QCD CORRECTIONS TO SEMILEPTONIC AND NONLEPTONIC DECAYS

Matteo Fael (CERN)

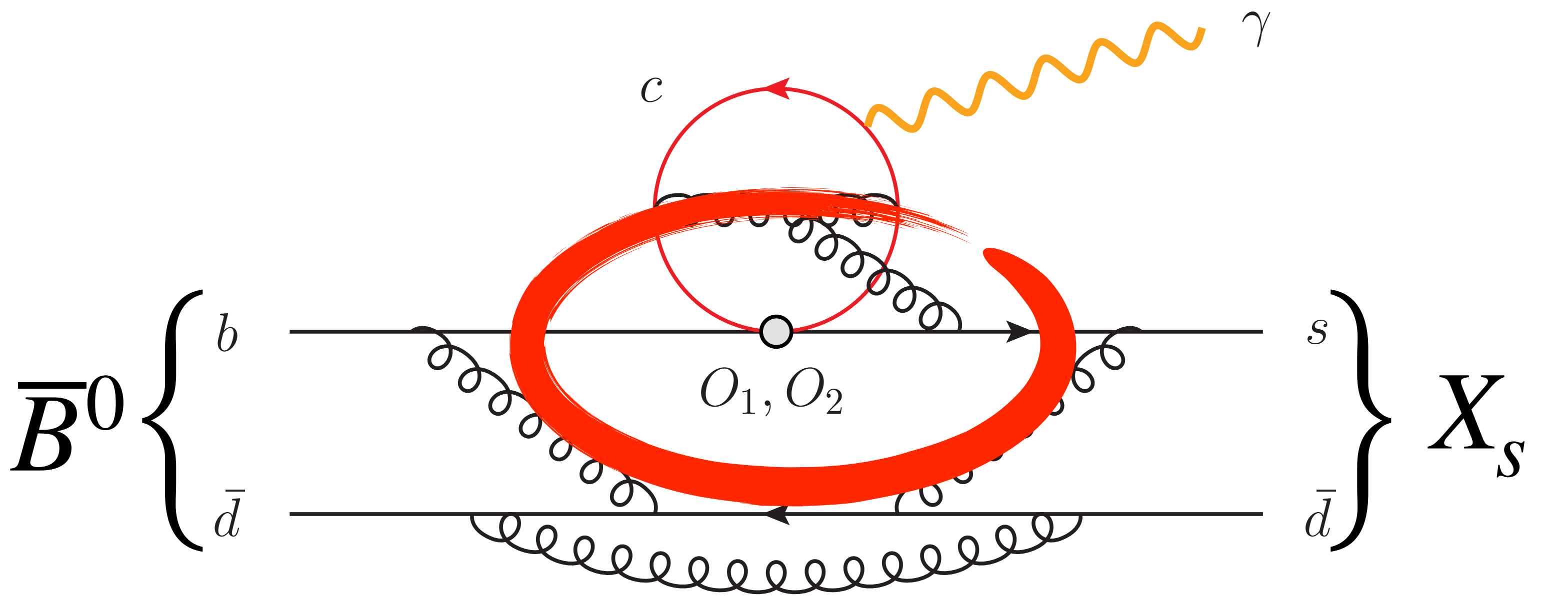
Christophest: Precision Predictions for FCNC Processes

BERN - October 25th, 2024

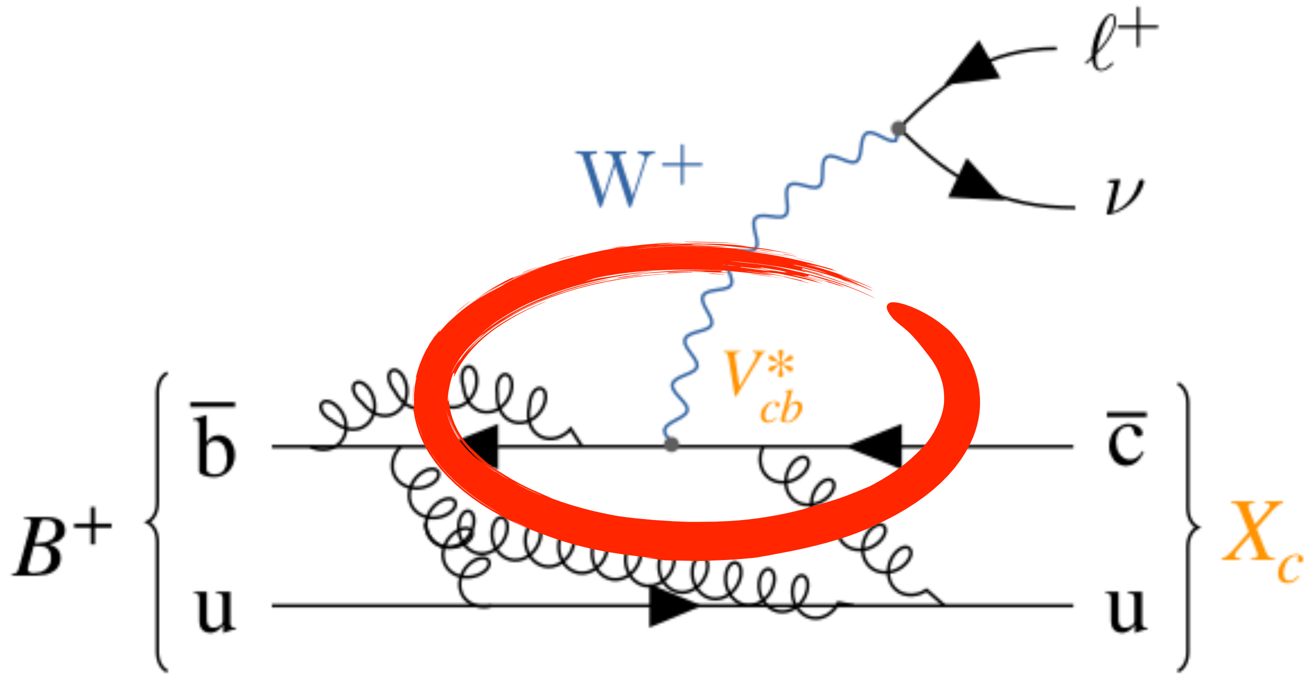
with M. Egner, K. Schönwald and M. Steinhauser

Funded by
the European Union

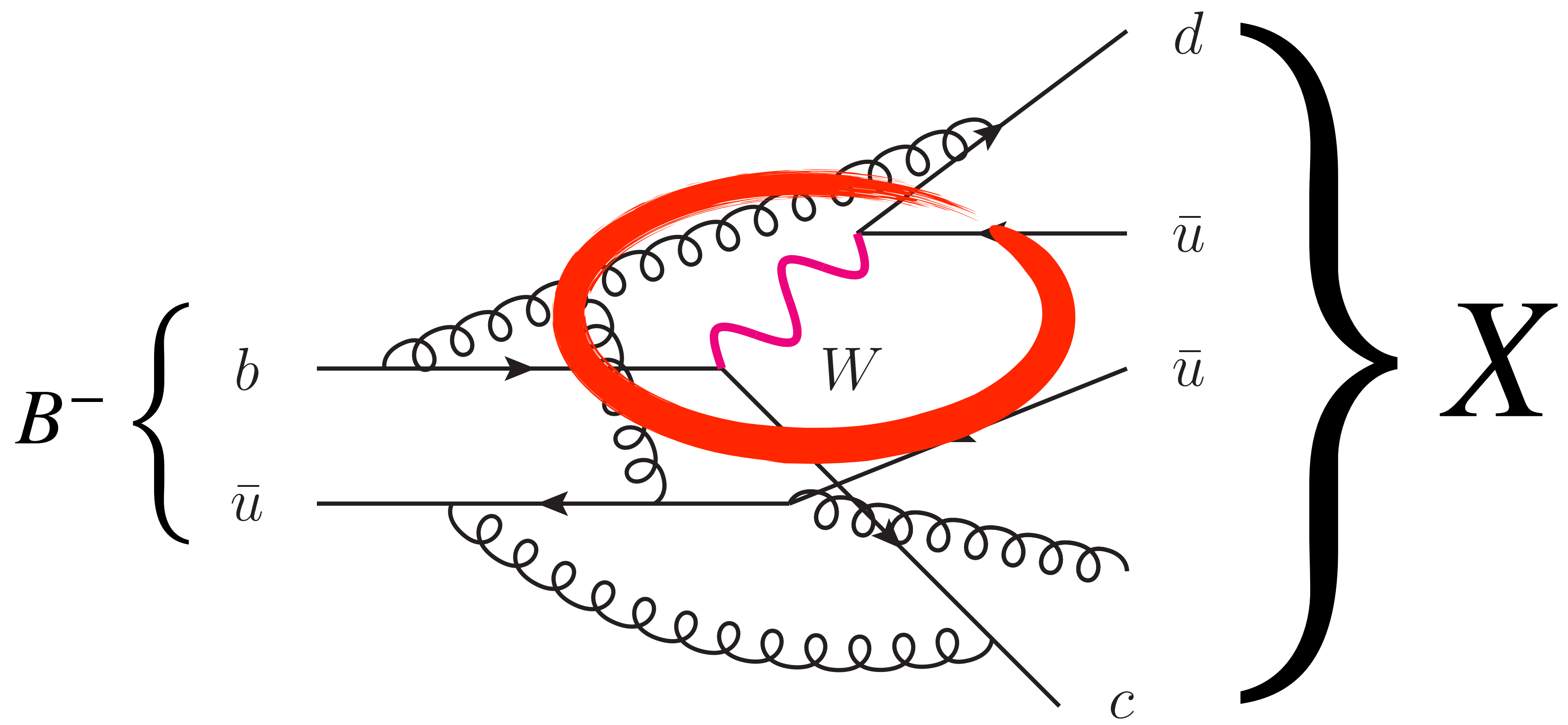
INCLUSIVE DECAYS OF B MESONS



Rare decay $B \to X_s\gamma$



Semileptonic $B \to X_c l \bar{\nu}_l$

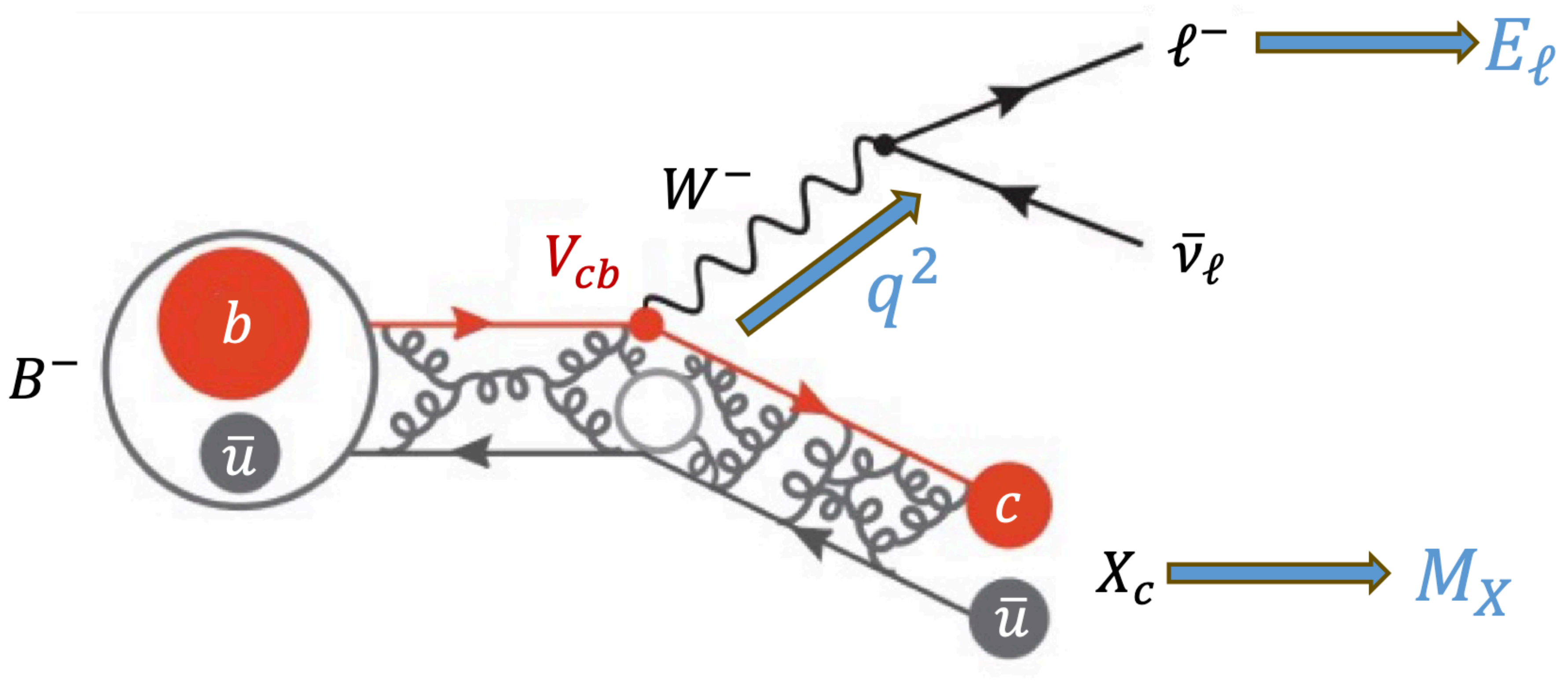


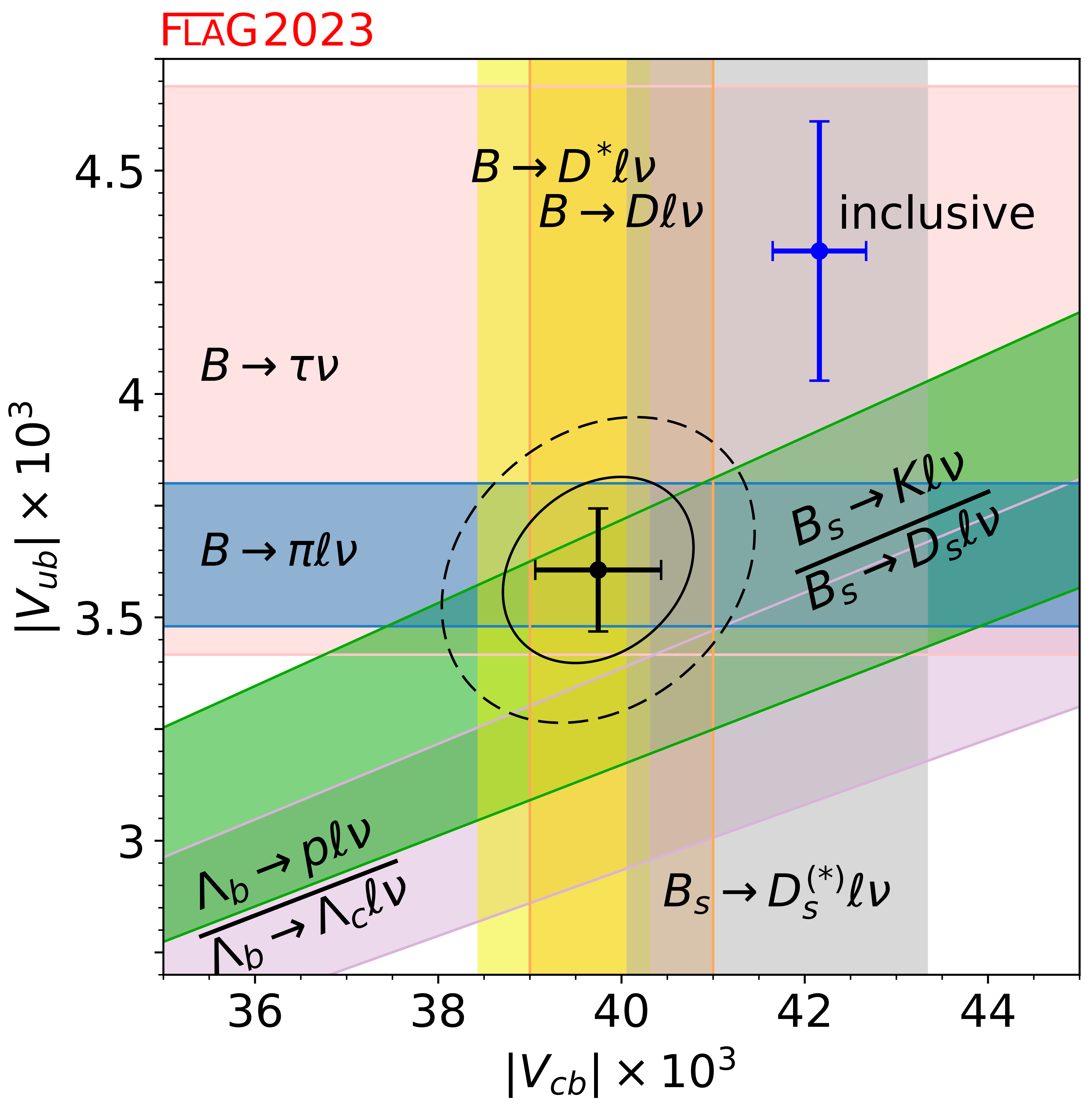
Nonleptonic decays

Lifetime of B mesons

We need precise predictions in the SM, often at the 1-2% level!

