Rotating gluodynamics and QCD: sign problem, mixed inhomogeneous phase and moment of inertia

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in collaboration with

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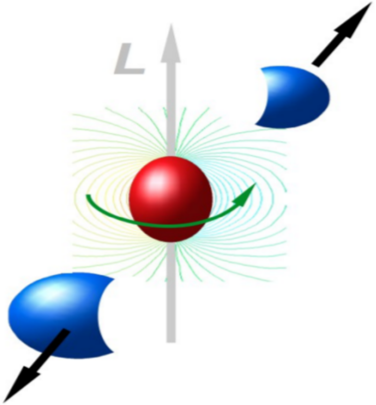

Introduction

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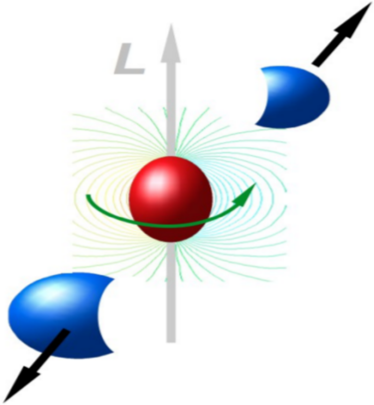
- The rotation occurs with relativistic velocities.



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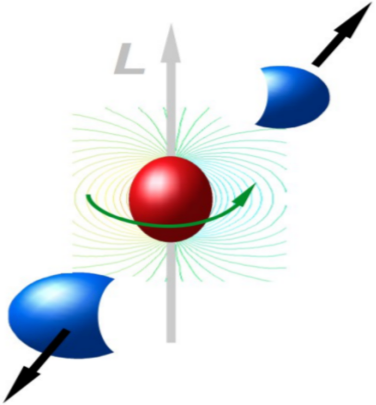

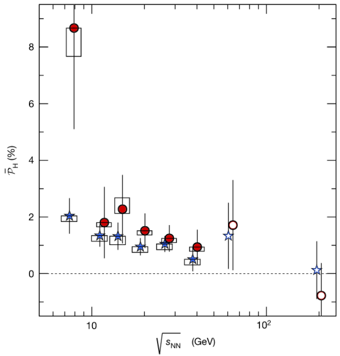

$\omega = 10$ MeV $\sim 0.05\ \mathrm{fm}^{-1}$ $\quad v \sim c$ at $r \sim 20$ fm

[L. Adamczyk et al. (STAR), Nature **548**, 62–65 (2017), arXiv:1701.06657 [nucl-ex]]

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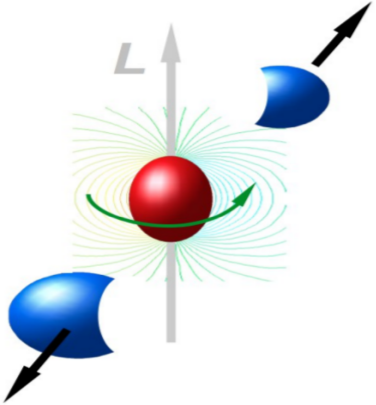

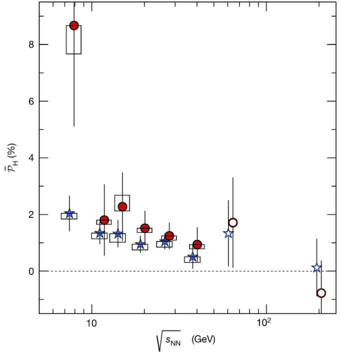

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$\langle\omega\rangle \sim 7$ MeV ($\sqrt{s_{NN}}$-averaged)

- How does the rotation affect QCD properties?

Lattice study of rotating QCD properties

Formulation of rotating QCD on the lattice

- A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]

Bulk-averaged critical temperature in rotating gluodynamics:

- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, JETP Lett. **112**, 6–12 (2020)
- V. Braguta, A. Kotov, D. Kuznedelev, and A. Roenko, Phys. Rev. D **103**, 094515 (2021), arXiv:2102.05084 [hep-lat]

Bulk-averaged critical temperature in rotating QCD:

- V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]
- J.-C. Yang and X.-G. Huang, (2023), arXiv:2307.05755 [hep-lat]

Thermodynamical properties and moment of inertia of rotating gluon plasma:

- V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]
- V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)
- V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]

Mixed inhomogeneous phase in rotating gluon plasma:

- V. V. Braguta, M. N. Chernodub, and A. A. Roenko, Phys. Lett. B **855**, 138783 (2024), arXiv:2312.13994 [hep-lat]
- V. V. Braguta, M. N. Chernodub, Y. A. Gershtein, and A. A. Roenko, (2024), arXiv:2411.15085 [hep-lat]

Rotating QCD in Minkowksi space

It is convenient to describe the system in the co-rotating reference frame, $x^\mu = (t, x, y, z)$,

$$\varphi = [\varphi_{\rm lab} - \Omega t]_{2\pi}\,, \quad t = t_{\rm lab}, \quad z = z_{\rm lab}, \quad r = r_{\rm lab}, \tag{1}$$

with the metric

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{2}$$

The Dirac Lagrangian in curved space is given by

$$\mathcal{L}_\psi = \bar{\psi}\left(i\gamma^\mu\left(D_\mu + \Gamma_\mu\right) - m\right)\psi \tag{3}$$

and the Lagrangian of Yang-Mills theory in the Minkowski curved spacetime is

$$\mathcal{L}_G = -\frac{1}{4g_{YM}^2} g^{\mu\nu} g^{\alpha\beta} F^a_{\mu\alpha} F^a_{\nu\beta} \tag{4}$$

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where $\mathcal{L}^{(n)} \propto \Omega^n$, and $\Omega = \partial_t \varphi_{\mathrm{lab}}$. The causality restriction: $\boxed{\Omega r < 1}$.

Rotating QCD in Euclidean space

The rotating system at thermal equilibrium is studied on the lattice. The partition function is

$$\mathcal{Z} = \mathrm{Tr}\left[e^{-\hat{H}/T_0}\right] = \int \mathcal{D}[U]\,\mathcal{D}[\psi,\bar{\psi}]\; e^{-S_G[U,\Omega]-S_F[U,\psi,\bar{\psi},\Omega]}, \tag{5}$$

where the Euclidean action, $S_G + S_F$, is formulated in curved space $(t \to -i\tau)$, $x^\mu = (x, y, z, \tau)$,

$$g^E_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & -y\Omega_I \\ 0 & 1 & 0 & x\Omega_I \\ 0 & 0 & 1 & 0 \\ -y\Omega_I & x\Omega_I & 0 & 1 + r^2\Omega_I^2 \end{pmatrix}, \tag{6}$$

and the angular velocity is imaginary, $\Omega_I = \partial_\tau \varphi_{\mathrm{lab}} = -i\partial_t \varphi_{\mathrm{lab}} = -i\Omega$, to avoid the sign problem.

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- The inverse temperature $1/T_0$ sets the system length in τ-direction.
- Ehrenfest–Tolman (TE) law: the local temperature *depends on the coordinates*

$$T(r)\sqrt{g_{00}} = T(r)\sqrt{1 - r^2\Omega^2} = T(r)\sqrt{1 + r^2\Omega_I^2} = T_0\,.$$

- We denote by $T \equiv T_0$ the temperature at the rotation axis $(r = 0)$.

