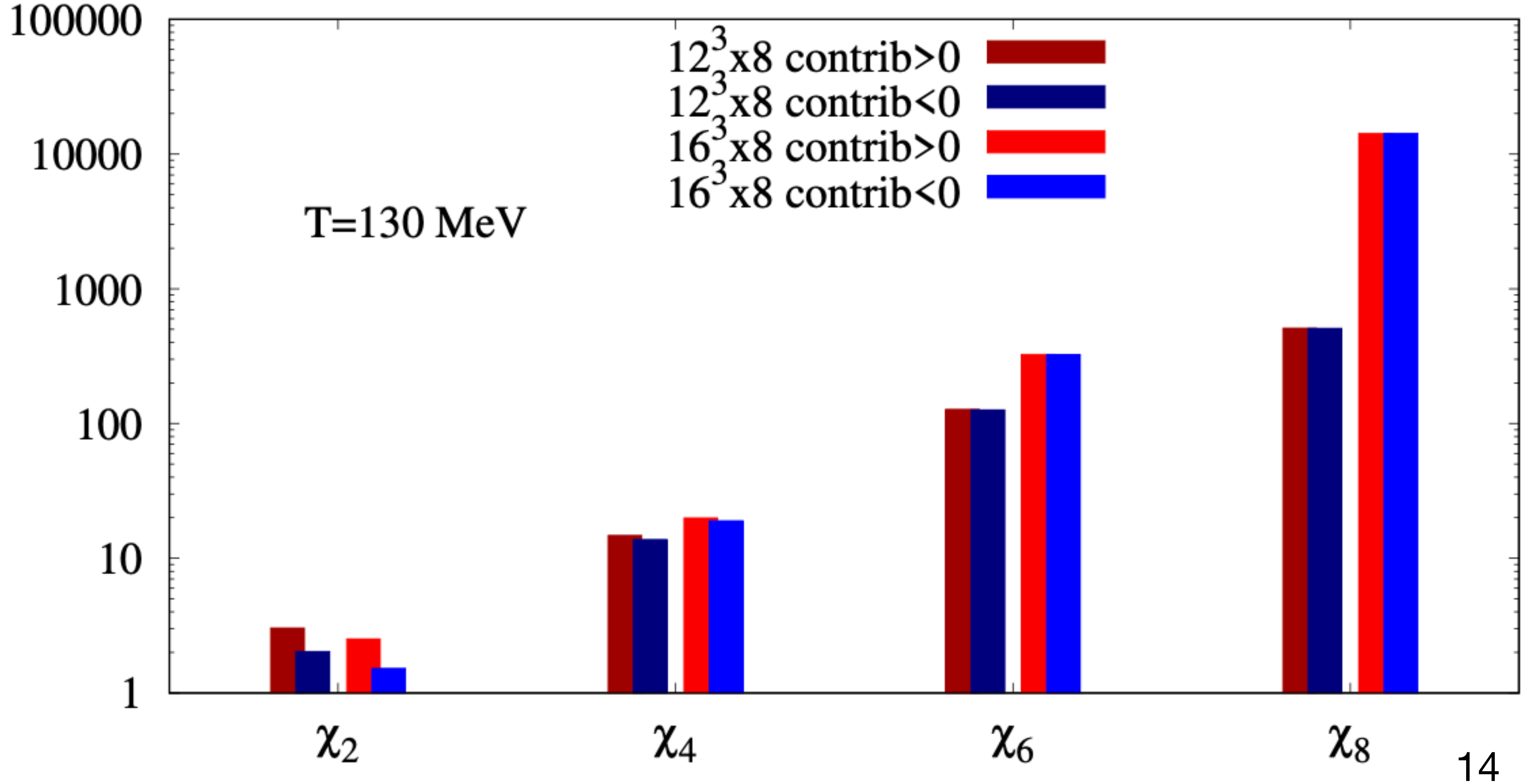
$$\chi_{ijk}^{uds}(T,\mu_u,\mu_d,\mu_s) = \frac{T}{V}\frac{\partial^{i+j+k}\log Z}{(\partial_u)^i(\partial_d)^j(\partial_s)^k}$$

- $\chi_1^u \sim < u > \sim n_u$
- $\chi_2^u \sim \frac{1}{V} < u^2 > - < u >^2$: a variance / V, we except $O(1)$
- χ_2^u contains terms like $\frac{1}{V}(< A^2 > - < A >^2) \rightarrow \ < A^2 > \sim V$



- Inside χ_4^u we have couples of terms like $\frac{1}{V}(\underbrace{<A^4>}_{O(V^2)} - 3\underbrace{<A^2>^2}_{O(V^2)}) \sim O(1)$ $\quad O(V^2) \leftarrow O(V^2)$
- the cancellation is $O(V)$
- Inside χ_6^u: $\frac{1}{V}(<A^6> - 15<A^2>^3) \sim O(1)$
- the cancellation is $O(V^2)$

contributions normalised to sum

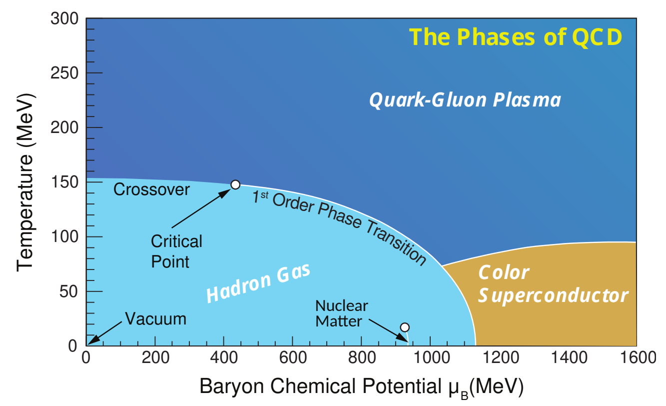


- χ_{2n}: cancellation is $O(V^{n-1})$
- we need small volumes!!

How small can the volume be?

- Small enough to take care of the cancellations

- Big enough to study phase transitions



- It depends on the observables: arXiv:2405.12320. (2+1+1 4stout staggered fermions)

- Two groups of observables related to QCD transition between hadron and Quark Gluon Plasma phases:

  - chiral observables ($SU(2) \times SU(2)$ symmetry in limit $m_q \to 0$)

  - deconfinement observables (Z_3 symmetry in limit $m_q \to \infty$)

Chiral observables

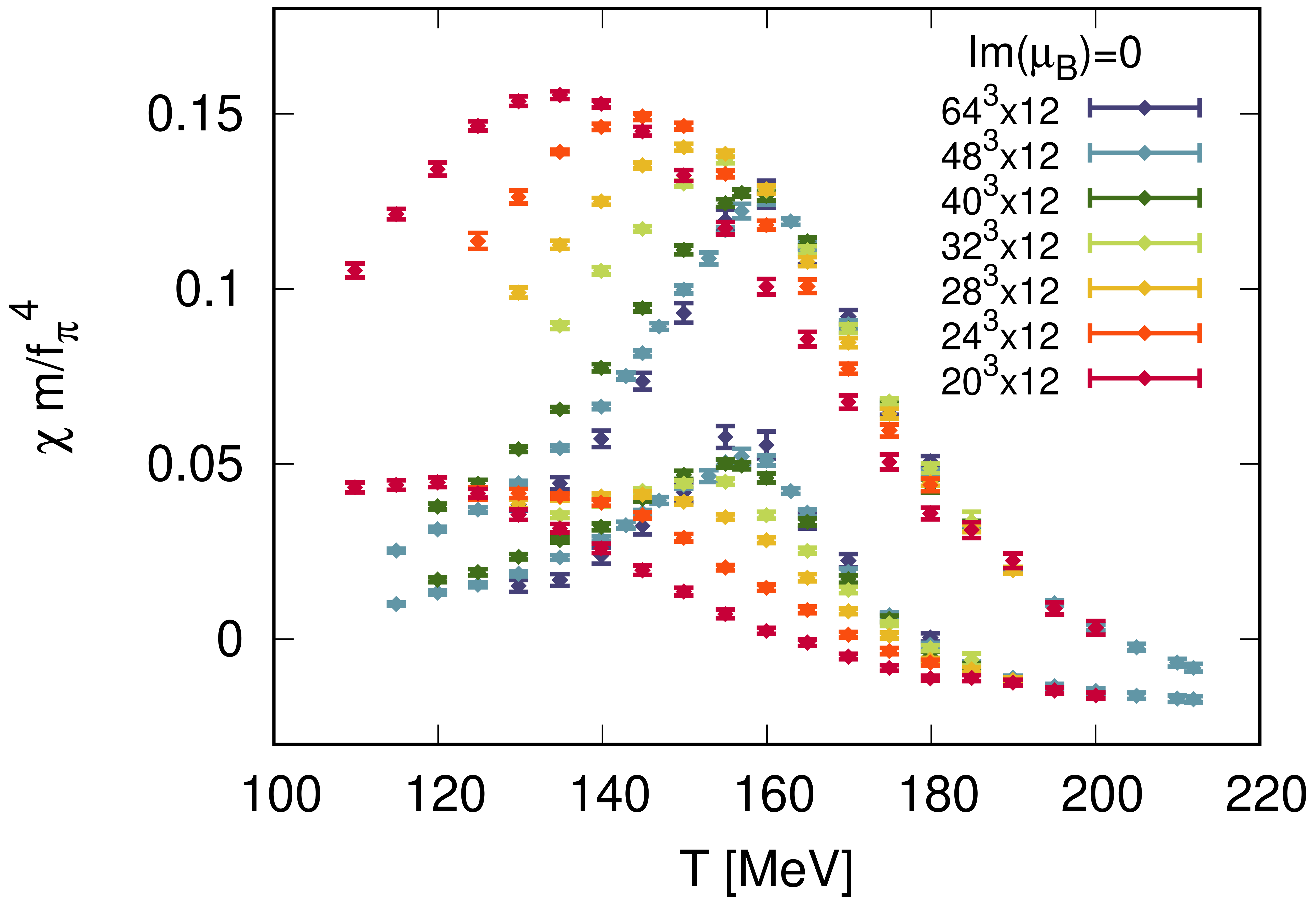
- **order parameter:** chiral condensate

$$< \bar{\psi}\psi > = \frac{T}{V}\frac{\partial \log Z}{\partial m_{ud}}$$

- chiral susceptibility $\chi = \frac{T}{V}\frac{\partial^2 \log Z}{\partial m_{ud}^2}$