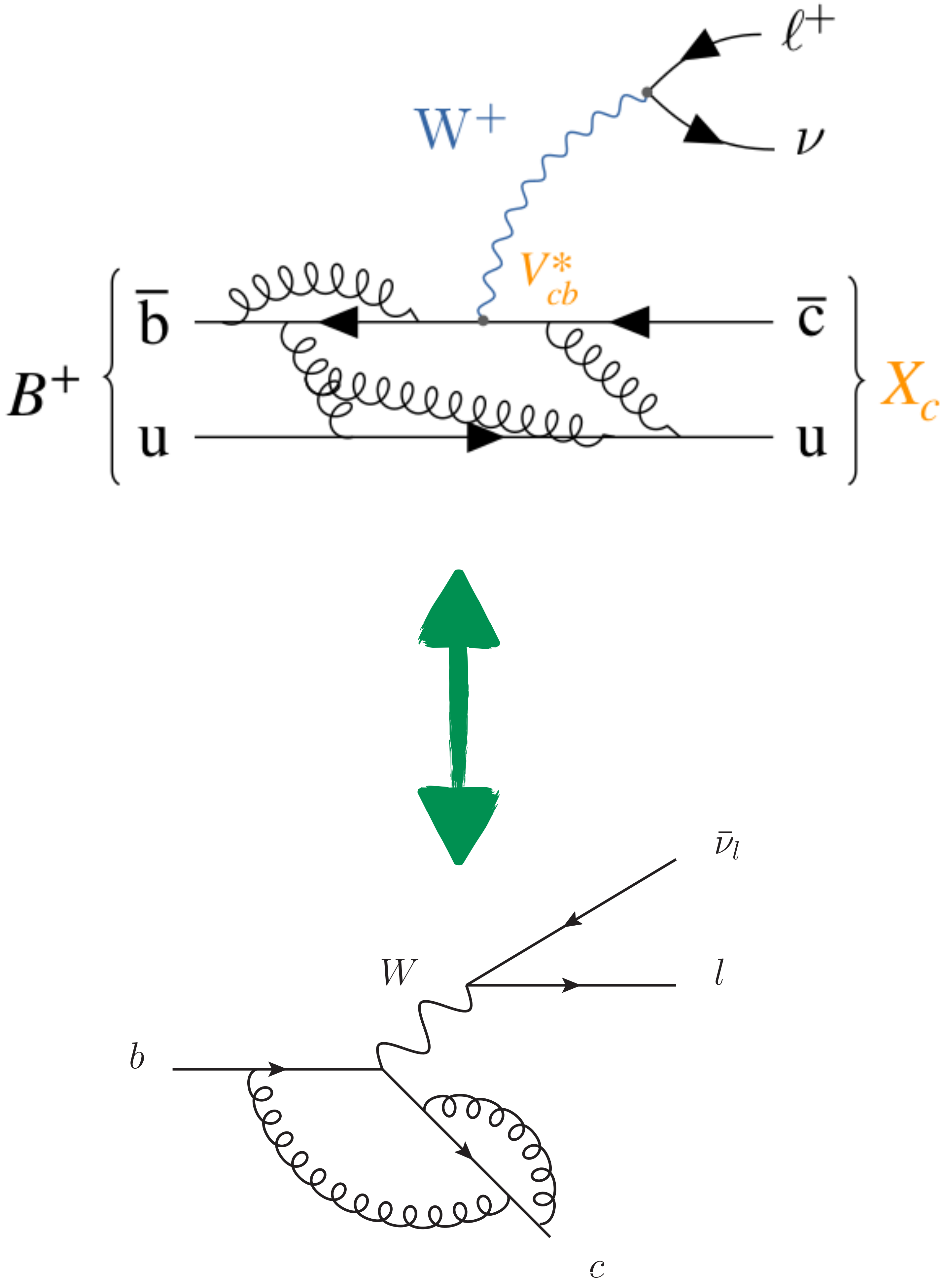
SEMILEPTOINC B DECAYS





- Extraction of the CKM element $|V_{cb}|$.
- Determination of the non-perturbative matrix elements from experimental data.
- Predictions for processes with FCNC crucially depend on these SM inputs.
  - $|V_{tb}V_{ts}^\star| \simeq |V_{cb}|^2(1 + O(\lambda^2))$
  - $\epsilon_K \simeq |V_{cb}|^4 x$

THE HEAVY QUARK EXPANSION



Use optical theorem

$$\Gamma_{\mathrm{sl}} = \frac{1}{2m_B} \sum_X \left| \langle X | \mathscr{H}_{\mathrm{eff}} | B \rangle \right|^2$$

$$= C_3 + \frac{C_5}{m_b^2} \langle B | O_5 | B \rangle + \frac{C_6}{m_b^3} \langle B | O_6 | B \rangle + \ldots$$

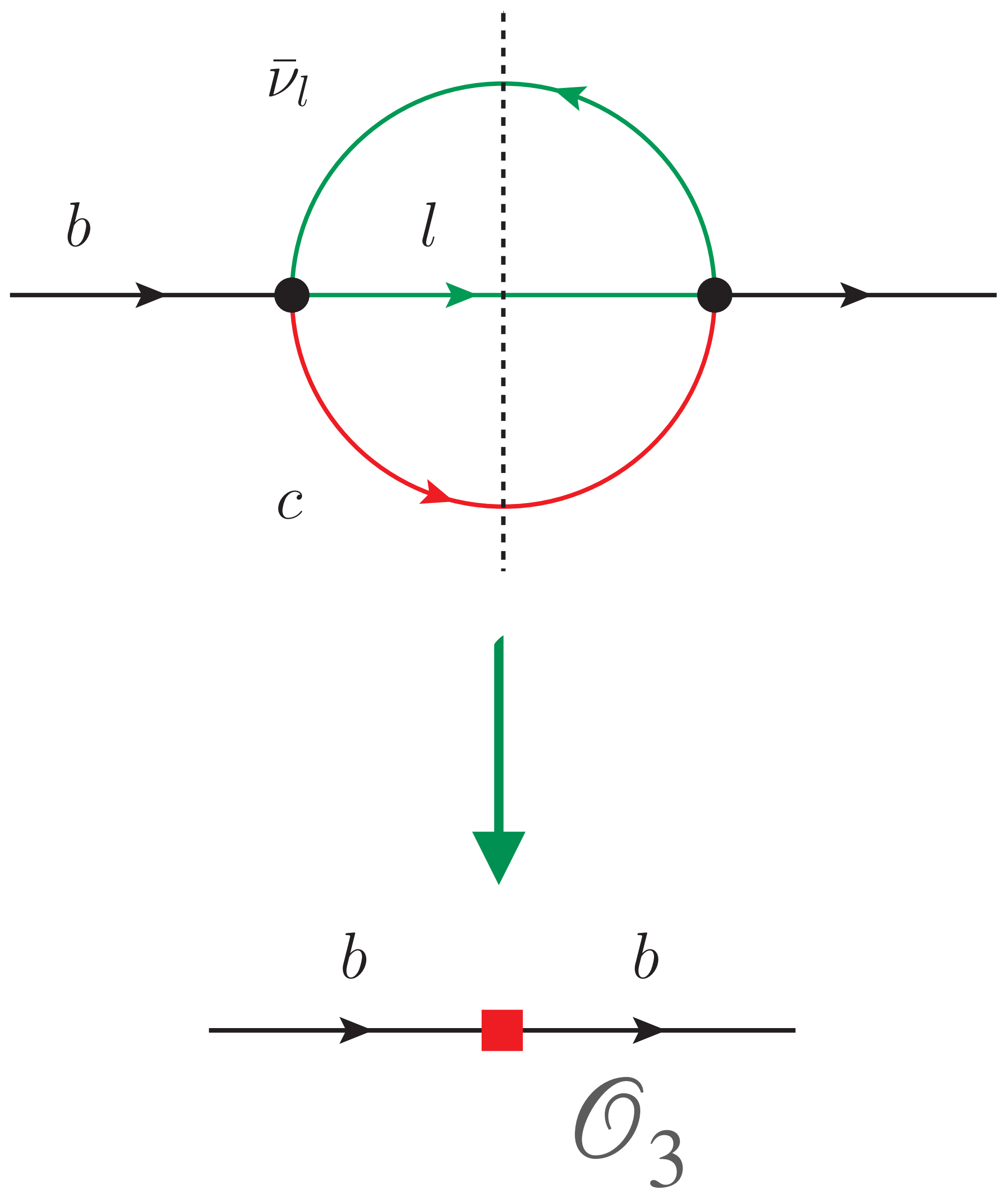
Calculable in perturbative QCD

No non-perturbative matrix element at leading power!

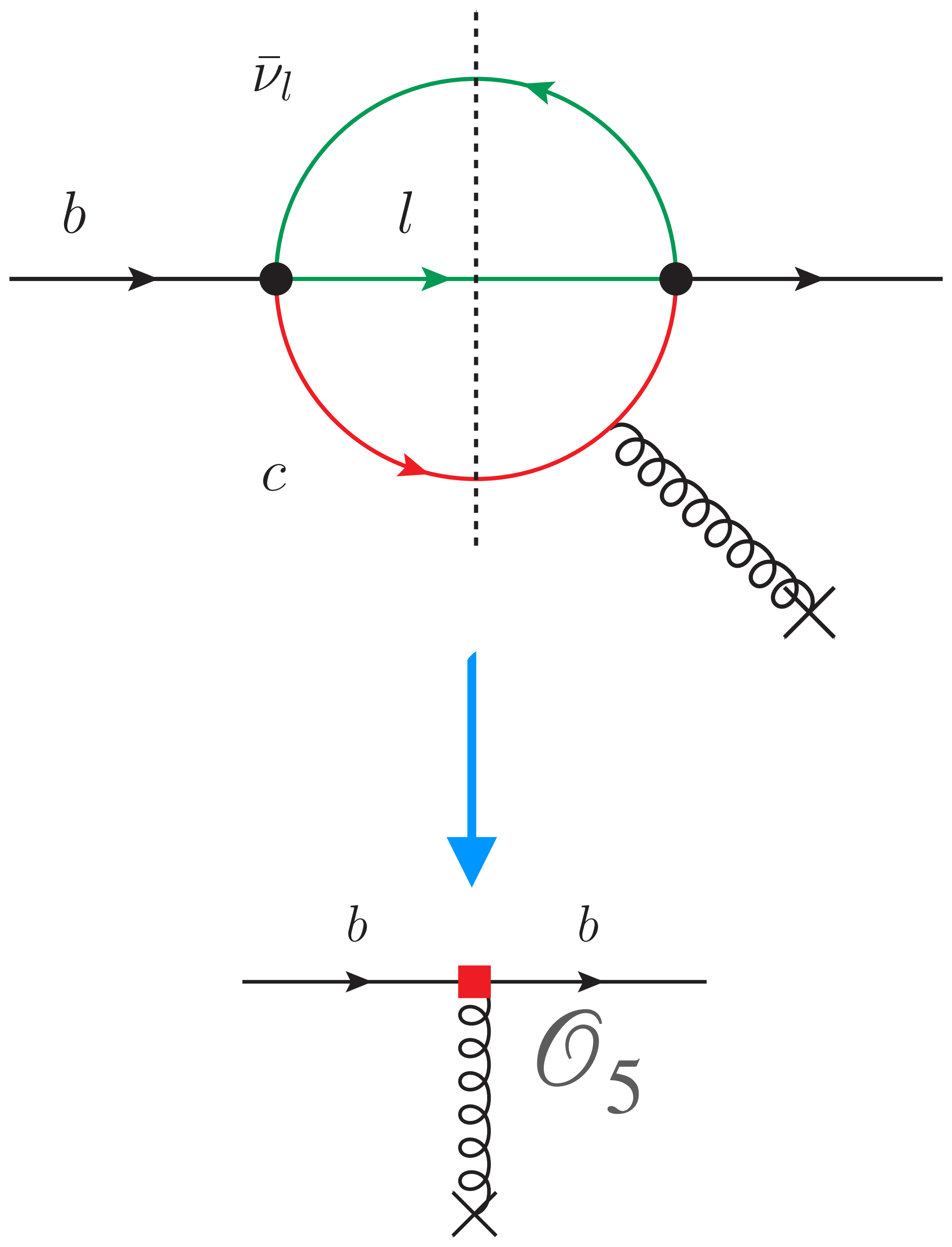
In a first approximation we can consider the **decay of a free bottom quark**

THE HEAVY QUARK EXPANSION

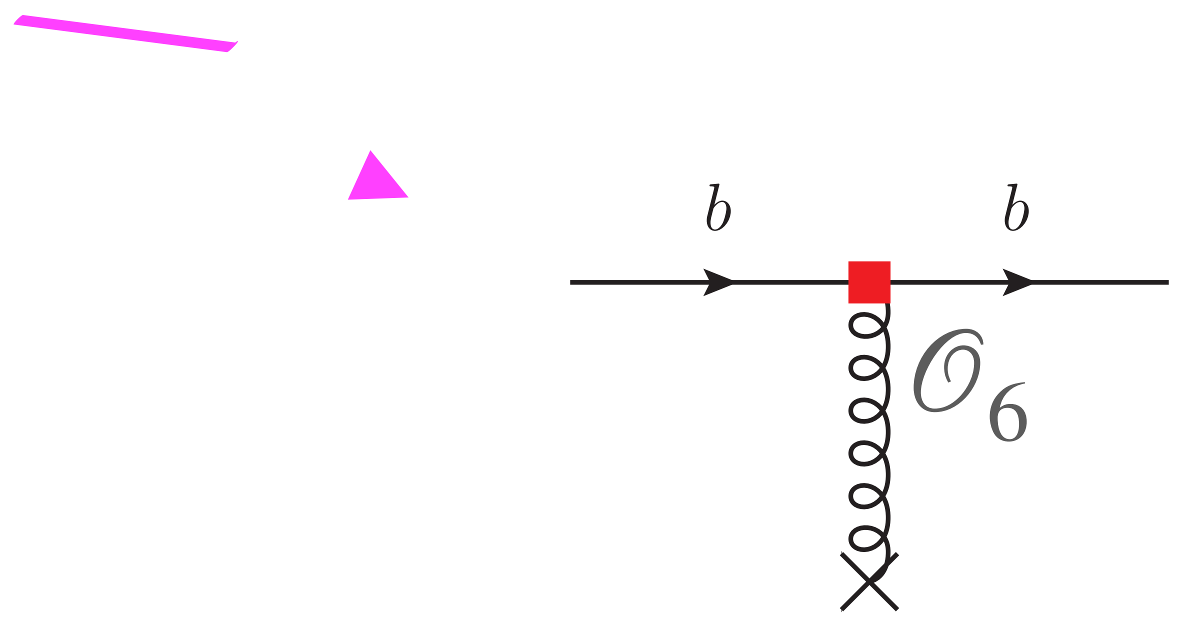
$$\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B | \mathcal{O}_5 | B \rangle}{m_b^2} + \Gamma_6 \frac{\langle B | \mathcal{O}_6 | B \rangle}{m_b^3} + \ldots$$



Free quark decay



Kinetic term μ_π^2, chromomagnetic term μ_G^2



Darwin term ρ_D^3

Spin-Orbit term ρ_{LS}^3

LIFETIMES

Total width

$$\frac{1}{\tau(B_q)} = \Gamma_b + \delta\Gamma_{B_q}$$
$$= \Gamma_{\text{non leptonic}} + \sum_{l=e,\mu,\tau} \Gamma(B \to X l \bar{\nu}_l) + \dots$$

- Nonleptonic decays (dominant)
  - $b \to c\bar{u}d$
  - $b \to c\bar{c}s$

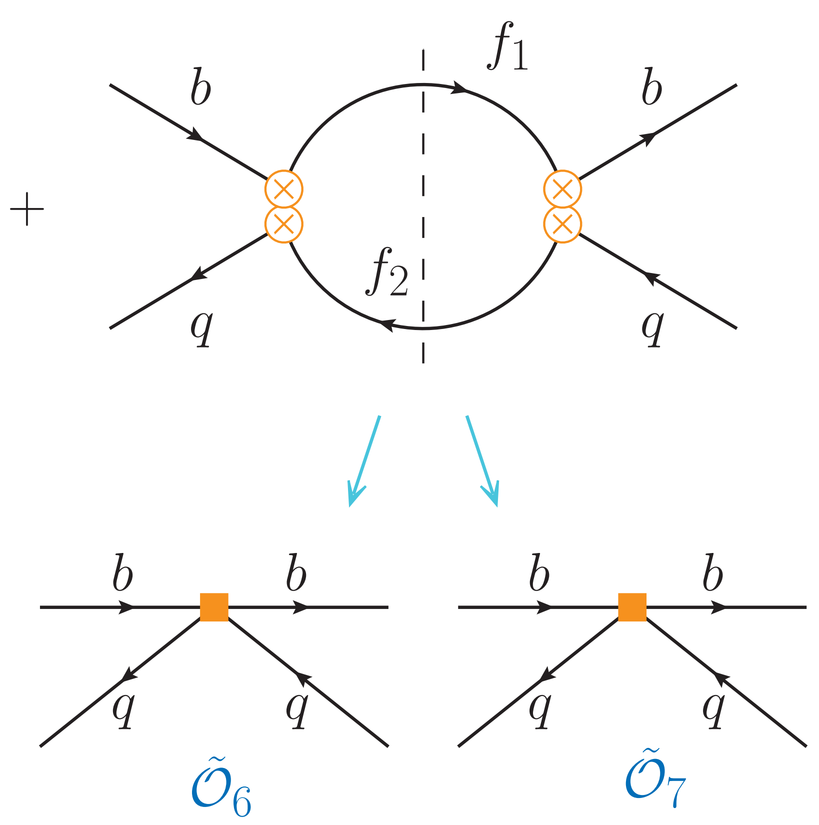
Lifetime ratios

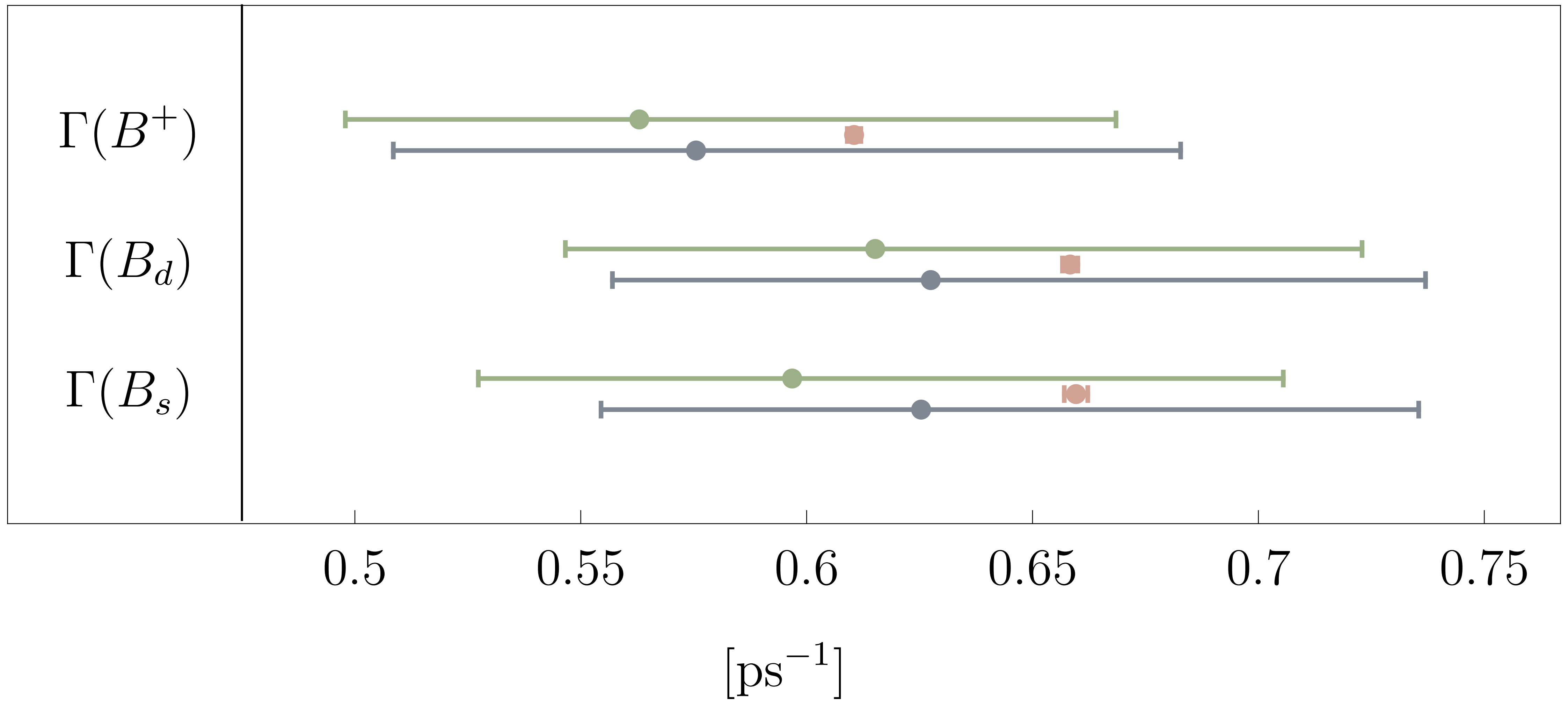
$$\frac{\tau(B_q)}{\tau(B_{q'})} = 1 + (\delta\Gamma_{B_q} - \delta\Gamma_{B'_q})\,\tau(B_q)$$