Rotating QCD in Euclidean space: quark action

The quark action is a linear function in angular velocity:

$$S_F = \int d^4x \sqrt{g_E}\, \bar\psi \left(\gamma^\mu(\partial_\mu + \Gamma_\mu) + m\right)\psi =$$
$$= \int d^4x\, \bar\psi \left(\left(\gamma^1 + y\Omega_I\gamma^4\right) D_x + \left(\gamma^2 - x\Omega_I\gamma^4\right) D_y + \gamma^3 D_z + \gamma^4 \left(D_\tau + i\Omega_I \frac{\sigma^{12}}{2}\right) + m \right)\psi\,, \quad (7)$$

where

$$\gamma^\mu = \gamma^i e_i^\mu, \qquad \Gamma_\mu = -\frac{i}{4}\omega_{\mu ij}\sigma^{ij}, \qquad \omega_{\mu ij} = g_{\alpha\beta} e_i^\alpha \left(\partial_\mu e_j^\beta + \Gamma^\beta_{\nu\mu} e_j^\nu\right), \qquad \sigma^{ij} = \frac{i}{2}\left(\gamma^i\gamma^j - \gamma^j\gamma^i\right), \quad (8)$$

with $e_1^x = e_2^y = e_3^z = e_4^\tau = 1, \quad e_4^x = y\Omega_I\,, \quad e_4^y = -x\Omega_I\,.$

The quark action contains the orbit-rotation coupling term $\gamma^\tau\Omega_I(yD_x - xD_y)$ and the spin-rotation coupling term $i\gamma^\tau\Omega_I\sigma^{12}/2$, i.e. $\mathcal{L}_\psi^{(1)} = \bar\psi(\boldsymbol{\Omega}\cdot\hat{\boldsymbol{J}})\psi$.

On the lattice, the spin-rotation coupling term is exponentiated like chemical potential.

Rotating QCD in Euclidean space: gluon action

The gluon action is a quadratic function in angular velocity:

$$S_G = \frac{1}{4g_{YM}^2} \int d^4x \sqrt{g_E}\, g_E^{\mu\nu} g_E^{\alpha\beta} F_{\mu\alpha}^a F_{\nu\beta}^a \equiv S_0 + S_1 \Omega_I + S_2 \frac{\Omega_I^2}{2}\,, \tag{9}$$

where

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$$S_1 = \frac{1}{g_{YM}^2} \int d^4x \left[-y F_{xy}^a F_{y\tau}^a - y F_{xz}^a F_{z\tau}^a + x F_{yx}^a F_{x\tau}^a + x F_{yz}^a F_{z\tau}^a\right], \tag{11}$$

$$S_2 = \frac{1}{g_{YM}^2} \int d^4x \left[r^2 (F_{xy}^a)^2 + y^2 (F_{xz}^a)^2 + x^2 (F_{yz}^a)^2 + 2xy F_{xz}^a F_{zy}^a\right], \tag{12}$$

i.e. $\mathcal{L}_G^{(1)} = \boldsymbol{\Omega} \cdot \boldsymbol{J}_G$ and $\mathcal{L}_G^{(2)} \propto B^2$.

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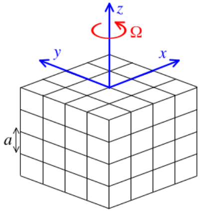
Sign problem

- The sign problem is due to the linear terms (both for quarks and for gluons, $S_1 \neq 0$)
- The Monte–Carlo simulation is conducted with imaginary angular velocity $\Omega_I = -i\Omega$
- The results are analytically continued to real angular velocity, $\Omega^2 \leftrightarrow -\Omega_I^2$

Causality restriction

- Analytic continuation is allowed only for bounded system with $\Omega r < 1$
- Boundary conditions are important! (they influence the result in all approaches)



[A. Yamamoto and Y. Hirono, Phys. Rev. Lett. **111**, 081601 (2013), arXiv:1303.6292 [hep-lat]]

- Lattice size: $N_t \times N_z \times N_s^2$ $(N_x = N_y = N_s)$
- "Radius" of the square cylinder: $R = a(N_s - 1)/2$
- Boundary velocity: $v_I^2 = (\Omega_I R)^2 < 1/2$
- periodic b.c. in directions τ, z.
- different types of b.c. in directions x,y:
  open / periodic / Dirichlet / ...

We start from rotating gluons.

Observables

The Polyakov loop is an order parameter, in gluodynamics ($\mathbb{Z}_3$ symmetry).

$$L(x,y) = \frac{1}{N_z}\sum_z \mathrm{Tr}\left[\prod_{\tau=0}^{N_t-1} U_4(\vec{r},\tau)\right], \qquad L = \frac{1}{N_s^2}\sum_{x,y} L(x,y)\,. \tag{13}$$

In confinement $\langle L\rangle = 0$; in deconfinement $\langle L\rangle \neq 0$. $\langle L\rangle = e^{-F_Q/T}$

The local critical temperature is associated with the peak of the local Polyakov loop susceptibility

$$\chi_L(r) = \langle |L(r)|^2\rangle - \langle |L(r)|\rangle^2\,. \tag{14}$$

We use tree-level improved (Symanzik) lattice action for S_0 and chair/plaquette discretization for S_1, S_2.[1]

The temperature is $T = 1/N_t a$. It coincides with the temperature on the rotation axis T_0.

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Inhomogeneous phases for imaginary rotation

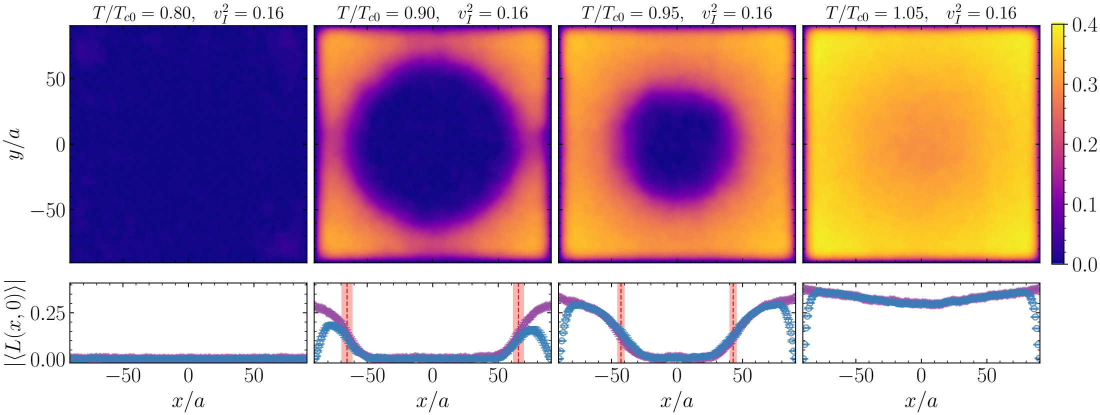


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed imaginary velocity at the boundary $v_I^2 \equiv (\Omega_I R)^2 = 0.16$ and different on-axis temperatures, $T = 1/N_t a$.

- As the (on-axis) temperature increases, the radius of the inner confining region shrinks.
- Boundary is screened; Rotating symmetry is restored.
- Local thermalization takes place; Phase transition occurs as a vortex evolution.