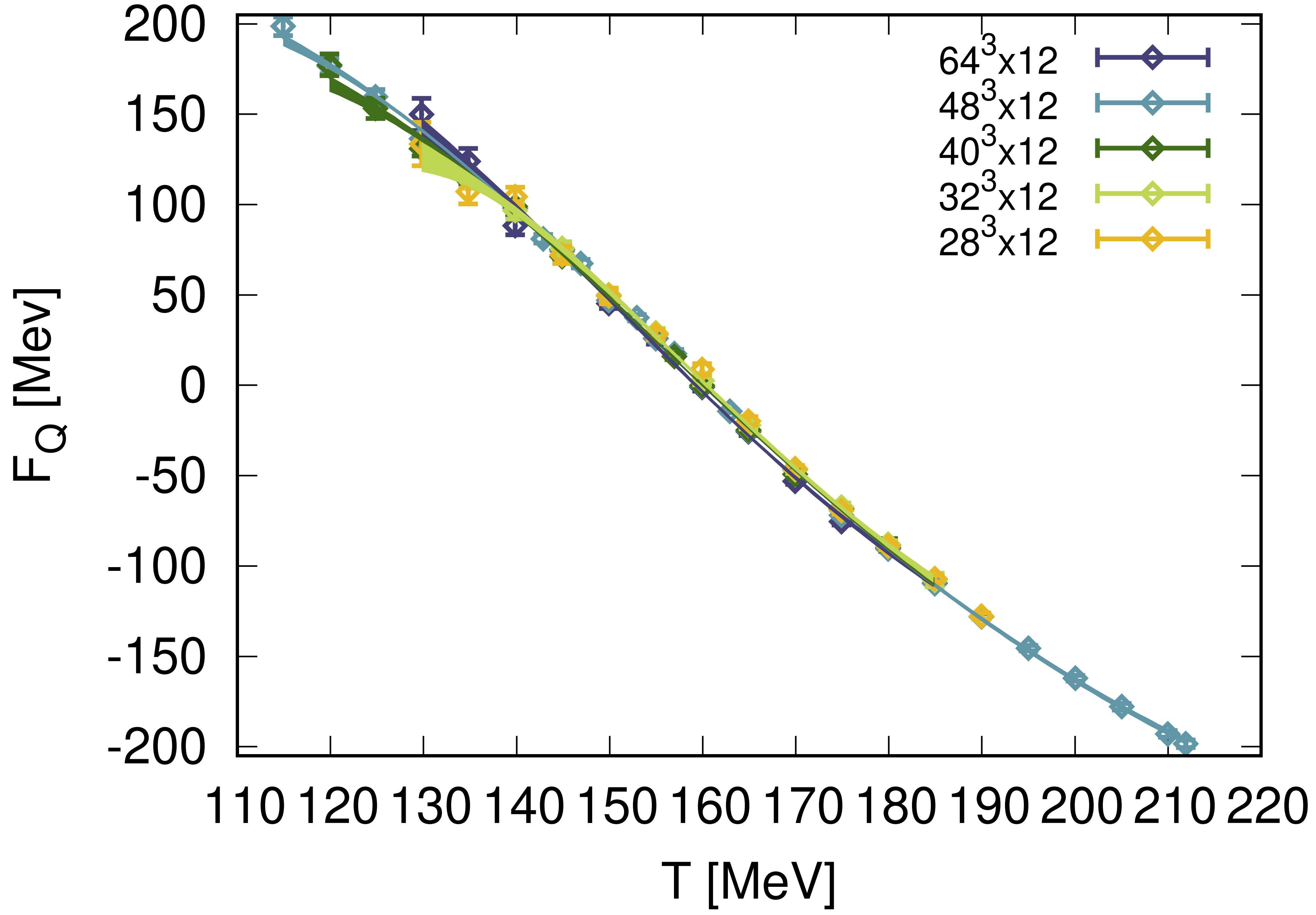
- disconnected chiral susceptibility

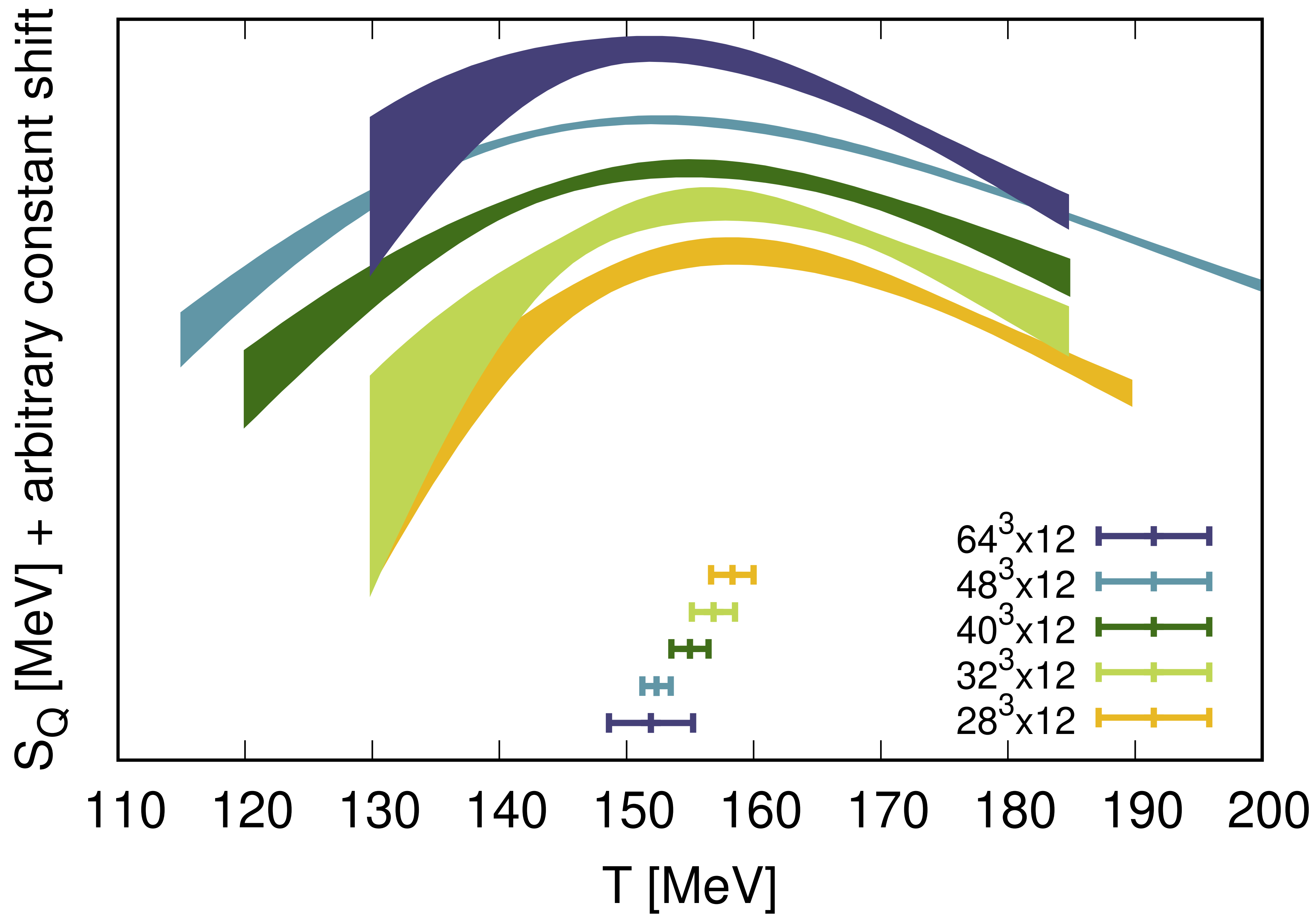
$$\chi_{\text{disc}} = \frac{T}{V}\left(\frac{\partial^2 \log Z}{\partial m_u \partial m_d}\right)_{m_u=m_d}$$



Deconfinement observables

- **order parameter**: Polyakov loop $P \sim e^{-F_Q/T}$
- $$P(\vec{x}) = \prod_{x_4=0}^{N_t-1} U_4(\vec{x}, x_4)$$
- F_Q: static quark free energy





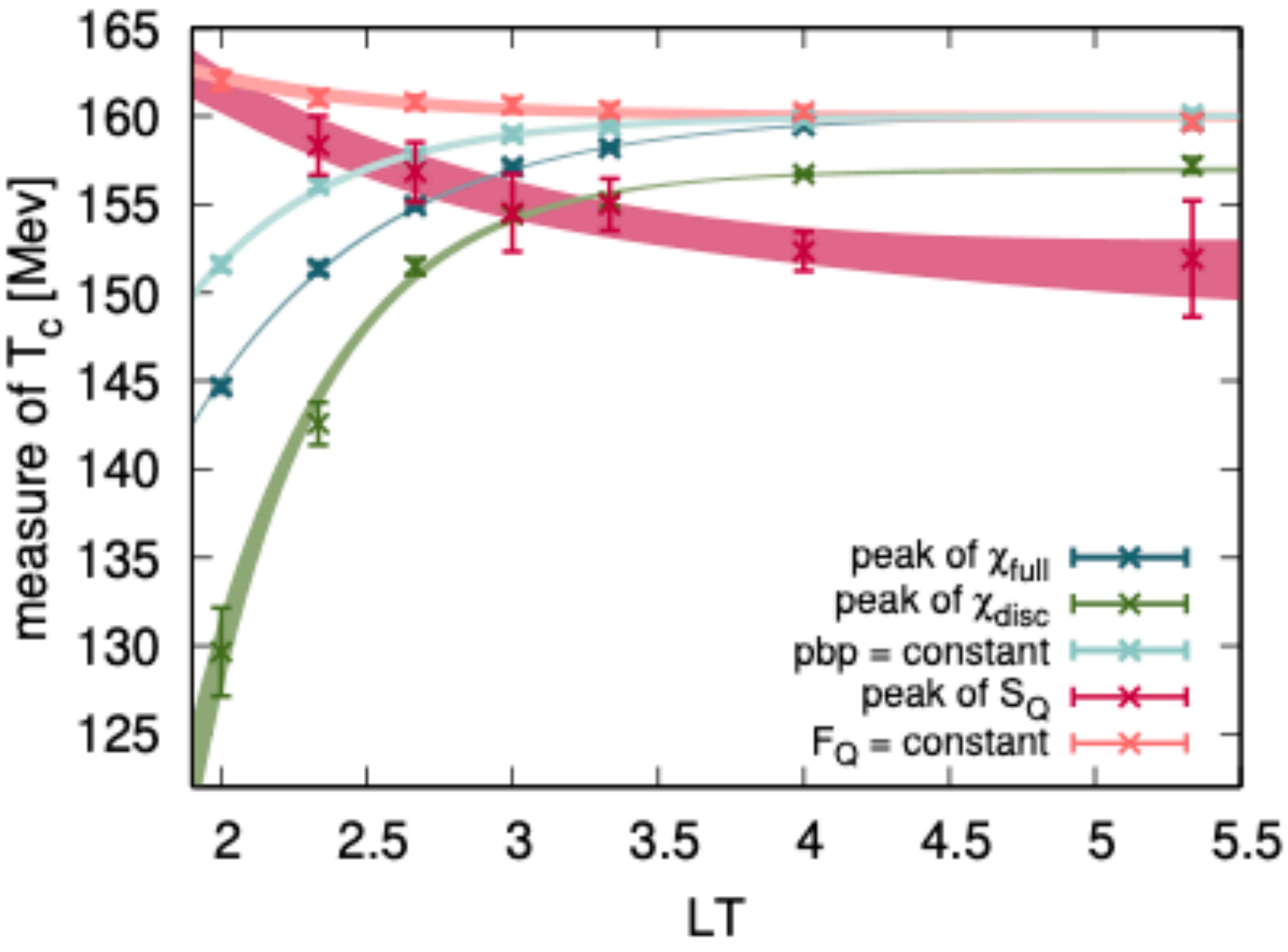
- static quark entropy $S_Q = -\frac{\partial F_Q}{\partial T}$
- scheme-independent peak position

introduced by TUMQCD Collaboration
arXiv:1603.06637

T_c vs LT $(\mu_B = 0)$

- $T_c^{(S_Q)} < T_c^{(\chi^R_{\rm disc})} < T_c^{(\chi^R)}$ for $LT \geq 3$

Previous result of TUMQCD Collaboration [1603.06637]: agreement within errors

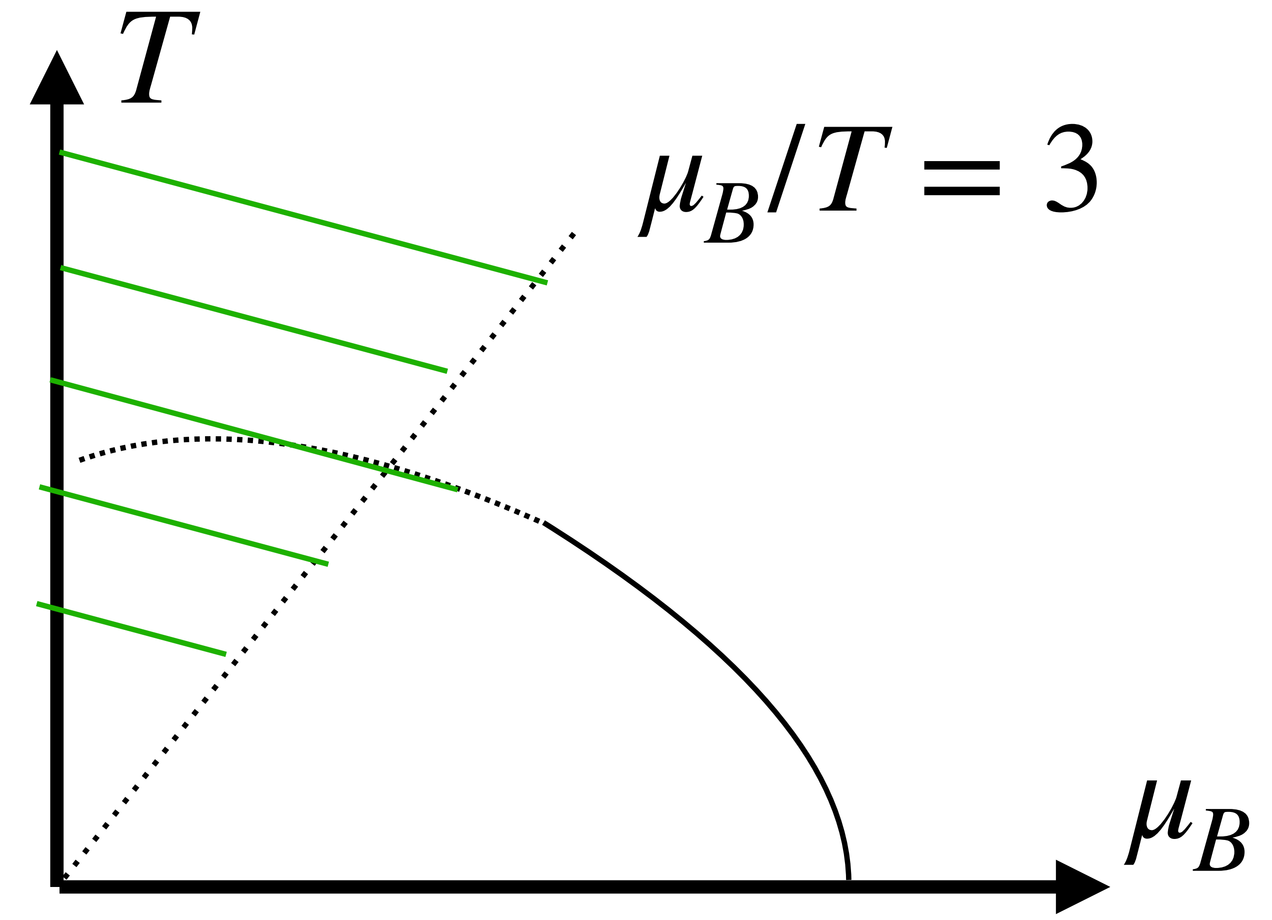


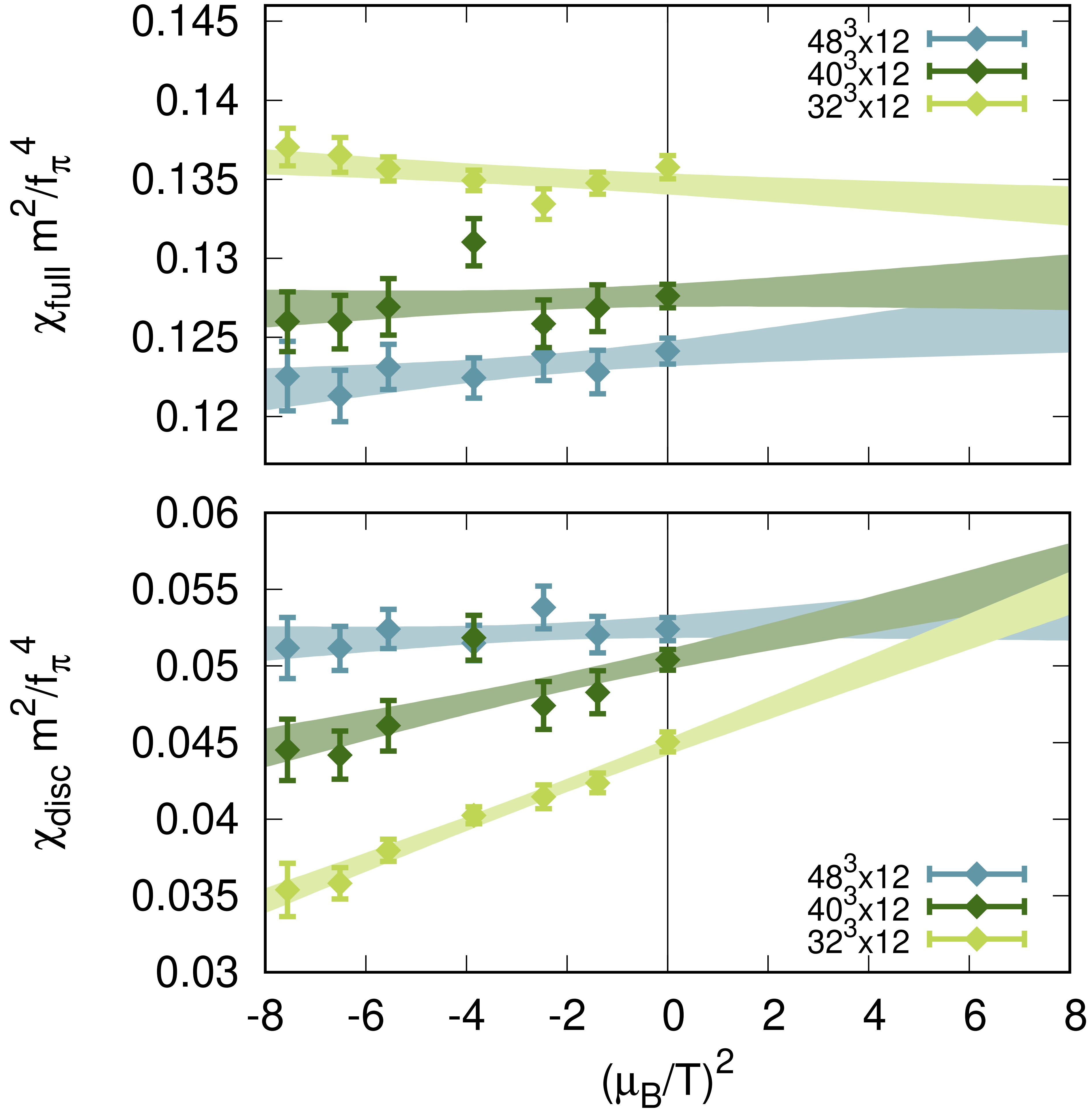
- $T_c^{(\chi^R_{\rm disc})}, T_c^{(\chi^R)}, T_c^{(<\bar{\psi}\psi>)}$ **increase** with the volume