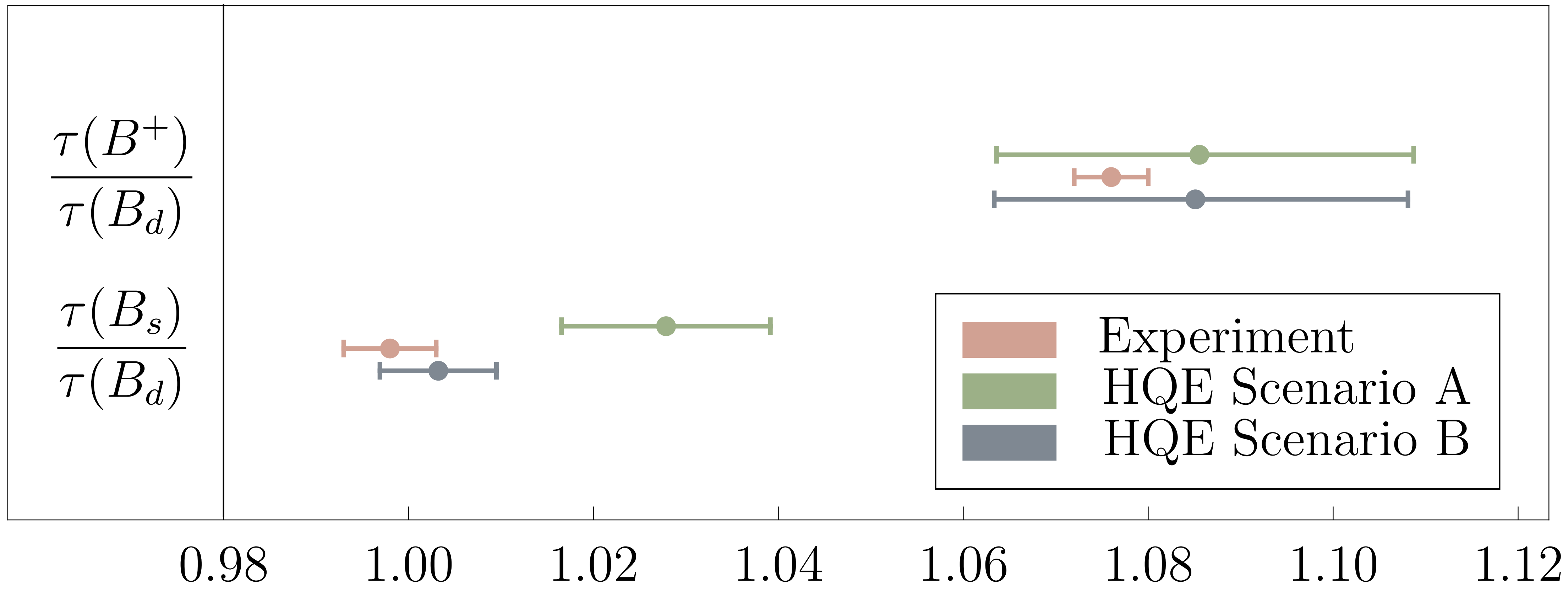
- Test the SM and framework used
- Perform indirect BSM searches

THE HEAVY QUARK EXPANSION

$$\Gamma = \Gamma_3 + \Gamma_5 \frac{\langle B | \mathcal{O}_5 | B \rangle}{m_b^2} + \Gamma_6 \frac{\langle B | \mathcal{O}_6 | B \rangle}{m_b^3} + 16\pi^2 \frac{\langle B | \tilde{\mathcal{O}}_6 | B \rangle}{m_b^3} + \ldots$$







Lenz, Piscopo, Rusov, JHEP 01 (2023) 004

SEMILEPTONIC DECAYS

- Total rate

NLO: Nir, *Phys.Lett.B* 221 (1989) 184
NNLO: Czarnecki, Pak, *Phys.Rev.Lett.* 100 (2008) 241807, *Phys.Rev.D* 78 (2008) 114015
N3LO: **MF**, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003, *JHEP* 08 (2022) 039
NLO at $1/m_b^2$: Mannel, Pivovarov, Rosenthal, *Phys.Rev.D* 92 (2015) 5, 054025

- E_ℓ and M_X^2 moments

NLO differential rate: Aquila, Gambino, Ridolfi, Uraltsev, *Nucl.Phys.B* 719 (2005) 77;
MF, Rahimi, Vos, *JHEP* 02 (2023) 086.
NNLO: Biswas, Melnikov, JHEP 02 (2010) 089; Gambino, JHEP 09 (2011) 055.

NLO at $1/m_b^2$: Alberti, Gambino, Nandi, *Nucl.Phys.B* 870 (2013) 16, *JHEP* 01 (2014) 147

- q^2

NLO: Jezebel, Kühn, *Nucl.Phys.B* 320 (1989) 20
NNLO: Fael, Herren,
NLO up to $1/m_b^2$: Mannel, Moreno, Pivovarov, *JHEP* 08 (2020) 089

- Kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017
Czarnecki, Melnikov, Uraltsev, Phys.Rev.Lett. 80 (1998) 3189
MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003, *Phys.Rev.D* 103 (2021) 1, 014005

NONLEPTONIC DECAYS

NLO: Altarelli, Petrarca, Phys.Lett.B 261 (1991) 303;
Bagan, Ball et al, Phys.Lett.B 351 (1995) 546, Nucl.Phys.B 432 (1994) 3
Lenz, Nierste, Ostermeier, Phys.Rev.D 56 (1997) 7228;
Krinner, Lenz, Rauh, Nucl.Phys.B 876 (2013) 31

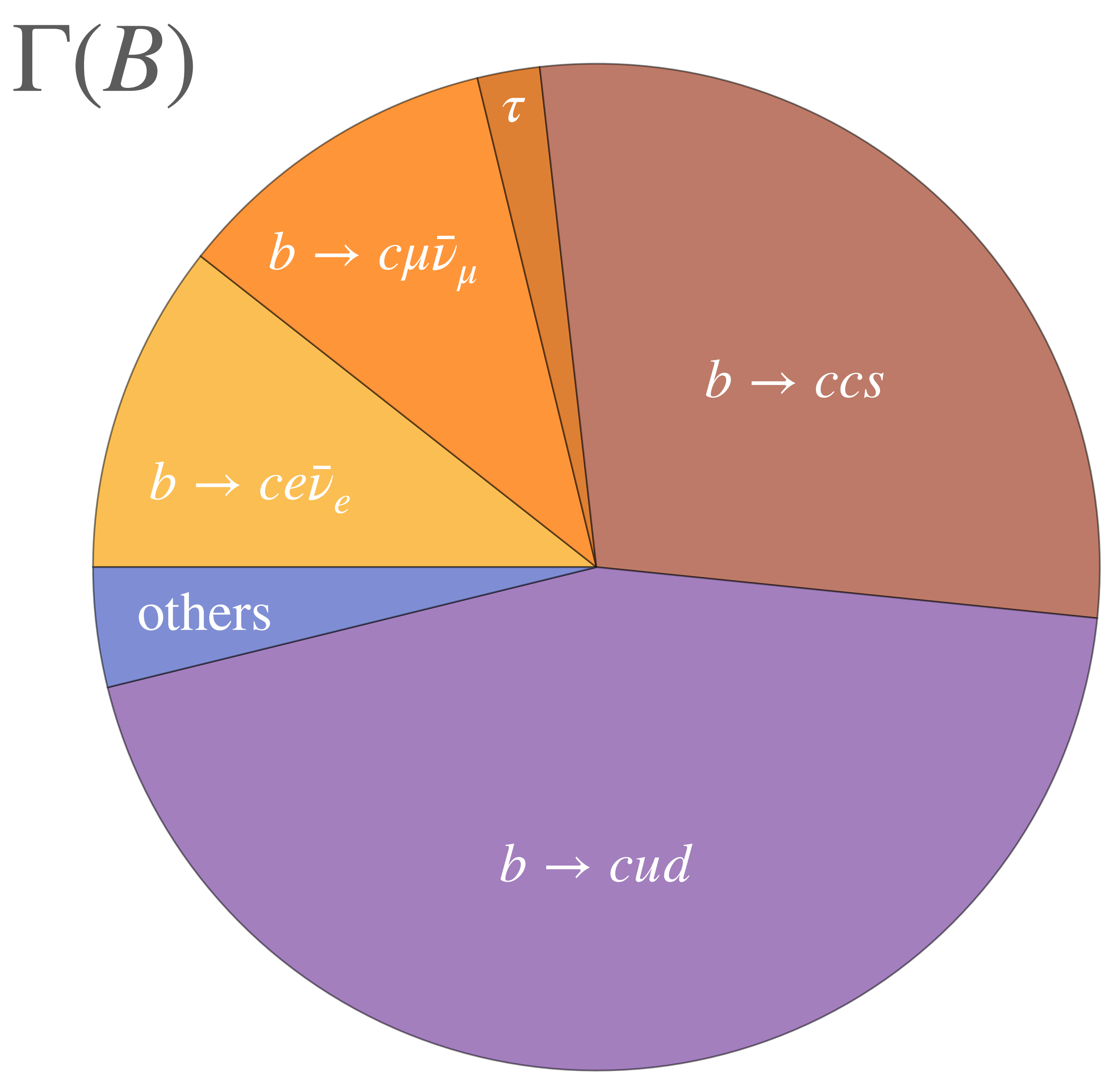
NNLO (massless and without resummation):
Czarnecki, Slusarczyk, Tkachov, Phys.Rev.Lett. 96 (2006) 171803

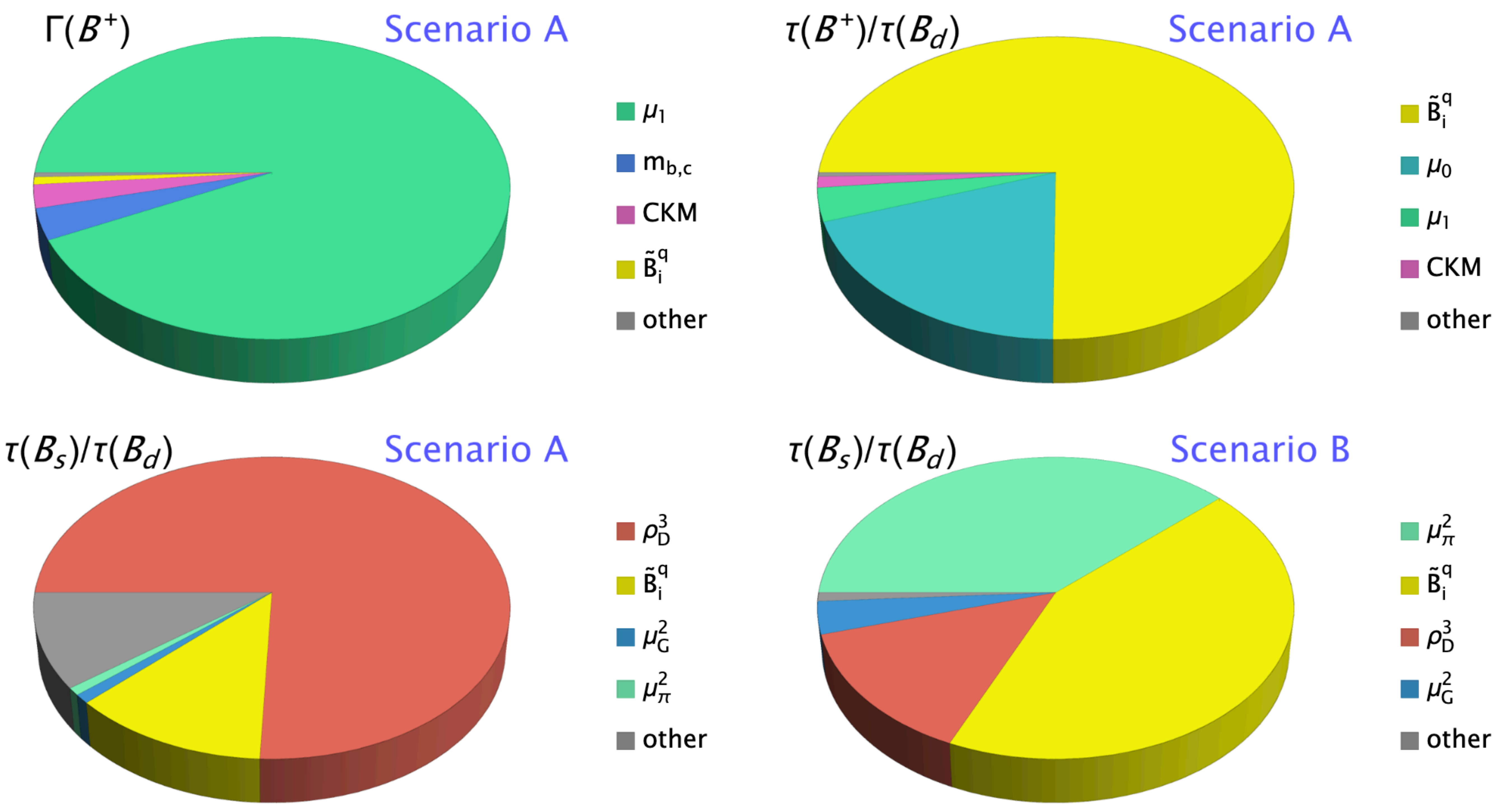
LO at $1/m_b^3$: Lenz, Piscopo, Rusov, JHEP 12 (2020) 199
Mannel, Moreno, Pivovarov, JHEP 08 (2020) 089
NLO at $1/m_b^2$ $(b \to c\bar{u}s)$: Mannel, Moreno, Pivovarov, 2408.06767 [hep-ph]

WA, PI at NLO: Beneke, Buchalla, Greub, Lenz, Nierste,
Phys.Lett.B 459 (1999) 631; Nucl.Phys.B 639 (2002) 389;
Franco, Lubicz, Mescia, Tarantino,
Nucl.Phys.B 625 (2002) 211; Nucl.Phys.B 633 (2002) 212

LO 4q at $1/m_b^4$: Gabbiani, Onishchenko, Petrov,
Phys.Rev.D 68 (2003) 114006; Phys.Rev.D 70 (2004) 094031

Error budget

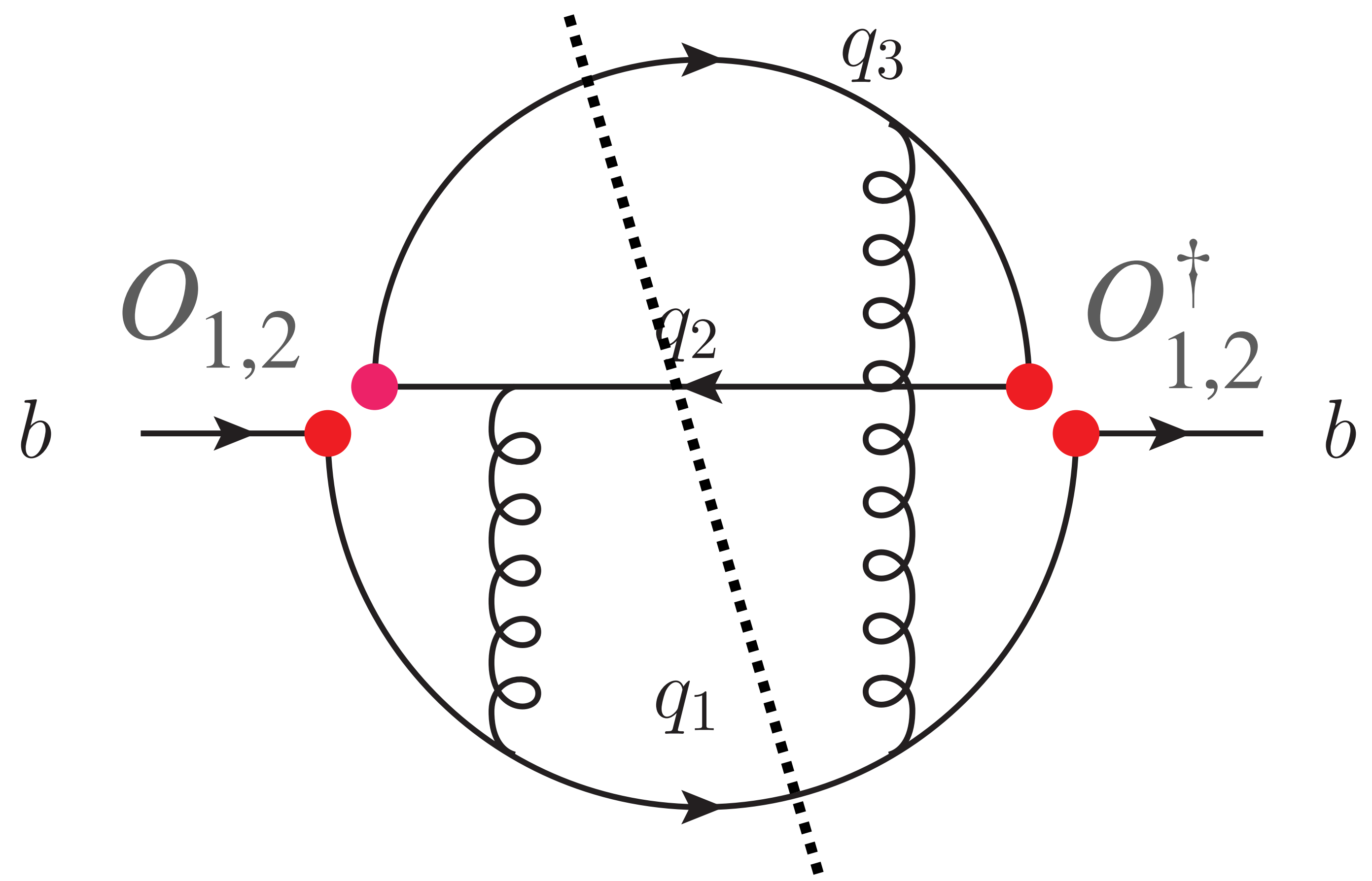




Lenz, Piscopo, Ruov, JHEP 01 (2023) 004

- Error on $\Gamma(B)$ dominated by theoretical uncertainties on Γ_3!
- GOAL: push accuracy for $\Gamma_3^{\text{non leptonic}}$ at NNLO