Inhomogeneous phases for imaginary rotation

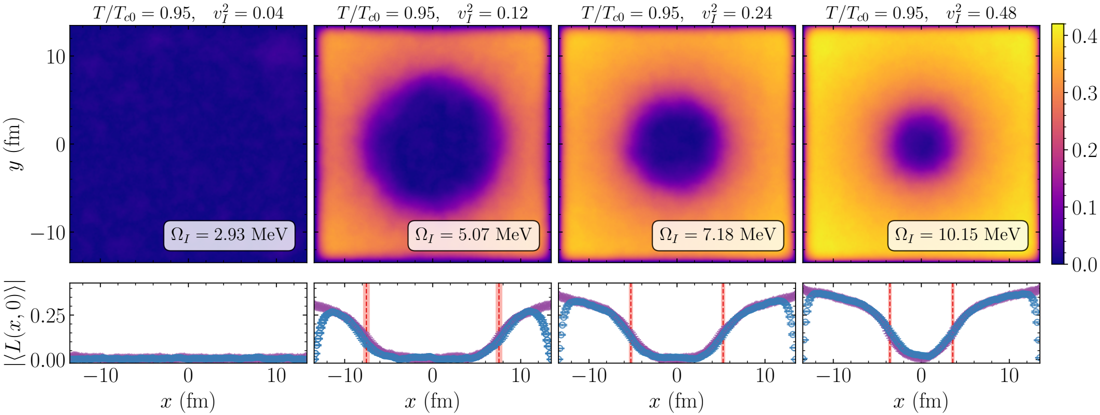


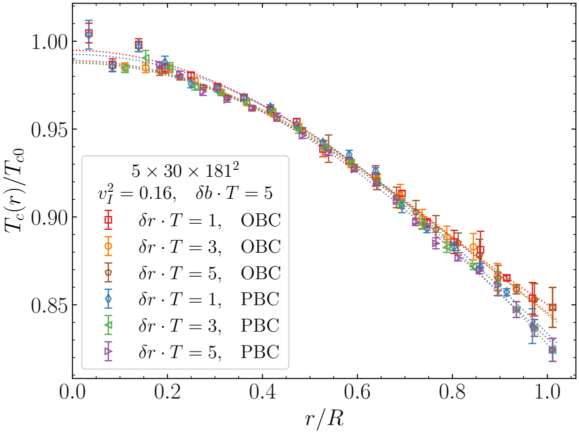
Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $5 \times 30 \times 181^2$ at the fixed temperature $T = 0.95\,T_{c0}$ and different Ω_I; System size $R = 13.5$ fm.

- Mixed inhomogeneous phase may be observed for $T \lesssim T_{c0}$. For imaginary rotation, deconfinement appears at the periphery; confinement is in the central regions.
- The confinement region shrinks with the increase in Ω_I;

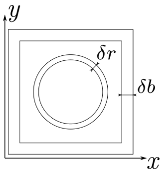
Local critical temperature

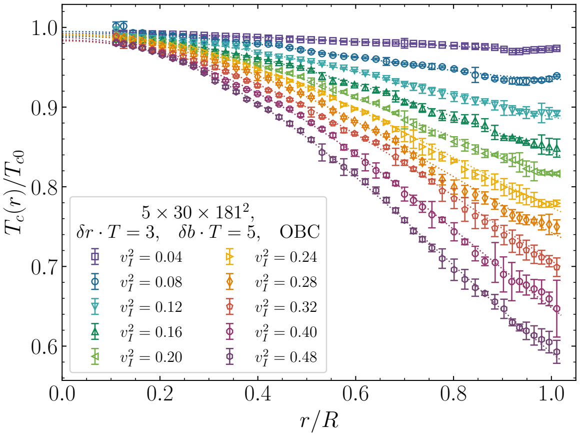
The local critical temperature $T_c(r)$ is the temperature at the rotation axis when the phase transition occurs at radius r.

- Technical details: We split the system into thin cylinders of width δr and measure $T_c(r)$.



- Results for different $\delta r \cdot T = 1, \ldots, 5$ are in agreement.
- δb is a width of ignored boundary layer
- Minor difference on b.c. appears at $r/R \sim 1$



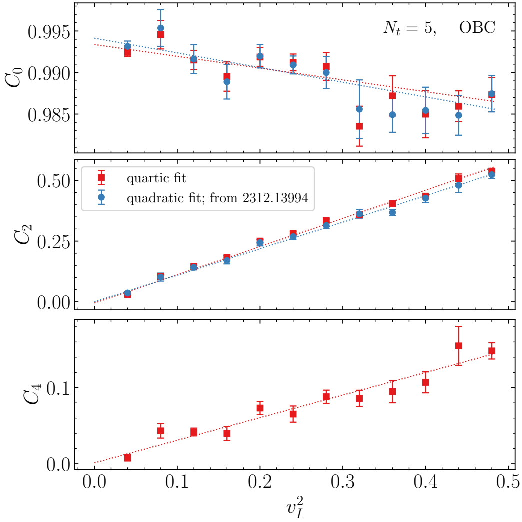


- The results in the whole region are well described by the quartic formula

$$\frac{T_c(r)}{T_{c0}} = C_0 - C_2\left(\frac{r}{R}\right)^2 + C_4\left(\frac{r}{R}\right)^4 . \quad (15)$$

- In the bulk, $r/R \lesssim 0.5$, quadratic fit is sufficient ($C_4 = 0$).

Local critical temperature



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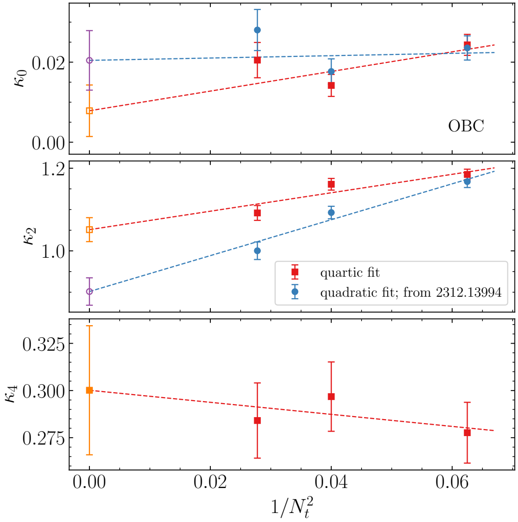
- We found numerically that

$$C_i(v_I^2) = a_i + \kappa_i v_I^2 . \quad (16)$$

- $T_c(0) \approx T_{c0}$ with few percent accuracy:
  - Effects of finite radius R.
  - Effects of averaging in layers of width δr.

Local critical temperature



- Results: The local critical temperature decreases with imaginary angular velocity.

$$\frac{T_c(r,\Omega_I)}{T_{c0}} = 1 - (\Omega_I r)^2 \left(\kappa_2 - \kappa_4 \left(\frac{r}{R}\right)^2\right). \quad (17)$$

- The vortical curvature in continuum limit from quadratic fit ($r/R \lesssim 0.5$) is universal

$$\kappa_2 = 0.902(33)\,, \quad (18)$$

- And from quartic fit (for OBC) there is

$$\kappa_2 = 1.051(29)\,, \qquad \kappa_4 = 0.300(34)\,, \quad (19)$$

  where κ_4 term is a finite volume correction;

- We can not distinguish $\sim \Omega^4$ term.

- How analytically continue inhomogeneous phase?