- $T_c^{(S_Q)}, T_c^{(F_Q)}$ **decrease** with the volume

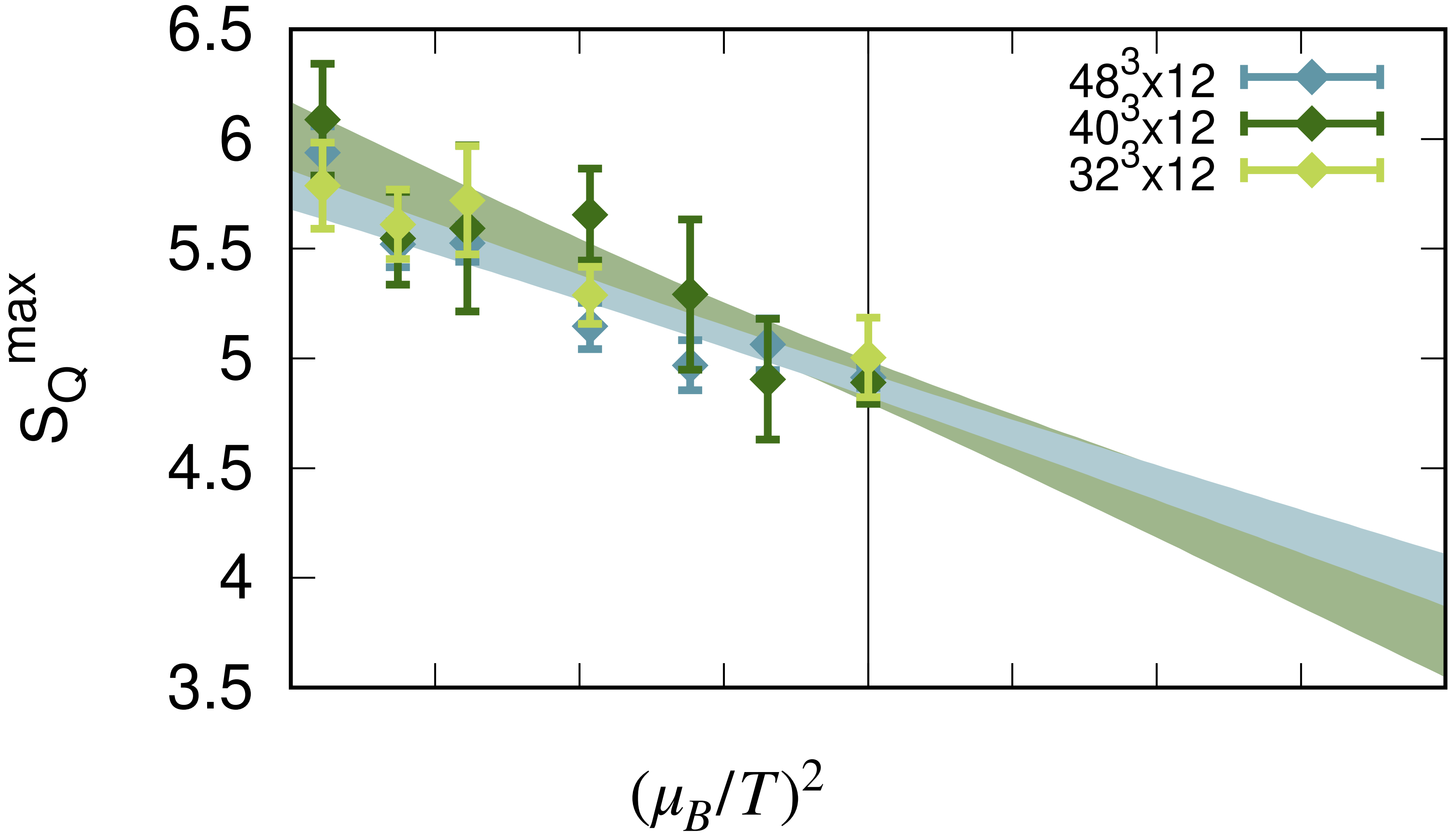
- $T_c^{(S_Q)}, T_c^{(F_Q)}$ have **milder volume effects** than $T_c^{(\chi^R_{\rm disc})}, T_c^{(\chi^R)}, T_c^{(<\bar{\psi}\psi>)}$

Volume effects at larger μ_B





chiral observables



deconfinement observable

no signs of CEP!

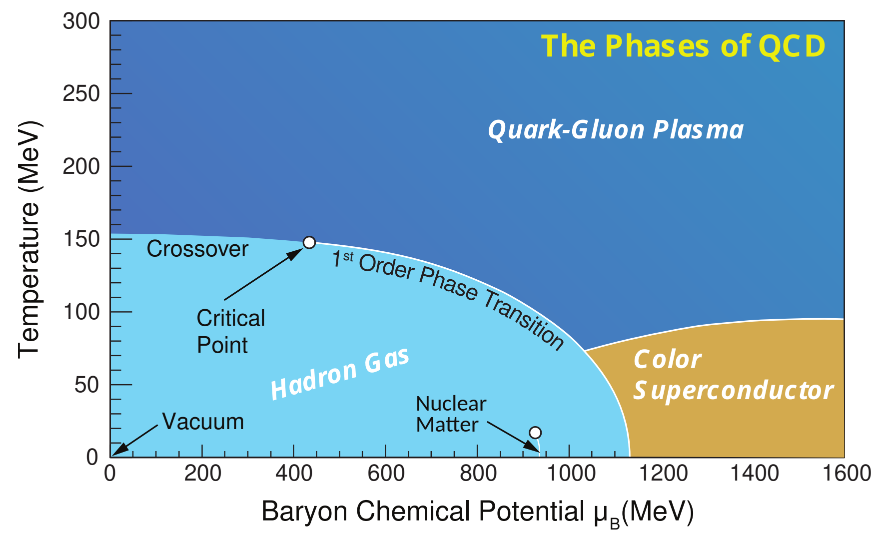
there are observables that have small volume effects

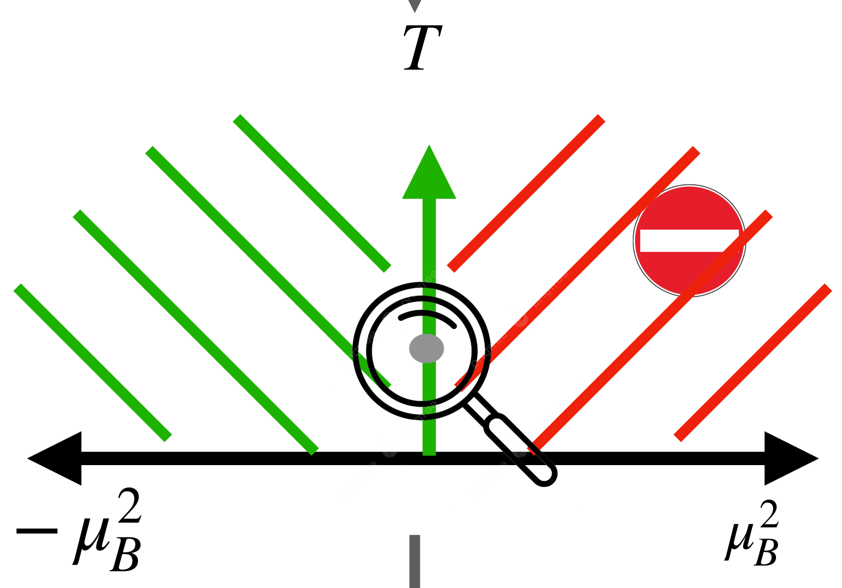
→ use them!

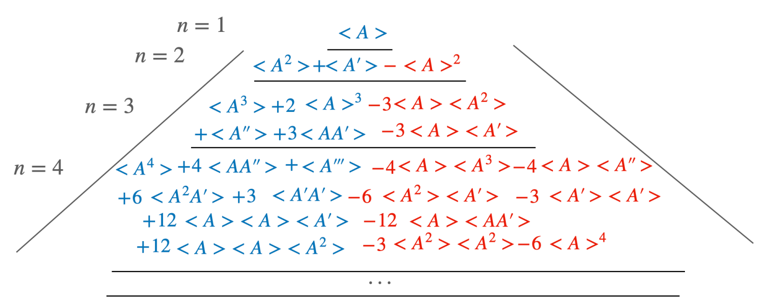
Coming to the last results

Going back over the path up here:

- Goal: explore phase diagram up to high μ_B
- $\rightarrow$ Sign problem: complex fermion determinant in MC simulations
- Chosen method: Taylor (no rooting ambiguities)
- Still cancellations problem! $\rightarrow$ we need small volumes
- For small volumes we can rely better on deconfinement observables
- Next step: QCD transition line up to μ_B=400 MeV from the peak position of S_Q in a $16^3 \times 8$ lattice.







