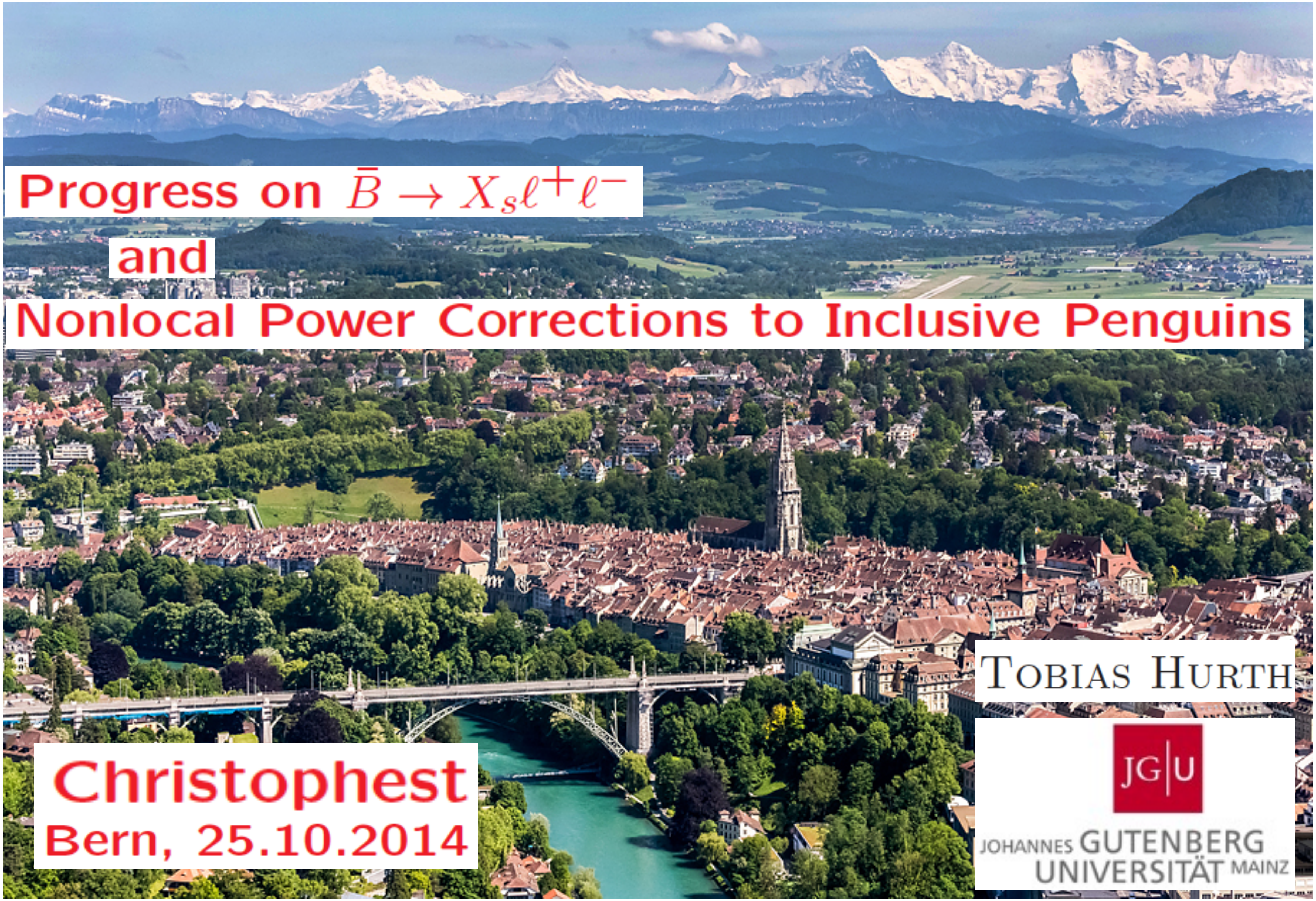

Progress on $\bar{B} \to X_s \ell^+ \ell^-$ and Nonlocal Power Corrections to Inclusive Penguins

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JOHANNES GUTENBERG UNIVERSITÄT MAINZ

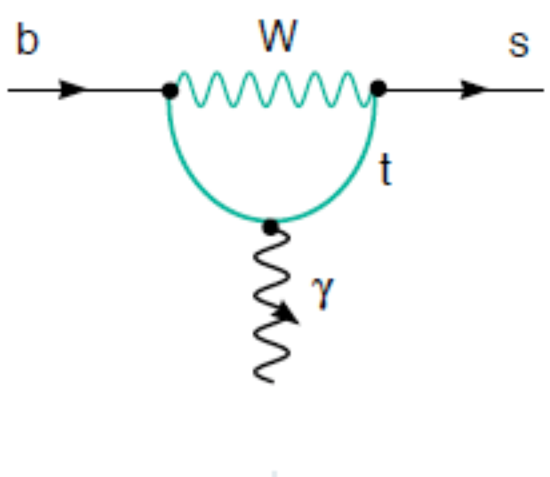
Christophest

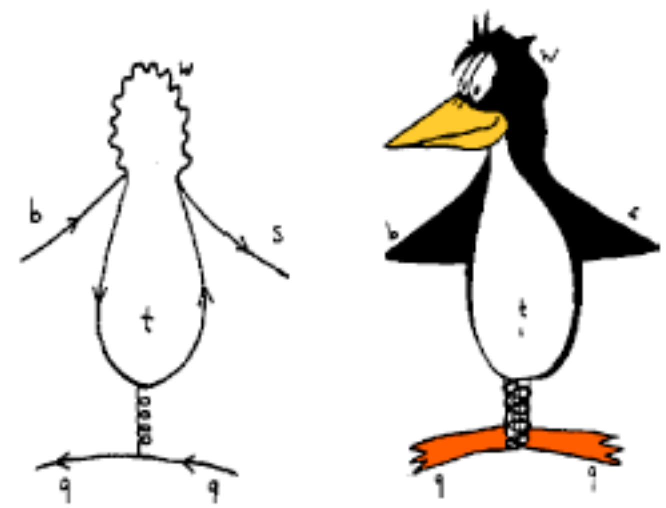
Bern, 25.10.2014

Prologue

Indirect exploration of higher scales via flavour

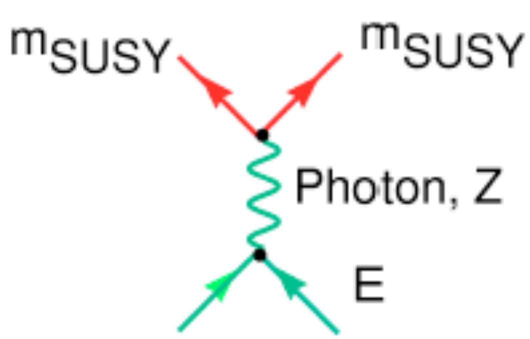
- Flavour changing neutral currrent processes like $b \to s\gamma$ or $b \to s\ell^+\ell^-$ directly probe the SM at the one-loop level.