The action of rotating gluons is a quadratic function in Ω_I,

$$S_G = S_0 + \quad S_1 \Omega_I + \quad S_2 \Omega_I^2 \,, \tag{20}$$

where S_1, S_2 are inhomogeneous;

Decomposition of rotating action

- How analytically continue inhomogeneous phase?

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$$S_G = S_0 + \lambda_1 S_1 \Omega_I + \lambda_2 S_2 \Omega_I^2 \,, \tag{20}$$

where S_1, S_2 are inhomogeneous; we introduce switching factors λ_1, λ_2.

- $S_1 \equiv S_{\rm mech}$ is an angular momentum of gluons (in laboratory frame) – "mechanical" coupling.
- $S_2 = S_{\rm magn}$ is related to the chromomagnetic fields F_{ij}^2 – "chromomagnetic" coupling.

The following regimes of the rotation are possible:

Im1) $\lambda_1 = 1, \quad \lambda_2 = 0; \quad \Omega_I^2 > 0$

Im2) $\lambda_1 = 0, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$

Im12) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$ (physical regime; it is already considered above)

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Im12) $\lambda_1 = 1, \quad \lambda_2 = 1; \quad \Omega_I^2 > 0$ (physical regime; it is already considered above)

Note that in the case $\lambda_1 = 0$ there is, actually, no sign problem:

Re2) $\lambda_1 = 0, \quad \lambda_2 = -1; \quad \Omega_I^2 < 0$ (real rotation) $\Rightarrow$ a.c. of mixed phase may be checked

Imaginary vs real rotation for different regimes

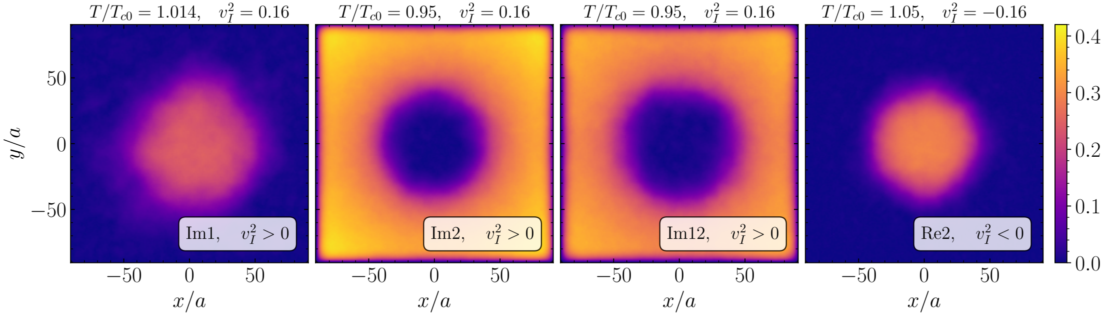
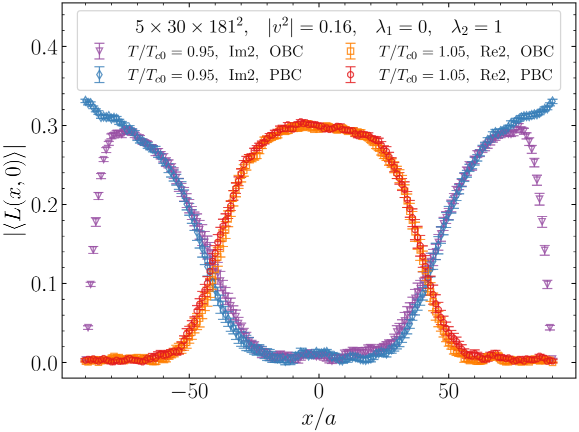


Figure: The distribution of the local Polyakov loop in x, y-plane for lattice size $5 \times 30 \times 181^2$, open boundary conditions (OBC) at fixed velocity $|v_I^2| = 0.16$ and different regimes. Temperature was chosen to see mixed phase.

- In the regimes Im1 and Re2, the rotation produces confinement phase in the outer region at $T > T_{c0}$. Regime Re2 realizes real rotation for S_2 system.
- Phase arrangement is the same in Im2- and Im12-regimes. The radius of the inner region in regime Im2 is slightly smaller, than in regime Im12.

Imaginary vs real rotation for different regimes

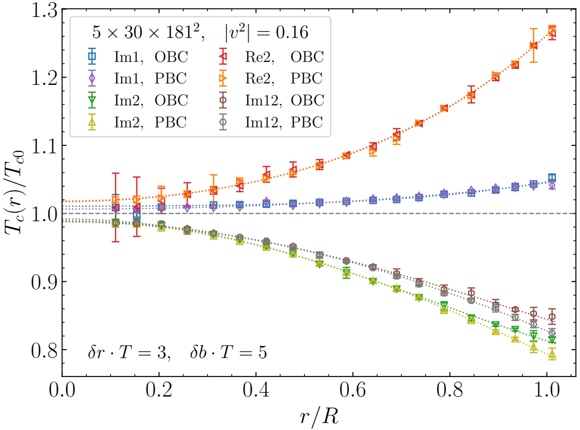
The distributions of the Polyakov loop for real and imaginary rotation (S_1 term is omitted).



- Re2: $T = T_{c0} + \Delta T$
  for real rotation $v^2 = 0.16$
- Im2: $T = T_{c0} - \Delta T$
  for imaginary rotation $v_I^2 = 0.16$

Confinement ↔ deconfinement
with approximately the same position.

The local critical temperature in these regimes has different behaviour.



- In the Im1-regime, $T_c(r) \nearrow$
- In the Im2-regime, $T_c(r) \searrow$
  the vortical curvature $\kappa_2^{(\text{Im2})} > \kappa_2^{(\text{Im12})}$

Imaginary vs real rotation for different regimes

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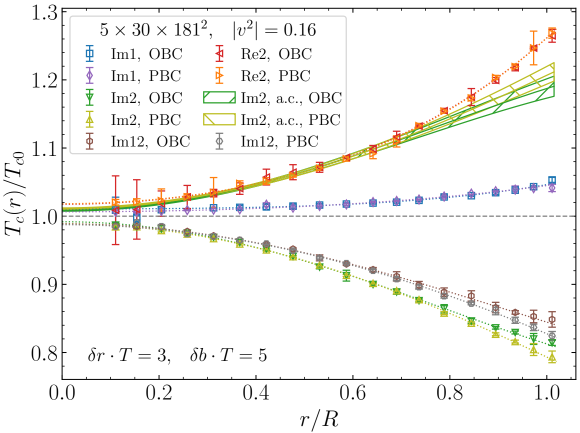

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- The Re2-regime is in agreement with a.c. of the Im2-results in a bulk (a.c. within Eq. (17))
- Contribution from $S_2 \equiv S_{\mathrm{magn}}$ dominates.

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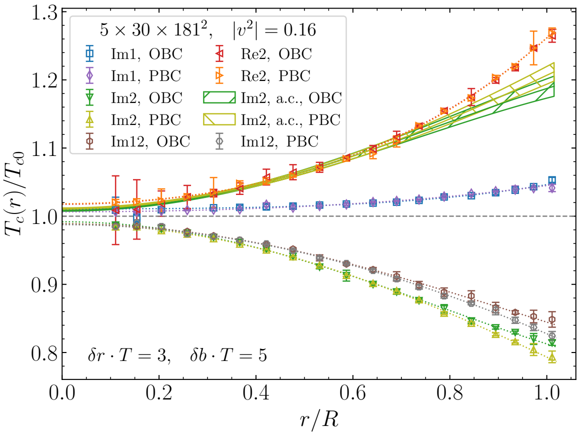

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  The results resemble the decomposition of I [V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]] (see below)

Local approximation for inhomogeneous action

Two major effects of rotation:

- Inhomogeneity: coefficients depend on coordinates (x, y).
- Anisotropy: chromoelectric and chromomagnetic components are affected differently by rotation.