arXiv:2410.06216

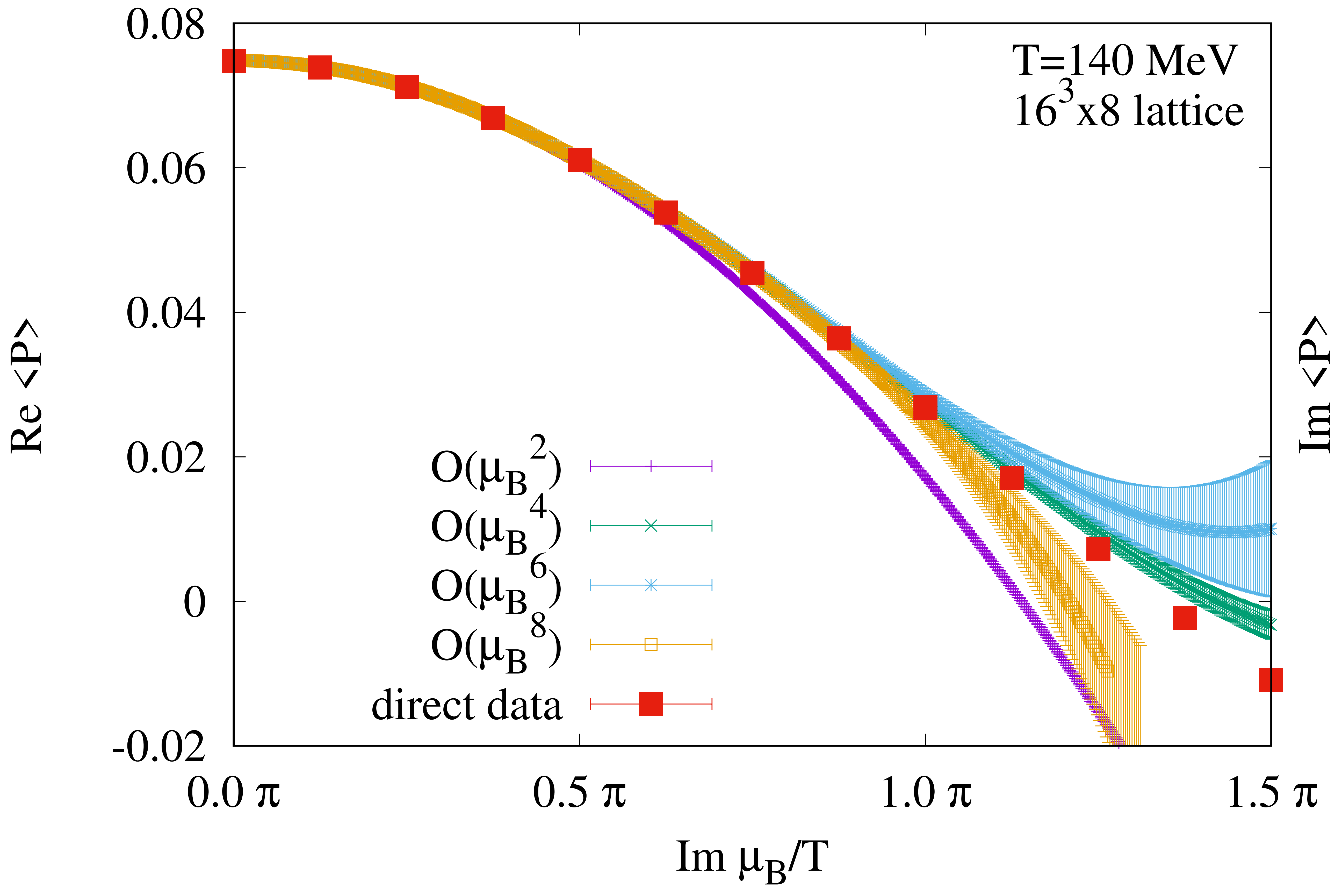
Results

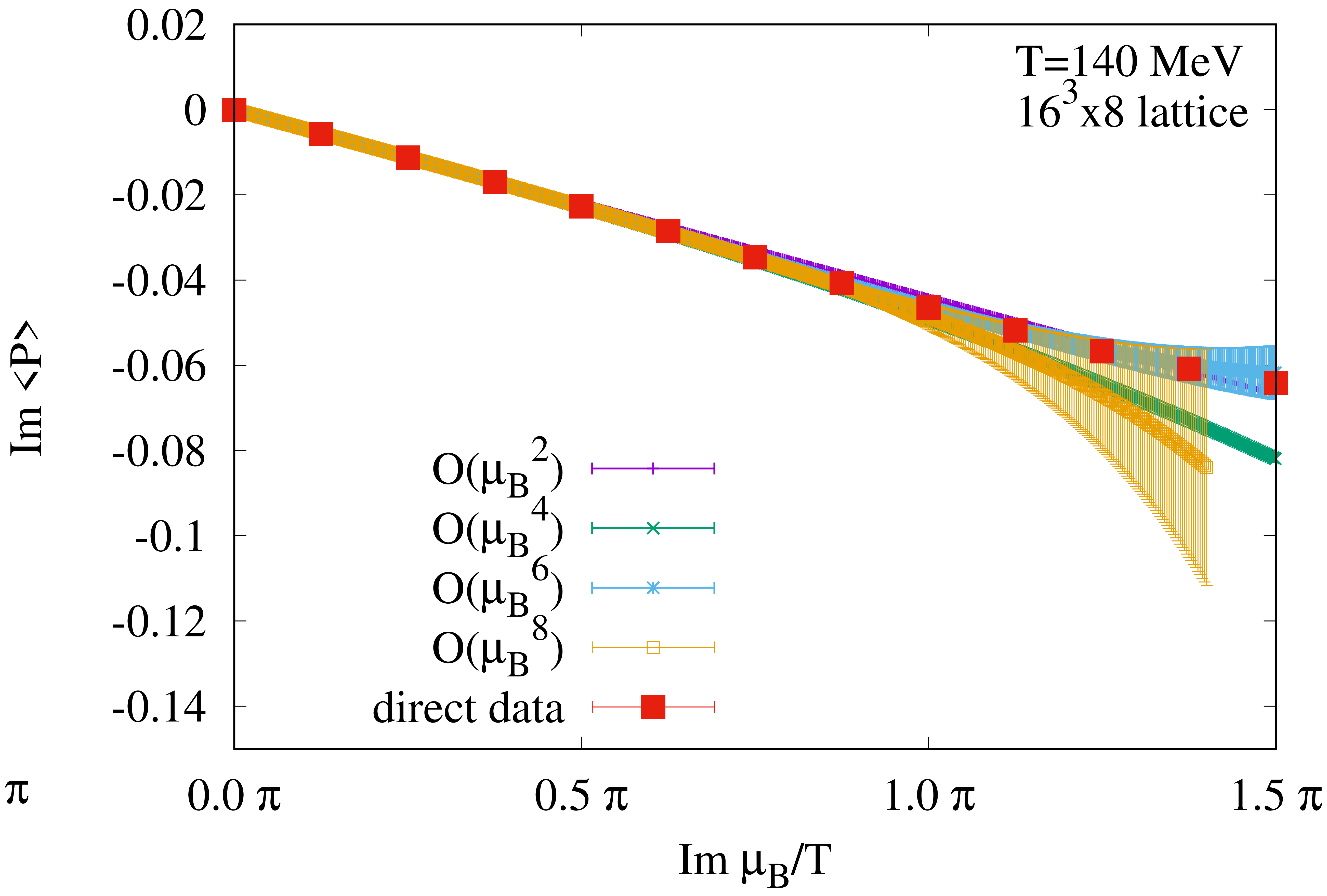
- First step: compute the derivatives of $Q = | < P > |^2$
- As said, it is difficult

$n = 6$

$$
\begin{aligned}
\partial_u^6 Q &= +2\langle P_R\rangle\langle F_u P_R\rangle + 20\langle P_R\rangle\langle C_u C_u P_R\rangle + 30\langle P_R\rangle\langle B_u D_u P_R\rangle + 30\langle P_R\rangle\langle B_u B_u B_u P_R\rangle + 12\langle P_R\rangle\langle A_u E_u P_R\rangle \\
&+120\langle P_R\rangle\langle A_u B_u C_u P_R\rangle + 30\langle P_R\rangle\langle A_u A_u D_u P_R\rangle + 90\langle P_R\rangle\langle A_u A_u B_u B_u P_R\rangle + 40\langle P_R\rangle\langle A_u A_u A_u C_u P_R\rangle \\
&+30\langle P_R\rangle\langle A_u A_u A_u A_u B_u P_R\rangle + 2\langle P_R\rangle\langle A_u A_u A_u A_u A_u A_u P_R\rangle - 20\langle C_u P_I\rangle\langle C_u P_I\rangle + 30\langle B_u P_R\rangle\langle D_u P_R\rangle \\
&+90\langle B_u P_R\rangle\langle B_u B_u P_R\rangle + 120\langle B_u P_R\rangle\langle A_u C_u P_R\rangle + 180\langle B_u P_R\rangle\langle A_u A_u B_u P_R\rangle - 12\langle A_u P_I\rangle\langle E_u P_I\rangle \\
&-120\langle A_u P_I\rangle\langle B_u C_u P_I\rangle - 60\langle A_u P_I\rangle\langle A_u D_u P_I\rangle - 180\langle A_u P_I\rangle\langle A_u B_u B_u P_I\rangle - 120\langle A_u P_I\rangle\langle A_u A_u C_u P_I\rangle \\
&-120\langle A_u P_I\rangle\langle A_u A_u A_u B_u P_I\rangle - 12\langle A_u P_I\rangle\langle A_u A_u A_u A_u A_u P_I\rangle - 120\langle A_u B_u P_I\rangle\langle C_u P_I\rangle - 180\langle A_u B_u P_I\rangle\langle A_u B_u P_I\rangle \\
&+30\langle A_u A_u P_R\rangle\langle D_u P_R\rangle + 90\langle A_u A_u P_R\rangle\langle B_u B_u P_R\rangle + 120\langle A_u A_u P_R\rangle\langle A_u C_u P_R\rangle + 180\langle A_u A_u P_R\rangle\langle A_u A_u B_u P_R\rangle \\
&+30\langle A_u A_u P_R\rangle\langle A_u A_u A_u A_u P_R\rangle - 40\langle A_u A_u A_u P_I\rangle\langle C_u P_I\rangle - 120\langle A_u A_u A_u P_I\rangle\langle A_u B_u P_I\rangle - 20\langle A_u A_u A_u P_I\rangle\langle A_u A_u A_u P_I\rangle \\
&+30\langle A_u A_u A_u A_u P_R\rangle\langle B_u P_R\rangle - 2\langle F_u\rangle\langle P_R\rangle\langle P_R\rangle - 60\langle D_u\rangle\langle P_R\rangle\langle B_u P_R\rangle - 60\langle D_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&+60\langle D_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 20\langle C_u C_u\rangle\langle P_R\rangle\langle P_R\rangle - 60\langle B_u\rangle\langle P_R\rangle\langle D_u P_R\rangle - 180\langle B_u\rangle\langle P_R\rangle\langle B_u B_u P_R\rangle \\
&-240\langle B_u\rangle\langle P_R\rangle\langle A_u C_u P_R\rangle - 360\langle B_u\rangle\langle P_R\rangle\langle A_u A_u B_u P_R\rangle - 60\langle B_u\rangle\langle P_R\rangle\langle A_u A_u A_u A_u P_R\rangle - 180\langle B_u\rangle\langle B_u P_R\rangle\langle B_u P_R\rangle \\
&+240\langle B_u\rangle\langle A_u P_I\rangle\langle C_u P_I\rangle + 720\langle B_u\rangle\langle A_u P_I\rangle\langle A_u B_u P_I\rangle + 240\langle B_u\rangle\langle A_u P_I\rangle\langle A_u A_u A_u P_I\rangle - 360\langle B_u\rangle\langle A_u A_u P_R\rangle\langle B_u P_R\rangle \\
&-180\langle B_u\rangle\langle A_u A_u P_R\rangle\langle A_u A_u P_R\rangle - 30\langle B_u D_u\rangle\langle P_R\rangle\langle P_R\rangle - 180\langle B_u B_u\rangle\langle P_R\rangle\langle B_u P_R\rangle - 180\langle B_u B_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&+180\langle B_u B_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 30\langle B_u B_u B_u\rangle\langle P_R\rangle\langle P_R\rangle - 12\langle A_u E_u\rangle\langle P_R\rangle\langle P_R\rangle - 240\langle A_u C_u\rangle\langle P_R\rangle\langle B_u P_R\rangle \\
&-240\langle A_u C_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle + 240\langle A_u C_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 120\langle A_u B_u C_u\rangle\langle P_R\rangle\langle P_R\rangle - 60\langle A_u A_u\rangle\langle P_R\rangle\langle D_u P_R\rangle \\
&-180\langle A_u A_u\rangle\langle P_R\rangle\langle B_u B_u P_R\rangle - 240\langle A_u A_u\rangle\langle P_R\rangle\langle A_u C_u P_R\rangle - 360\langle A_u A_u\rangle\langle P_R\rangle\langle A_u A_u B_u P_R\rangle \\
&-60\langle A_u A_u\rangle\langle P_R\rangle\langle A_u A_u A_u A_u P_R\rangle - 180\langle A_u A_u\rangle\langle B_u P_R\rangle\langle B_u P_R\rangle + 240\langle A_u A_u\rangle\langle A_u P_I\rangle\langle C_u P_I\rangle \\
&+720\langle A_u A_u\rangle\langle A_u P_I\rangle\langle A_u B_u P_I\rangle + 240\langle A_u A_u\rangle\langle A_u P_I\rangle\langle A_u A_u A_u P_I\rangle - 360\langle A_u A_u\rangle\langle A_u A_u P_R\rangle\langle B_u P_R\rangle \\
&-180\langle A_u A_u\rangle\langle A_u A_u P_R\rangle\langle A_u A_u P_R\rangle - 30\langle A_u A_u D_u\rangle\langle P_R\rangle\langle P_R\rangle - 360\langle A_u A_u B_u\rangle\langle P_R\rangle\langle B_u P_R\rangle \\
&-360\langle A_u A_u B_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle + 360\langle A_u A_u B_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 90\langle A_u A_u B_u B_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&-40\langle A_u A_u A_u C_u\rangle\langle P_R\rangle\langle P_R\rangle - 60\langle A_u A_u A_u A_u\rangle\langle P_R\rangle\langle B_u P_R\rangle - 60\langle A_u A_u A_u A_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&+60\langle A_u A_u A_u A_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 30\langle A_u A_u A_u A_u B_u\rangle\langle P_R\rangle\langle P_R\rangle - 2\langle A_u A_u A_u A_u A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&+90\langle B_u\rangle\langle D_u\rangle\langle P_R\rangle\langle P_R\rangle + 540\langle B_u\rangle\langle B_u\rangle\langle P_R\rangle\langle B_u P_R\rangle + 540\langle B_u\rangle\langle B_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&-540\langle B_u\rangle\langle B_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle + 270\langle B_u\rangle\langle B_u B_u\rangle\langle P_R\rangle\langle P_R\rangle + 360\langle B_u\rangle\langle A_u C_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&+540\langle B_u\rangle\langle A_u A_u B_u\rangle\langle P_R\rangle\langle P_R\rangle + 90\langle A_u A_u\rangle\langle D_u\rangle\langle P_R\rangle\langle P_R\rangle + 1080\langle A_u A_u\rangle\langle B_u\rangle\langle P_R\rangle\langle B_u P_R\rangle \\
&+1080\langle A_u A_u\rangle\langle B_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle - 1080\langle A_u A_u\rangle\langle B_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle + 270\langle A_u A_u\rangle\langle B_u B_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&+360\langle A_u A_u\rangle\langle A_u C_u\rangle\langle P_R\rangle\langle P_R\rangle + 540\langle A_u A_u\rangle\langle A_u A_u\rangle\langle P_R\rangle\langle B_u P_R\rangle + 540\langle A_u A_u\rangle\langle A_u A_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&-540\langle A_u A_u\rangle\langle A_u A_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle + 540\langle A_u A_u\rangle\langle A_u A_u B_u\rangle\langle P_R\rangle\langle P_R\rangle + 90\langle A_u A_u\rangle\langle A_u A_u A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&+90\langle A_u A_u A_u A_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle - 360\langle B_u\rangle\langle B_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle - 1080\langle A_u A_u\rangle\langle B_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&-1080\langle A_u A_u\rangle\langle A_u A_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle - 360\langle A_u A_u\rangle\langle A_u A_u\rangle\langle A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle \\
\partial_u^8 Q &= \text{has 405 terms}
\end{aligned}
$$

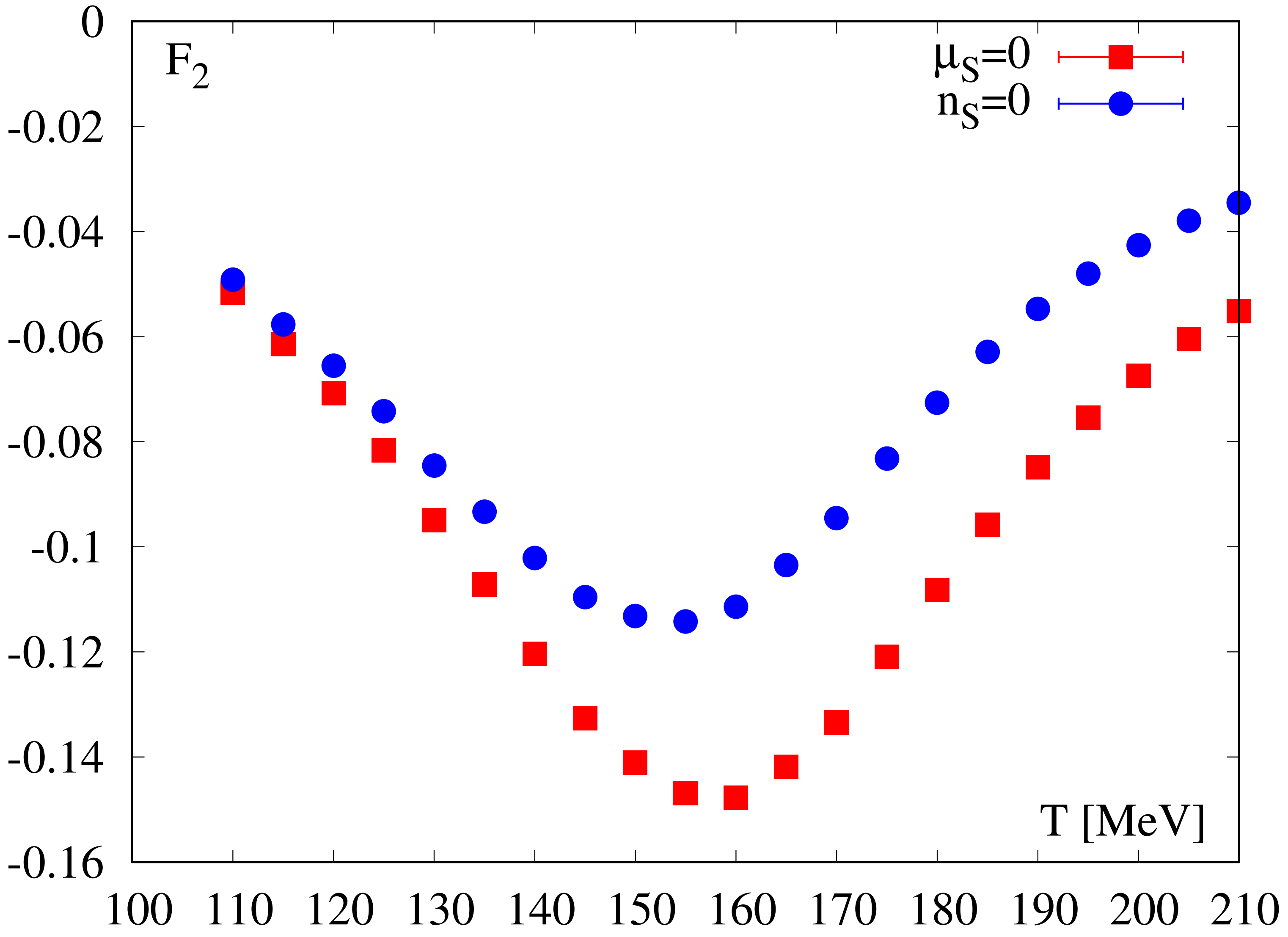
Is the expansion converging well?