NONLEPTONIC DECAYS AT NNLO: CHALLENGES



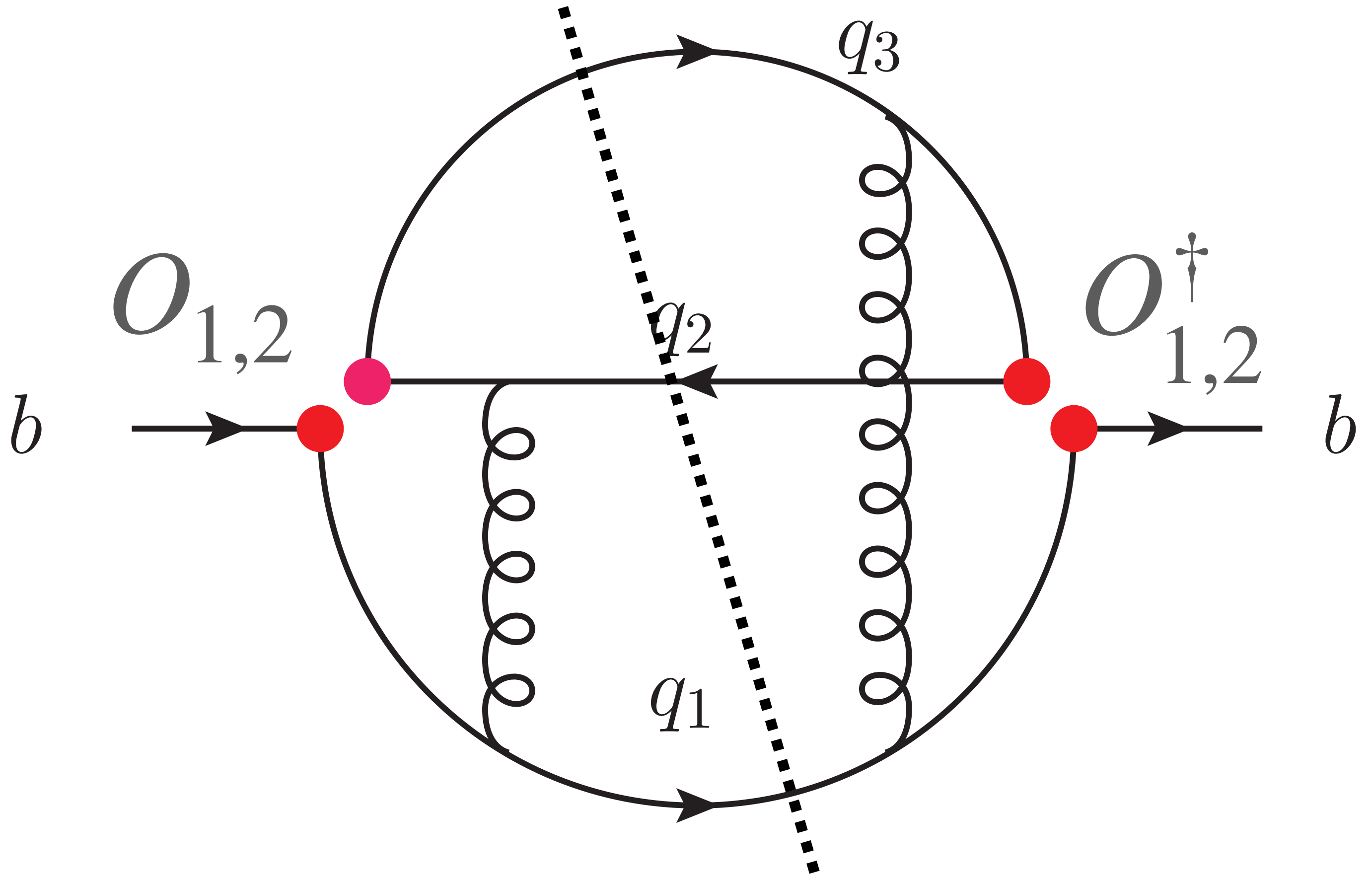
$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} \lambda_{q_1q_2q_2} \Big(C_1(\mu_b) O_1^{q_1q_2q_3} + C_2(\mu_b) O_2^{q_1q_2q_3} \Big) + \text{h.c.}$$

Four loop master integrals depending on $\rho = m_c/m_b$

Non-trivial renormalization of effective operators

Issues with γ_5 in dimensional regularisation

NONLEPTONIC DECAYS AT NNLO: CHALLENGES



$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} \lambda_{q_1q_2q_2}\Big(C_1(\mu_b)O_1^{q_1q_2q_3} + C_2(\mu_b)O_2^{q_1q_2q_3} \Big) + \text{h.c.}$$

Four loop master integrals depending on $\rho = m_c/m_b$

Non-trivial renormalization of effective operators

Issues with γ_5 in dimensional regularisation

DIVIDE ET IMPERA

STRATEGY

- Warm up exercise: recalculate $b \to cl\bar{\nu}_l$ at NNLO with "Expand & match" method. Compare and validate with known results in the literature. MF, Lange, Schönwald, Steinhauser JHEP 09 (2021) 152
- Attack the more complicated nonleptonic decays $b \to c\bar{u}d$ and $b \to c\bar{c}s$ with "Expand & match" method. Solve issue with γ_5 and renormalization.
- Update predictions for $\Gamma(B_q)$

NUMERICAL EVALUATION OF MASTER INTEGRALS

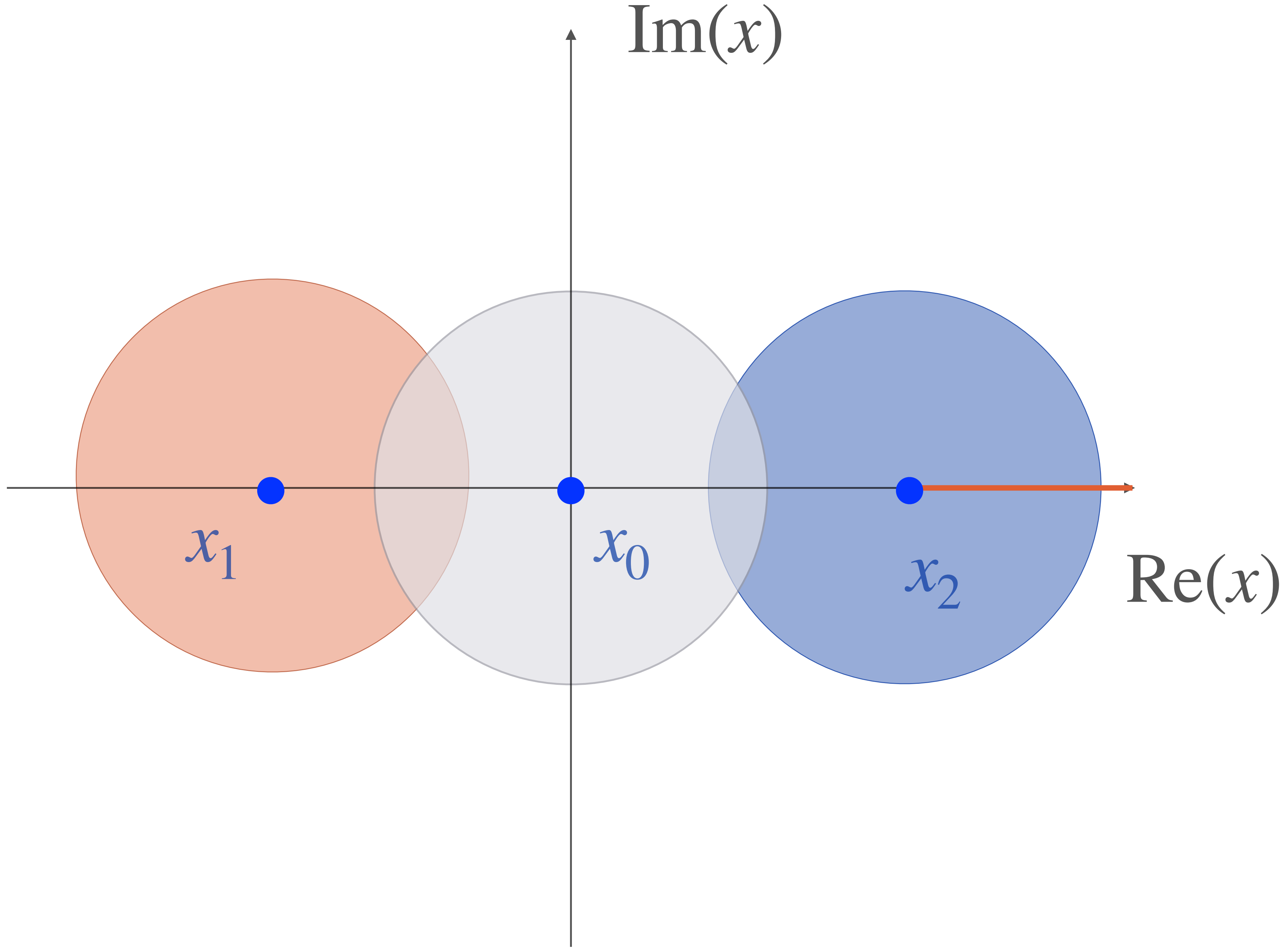
➤ Solving master integrals: method of differential equations

Kotikov, Phys. Lett. B 254 (1991) 158;
Gehrmann, Remiddi, Nucl. Phys. B 580 (2000) 485

Master integrals

$$\frac{\partial \vec{I}}{\partial x} = M(x, \epsilon)\vec{I}$$

➤ Construct a series expansion around some point x_0 [and $\epsilon = (d-4)/2$]



$$I_a(x,\epsilon) = \sum_{m=m_{\min}}^{m_{\max}} \sum_{n=0}^{n_{\max}} c_{a,mn}\, \epsilon^m (x - x_0)^n$$

S. Pozzorini and E. Remiddi, Comput. Phys. Commun. 175, 381 (2006).
X. Liu, Y.-Q. Ma, and C.-Y. Wang, Phys. Lett. B 779, 353 (2018).
R. N. Lee, A. V. Smirnov, and V. A. Smirnov, JHEP 03, 008 (2018).
M. K. Mandal and X. Zhao, JHEP 03, 190 (2019).
M. L. Czakon and M. Niggetiedt, JHEP 05, 149 (2020)..
F. Moriello, JHEP 01, 150 (2020).
MF, Lange, Schönwald, Steinhauser JHEP 09 (2021) 152
Hidding, Comput.Phys.Commun. 269 (2021) 108125
Armadillo, Bonciani, Devoto, Rana, Vicini, Comput.Phys.Commun. 282 (2023) 108545

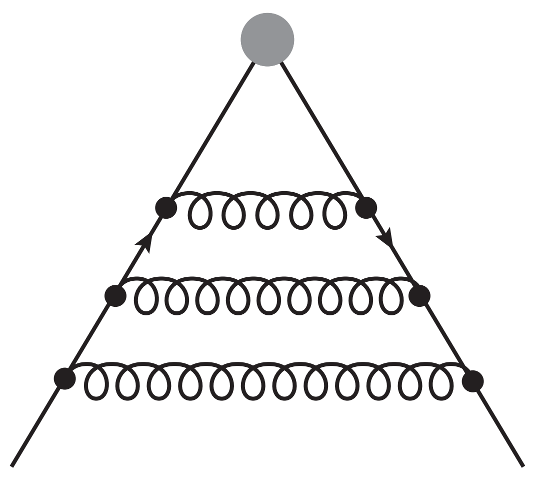
APPLICATIONS

Fix all external kinematics to numerical values
$s = 2, t = 1/10, m = 1$, etc

Several approaches

- DESS
  Lee, Smirnov, Smirnov, JHEP 03 (2018) 008
- DiffExp
  Hidding, Comput.Phys.Commun. 269 (2021) 108125
- SeaSide
  Armadillo, Bonciani, Devoto, Rana, Vicini, Comput.Phys.Commun. 282 (2023) 108545
- Expand and match
  MF, Lange, Schönwald, Steinhauser JHEP 09 (2021) 152

Heavy-quark form factors at $O(\alpha_s^3)$



MF, Lange, Schönwald, Steinhauser Phys.Rev.Lett. 128 (2022) 17; Phys.Rev.D 106 (2022) 3, 034029; Phys.Rev.D 107 (2023), 094017

also application to NRQCD

Egner, **MF,** Lange, Piclum, Schönwald, Steinhauser, Phys.Rev.D 104 (2021) 5, 054033, Phys.Rev.D 105 (2022) 11, 114007

- Auxiliary mass method
  Xiao Liu, Yan-Qing Ma, Comput.Phys.Commun. 283 (2023) 108565

$$I(\vec{n}) = \int \prod_{i=1}^{L} d^D\ell_i \frac{1}{D_1^{n_1}\dots D_N^{n_N}}$$

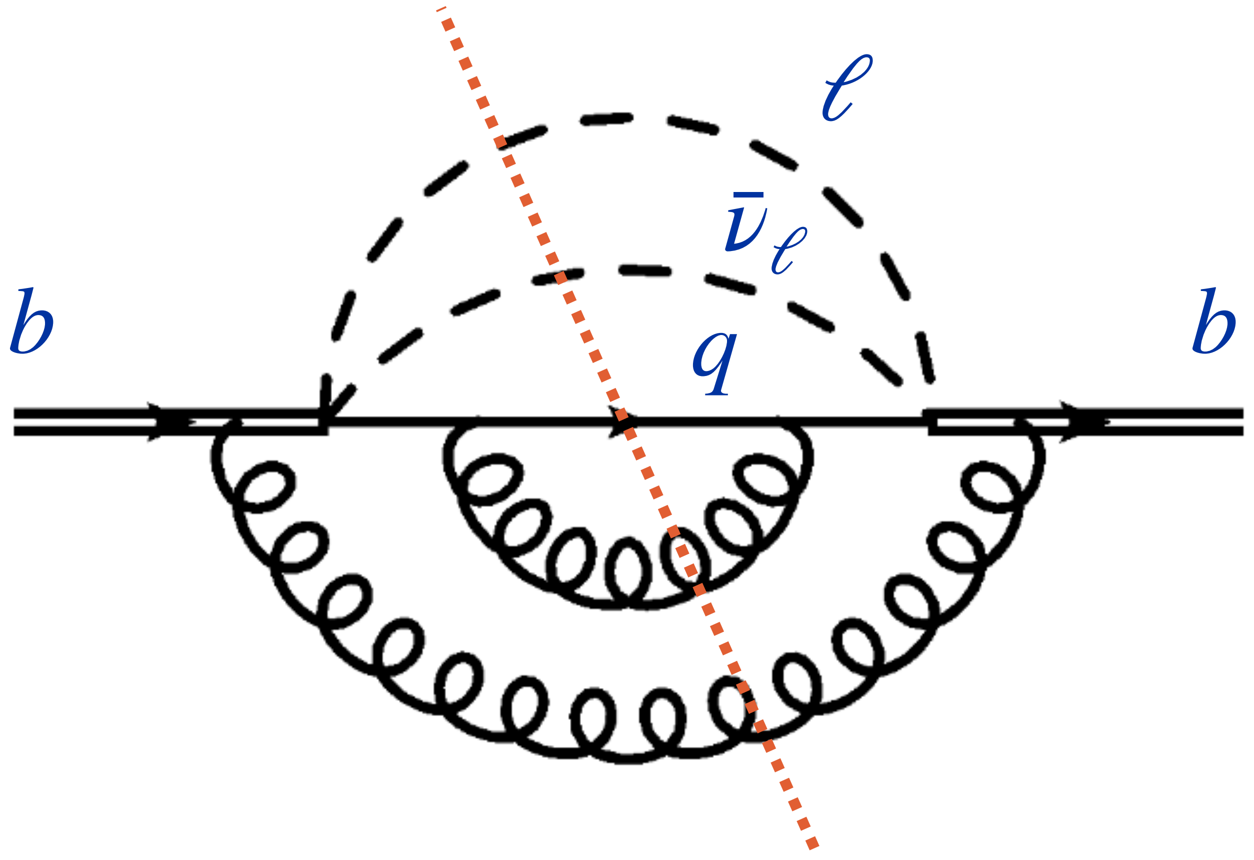
$$= \lim_{\eta \to i0^-} I_{\text{aux}}(\vec{n}, \eta)$$

- Precise numerical evaluation of boundary conditions

WARM UP: SEMILEPTONIC DECAYS AT NNLO

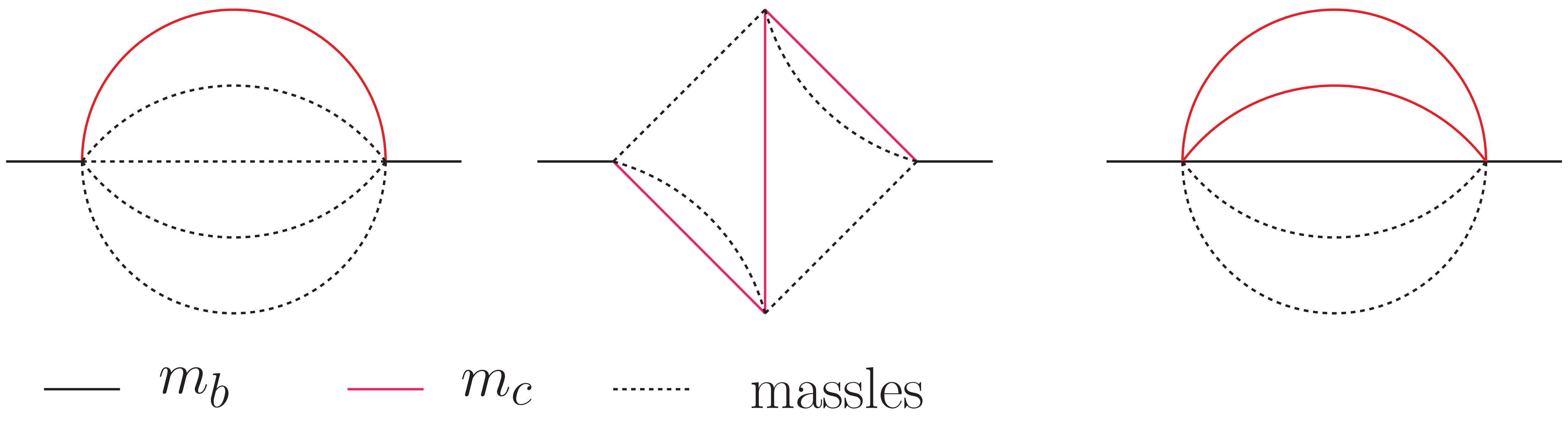
Egner, Fael, Schönwald, Steinhauser, *JHEP* 09 (2023) 112

$$\Gamma(B \to X_q \ell \bar{\nu}_\ell) = \frac{G_F^2 m_b^5 |V_{qb}|^2}{192\pi^3} \left[X_0(\rho) + \frac{\alpha_s}{\pi} X_1(\rho) + \left(\frac{\alpha_s}{\pi}\right)^2 X_2(\rho) + \ldots \right]$$