The approximation of *local thermalization*: We consider a small subsystem at distance r_0 from the rotation axis, $(x, y) = (r_0, 0)$, for which the coefficients in action is approximately constant.

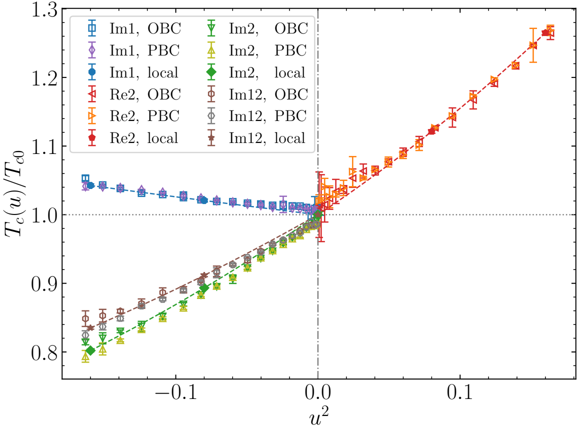
The homogeneous local action is

$$S_G = \frac{1}{2g_{YM}^2} \int d^4x \Big[F^a_{x\tau}F^a_{x\tau} + F^a_{y\tau}F^a_{y\tau} + F^a_{z\tau}F^a_{z\tau} + F^a_{xz}F^a_{xz} + \\ + \left(1 + u_I^2\right) F^a_{yz}F^a_{yz} + \left(1 + u_I^2\right) F^a_{xy}F^a_{xy} + 2u_I \left(F^a_{yx}F^a_{x\tau} + F^a_{yz}F^a_{z\tau}\right) \Big], \quad (21)$$

where $u_I = \Omega_I r_0$ is a local velocity.

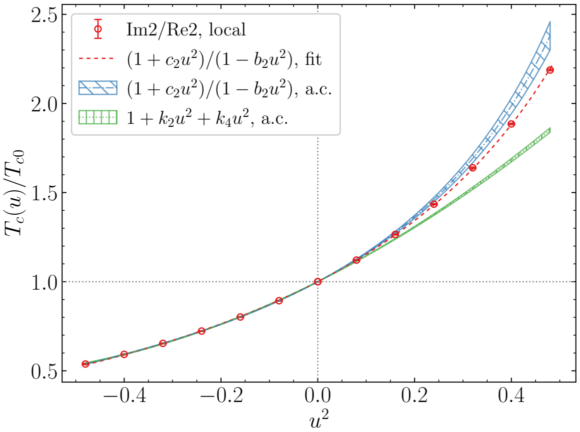
Local thermalization approximation

- The system (21) is simulated using standard lattice methods with PBC.
- Local approximation is free from the effects of finite R and the influence of boundary conditions.



- The results for *local* action and for full system are in a good agreement with each other in all regimes.
- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4 \,, \qquad (22)$$



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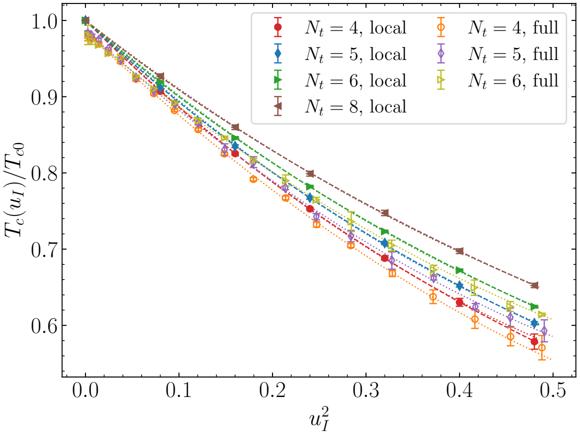
$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4 \,, \tag{22}$$

- Or, by the rational function:

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^2} \,. \tag{23}$$

- The function (23) better describe all data from regimes Im2/Re2.



Now, switch on the full rotation, Im12

- The data are well described by the polynomial:

$$\frac{T_c(u_I)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4 \,, \tag{24}$$

- And by the rational function:

$$\frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^2} \,. \tag{25}$$

- In continuum limit the coefficients are

$$k_2 = 0.869(31)\,, \qquad k_4 = 0.388(53)\,. \tag{26}$$

$$c_2 = 0.206(66)\,, \qquad b_2 = 0.694(101)\,. \tag{27}$$

- The local critical temperature increases with real velocity $u = \Omega r$.

Ehrenfest-Tolman effect in rotating (Q)GP

Ehrenfest-Tolman effect: In gravitational field the temperature isn't a constant in space at thermal equilibrium, $T(r)\sqrt{g_{00}} = T_0 = \text{const}$. In the co-rotating reference frame:

$$T(r) = \frac{T_0}{\sqrt{1-\Omega^2 r^2}} = \frac{T_0}{\sqrt{1+\Omega_I^2 r^2}}\,. \tag{28}$$

TE law suggests that the rotation effectively heats the periphery. Let's derive $T_c^{TE}(u)$ from an assumption $T(r) = T_{c0}$, then the local critical temperature decreases:

$$\frac{T_c^{TE}(u)}{T_{c0}} = \sqrt{1-u^2} \approx 1 - 0.5u^2 + \dots\,, \tag{29}$$

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External gravitational field generates asymmetry in the coupling constants of different components of the fields $(F_{\mu\nu})^2$, which influences the dynamics of gluons. This mechanism can not be accounted for by TE.

$$S_G = \int d^4x \left[\beta\left((F^a_{x\tau})^2 + (F^a_{y\tau})^2 + (F^a_{z\tau})^2 + (F^a_{xz})^2\right) + \tilde{\beta}\left((F^a_{yz})^2 + (F^a_{xy})^2\right)\right], \tag{30}$$

where $\beta = \frac{1}{2}g^2_{YM}$ and $\tilde{\beta} = \left(1-(\Omega r_0)^2\right)\beta \equiv \left(1+(\Omega_I r_0)^2\right)\beta$.

- $\tilde{\beta}/\beta > 1$ (imaginary rotation) $\Rightarrow$ T_c decreases;
- $\tilde{\beta}/\beta < 1$ (real rotation) $\Rightarrow$ T_c increases.