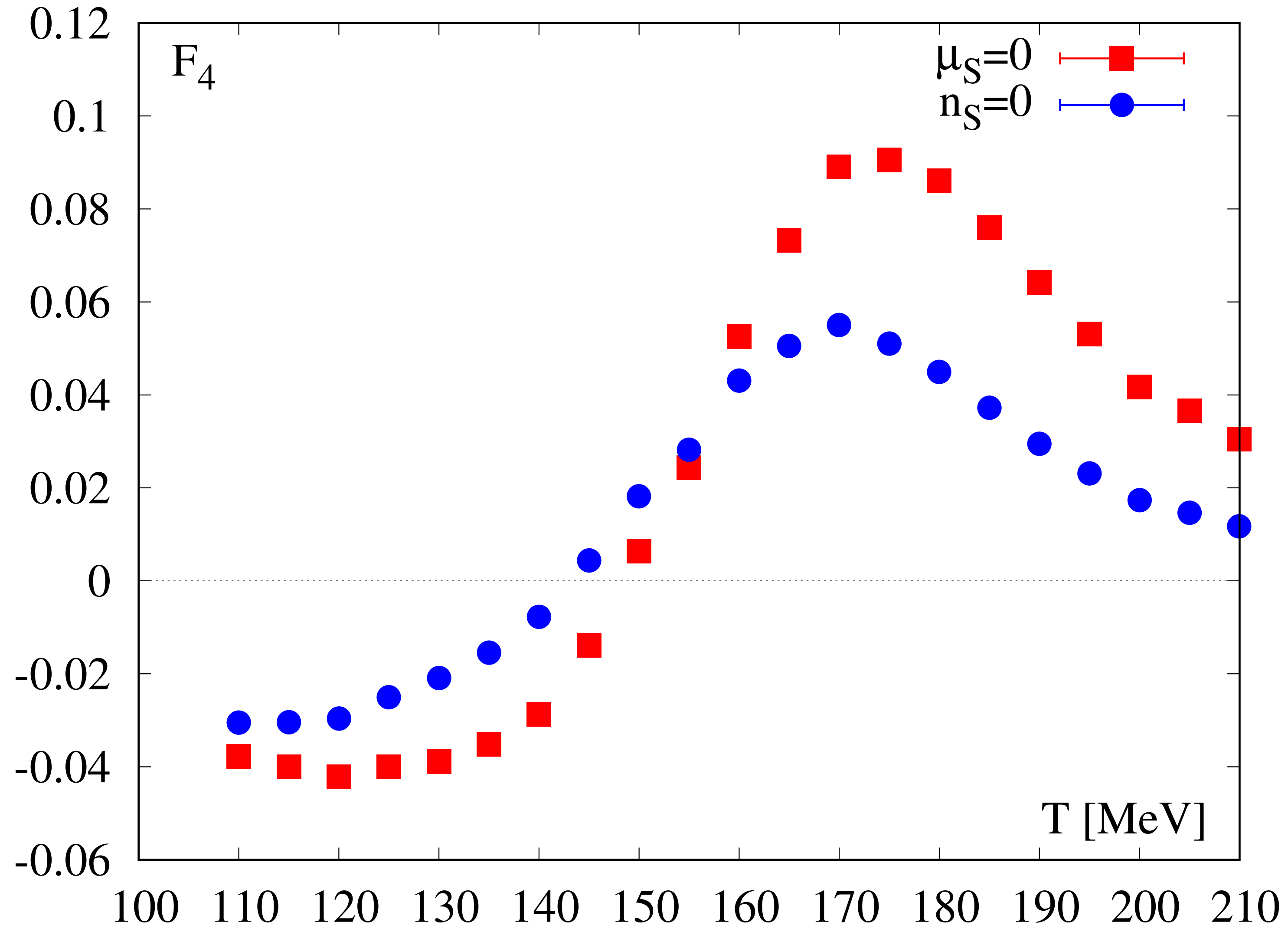


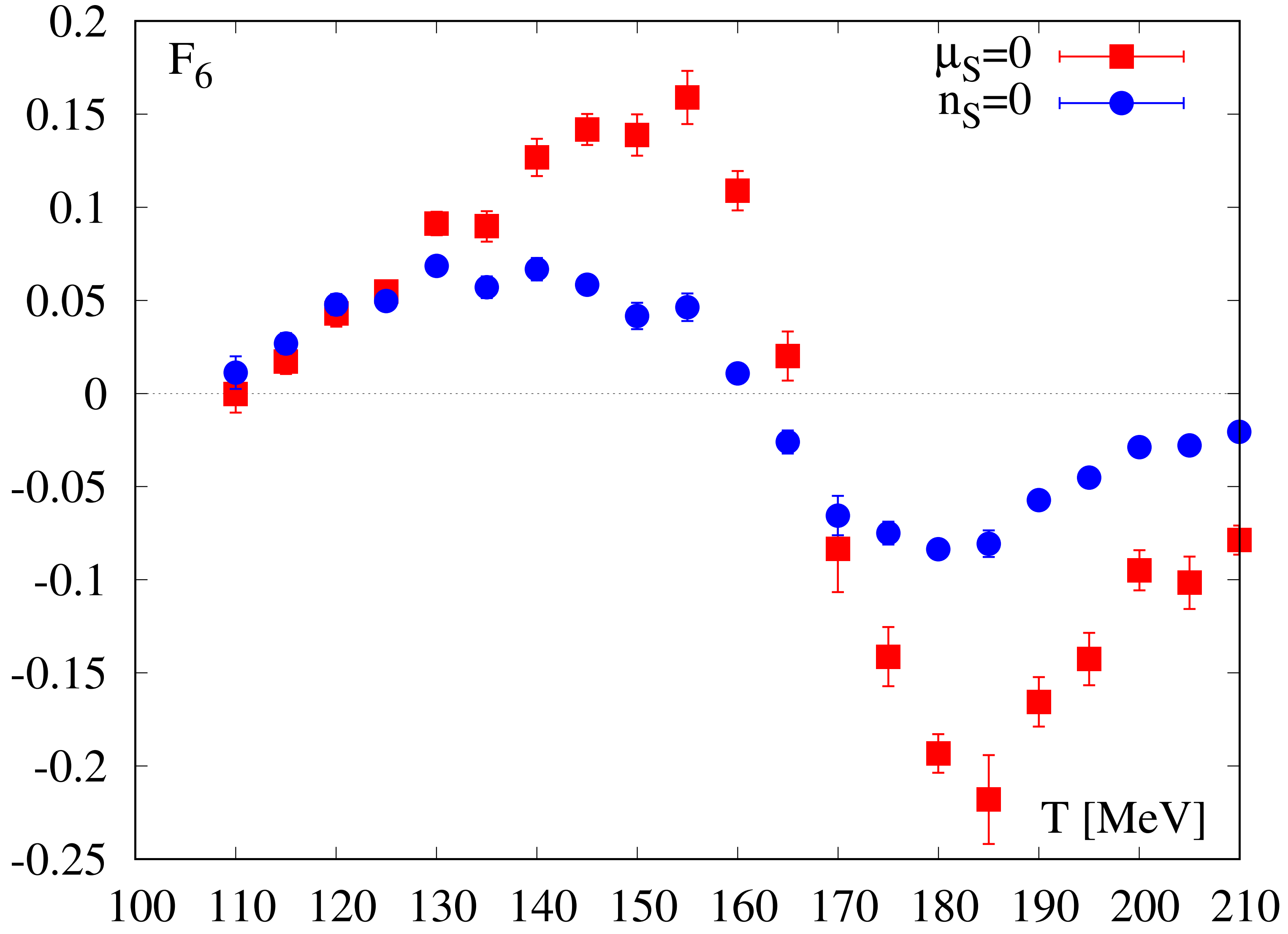


Results from previous reference up to order 2:
D'Elia, arXiv 1907.09461

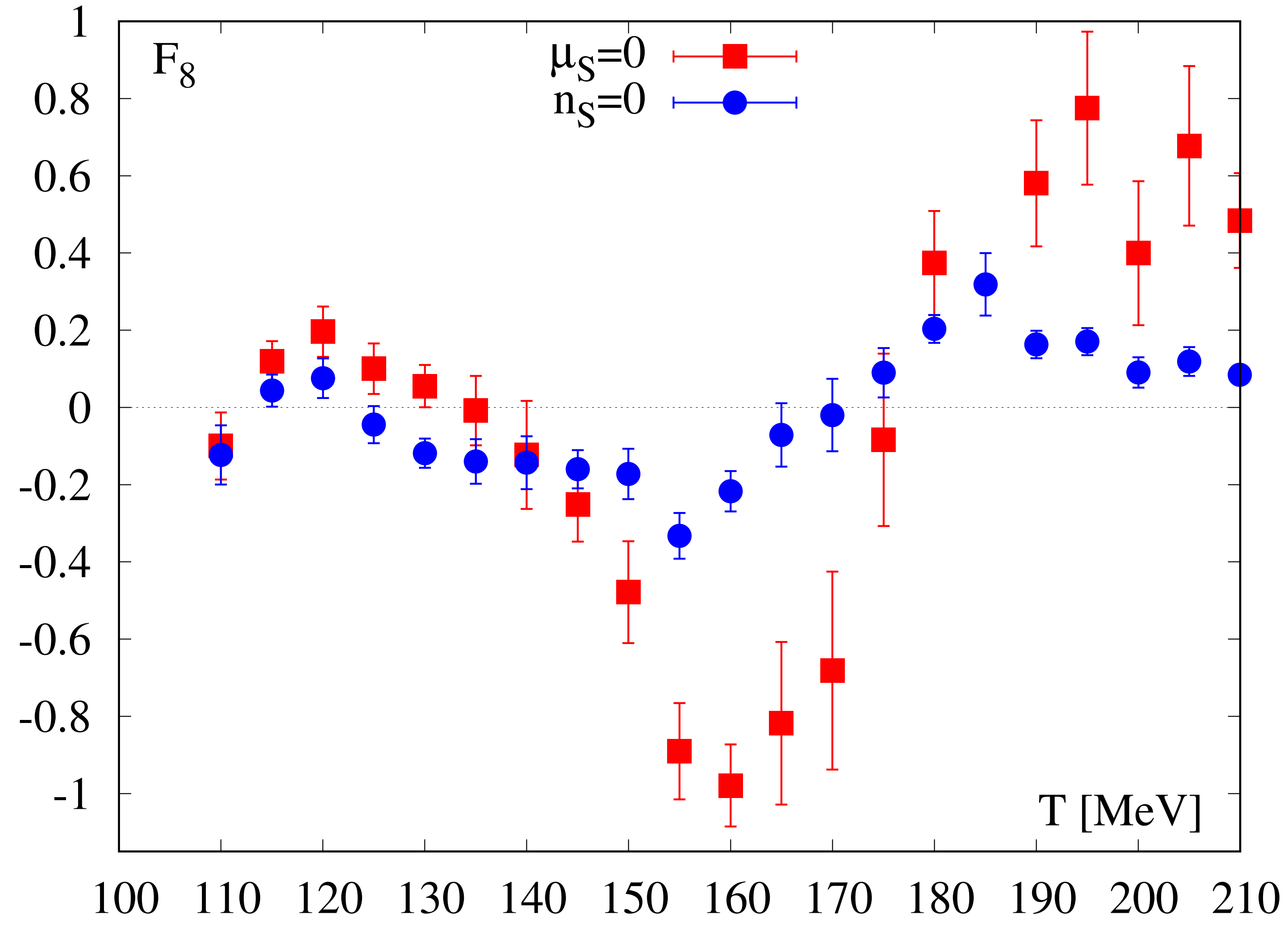
F_Q coefficients



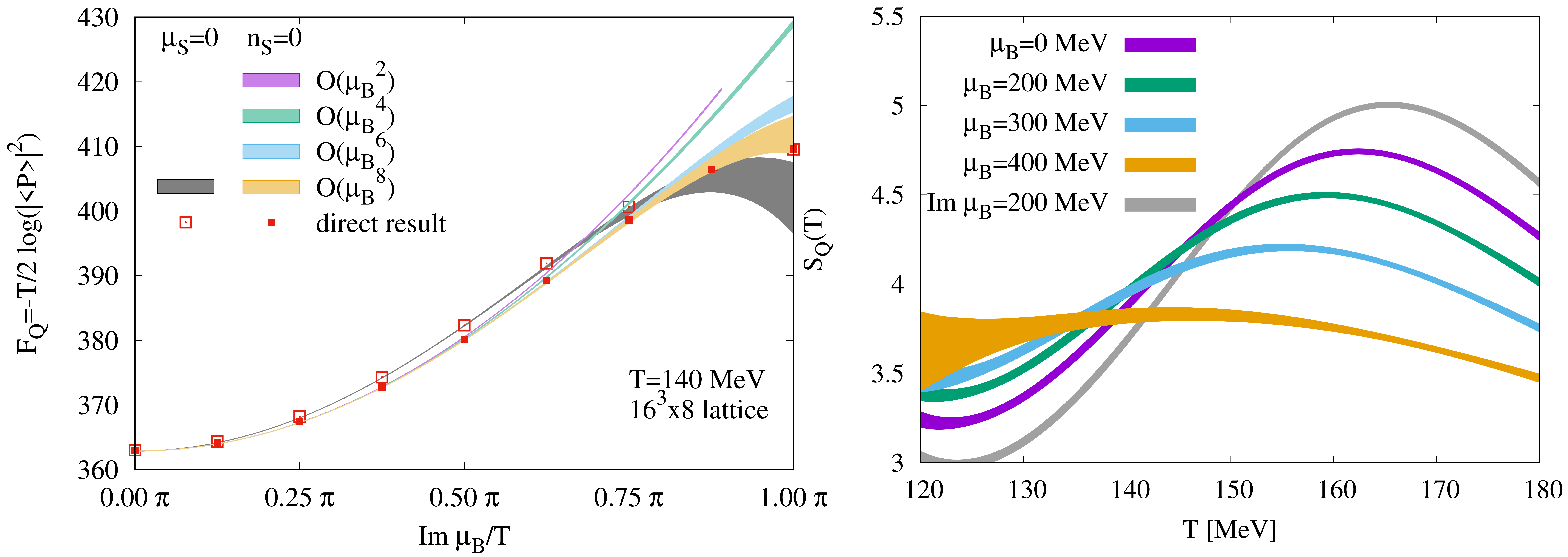




- $\mu_S = 0$ vs $n_S = 0$: null strangeness-chemical potential vs strangeness neutrality
- interpolate the coefficients, then **for each** μ_B compute $F_Q(\mu_B)$
- $$F_Q(T,\mu_B) = F_Q(T,0) + \sum_{n=2,4,\ldots,8} \frac{F_n(T)\hat{\mu}_B^n}{n!}$$