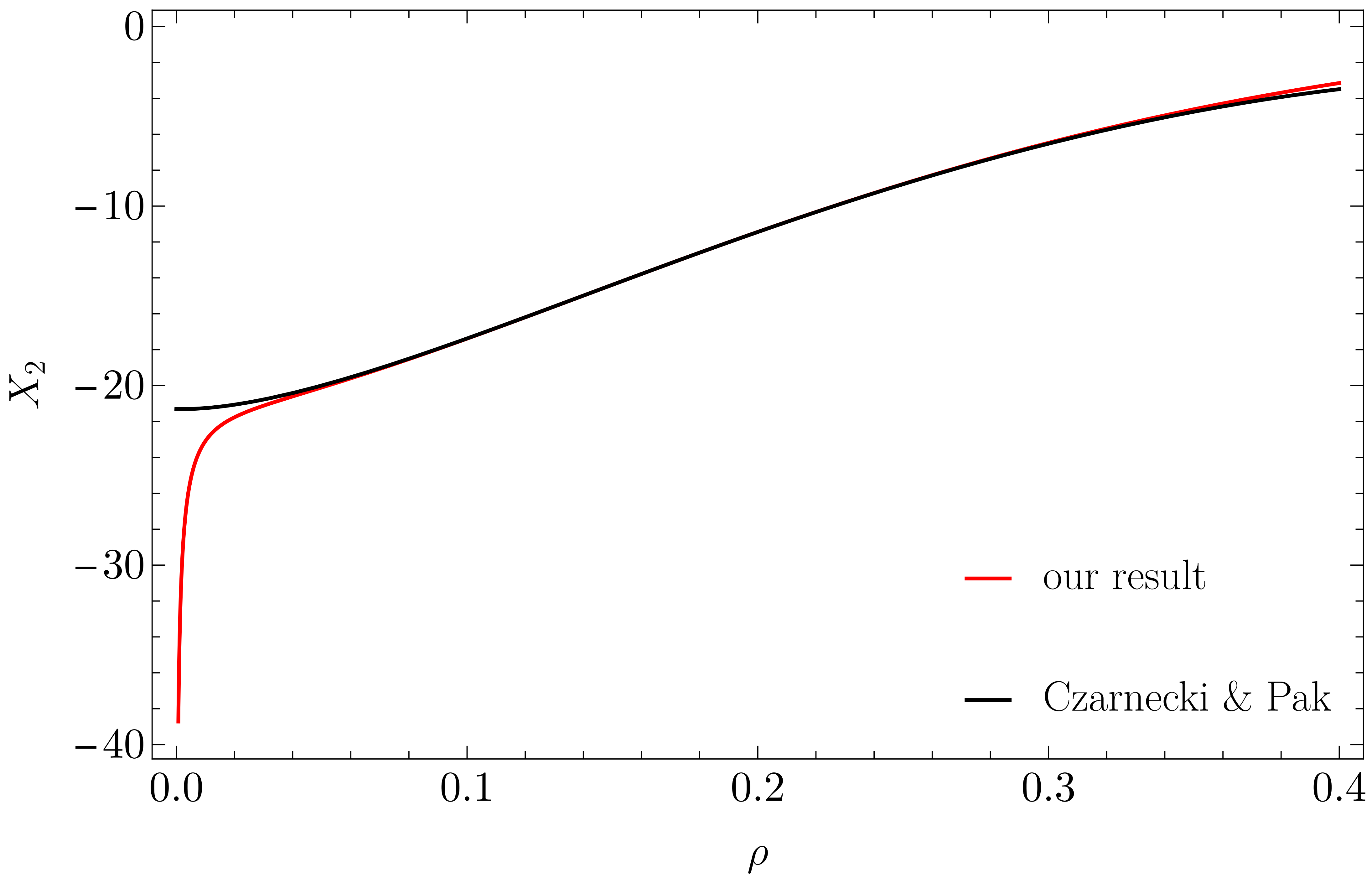


- Establish differential equations w.r.t. $\rho = m_c/m_b$
- Solve for the imaginary part of master integrals with "Expand & match"
- Compare with asymptotic expansion in the limit $\rho \to 0$

Czarnecki, Pak, Phys.Rev.D 78 (2008) 114015

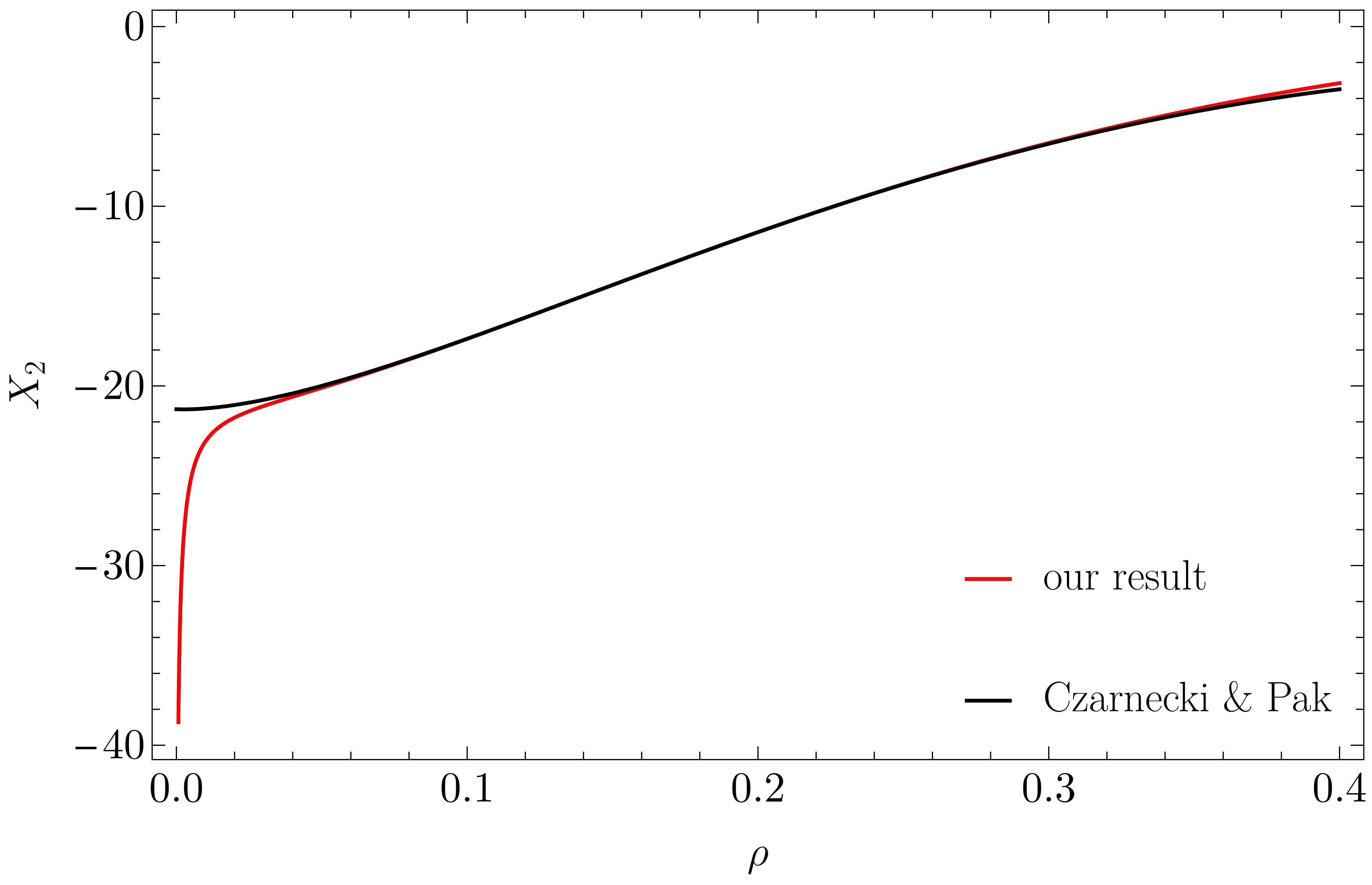


- Analytic boundary conditions can be easily calculated in the limit $\rho \to 1\,(m_c \simeq m_b)$
- "Expand and match" allows to extrapolate the solution at $\rho = 0$



Egner, Fael, Schönwald, Steinhauser, *JHEP* 09 (2023) 112
asymptotic expansion from: Czarnecki, Pak, Phys.Rev.D 78 (2008) 114015

- Analytic boundary conditions can be easily calculated in the limit $\rho \to 1\,(m_c \simeq m_b)$
- "Expand and match" allows to extrapolate the solution at $\rho = 0$



Egner, Fael, Schönwald, Steinhauser, *JHEP* 09 (2023) 112
asymptotic expansion from: Czarnecki, Pak, Phys.Rev.D 78 (2008) 114015

Our result manifests a mass singularity, what is going on here?