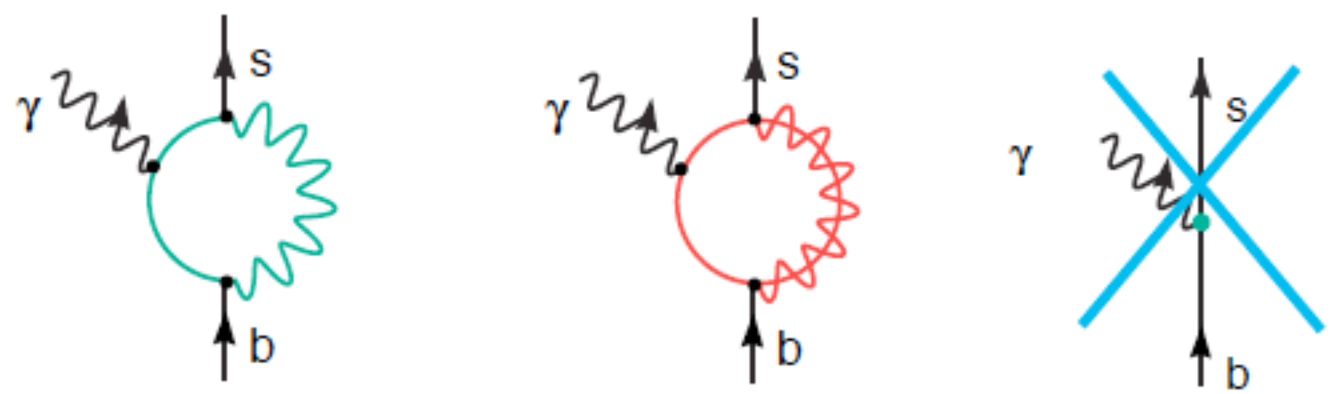


- Indirect search strategy for new degrees of freedom beyond the SM

Direct:

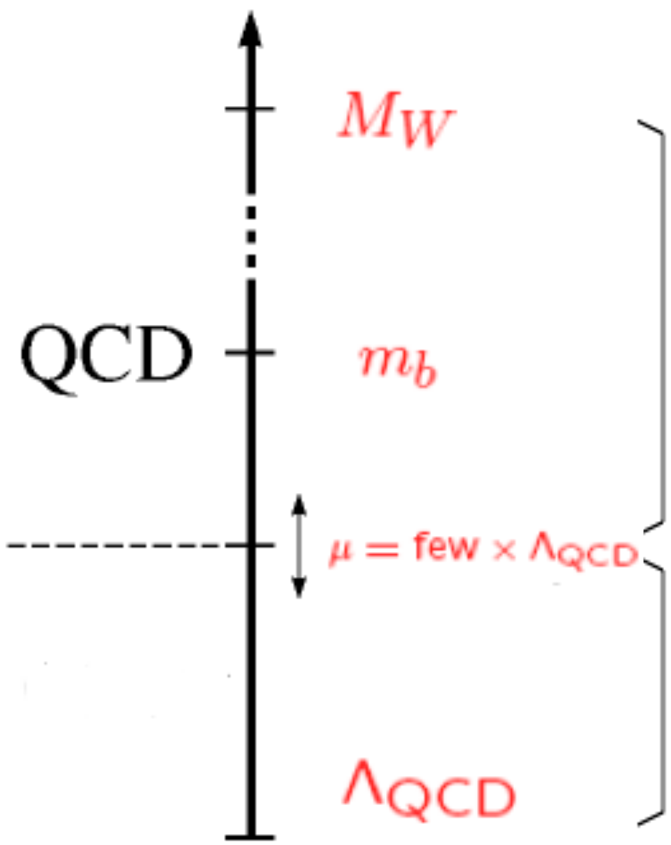


Indirect:



Theoretical Framework

Theoretical tools for flavour precision observables



short-distance physics
perturbative

long-distance physics
nonperturbative

Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \; \mathcal{O}_i(\mu)$

- $\mu^2 \approx M_{New}^2 >> M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

HQET, SCET, ...

Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

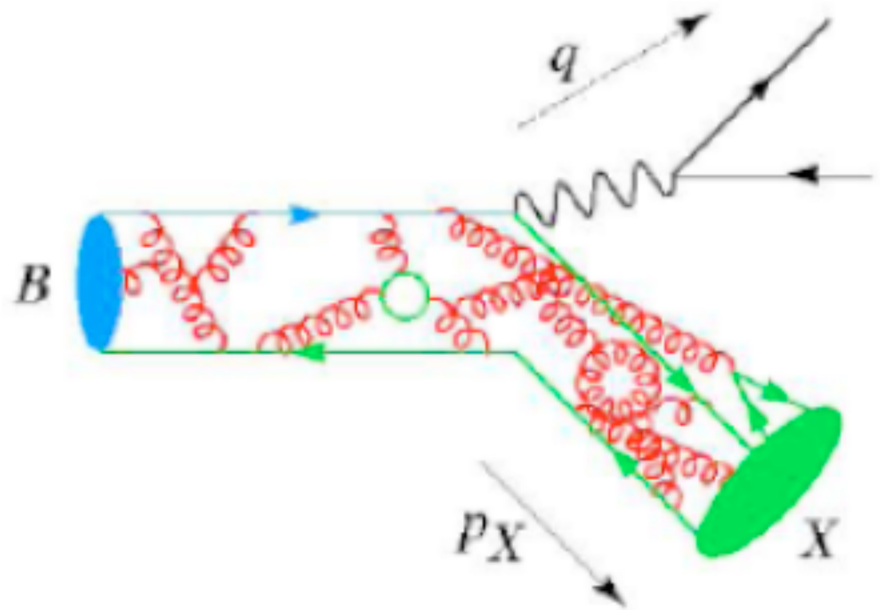
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda_{QCD}^2/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay,Georgi,Grinstein 1990



Inclusive modes $B \to X_s\gamma$ and $B \to X_s\ell^+\ell^-$

How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \to X_s\gamma) \stackrel{m_b\to\infty}{\longrightarrow} \Gamma(b \to X_s^{parton}\gamma)\,, \quad \Delta^{nonpert.} \sim \Lambda^2_{QCD}/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Old story:

– If one goes beyond the leading operator ($\mathcal{O}_7$, $\mathcal{O}_9$):
  breakdown of local expansion

Dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

$b \to s\gamma$: Benzke,Lee,Neubert,Paz,arXiv:1003.5012

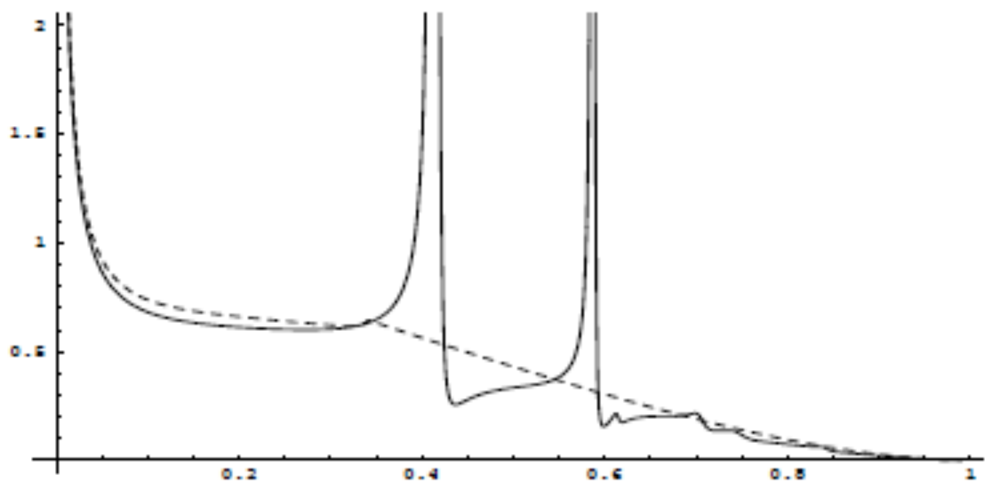
$b \to s\ell\ell$: Benzke,Hurth,Turczyk,arXiv:1705.10366

Inclusive semi-leptonic penguins

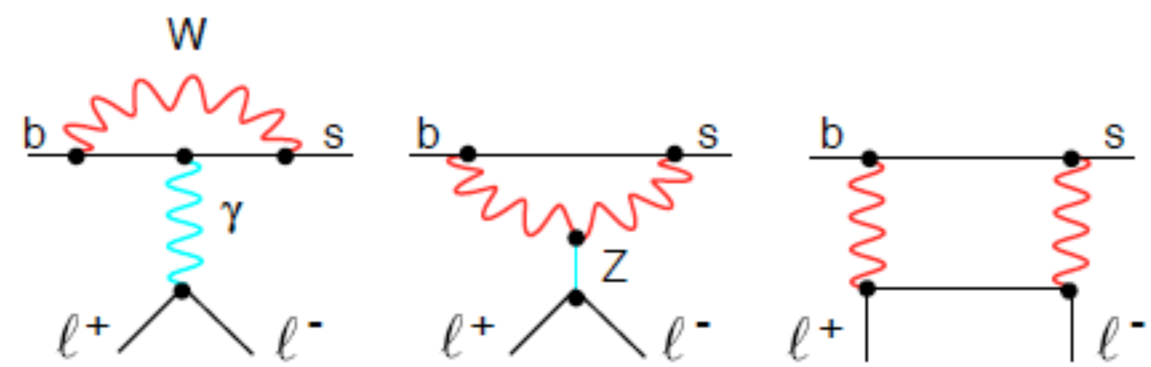
Review of previous calculations for $B \to X_s \ell\ell$

- On-shell-$c\bar{c}$-resonances $\Rightarrow$ cuts in dlepton mass spectrum necessary :
  $1\text{GeV}^2 < q^2 < 6\text{GeV}^2$ and $14.4\text{GeV}^2 < q^2$ $\Rightarrow$ perturbative contributions dominant

$\frac{d}{d\hat{s}} BR(\bar{B} \to X_s l^+ l^-) \times 10^{-5}$



$\hat{s} = q^2/m_b^2$



- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: dilepton mass spectrum
  Asatryan,Asatrian,Greub,Walker,hep-ph/0204341
  Ghinculov,Hurth,Isidori,Yao,hep-ph/0312128

$$BR(\bar{B} \to X_s \ell^+ \ell^-)_{Cut:\ q^2 \in [1GeV^2, 6GeV^2]} = (1.63 \pm 0.20) \times 10^{-6}$$

$$BR(\bar{B} \to X_s l^+ l^-)_{Cut:\ q^2 > 14.4GeV^2} = (4.04 \pm 0.78) \times 10^{-7}$$

NNLL QCD corrections $q^2 \in [1GeV^2, 6GeV^2]$

central value: -14%, perturbative error: $13\% \to 6.5\%$

- Further refinements:
  - Completing NNLL QCD corrections:
    Mixing into $\mathcal{O}_9$ ($+1\%$), NNLL matrixelement of $\mathcal{O}_9$ (-4%)
  - NLL QED two-loop corrections to Wilson coefficients
    -1.5% shift for $\alpha_{em}(\mu = m_b)$, -8.5% for $\alpha_{em}(\mu = m_W)$
    Bobeth,Gambino,Gorbahn,Haisch,hep-ph/0312090
  - QED two-loop corrections to matrix elements
    Large collinear logarithm $Log(m_b/m_\ell)$ which survive intregration
    if a restricted part of the dilepton mass spectrum is considered
    $+2\%$ effect in the low-q^2 region for muons, for the electrons
    the effect depends on the experimental cut parameters
    Huber,Lunghi,Misiak,Wyler,hep-ph/0512066 Huber,Hurth,Lunghi, arXiv:0712.3009

- NNLL prediction of $\bar{B} \to X_s \ell^+ \ell^-$: forward-backward-asymmetry (FBA)
  Asatrian,Bieri,Greub,Hovhannisyan,hep-ph/0209006
  Ghinculov,Hurth,Isidori,Yao,hep-ph/0208088,hep-ph/0312128

$$A_{\rm FB} \equiv \frac{1}{\Gamma_{semilep}} \left(\int_0^1 d(\cos\theta)\, \frac{d^2\Gamma}{dq^2 d\cos\theta} - \int_{-1}^0 d(\cos\theta)\, \frac{d^2\Gamma}{dq^2 d\cos\theta} \right)$$

(θ angle between ℓ^+ and B momenta in dilepton CMS)

$$A_{FB}(q_0^2) = 0 \quad \text{for} \quad q_0^2 \sim C_7/C_9 \quad q_0^2 = (3.90 \pm 0.25) GeV^2$$

Complete angular analysis of inclusive $B \to X_s \ell\ell$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d^2\Gamma}{dq^2\, dz} = \frac{3}{8}\left[(1+z^2)\, H_T(q^2) + 2\, z\, H_A(q^2) + 2\,(1-z^2)\, H_L(q^2)\right] \qquad (z = \cos\theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2) \qquad \frac{dA_{\rm FB}}{dq^2} = 3/4\, H_A(q^2)$$

- Dependence on Wilson coefficients

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2\left[\left|C_9 + \frac{2}{s}\, C_7\right|^2 + |C_{10}|^2\right]$$

$$H_A(q^2) \propto -4s(1-s)^2\ \mathrm{Re}\left[C_{10}\left(C_9 + \frac{2}{s}\, C_7\right)\right]$$

$$H_L(q^2) \propto (1-s)^2\left[\left|C_9 + 2\, C_7\right|^2 + |C_{10}|^2\right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\rm em}\, log(m_b^2/m_\ell^2) \qquad q^2 = (p_{\ell+} + p_{\ell-}) \Rightarrow q^2 = (p_{\ell+} + p_{\ell-} + p_\gamma)$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- In the ratio of the inclusive $b \to s\ell\ell$ decay rate in the high-q^2 region and the semileptonic decay rate large part of the nonperturbative effects cancel out:

Ligeti,Tackmann, arXiv:0707.1694

$$R_{\rm incl}^{(\ell)}(q_0^2) = \frac{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\displaystyle\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}}$$

Intermezzo

Tensions in the inclusive high q^2 decay rate ??

Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612

$$R^{SM}_{\rm incl}(15) = \frac{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_s \bar{\ell}\ell)}{dq^2}}{\int_{15}^{m_B^2} dq^2 \frac{d\Gamma(B \to X_u \bar{\ell}\nu)}{dq^2}} \quad x \quad \mathcal{B}(B \to X_u \bar{\ell}\nu)^{\rm exp}_{[15]} = (1.50 \pm 0.24) \times 10^{-4}$$

Belle,arXiv:2107.13855

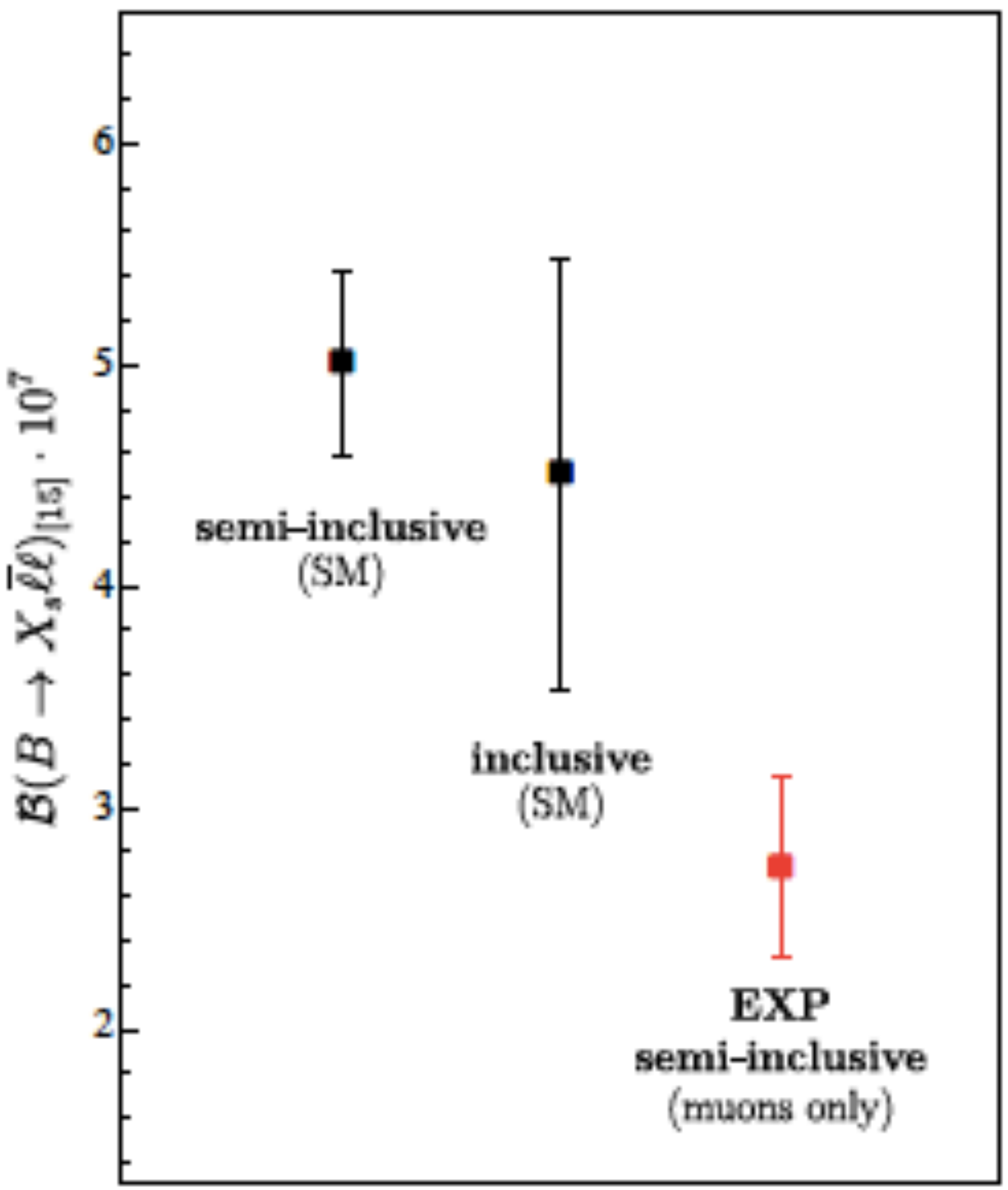
$$= \quad ''\mathcal{B}(B \to X_s \bar{\ell}\ell)^{SM}_{[15]}'' \quad \stackrel{!}{=} \quad \sum_i \mathcal{B}(B \to X_s^i \bar{\mu}\mu)^{\rm exp}_{[15]} = (2.74 \pm 0.41) \times 10^{-7}$$

Isidori,Polonsky,Tinari,arXiv:2305.03076

- Experimental semi-inclusive rate is estimated by the sum of the $B \to K$ and $B \to K^*$ modes and a correction factor for the two-body final states $B \to K\pi$.

- Isidori et al. claim a tension up to 2σ – confirming analogous results in the exclusive modes.