From F_Q to S_Q

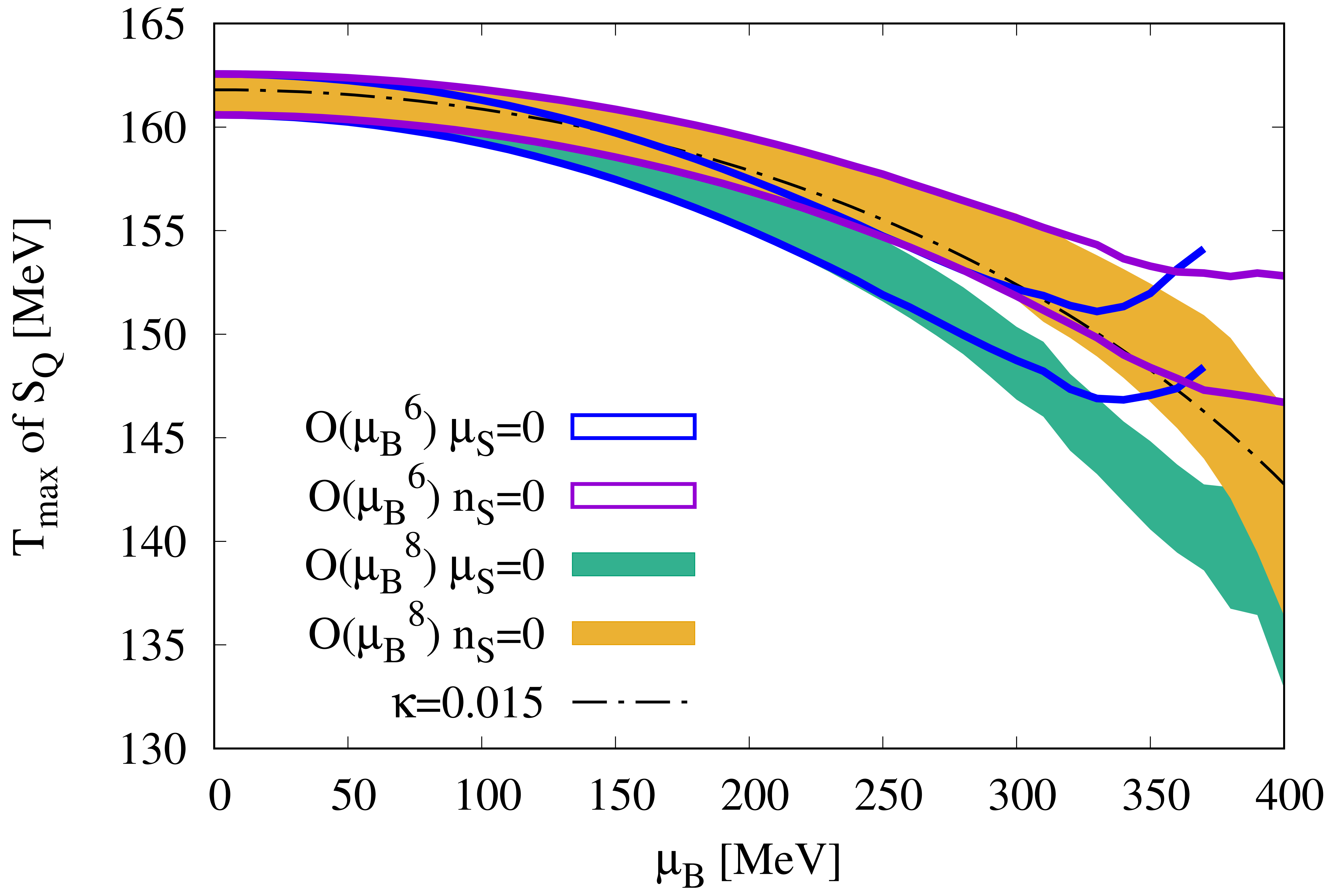


- From the interpolations of F_Q: $S_Q(T, \mu_B = \text{fixed}) = -\dfrac{\partial F_Q(T, \mu_B = \text{fixed})}{\partial T}$

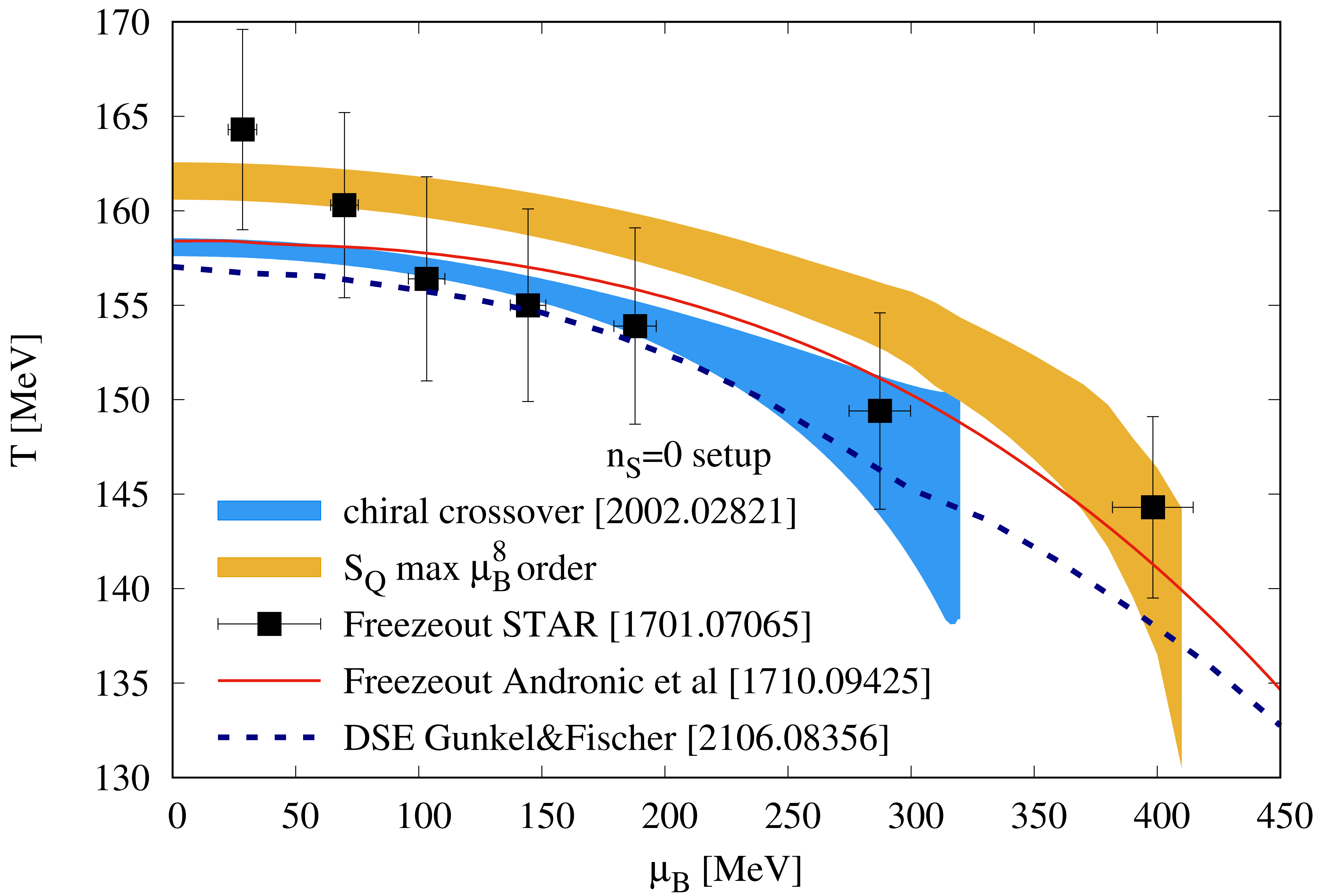
Phase diagrams to different orders in Taylor expansion

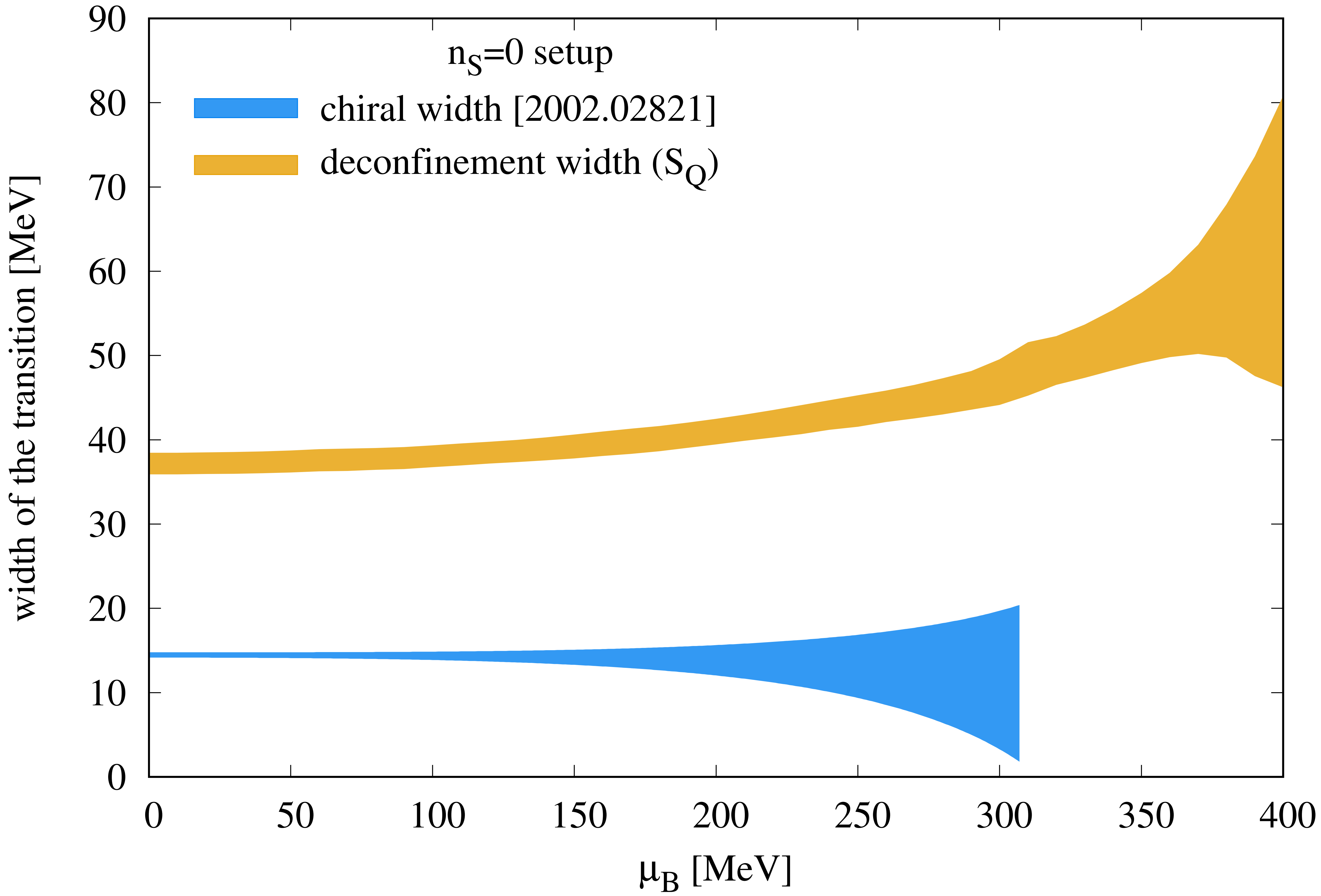
- 8th order negligible up to $T \leq 300$ MeV for $n_S = 0$ and $T \leq 250$ MeV for $\mu_S = 0$

- 1 sigma errorbars of 8th and 6th order touch at $T \sim 400$ MeV for $n_S = 0$ and $T \sim 330$ MeV for $\mu_S = 0$



And, the CEP?