Equation of State and Moment of Inertia

A mechanical response of a thermodynamic ensemble to rigid rotation $\boldsymbol{\Omega} = \Omega \boldsymbol{e}$ is described in terms of the total angular momentum $\boldsymbol{J}$. The energy in co-rotating reference frame is

$$E = E^{(lab)} - \boldsymbol{J} \cdot \boldsymbol{\Omega} \,, \qquad F = E - TS \,, \qquad dF = -S dT - \boldsymbol{J} \cdot d\boldsymbol{\Omega} + \ldots \,,$$

The moment of inertia is a scalar quantity, $\boldsymbol{J} = I(T,\Omega)\boldsymbol{\Omega}$,

$$I(T,\Omega) = \frac{J(T,\Omega)}{\Omega} = -\frac{1}{\Omega}\left(\frac{\partial F}{\partial \Omega}\right)_T ,$$

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For a classical system with characteristic radius R the moment of inertia is given by

$$I(T,\Omega) = \int_V d^3x\, x_\perp^2 \rho(T, x_\perp, \Omega) \simeq \alpha\, \rho_0(T) V R^2\,,$$

The free energy may be represented as a series in angular velocity (or linear velocity $v_R = \Omega R$)

$$F(T,V,\Omega) = F_0(T,V) - \frac{F_2(T,V)}{2}\Omega^2 + \mathcal{O}(\Omega^4) \equiv f_0(T)V - \frac{\mathrm{i}_2(T)}{2} V v_R^2 + \mathcal{O}(v_R^4)\,,$$

where $F_2(T,V) = f_2(T)V = I(T,V,\Omega=0) \equiv \mathrm{i}_2(T) V R^2$, and $\mathrm{i}_2(T)$ is a *specific* moment of inertia; $K_2 \equiv -\mathrm{i}_2/f_0$

Results of lattice simulation with non-zero imaginary angular velocity

Symanzik gauge action; we calculate $f = F/V$ using standard relations

$$\frac{f(T)}{T^4} = -N_t^4 \int_{\beta_0}^{\beta} d\beta' \Delta s(\beta')\,,$$

where $\Delta s(\beta) = \langle s(\beta)\rangle_{T=0} - \langle s(\beta)\rangle_T \equiv -\langle\langle s\rangle\rangle$.

- $N_t \times 40 \times 41^2$ lattices with $N_t = 5, 6, 7, 8$;
- $N_t^{(T=0)} = 40$ for $T = 0$ subtraction;
- $v_I^2 \ll 1$, where $v_I = \Omega_I R$, $R = a(N_s - 1)/2$.
- $v_I = \text{const} \quad \Leftrightarrow \quad \Omega_I/T = v_I/RT = \text{const}$.

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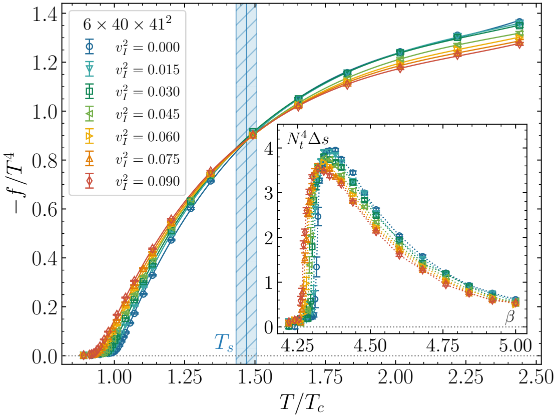
- $v_I = \text{const} \quad \Leftrightarrow \quad \Omega_I/T = v_I/RT = \text{const}$.
- $T_c \searrow$ with the imaginary angular velocity.
- Fit by the quadratic function ($f_0 = -p < 0$):

$$f(T, v_I) = f_0(T)\left(1 - \frac{1}{2} K_2(T) v_I^2\right).$$



[V. V. Braguta, M. N. Chernodub, A. A. Roenko, and D. A. Sychev, Phys. Lett. B **852**, 138604 (2024), arXiv:2303.03147 [hep-lat]]

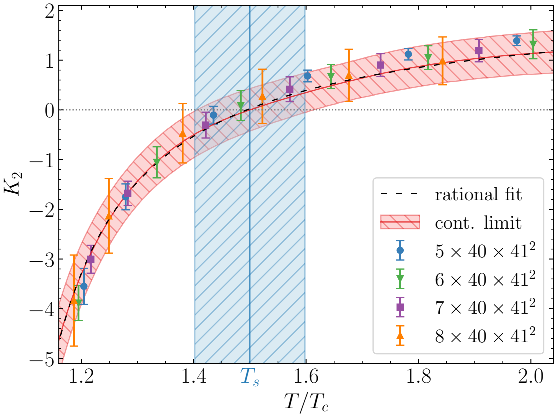
- The moment of inertia of gluon plasma

$$I(T)|_{\Omega=0} = -K_2 F_0 R^2 ,$$

becomes zero at "supervortical" temperature

$$T_s = 1.50(10) T_c .$$

and it is negative for $T < T_s$.



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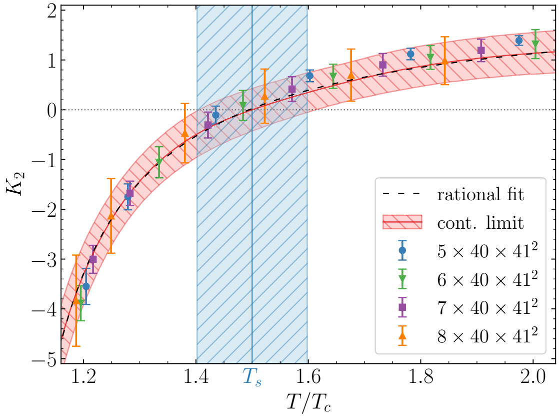
- The result for the system with OBC is

$$T_s = 1.53(15) T_c$$



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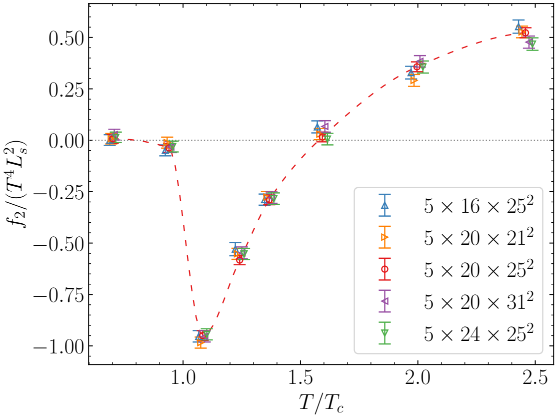
Results of lattice simulations with zero angular velocity

Taking the derivative at $\Omega = 0$, we obtain:

$$I = F_2 = T \frac{\partial^2 \log Z}{\partial \Omega^2}\bigg|_{\Omega=0} = T\big(\langle\!\langle S_1^2 \rangle\!\rangle_T + \langle\!\langle S_2 \rangle\!\rangle_T\big)\,,$$

where $\langle\!\langle \mathcal{O} \rangle\!\rangle_T = \langle \mathcal{O} \rangle_T - \langle \mathcal{O} \rangle_{T=0}$ corresponds to the thermal contribution to $\langle \mathcal{O} \rangle$.

$$f_2/(T^4 L_s^2) \equiv \mathfrak{i}_2/T^4\,,$$



[V. V. Braguta et al., JETP Lett. **117**, 639–644 (2023)]

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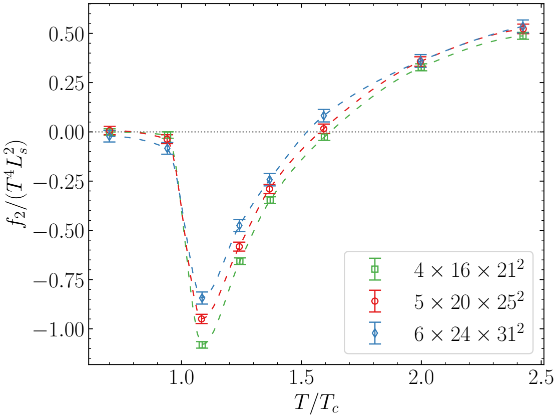

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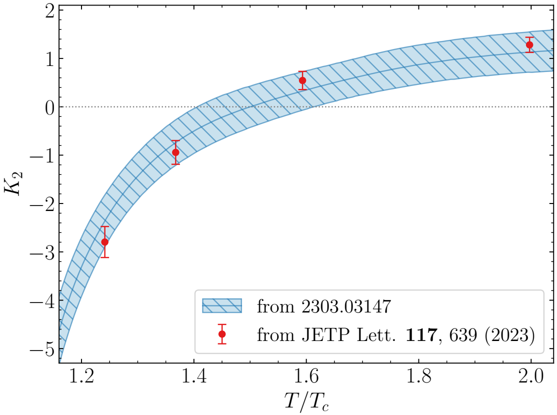
$$f_2/(T^4 L_s^2) \equiv \mathsf{i}_2/T^4\,, \qquad K_2 = \mathsf{i}_2/(-f_0)$$

Results of two methods (a.c. from Ω_I and $\partial_\Omega|_{\Omega=0}$) are in agreement.