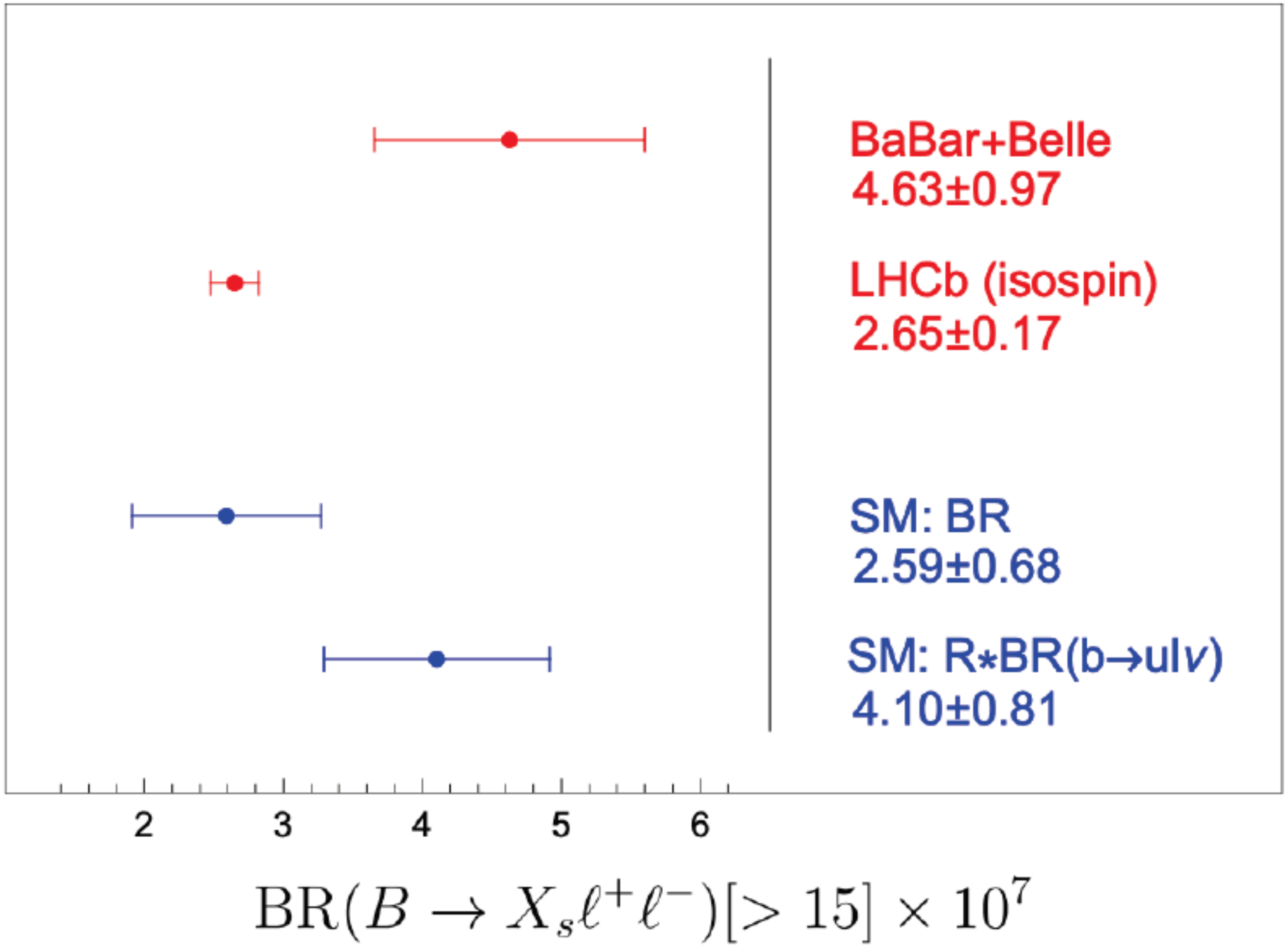
Isidori,Polonsky,Tinari,arXiv:2305.03076
Isidori, arXiv:2308.11612



- We do not find any tension if we also consider our direct result for the branching $\mathcal{B}(B \to X_s \ell\ell)^{\text{SM}}_{[15]}$ and the Babar/Belle measurements.

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Talk by T.H. at FPCP23 and arXiv:2404.03517



- We find a slight tension between the two theoretical and also between the two experimental results. We have to be patient!

New Physics Reach of Semi-leptonic Penguin Decays

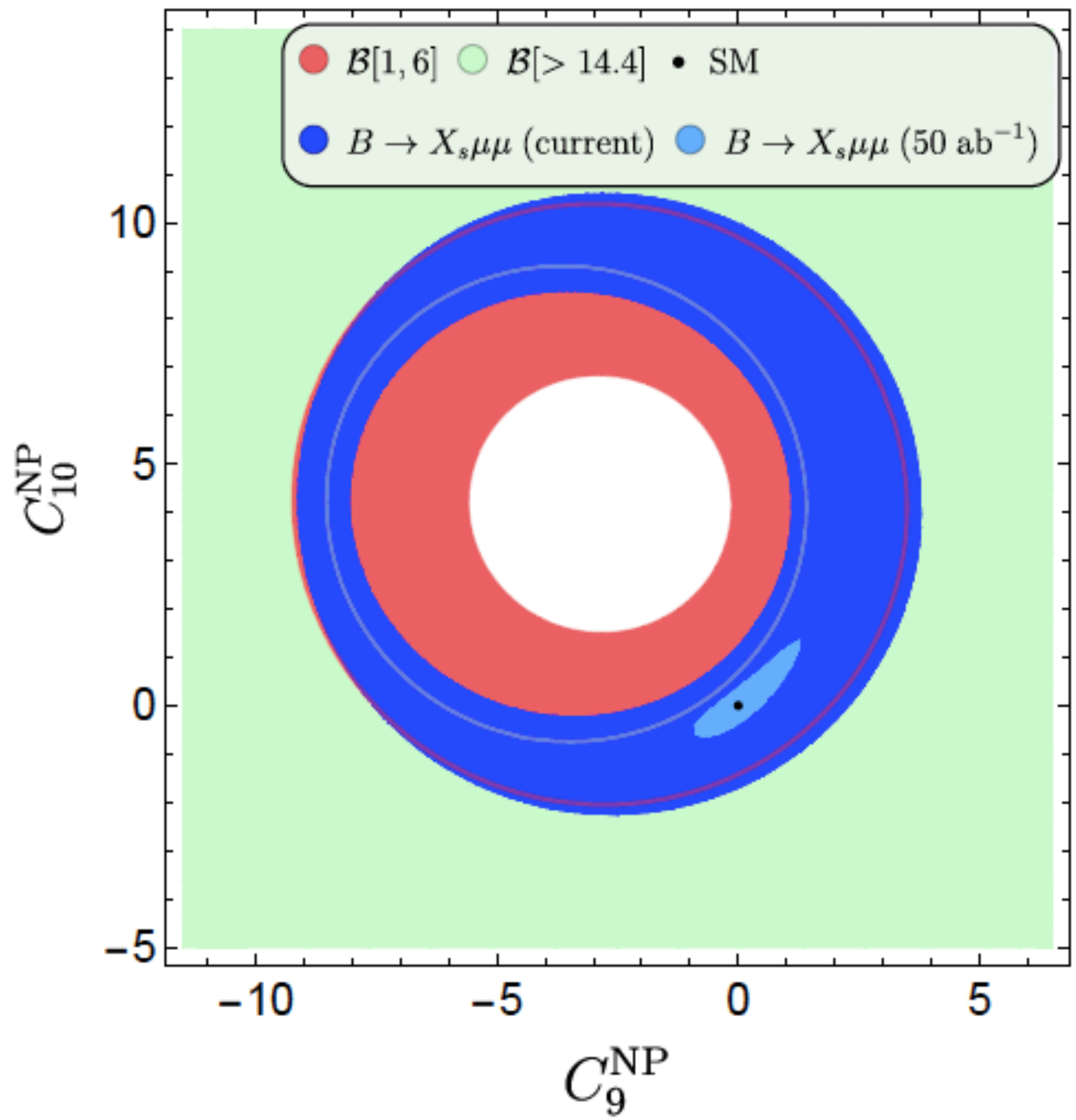
New physics sensitivity

Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Constraints on Wilson coefficients C_9^{NP} and C_{10}^{NP}