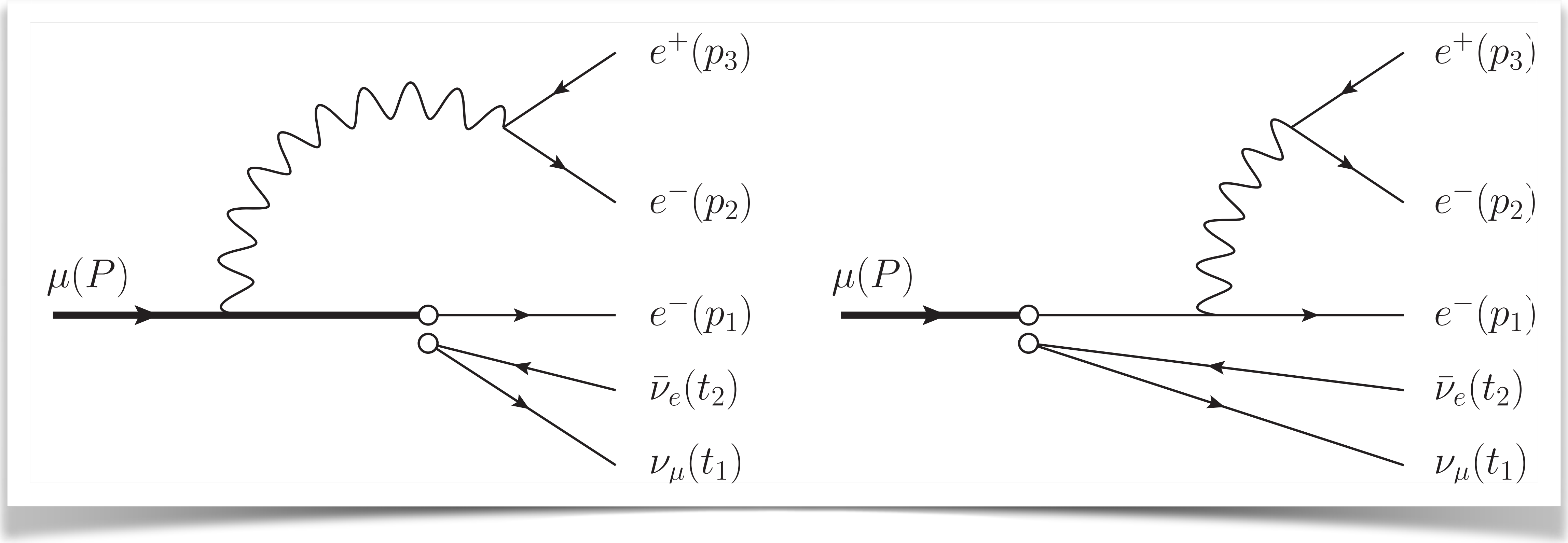
INTERLUDE: FIVE-BODY DECAY OF THE MUON

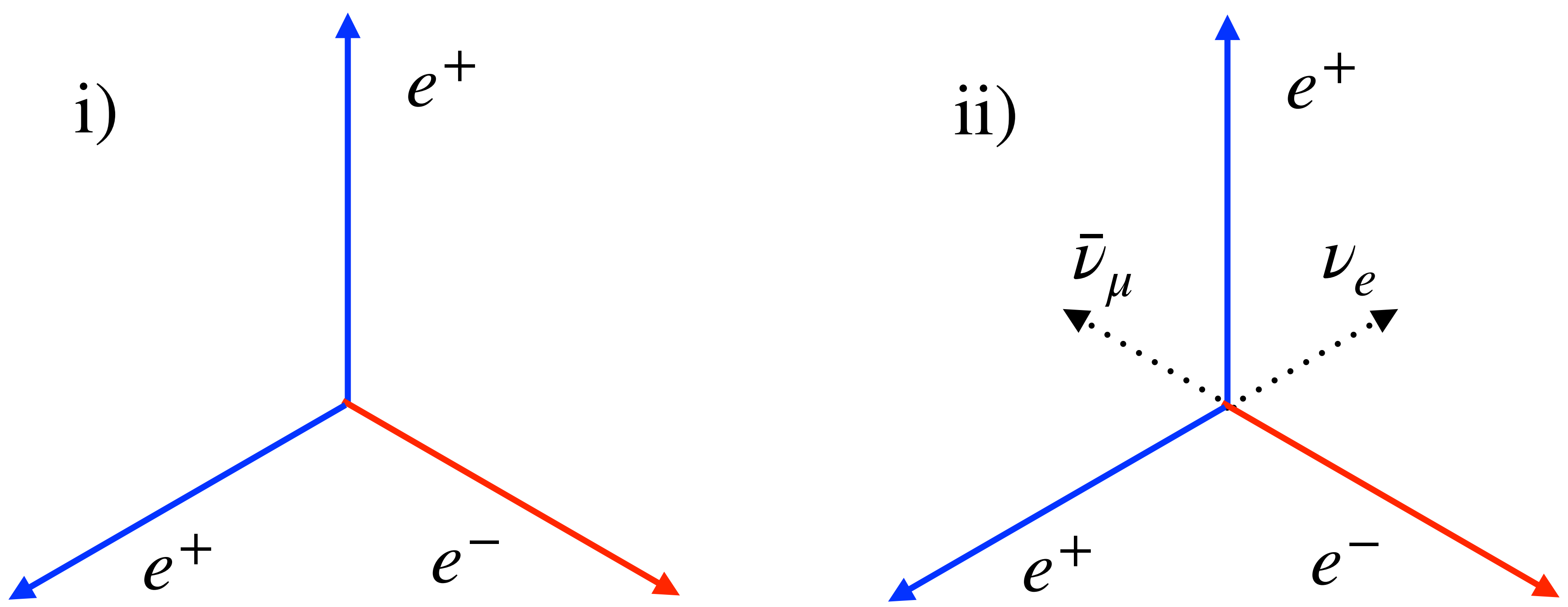
- Search for CLFV at Mu3e experiment

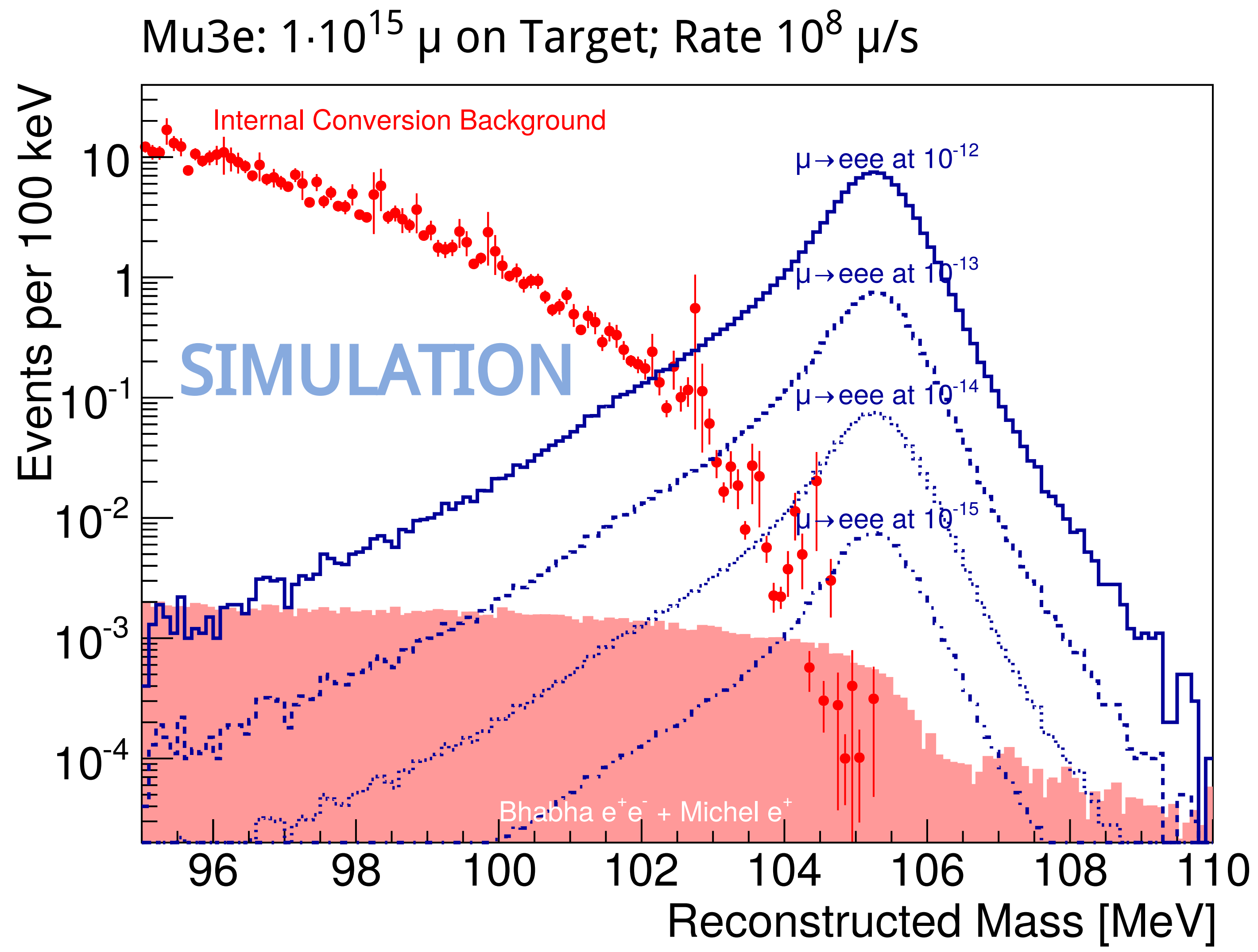
$$\mu^+ \to e^+ e^- e^+$$

- Dominant source of background

$$\mu^+ \to e^+ e^- e^+ \bar{\nu}_e \nu_e$$



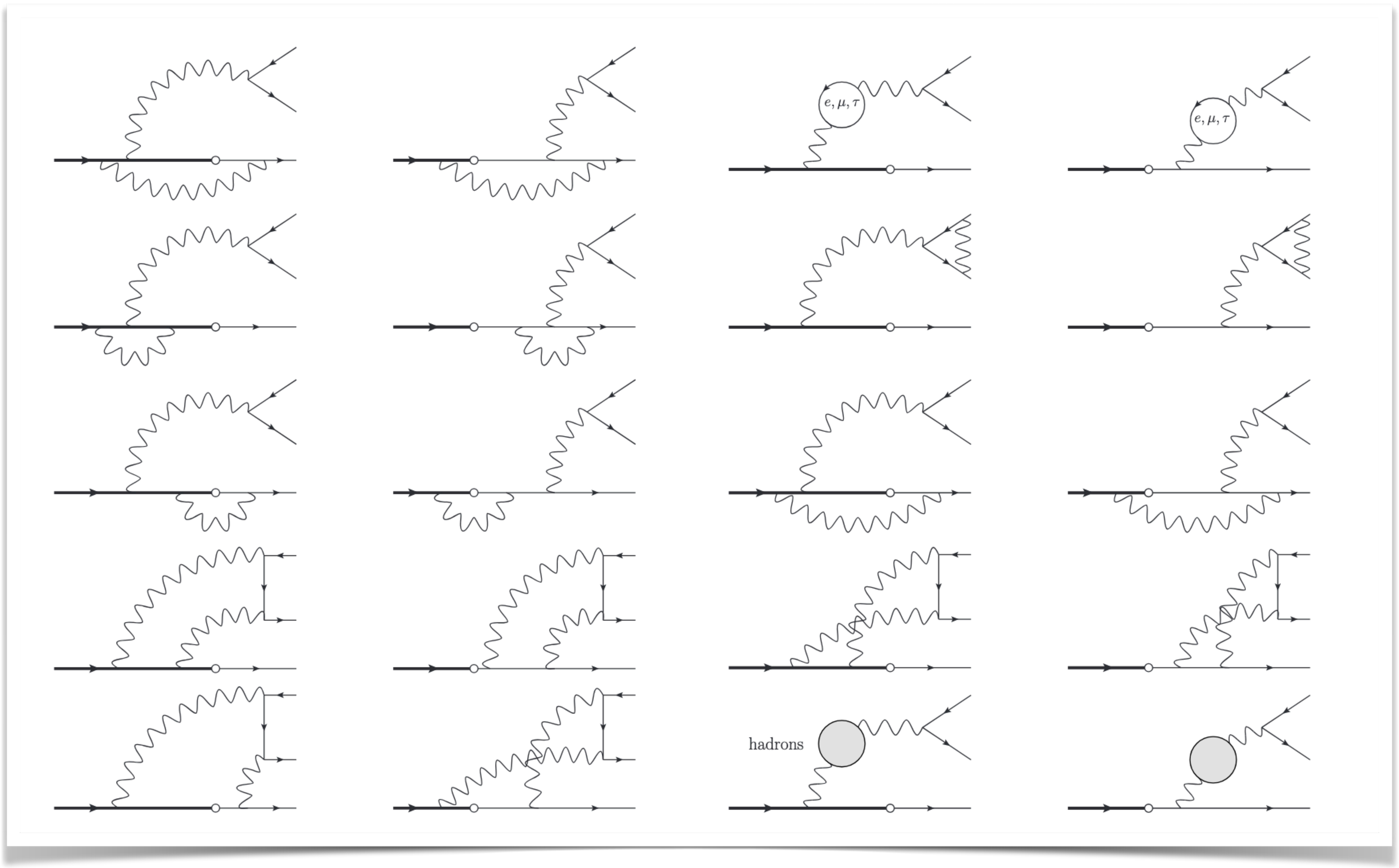


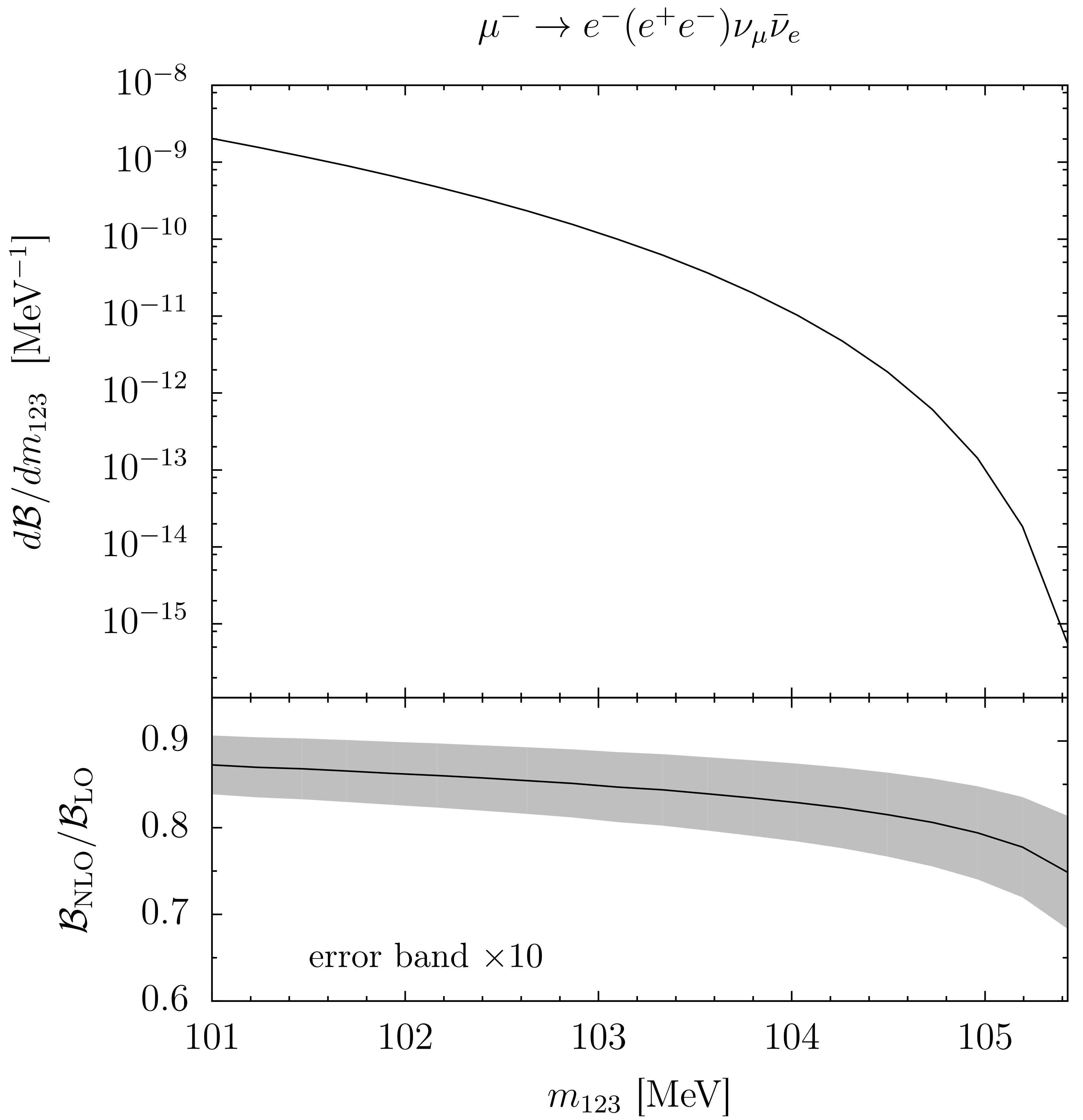


K. Arndt et al. [Mu3e], Nucl. Instrum. Meth. A 1014, 165679 (2021)

NLO CORRECTIONS TO FIVE-BODY DECAY

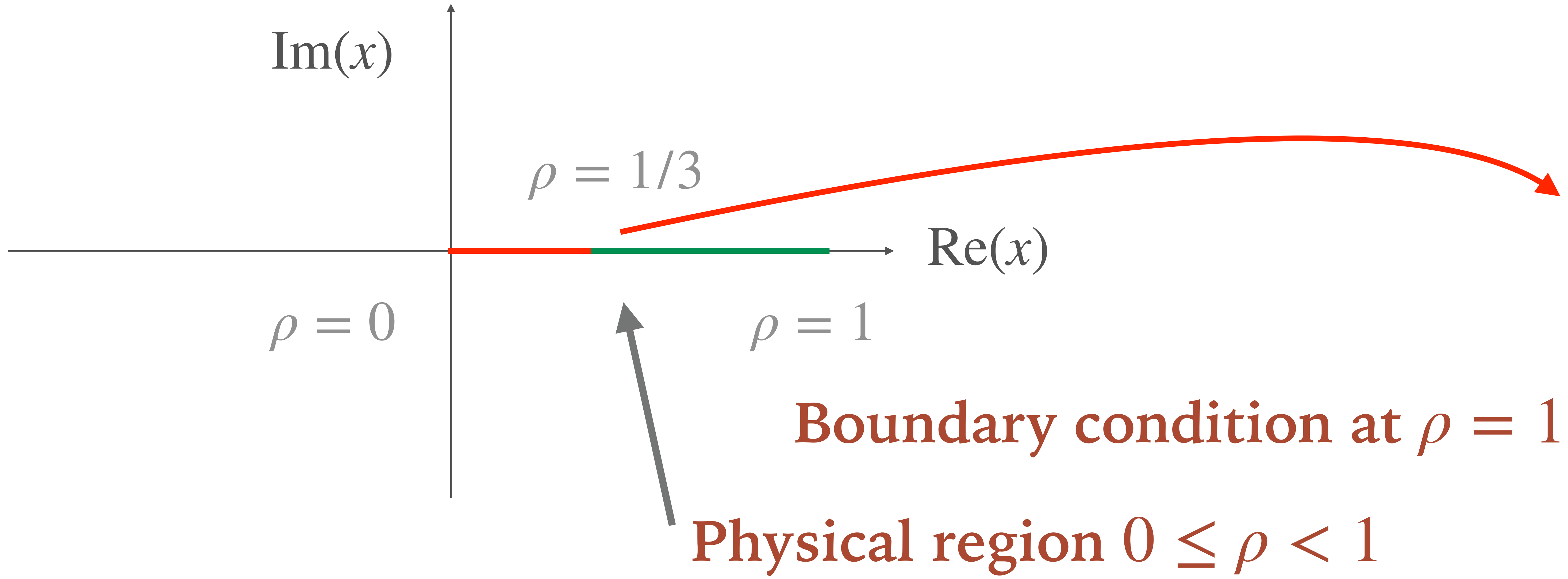
Fael, Greub, JHEP 01 (2017) 084



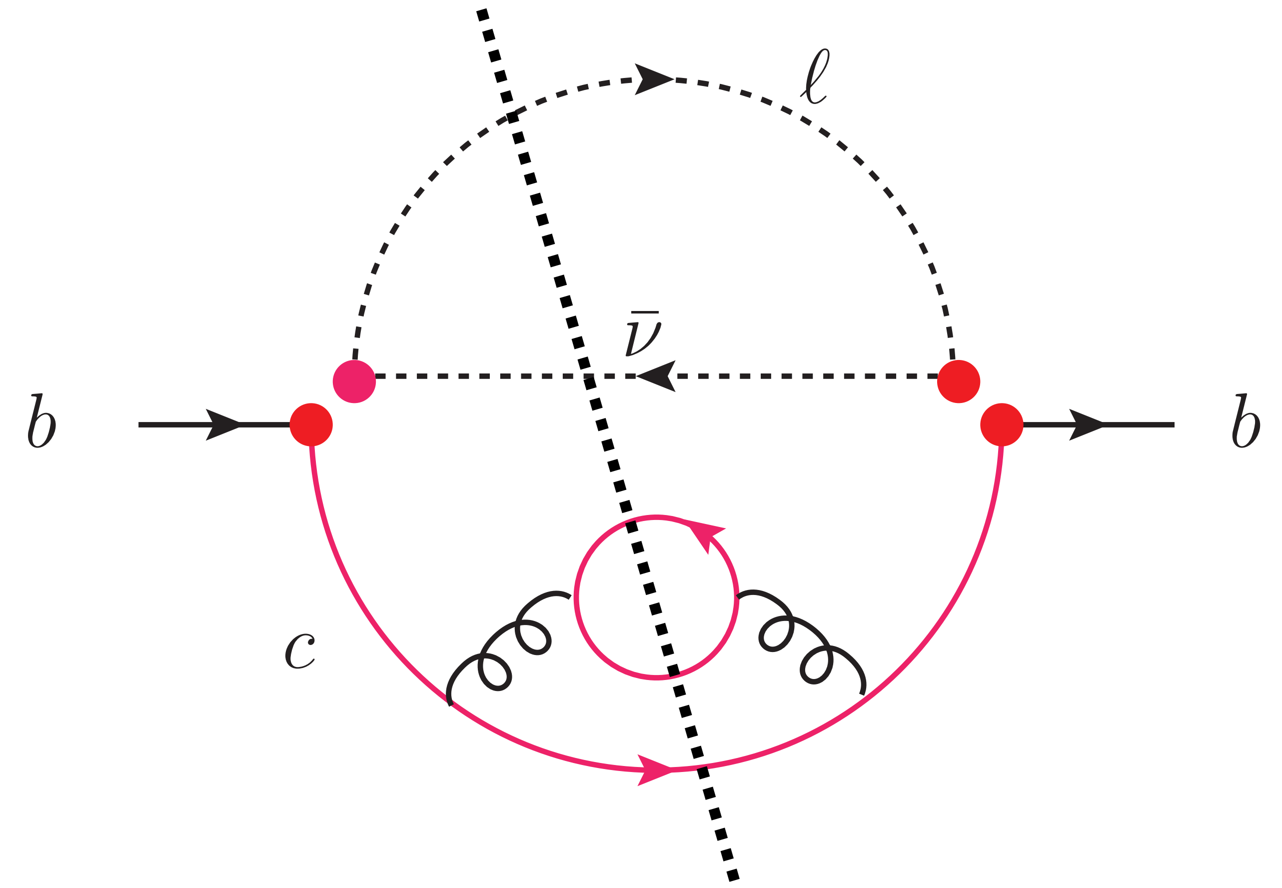


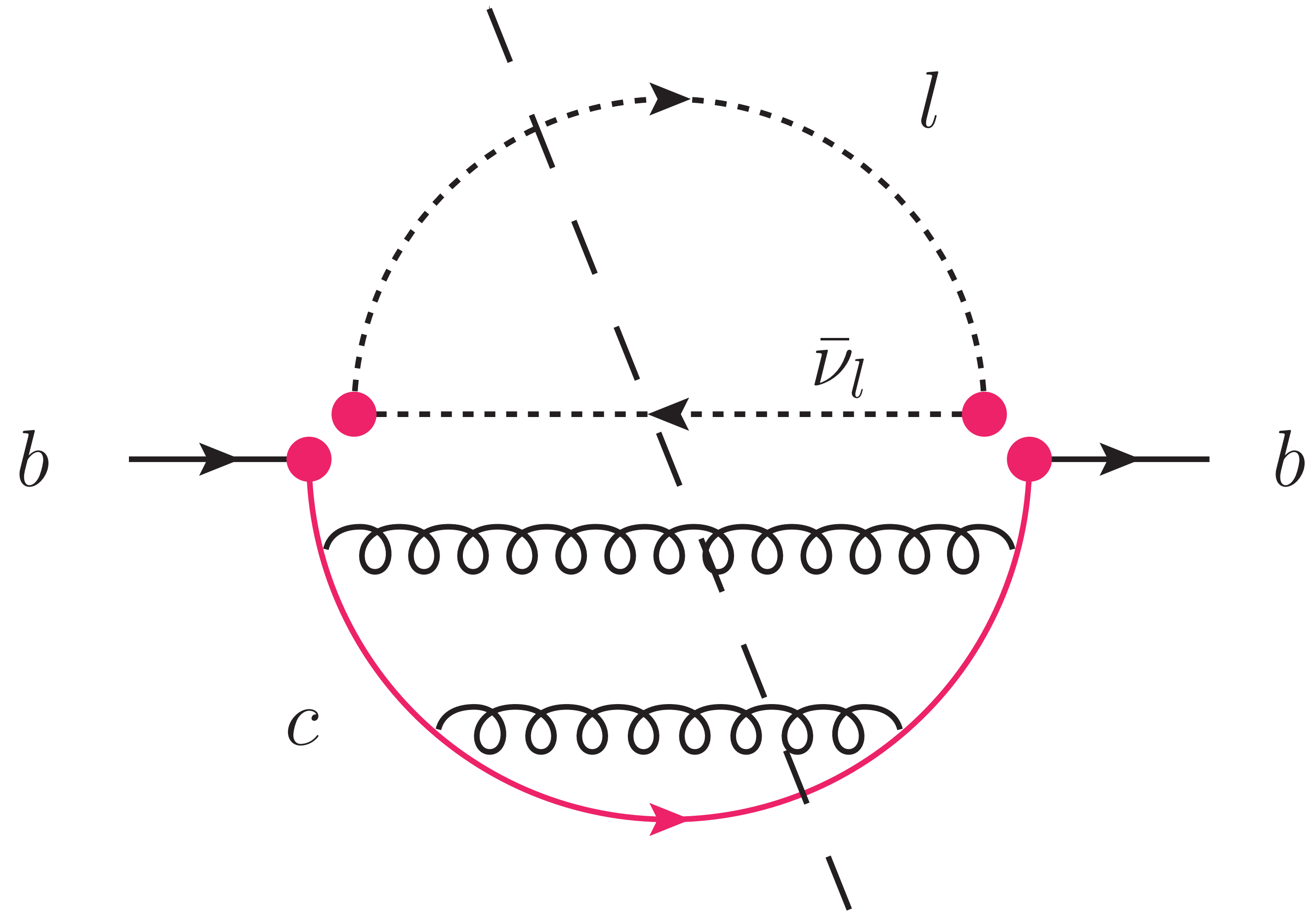
- Very stringent selection cuts on m_{123}
- QED corrections can reach 15-20% effect