[V. V. Braguta et al., PoS **LATTICE2023**, 181 (2024), arXiv:2311.03947 [hep-lat]]

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Using the exact forms of S_1, S_2, we get

$$I = I_{\mathrm{mech}} + I_{\mathrm{magn}}$$

where ($\langle J \rangle = 0$ for any T) and

$$I_{\mathrm{mech}} = \frac{1}{T}\left(\langle\!\langle J^2 \rangle\!\rangle_T - \langle\!\langle J \rangle\!\rangle_T^2\right) \geq 0,$$

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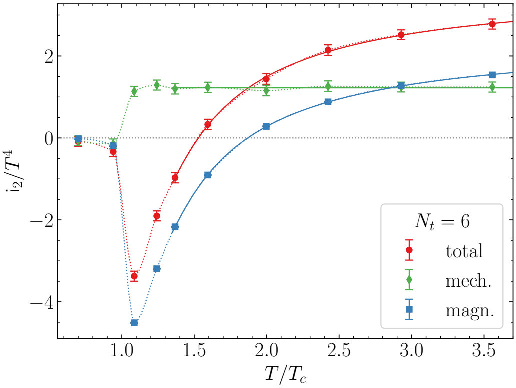
$J \equiv J_G$ is the total angular momentum of gluon field.

- Mass density $\rho_0(T) \leftrightarrow \langle\!\langle (G_{\mathrm{magn}})^2 \rangle\!\rangle_T / 3$.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Negative moment of inertia and magnetic gluon condensate

Taking the derivative at $\Omega = 0$, we obtain:

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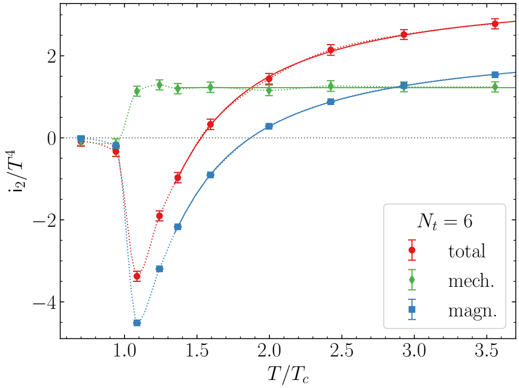

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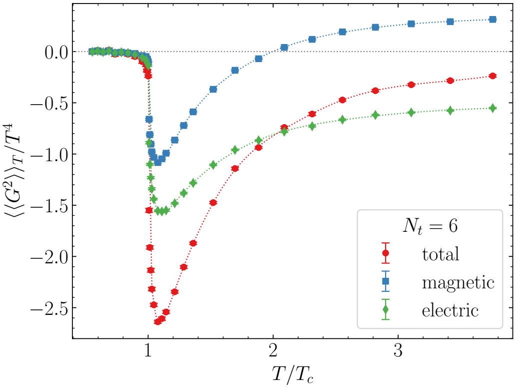
$J \equiv J_G$ is the total angular momentum of gluon field.

- Mass density $\rho_0(T) \leftrightarrow \langle\langle (G_{\text{magn}})^2 \rangle\rangle_T / 3$.
- Magnetic gluon condensate reverse its sign at $\sim 2T_c$.
- In QCD fermionis (J_ψ) contribute only to I_{mech}.



[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

Total angular momentum $\boldsymbol{J} = I\boldsymbol{\Omega}$ is a sum of the orbital and spin parts:

$$\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}\,, \tag{31}$$

and $I < 0$. The *possible* physical picture: instability, or *negative* Barnett effect for gluon .

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In the temperature range $T_c \lesssim T < T_s \simeq 1.5 T_c$:

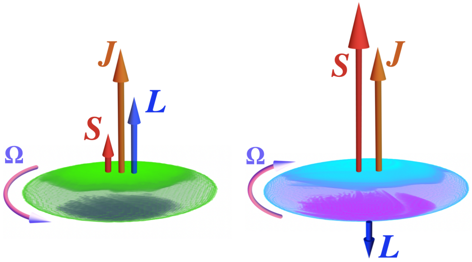
(i) a sizable fraction of the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ is accumulated in the spin of gluons $\boldsymbol{S}$;

(ii) therefore, $\boldsymbol{S} \uparrow\uparrow \boldsymbol{J}$ and $\boldsymbol{S} \uparrow\downarrow \boldsymbol{L}$.

Let's introduce $\boldsymbol{L} = I_L \boldsymbol{\Omega}$, $\boldsymbol{S} = I_S \boldsymbol{\Omega}$, therefore

$$I_L > 0\,, \quad I_S < 0\,, \quad I = I_L + I_S < 0\,.$$

Fof classical system $I_S = 0$.



(left) usual Barnett effect
(right) negative Barnett effect
[V. V. Braguta et al., Phys. Rev. D **110**, 014511 (2024), arXiv:2310.16036 [hep-ph]]

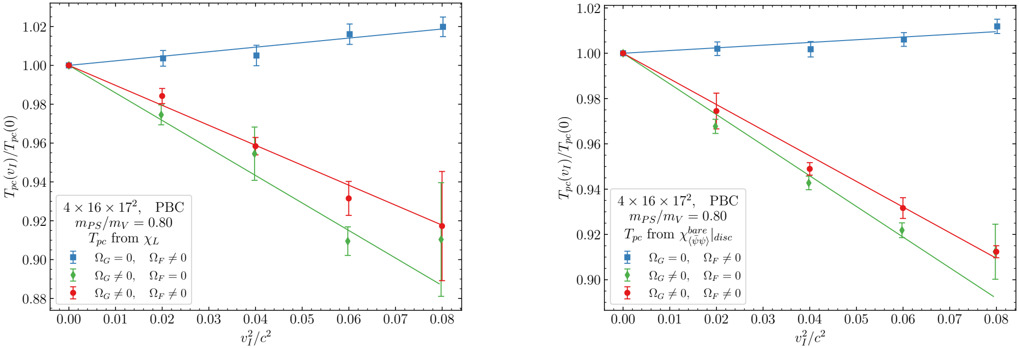


Figure: The (bulk-averaged) pseudo-critical temperature as a function of imaginary linear velocity on the boundary for various rotation regimes (full, only gluons, only fermions). [V. Braguta, A. Kotov, A. Roenko, and D. Sychev, PoS **LATTICE2022**, 190 (2023), arXiv:2212.03224 [hep-lat]]

QCD action: $S = S_G(\Omega_G) + S_F(\Omega_F)$

Rotation in fermionic and gluonic sectors have different influence on (bulk-averaged) T_{pc}. Gluons dominate.