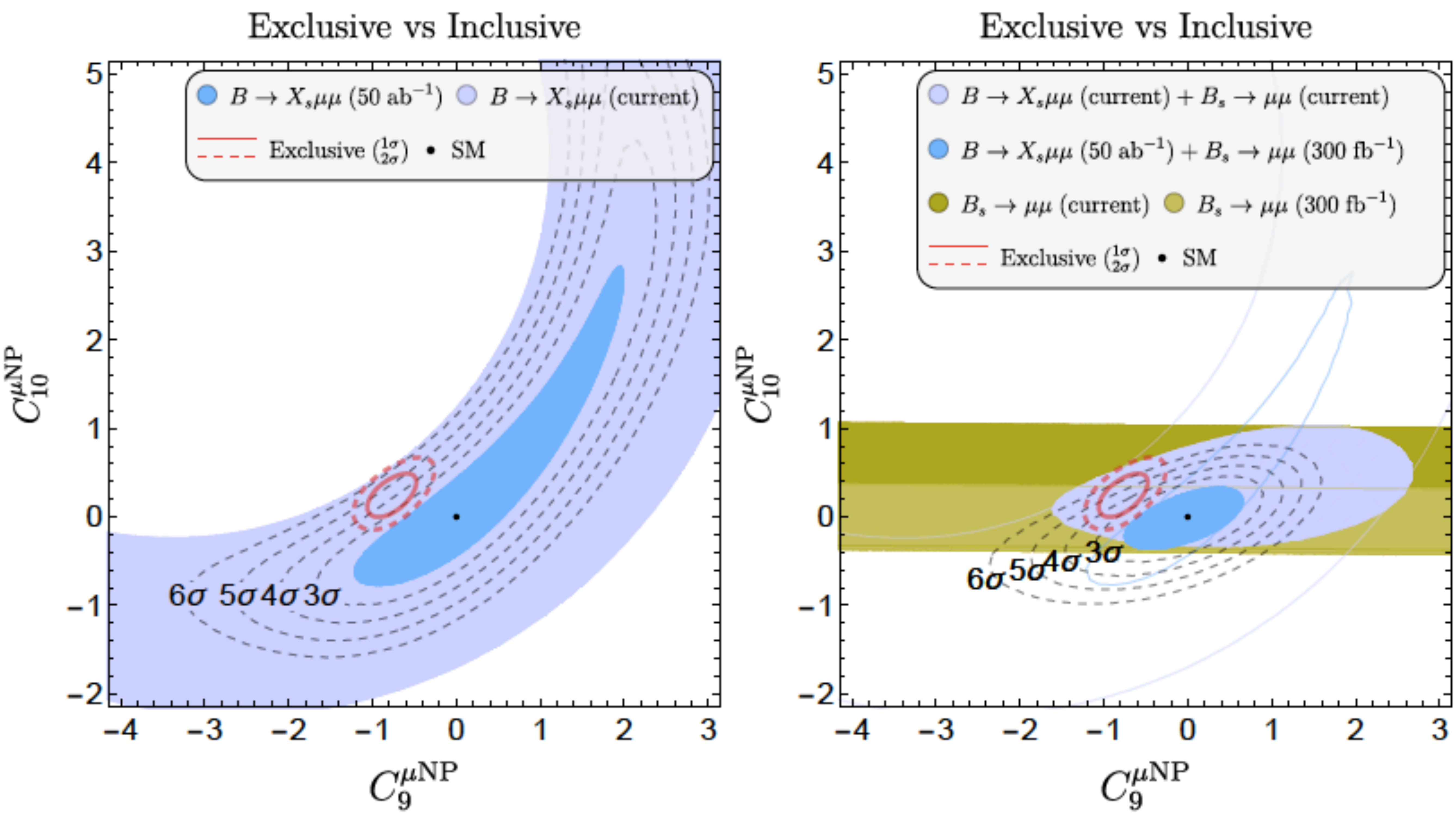
that we obtain at 95% C.L. from present experimental data (red low q^2, green high q^2)

that we will obtain at 95% C.L. from $50ab^{-1}$ data at Belle-II (light blue)