FIVE-BODY DECAY OF THE BOTTOM

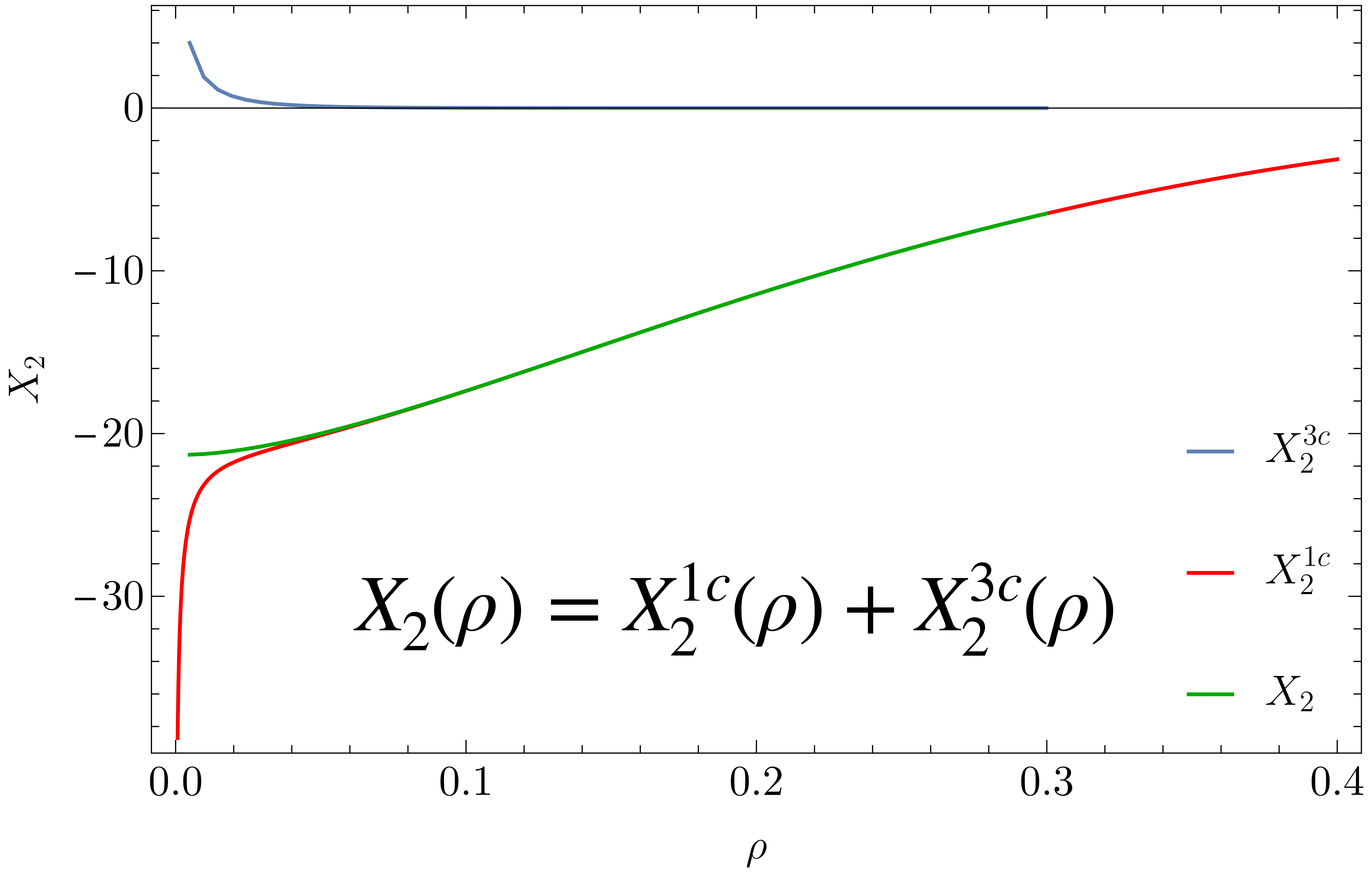


Threshold for 3 charm quarks





- Boundary conditions at $\rho = 1/2$ with AMFlow
- For all master integrals consider real and imaginary part.



Egner, Fael, Schönwald, Steinhauser, *JHEP* 09 (2023) 112

NONLEPTONIC DECAYS AND γ_5

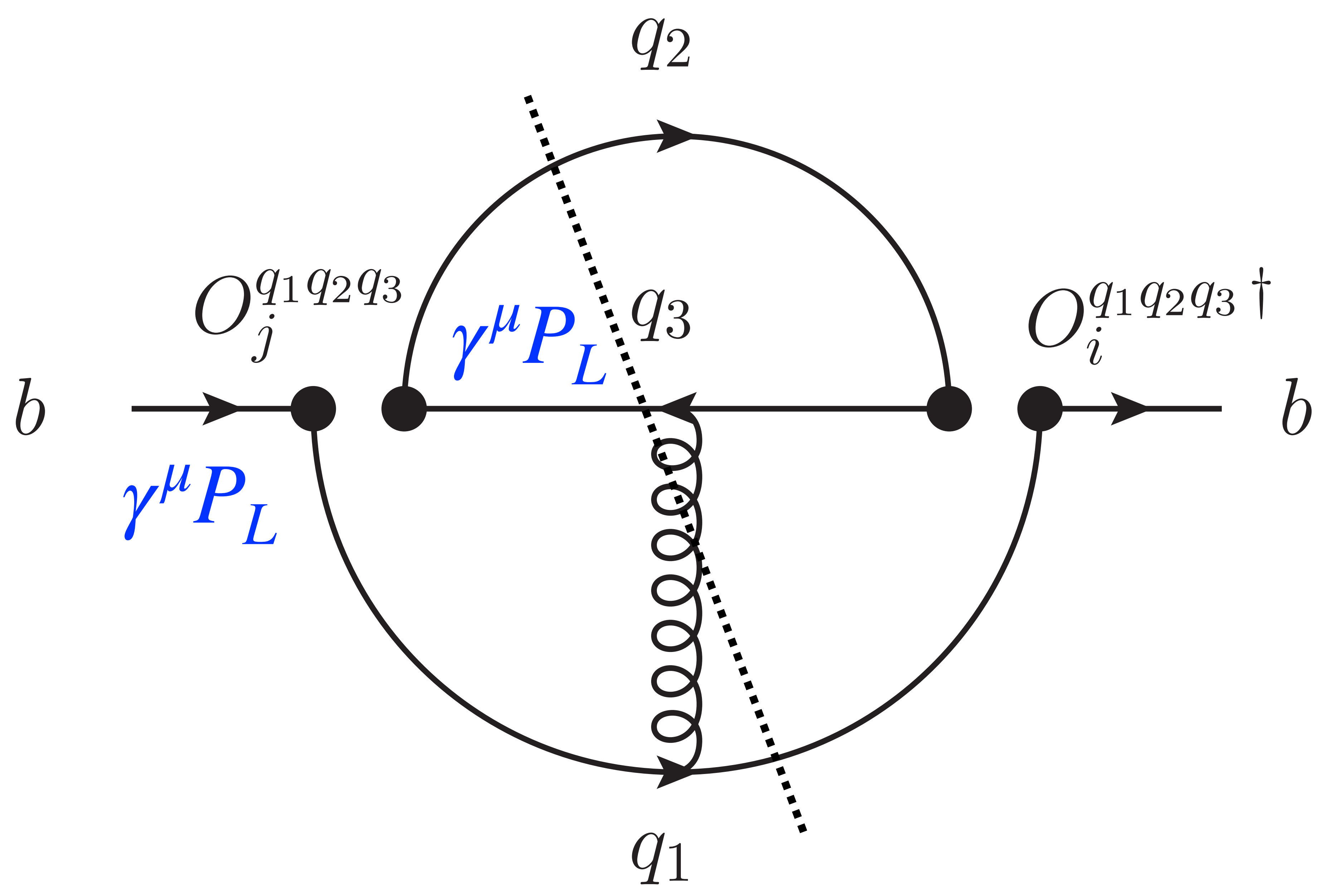
$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{q_{1,3}=u,c} \sum_{q_2=d,s} \lambda_{q_1q_2q_2}\Big(C_1(\mu_b)O_1^{q_1q_2q_3} + C_2(\mu_b)O_2^{q_1q_2q_3}\Big) + \mathrm{h.c.}$$

Traditional basis

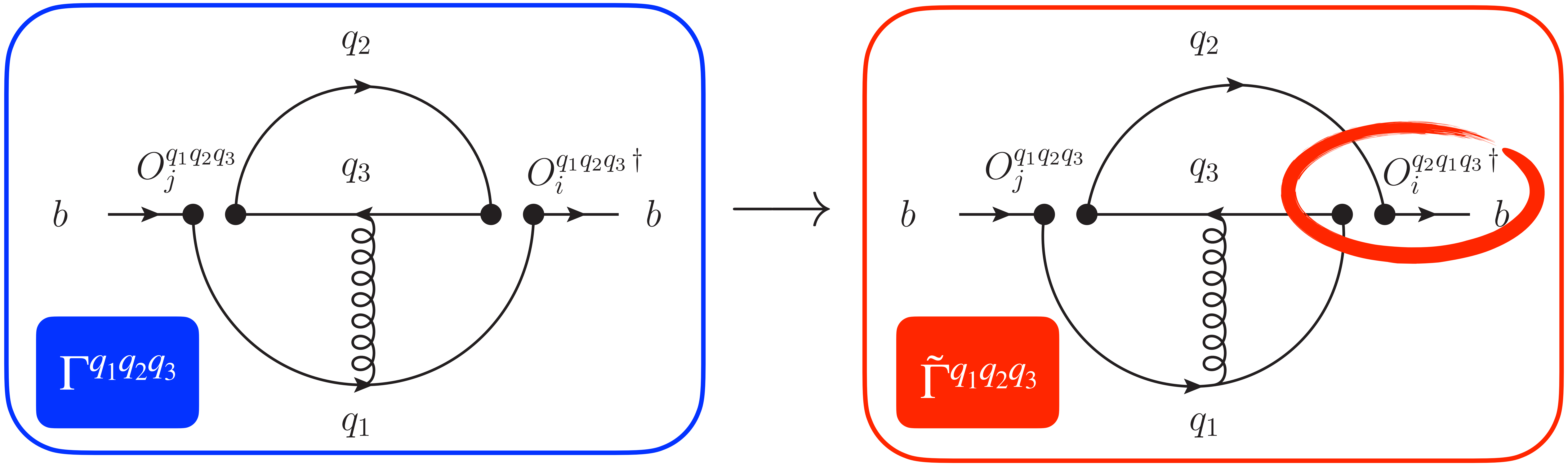
Buras, Weisz, NPB 333 (1990) 66

$$O_1^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha),$$

$$O_2^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\alpha)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\beta),$$



$$\simeq \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5)\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5)$$



Fierz identity in $d = 4$

$$O_1^{q_1q_2q_3} = (\bar{q}_1^\alpha \gamma^\mu P_L b^\beta)(\bar{q}_2^\beta \gamma_\mu P_L q_3^\alpha)$$

$$= (\bar{q}_2^\alpha \gamma^\mu P_L b^\alpha)(\bar{q}_1^\beta \gamma_\mu P_L q_3^\beta) = O_2^{q_2q_1q_3}$$

$$\Gamma^{q_1q_2q_3}(\rho) = \tilde{\Gamma}^{q_1q_2q_3}(\rho)\Big|_{\tilde{C}_1 \to C_2, \tilde{C}_2 \to C_1}$$

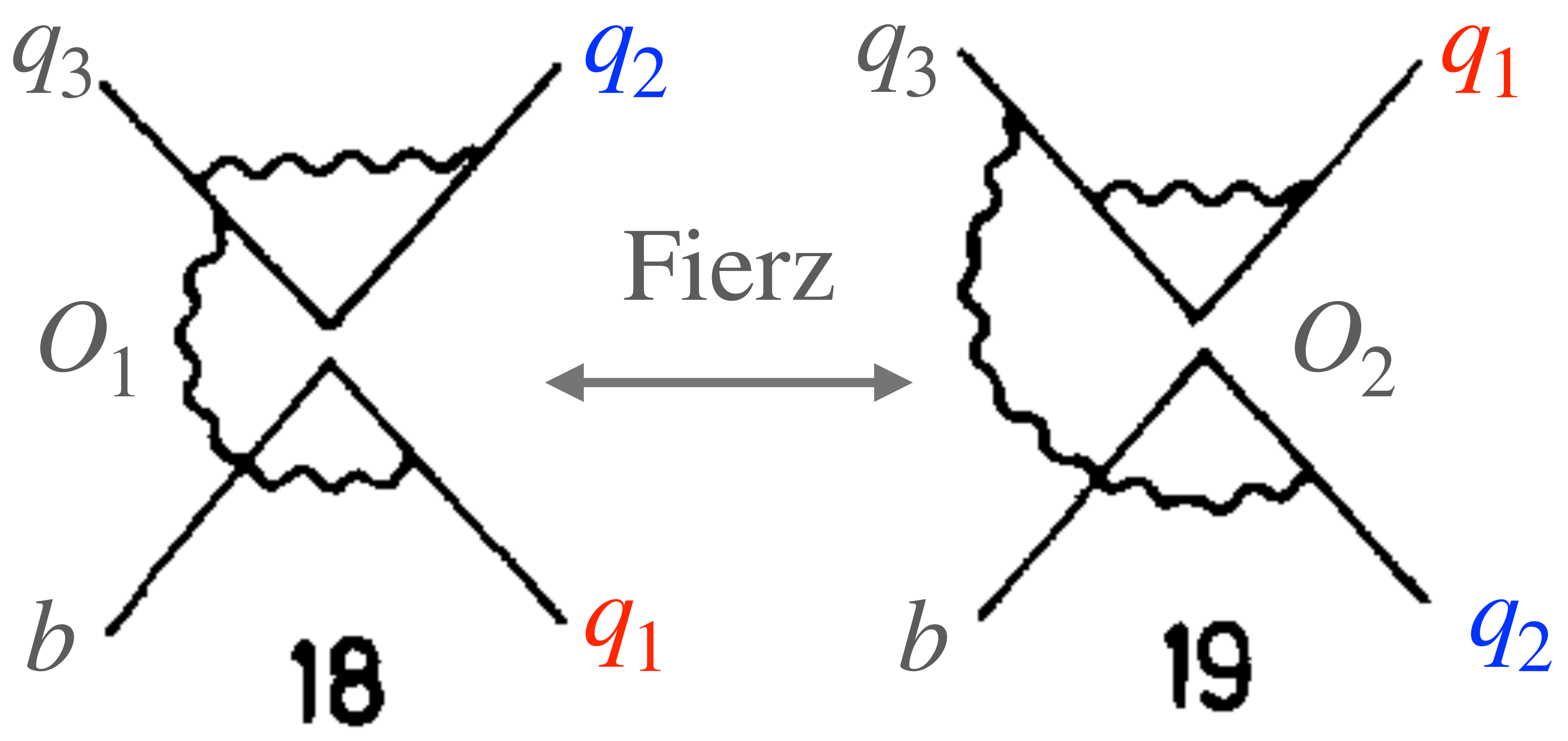
PRESERVING FIERZ IDENTITY IN $d \neq 4$

- Fierz identity can be restored order by order in perturbation theory
- Use definition of evanescent operator which preserves a symmetric ADM

Buras, Weisz, NPB 333 (1990) 66

$$\gamma_{11} = \gamma_{22} \qquad \gamma_{12} = \gamma_{21}$$

- Equivalent to require that $O_\pm = (O_1 \pm O_2)/2$ do not mix under renormalization.



EVANESCENT OPERATORS

$$E_1^{(1),q_1q_2q_3} = (\bar{q}_1^{\alpha}\gamma^{\mu_1\mu_2\mu_3}P_L b^{\beta})(\bar{q}_2^{\beta}\gamma_{\mu_1\mu_2\mu_3}P_L q_3^{\alpha}) - (16 - 4\epsilon + A_2\epsilon^2)O_1^{q_1q_2q_3}$$

$$E_2^{(1),q_1q_2q_3} = (\bar{q}_1^{\alpha}\gamma^{\mu_1\mu_2\mu_3}P_L b^{\alpha})(\bar{q}_2^{\beta}\gamma_{\mu_1\mu_2\mu_3}P_L q_3^{\beta}) - (16 - 4\epsilon + A_2\epsilon^2)O_2^{q_1q_2q_3}$$

$$E_1^{(2),q_1q_2q_3} = (\bar{q}_1^{\alpha}\gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}P_L b^{\beta})(\bar{q}_2^{\beta}\gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5}P_L q_3^{\alpha}) - (256 - 224\epsilon + B_1\epsilon^2)O_1^{q_1q_2q_3}$$

$$E_2^{(2),q_1q_2q_3} = (\bar{q}_1^{\alpha}\gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}P_L b^{\alpha})(\bar{q}_2^{\beta}\gamma_{\mu_1\mu_2\mu_3\mu_4\mu_5}P_L q_3^{\beta}) - (256 - 224\epsilon + B_2\epsilon^2)O_2^{q_1q_2q_3}$$

$\hat{\gamma}^{(2)}$ in the CMM basis → $\hat{\gamma}^{(2)}$ in the Traditional basis → Impose $\gamma_{11} = \gamma_{22}, \gamma_{12} = \gamma_{21}$

Chetyrkin, Misiak, Munz, hep-ph/9711280;
Gorbahn, Heisch, hep-ph/0411071

$$B_1 = -\frac{4384}{115} - \frac{32}{5}n_f + A_2\left(\frac{10388}{115} - \frac{8}{5}n_f\right)$$

$$B_2 = -\frac{38944}{115} - \frac{32}{5}n_f + A_2\left(\frac{19028}{115} - \frac{8}{5}n_f\right)$$