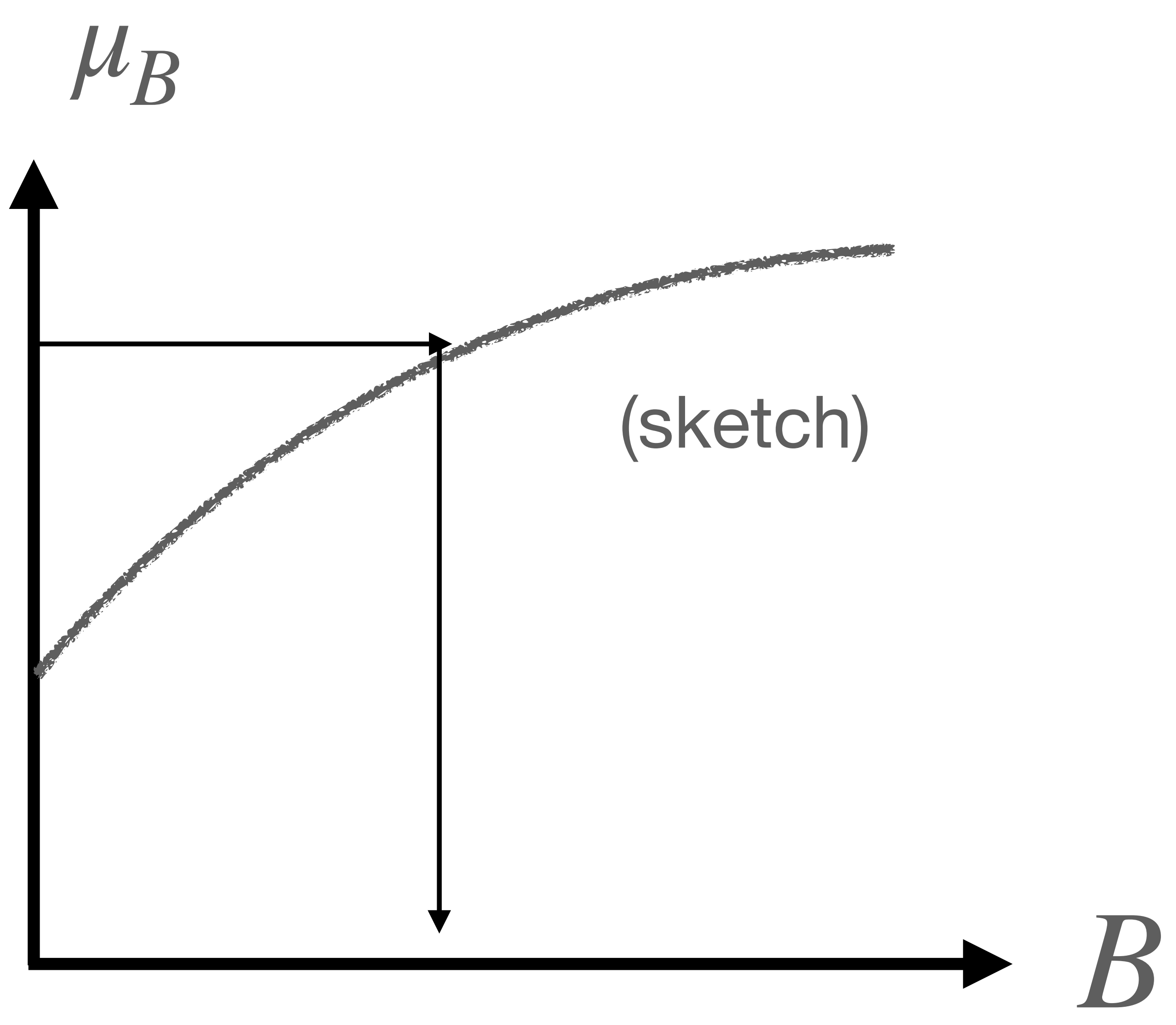




- deconfinement width increases in μ_B
- no sign of CEP up to these μ_B values

What next?

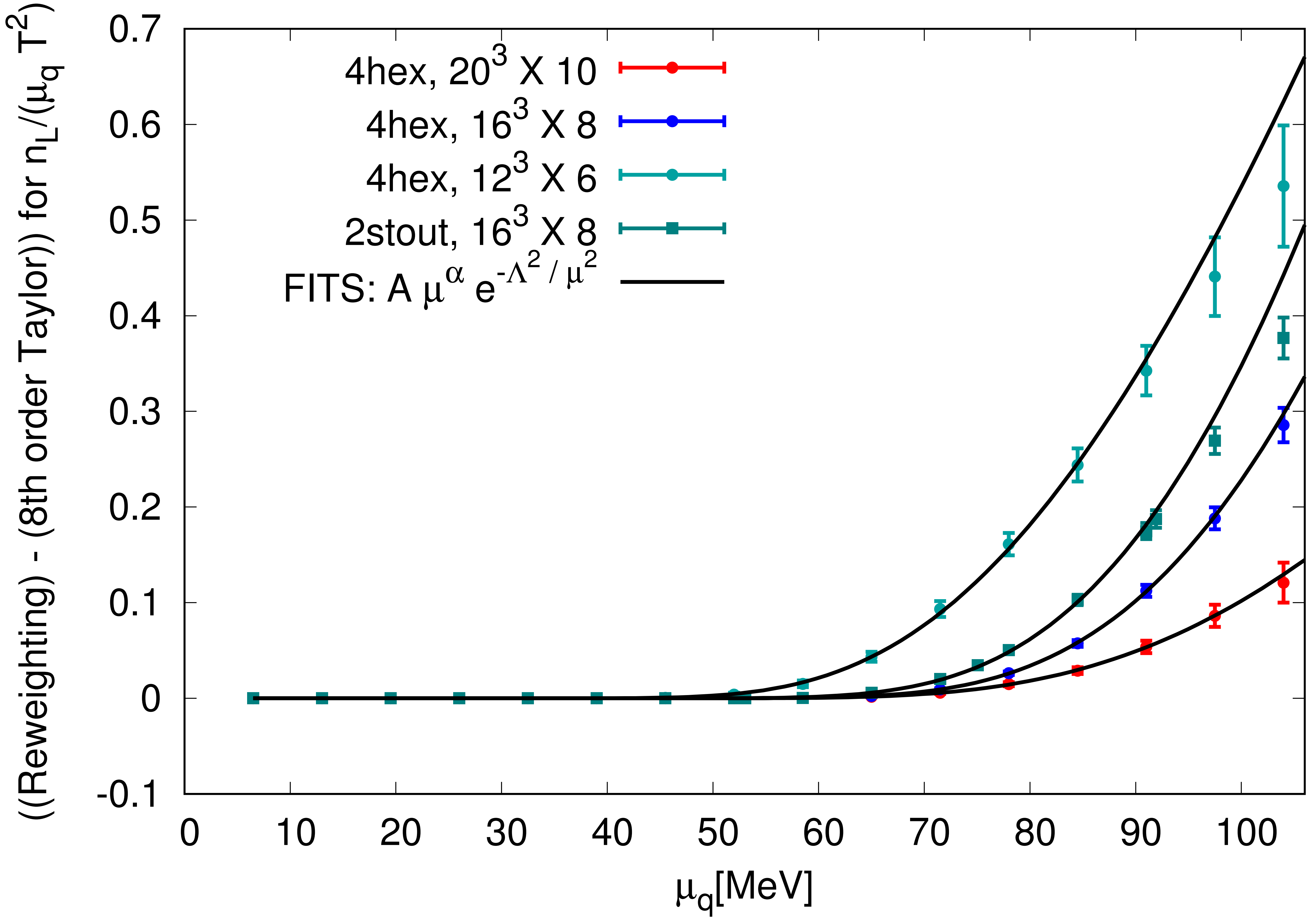
- Go on with Taylor up to order…? Which order?
- The truncation error in μ_B is uncontrolled. We can't really know if at $\mu_B = 400$ MeV we should stop at 8th order, or at 10th, or at 12th: it depends only on the available statistics
    - In a canonical formulation that would be controlled
    - Baryon number B is fixed, μ_B is computed
    - Several attempts in the past by various collaborations: arXiv:0507020, 0602024, 0906.1088,…
    - Hope to have some new results for that by Quark Matter 2025!!



Backup slides

Reweighting for staggered fermions

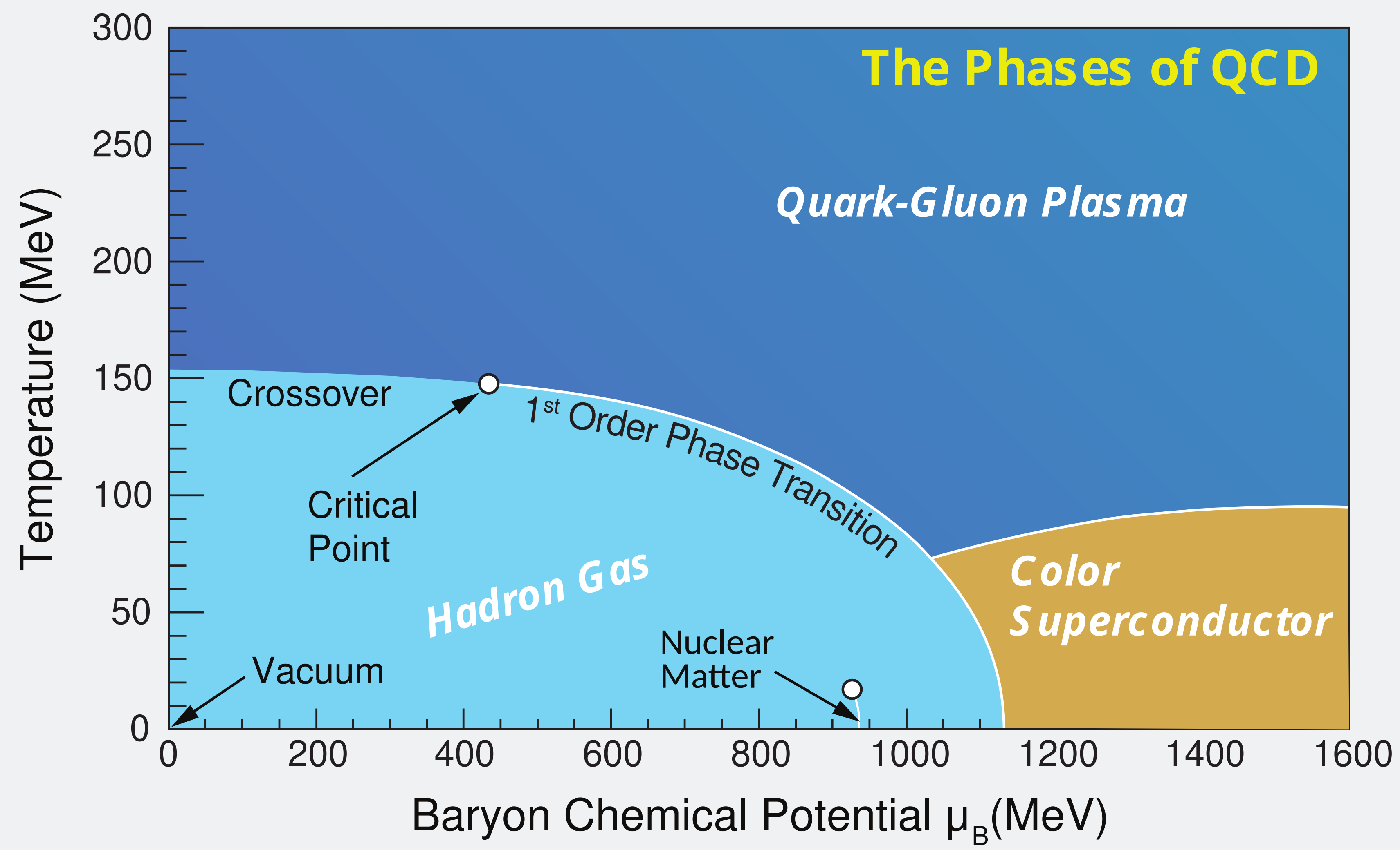
- Difference between between the full reweighted result and the 8th-order Taylor expansion



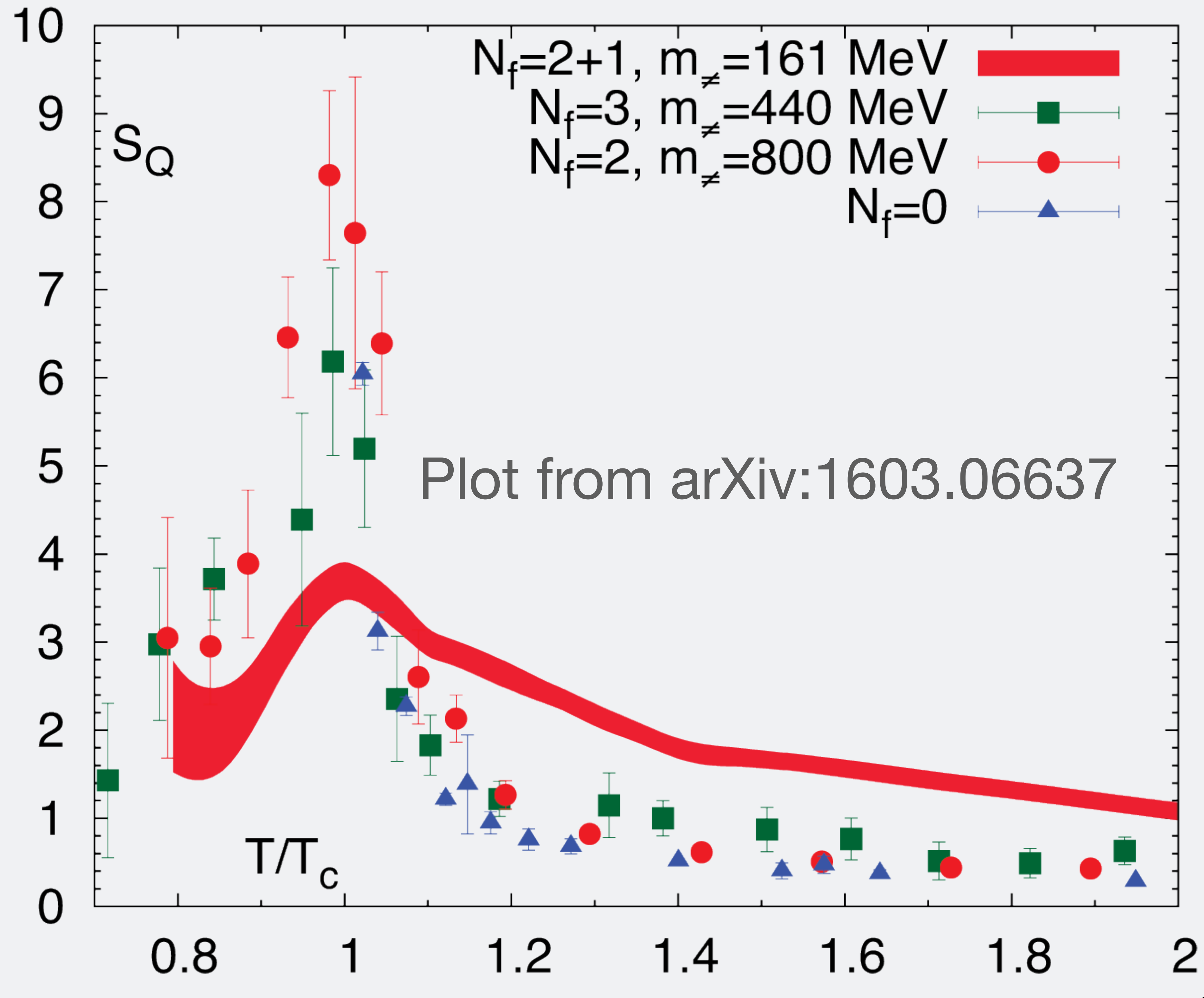
- We are interested in the **deconfinement transition**
- **Order parameter:** Polyakov loop $\mathbf{P} \sim e^{-F_Q/T}$
- Problem: **scheme-dependent**
- $P^{\text{ren}} \sim c_P P^{\text{bare}}$, where c_P depends on the scheme
- $F_Q = -T \log P \sim F_Q^{bare} - c_F \rightarrow c_F$ depends on the scheme, but the inflection point does not
- **Better** way to find deconfinement temperature $\mathbf{T_c}$: **peak of** static quark **entropy**