Inhomogeneous phase in QCD (preliminary)

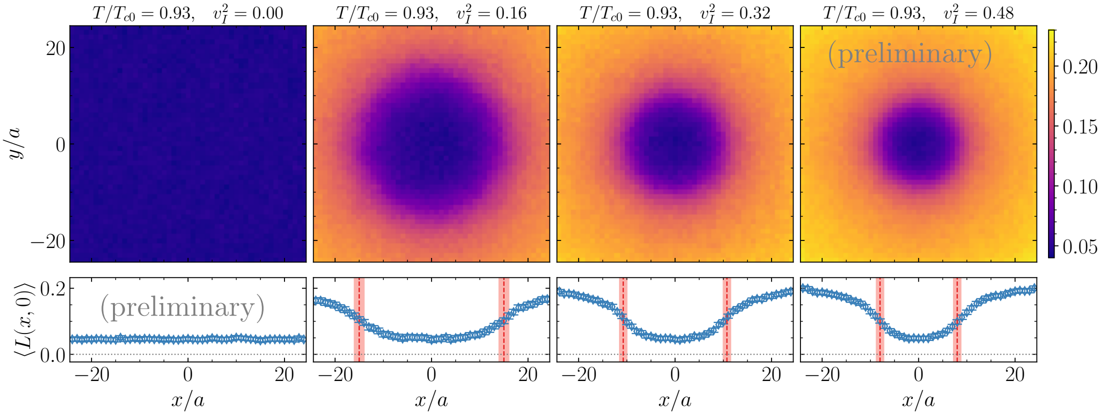


Figure: The distribution of the local Polyakov loop in x, y-plane for the lattice of size $4 \times 20 \times 49^2$ at the fixed temperature $T = 0.93\, T_{c0}$ and different v_I; QCD with Wilson fermions (Iwasaki action), $m_\pi / m_\rho = 0.80$.

- Mixed inhomogeneous phase takes place also in QCD! (work in progress . . .)

Conclusions

- Using lattice simulation with *imaginary* angular velocity, we found the mixed phase in rotating gluodynamics at thermal equilibrium. For *imaginary* rotation, it takes place for $T < T_{c0}$ with confinement phase in the center and deconfinement at the periphery.
- The local critical temperature in rotating gluodynamicsdepends on the local velocity $u = \Omega r$:

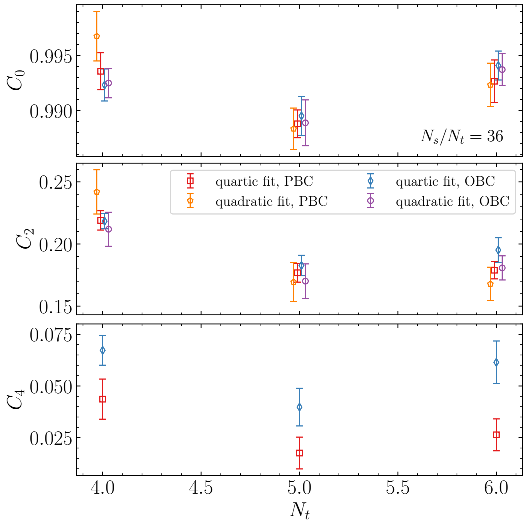
$$\frac{T_c(r,\Omega)}{T_{c0}} = 1 + \kappa_2 (\Omega r)^2 \qquad \text{[bulk of full rotating system]}, \tag{32}$$

$$\frac{T_c(u)}{T_{c0}} = 1 + k_2 u^2 + k_4 u^4, \quad \text{or} \quad \frac{T_c(u)}{T_{c0}} = \frac{1 + c_2 u^2}{1 - b_2 u^4}, \qquad \text{[local action]}, \tag{33}$$

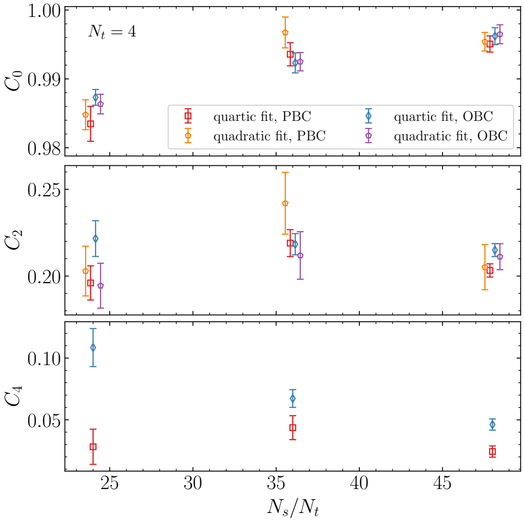
  The approximation of local thermalization gives consistent results. Note that $T_c(0) \approx T_{c0}$.
- For *real* rotation, the inhomogeneous phase may arise for $T > T_{c0}$ with confinement (deconfinement) at the periphery (center).
- We demonstrate the validity of analytic continuation using Im2/Re2-regimes.
- The magnetovortical coupling generates asymmetry in the action for chromomagnetic fields. Linear coupling play subleading role. This mechanism can not be accounted for by TE.
- Gluon plasma has $I < 0$ below the supervortical temperature T_s. Possible physical explanation: NBE. Results for a.c. from Ω_I and $\partial_\Omega|_{\Omega=0}$ are in agreement.
- We expect similar picture for QCD (work in progress).

Thank you for your attention!

Backup

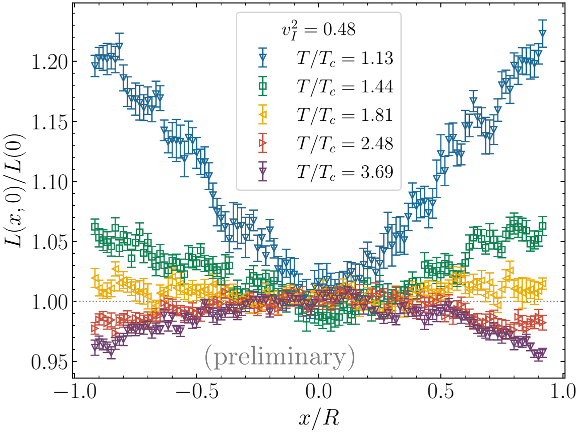


C_0
0.995
0.990
$N_s/N_t = 36$
C_2
0.25
0.20
0.15
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_4
0.075
0.050
0.025
4.0
4.5
5.0
5.5
6.0
N_t



1.00
$N_t = 4$
C_0
0.99
0.98
quartic fit, PBC
quadratic fit, PBC
quartic fit, OBC
quadratic fit, OBC
C_2
0.25
0.20
C_4
0.10
0.05
25
30
35
40
45
N_s/N_t

Local Polyakov loop at high temperatures



- $T > T_s \simeq 1.5 T_{c0}$: $I > 0$
- $T \gtrsim 2T_{c0}$: $\langle\langle \mathcal{B}^2 \rangle\rangle > 0$
- Local Polyakov loop decreases with r at high temperatures $T \gtrsim 2T_{c0}$

  (local temperature from TE decreases with r for imaginary Ω_I)