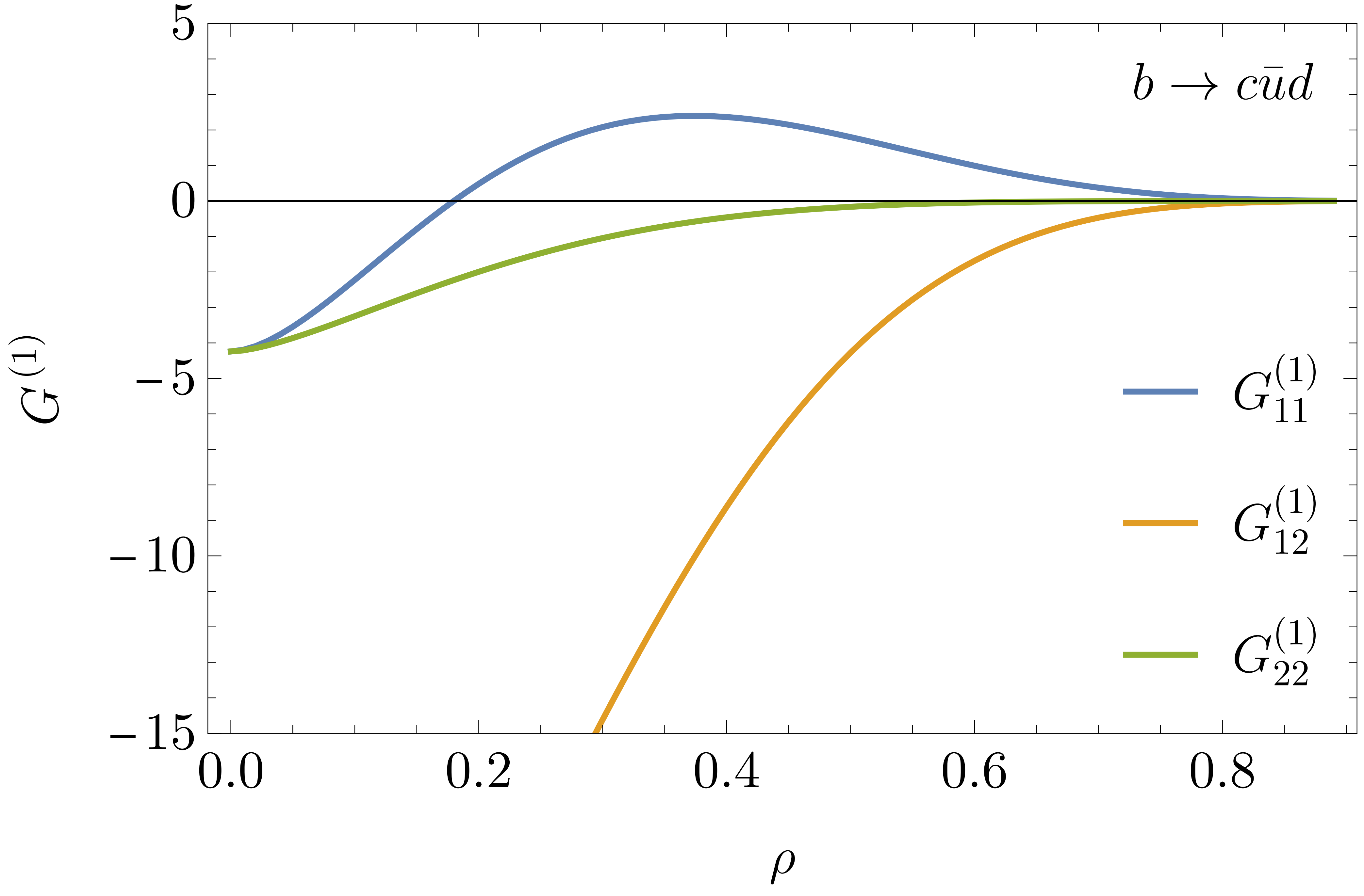
RESULTS IN THE ON SHELL SCHEME

Egner, Fael, Schönwald, Steinhauser, JHEP10(2024)144

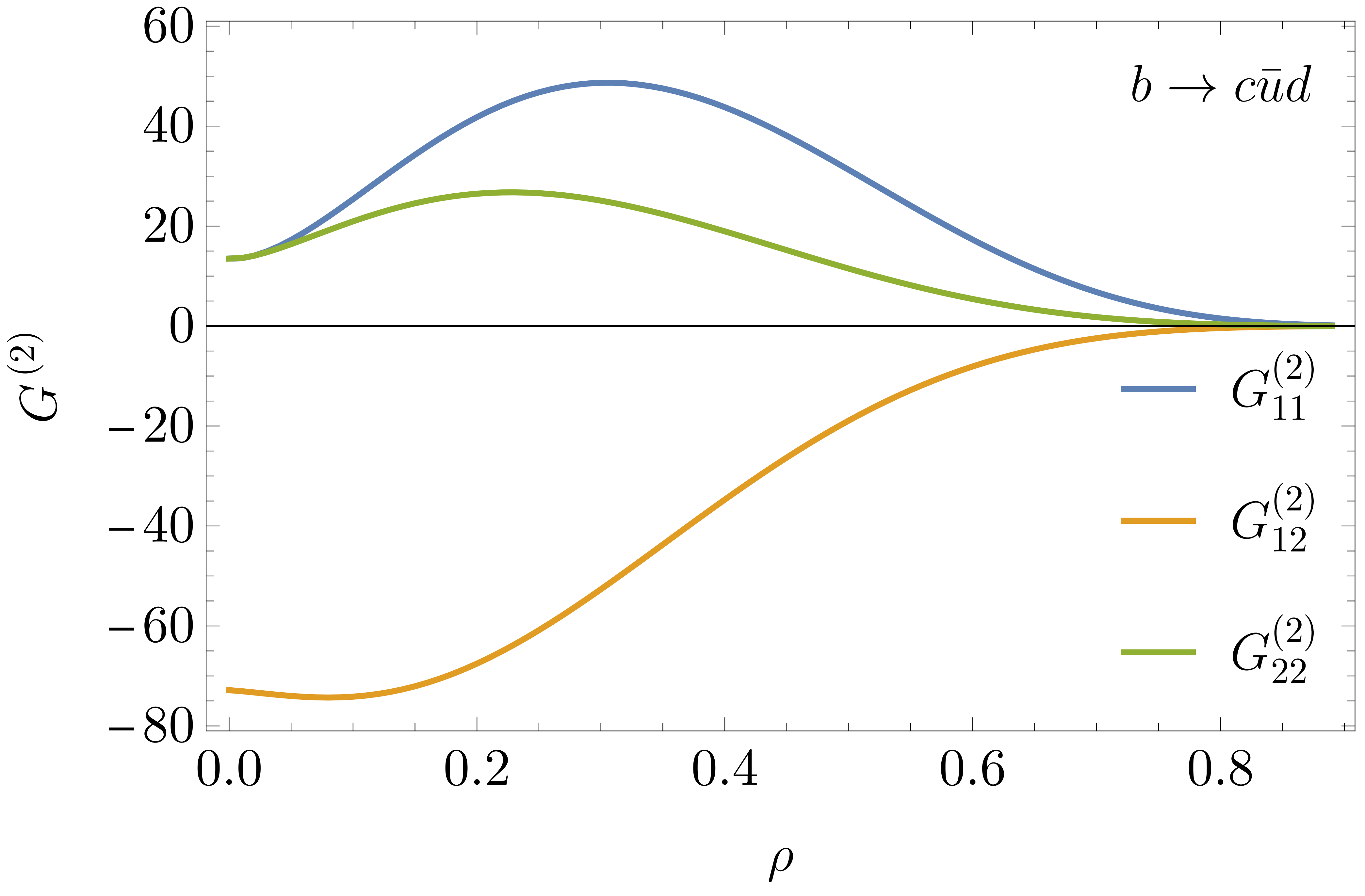
Note: the functions G_{ij} are scheme dependent!

$$\Gamma^{q_1 q_2 q_3} = \frac{G_F^2 m_b^5 \lambda_{q_1 q_2 q_3}}{192\pi^3}\left[C_1^2(\mu_b)G_{11}^{q_1 q_2 q_3} + C_1(\mu_b)C_2(\mu_b)G_{12}^{q_1 q_2 q_3} + C_2^2(\mu_b)G_{22}^{q_1 q_2 q_3}\right]$$

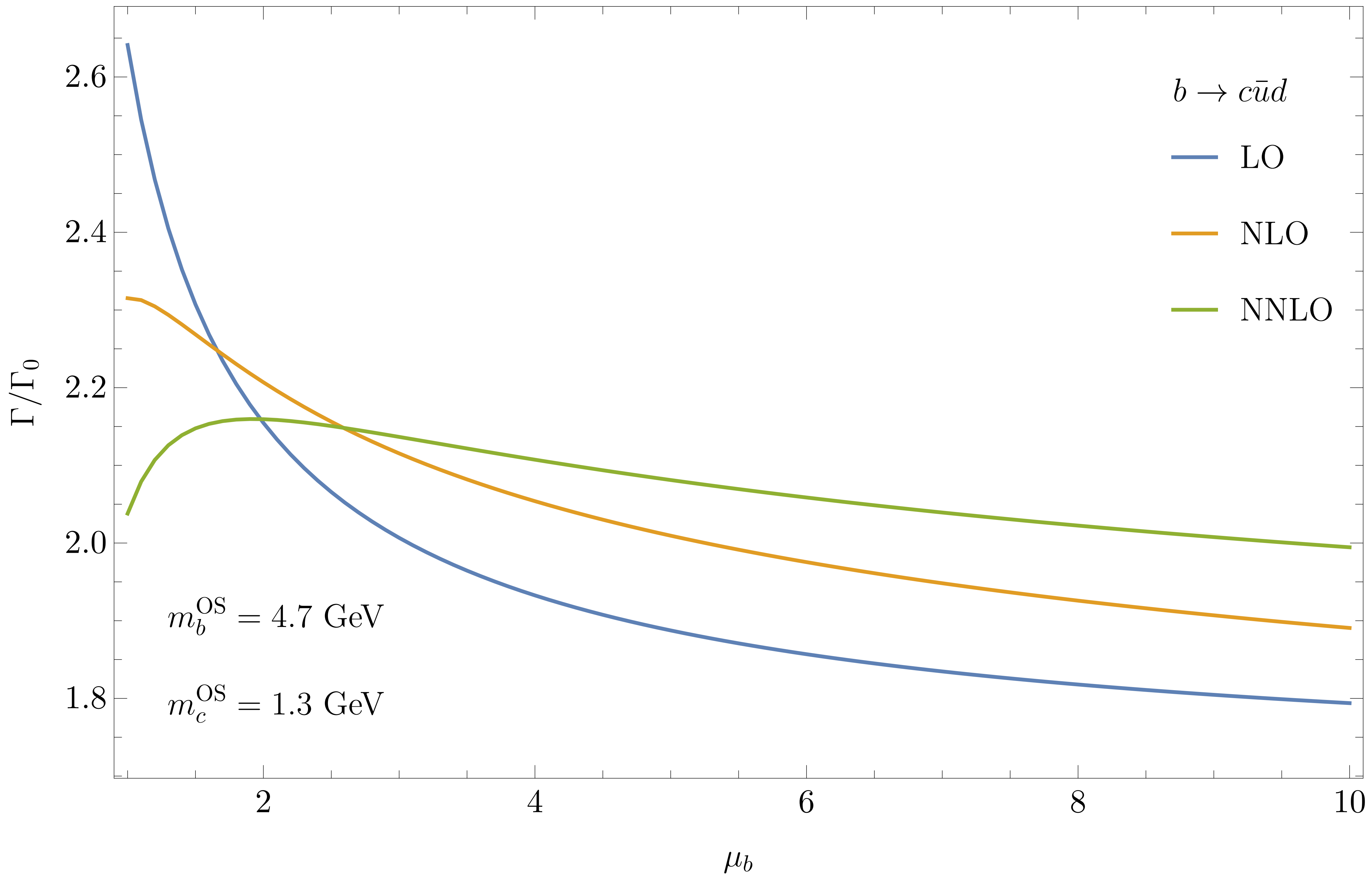
Coefficient of $\alpha_s(m_b)/\pi$



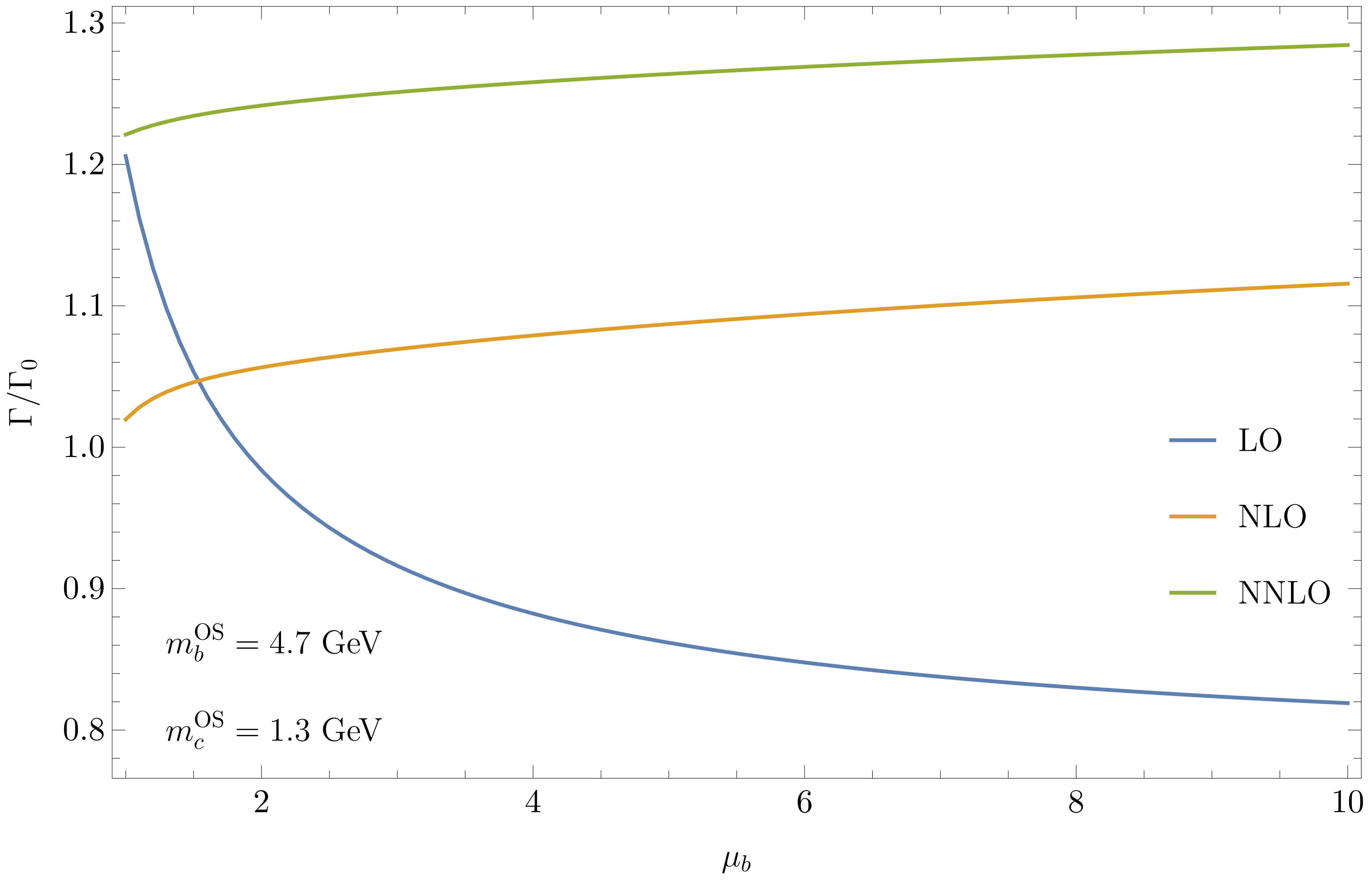
Coefficient of $(\alpha_s(m_b)/\pi)^2$



$b \to c\bar{u}d$



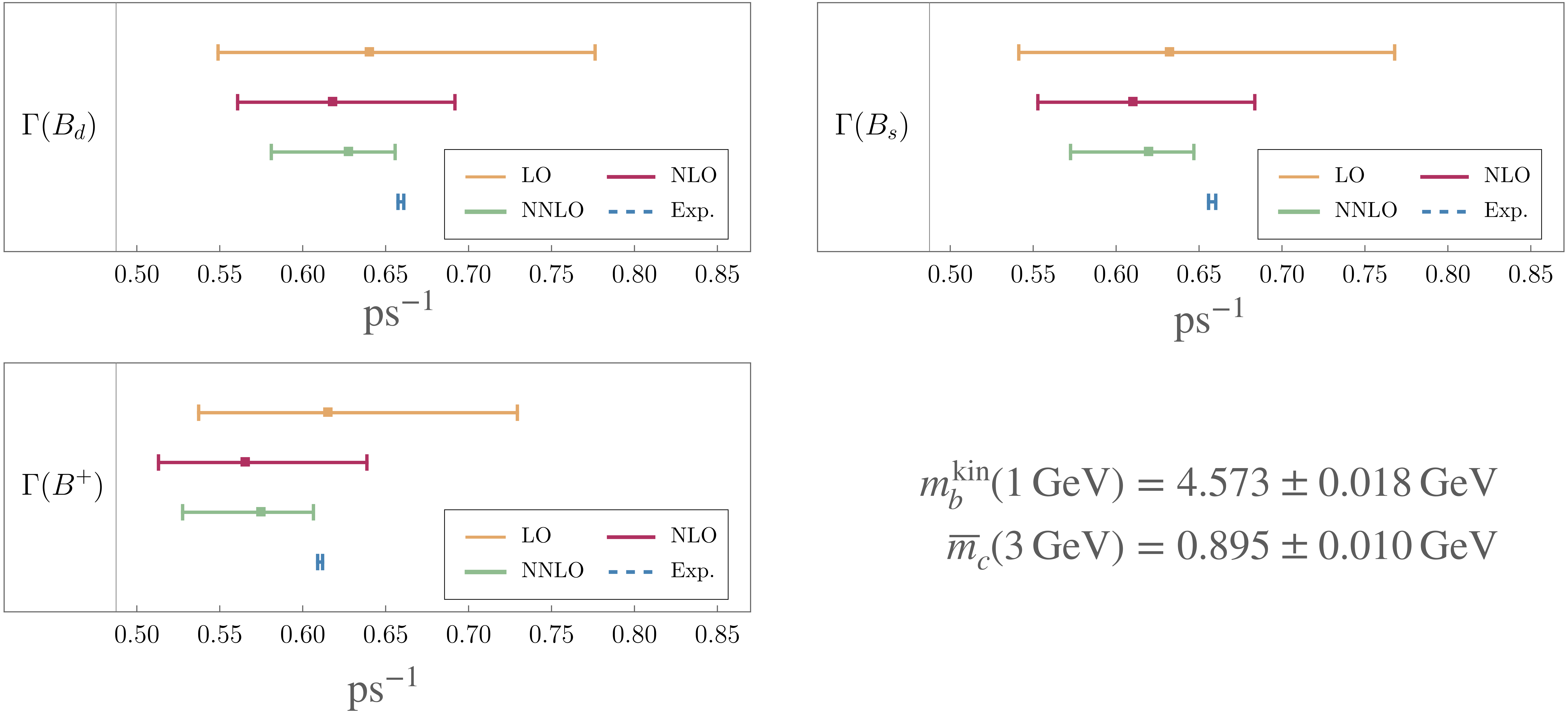
$b \to c\bar{c}s$



Egner, Fael, Schönwald, Steinhauser, JHEP10(2024)144

UPDATING THE LIFETIMES OF B MESONS

Egner, **MF**, Lenz, Piscopo, Rusov, Schönwald, Steinhauser, in preparation



$$m_b^{\text{kin}}(1\,\text{GeV}) = 4.573 \pm 0.018\,\text{GeV}$$

$$\overline{m}_c(3\,\text{GeV}) = 0.895 \pm 0.010\,\text{GeV}$$

CONCLUSIONS

- Calculation of the NNLO QCD corrections to nonleptonic decays (O_1 and O_2)
- This calculation was made possible by recent developments in multi-loop calculation
  - Numerical methods for solving master integrals
  - Auxiliary mass flow (AMFlow)
- Quite significant reduction of the theoretical uncertainties from scale variation
- SOON: update of the lifetime predictions
- Improved accuracy opens the possibility to use $\tau(B)$ in the global fits for V_{cb}

Funded by
the European Union

This research was supported by the European Union's Horizon 2020 research and innovation program under the Marie Sklodowska-Curie grant agreement No. 101065445 - PHOBIDE.