$$\mathbf{S_Q} = -\frac{\partial F_Q(T)}{\partial T}$$

first used in arXiv:1603.06637, first used by this collaboration in 2405.12320



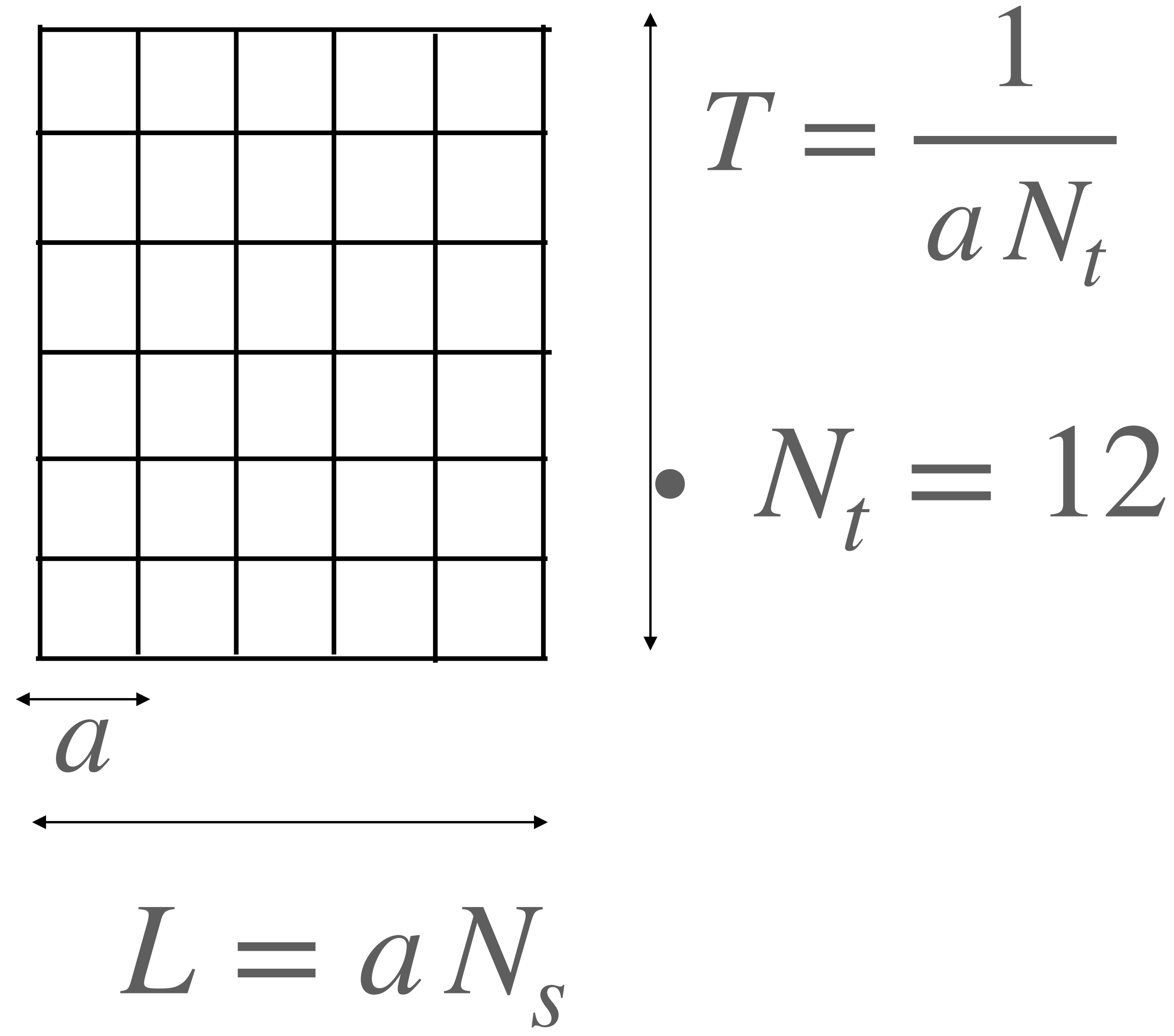
Plot from arXiv:1906.00936



Plot from arXiv:1603.06637

Lattice setup for 2405.12320

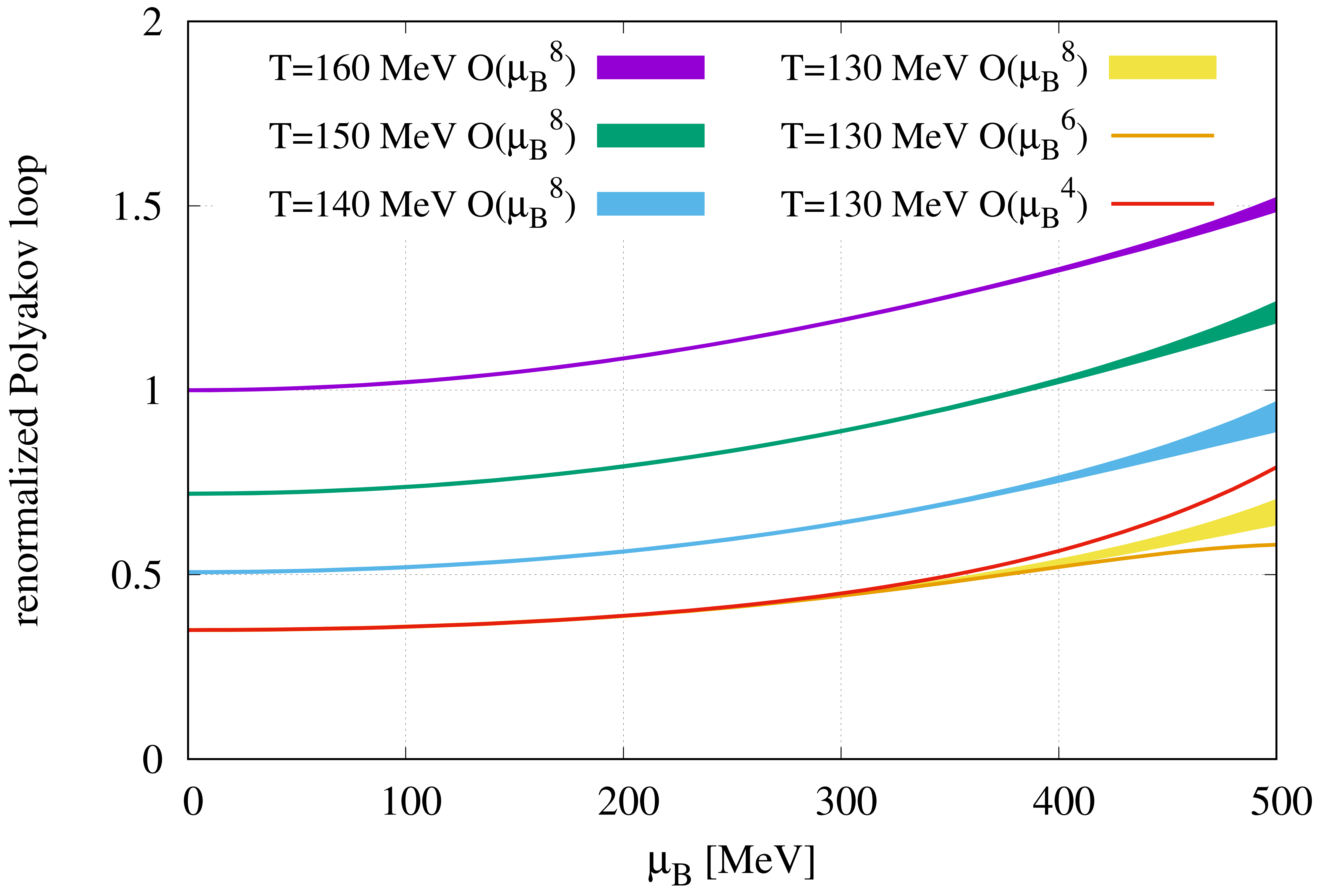
- tree-level Symanzik improved gauge action
- $N_f = 2 + 1 + 1$ **staggered fermions** with 4stout smearing
- Details in Phys. Rev., D92(11):114505, 2015
- **simulations** at **imaginary** $\mu_\mathbf{B}$ → extrapolations to real values
- For $N_s = 32, 40\,, 48$ simulations also at $\mathrm{Im}\frac{\mu_\mathrm{B}}{\mathrm{T}}\frac{\pi}{8}$ = 0, 3, 4, 5, 6, 6.5, 7
- **strangeness neutrality** setting: $< N_S > = \ 0$

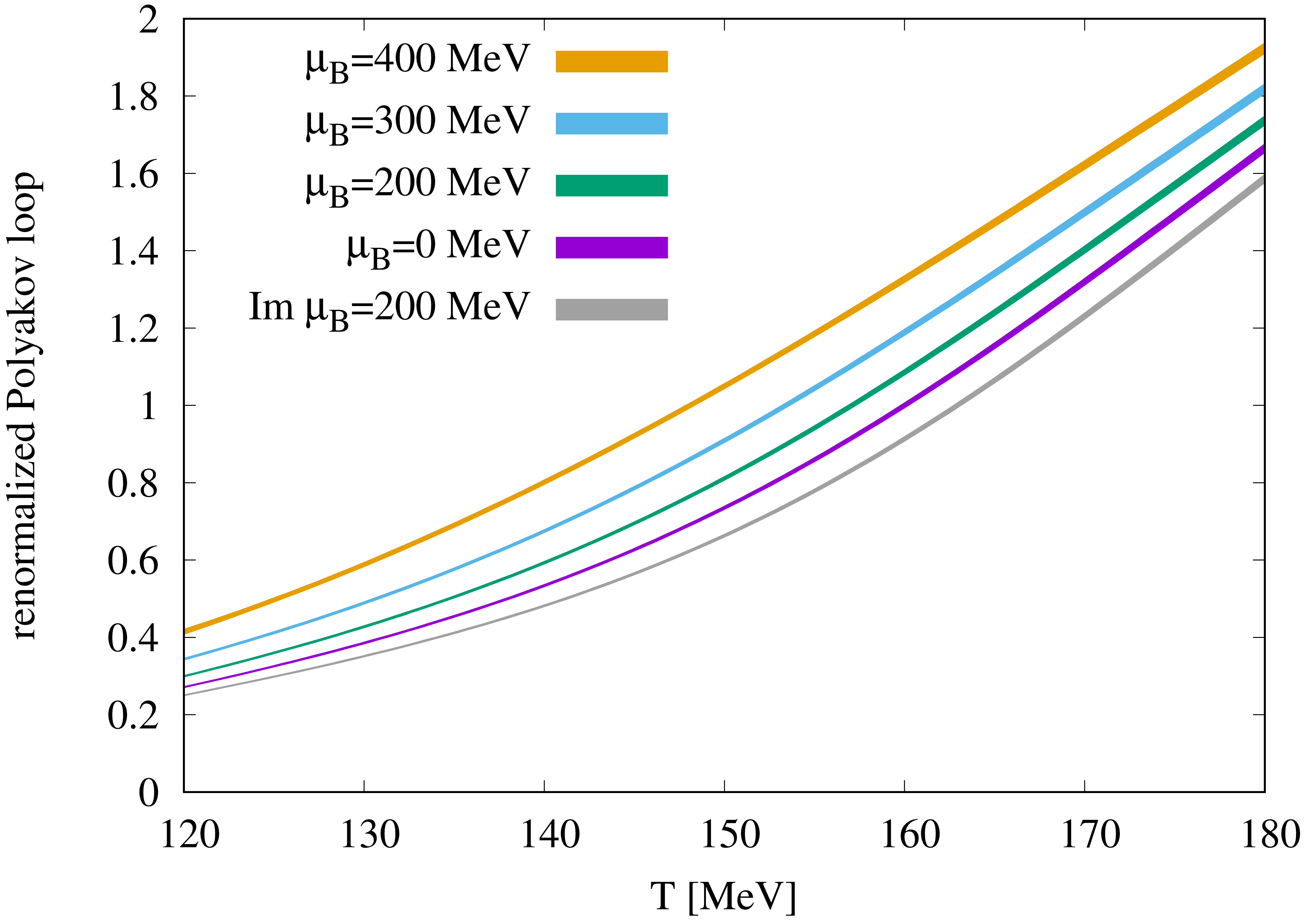


- $N_t = 12$
- N_s = 20, 24, 28, 32, 40, 48, 64 $(\mu_B = 0)$

Statistics for 2410.06216

T [MeV]	β	m_l	m_s	# configs
110	0.5236	0.00432111	0.1193920	410816
115	0.5406	0.00409845	0.1132400	1036373
120	0.5560	0.00390982	0.1080280	1080141
125	0.5700	0.00374705	0.1035310	1500967
130	0.5829	0.00360381	0.0995733	1887321
135	0.5947	0.00347548	0.0960274	1216195
140	0.6056	0.00335869	0.0928007	1912628
145	0.6158	0.00325107	0.0898270	1383987
150	0.6252	0.00315088	0.0870590	1338744
155	0.6341	0.00305689	0.0844619	1005178
160	0.6425	0.00296817	0.0820105	2215412
165	0.6504	0.00288403	0.0796857	1596043
170	0.6579	0.00280394	0.0774727	595253
175	0.6651	0.00272748	0.0753604	1131649
180	0.6719	0.00265434	0.0733394	1240884
185	0.6785	0.00258424	0.0714026	436002
190	0.6848	0.00251696	0.0695436	317895
195	0.6909	0.00245231	0.0677573	361870
200	0.6968	0.00239013	0.0660393	323968
205	0.7025	0.00233028	0.0643856	158703
210	0.7080	0.00227263	0.0627928	260064


T=160 MeV O(μB8)
T=150 MeV O(μB8)
T=140 MeV O(μB8)
T=130 MeV O(μB8)
T=130 MeV O(μB6)
T=130 MeV O(μB4)
renormalized Polyakov loop
μB [MeV]
0
0.5
1
1.5
2
0
100
200
300
400
500


μB=400 MeV
μB=300 MeV
μB=200 MeV
μB=0 MeV
Im μB=200 MeV
renormalized Polyakov loop
T [MeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
120
130
140
150
160
170
180