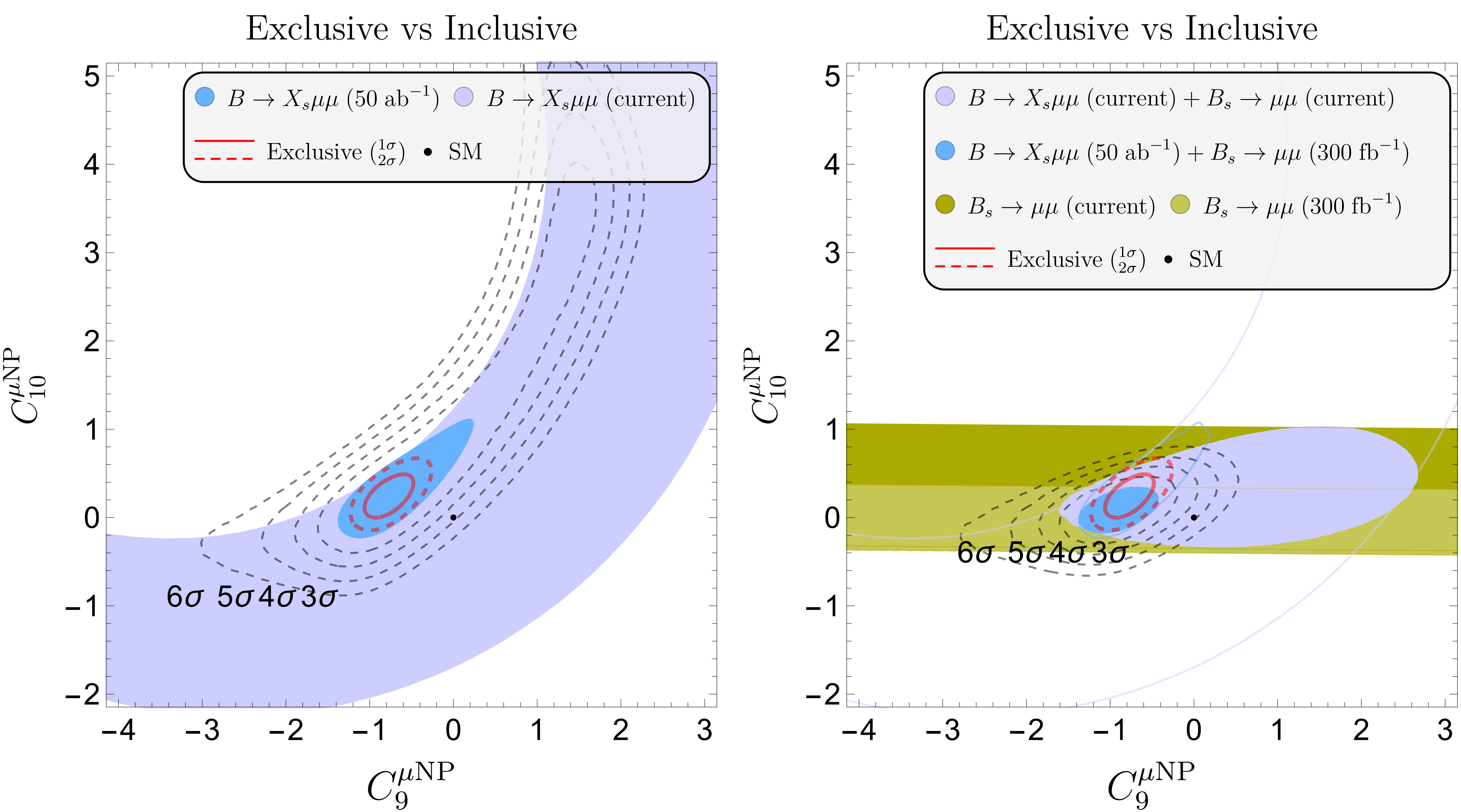
Assuming Belle II measures SM values

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191



Assuming Belle II measures best fit point of exclusive fit

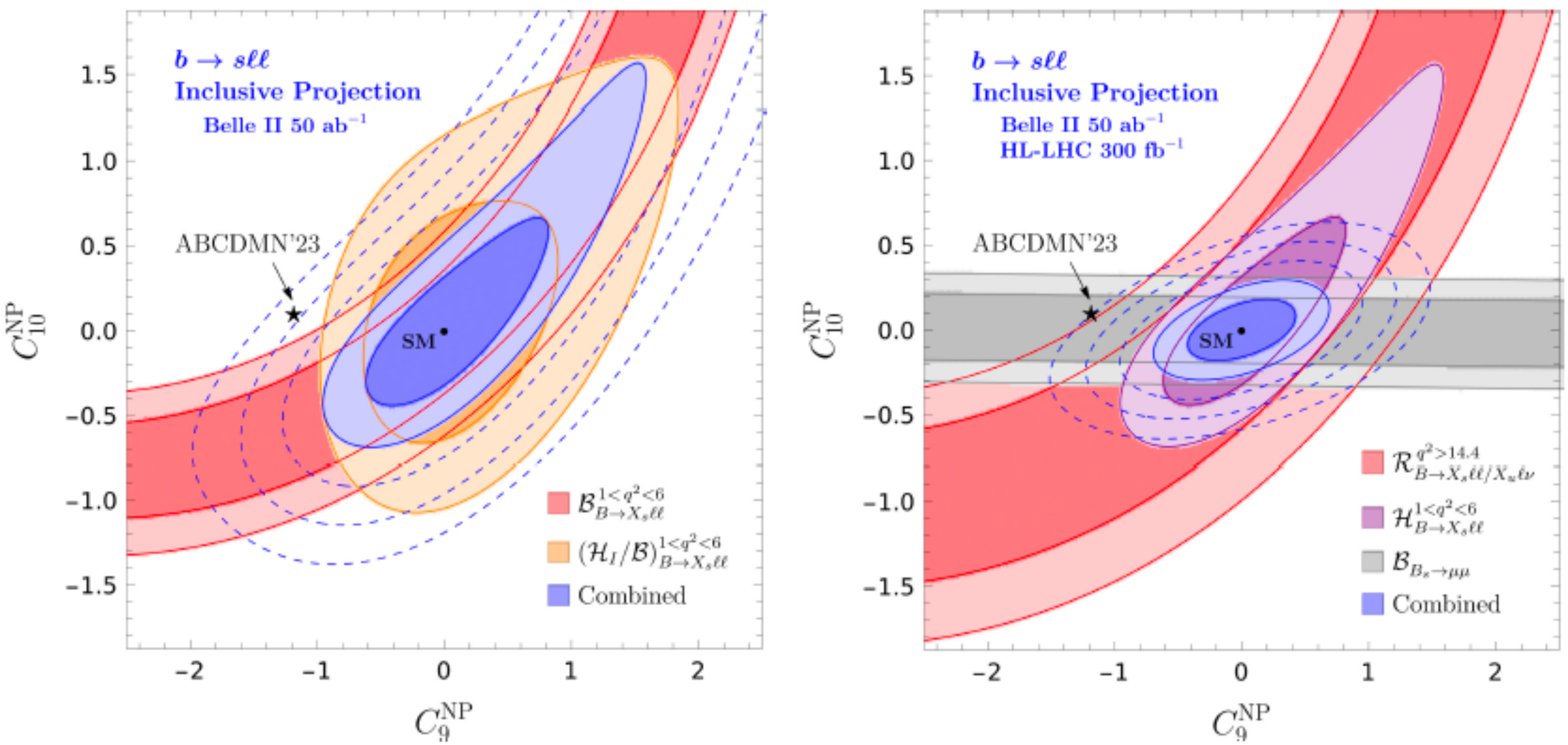
Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191



Assuming Belle II measures SM values

Huber,Hurth,Jenkins,Lunghi,Qin Qin,Vos,arXiv:2007.04191

Update for post-R_K era arXiv:2404.03517



Belle-II Extrapolations

Akimasa Ishikawa, B2TiP

Error of Branching ratio $\bar{B} \to X_s \ell^+ \ell^-$

BF (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
$\geq$ 14.4	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

Error of Normalized Forward-Backward-Asymmetry

AFBn (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
$\geq$ 14.4	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

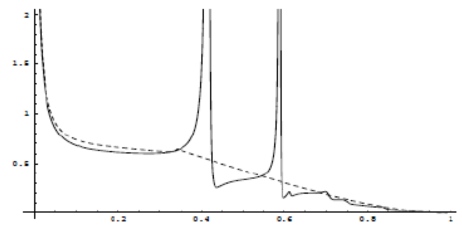
$B \to (\pi, \rho)\ell^+\ell^-$, semi-inclusive $\bar{B} \to X_d \ell^+ \ell^-$ at 50/ab
(uncertainties like $\bar{B} \to X_s \ell^+ \ell^-$ at 0.7/ab)

Nonlocal subleading contributions

Subleading power factorization in $B \to X_s \ell^+ \ell^-$

Benzke,Hurth,Turczyk,arXiv:1705.10366; Benzke,Hurth,arXiv:2006.00624

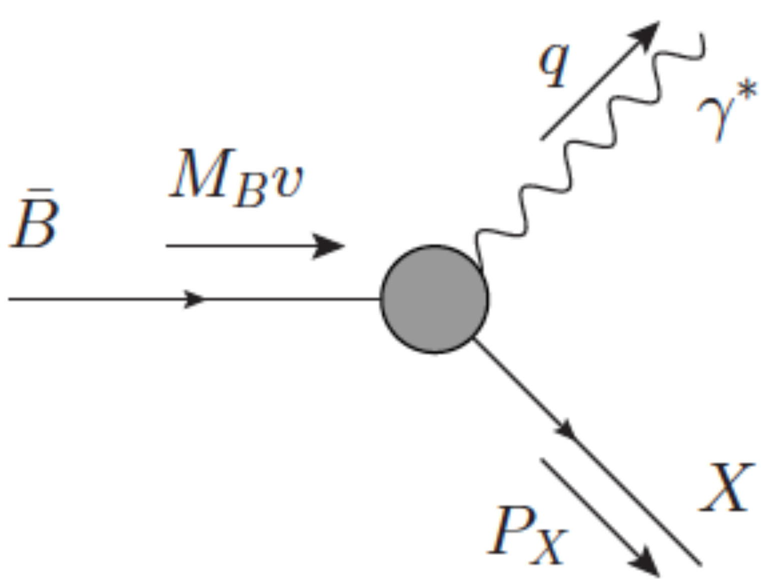
- Cuts in the dilepton mass spectrum necessary due to $c\bar{c}$ resonances



- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Kinematics: X_s is jetlike and $m_x^2 \leq m_b \Lambda_{QCD}$ (shapefunction region)

- Multiscale problem $\Rightarrow$ SCET with scaling Λ_{QCD}/m_b



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{\rm QCD} m_b \gg \Lambda_{\rm QCD}^2$$

Little calculation

- B meson rest frame $\quad q = p_B - p_X \qquad 2\, m_B\, E_X = m_B^2 + M_X^2 - q^2$

  X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

- $p_X^- p_X^+ = m_X^2 \qquad$ two light-cone components

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\rm QCD})$$

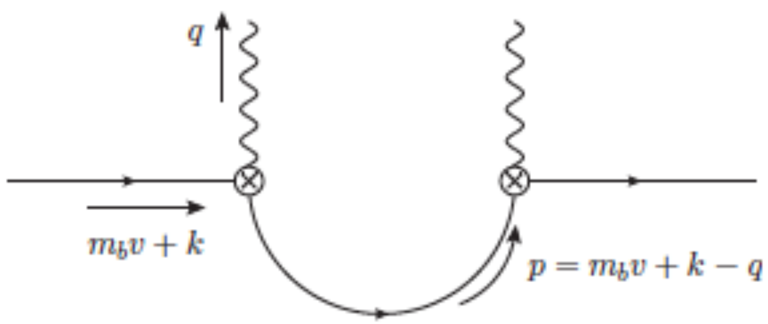
- $q^+ = nq = m_B - p_X^+ \qquad q^- = \bar{n} q = m_B - p_X^-$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

$$\lambda = \Lambda_{\rm QCD}/m_b \qquad m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q}\left(1 - \frac{n \cdot k}{m_b - n \cdot q} + \ldots\right)\frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \to X_s \gamma$)

Factorization theorem

$$d\Gamma \sim H \cdot J \otimes S$$

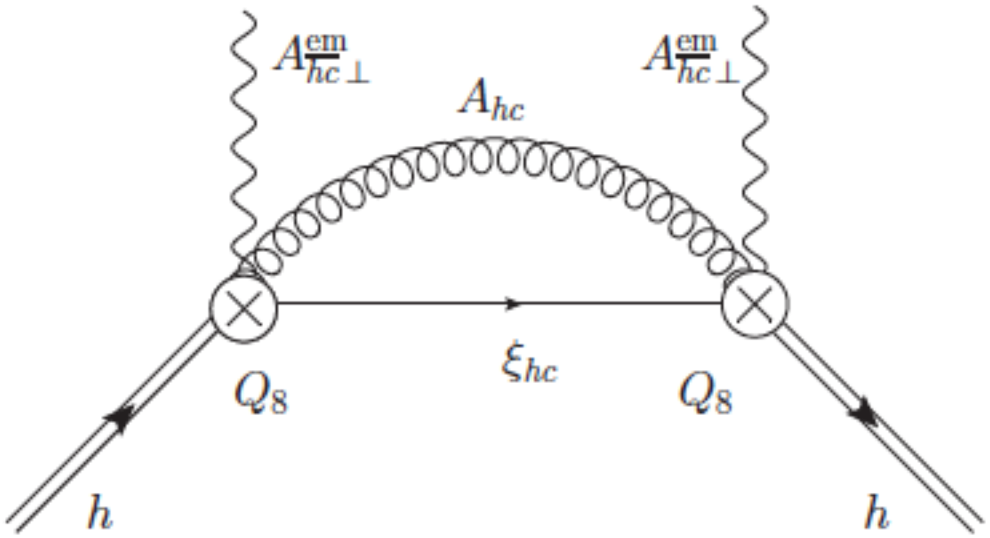
The hard function H and the jet function J are perturbative quantities.

The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

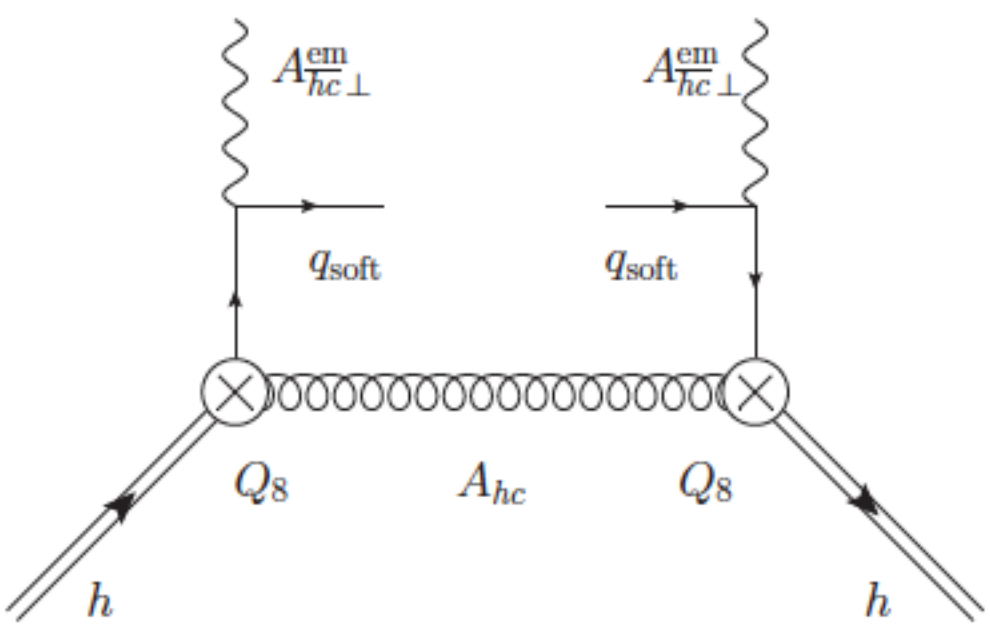
Calculation at subleading power

Example of **direct** photon contribution which factorizes $d\Gamma \sim H \cdot j \otimes S$



$\rightarrow \frac{\alpha_s}{m_b}$ in low m_X^2 region

Example of **resolved** photon contribution (double-resolved) which factorizes



$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$

: Benzke,Lee,Neubert,Paz,arXiv:1003.5012

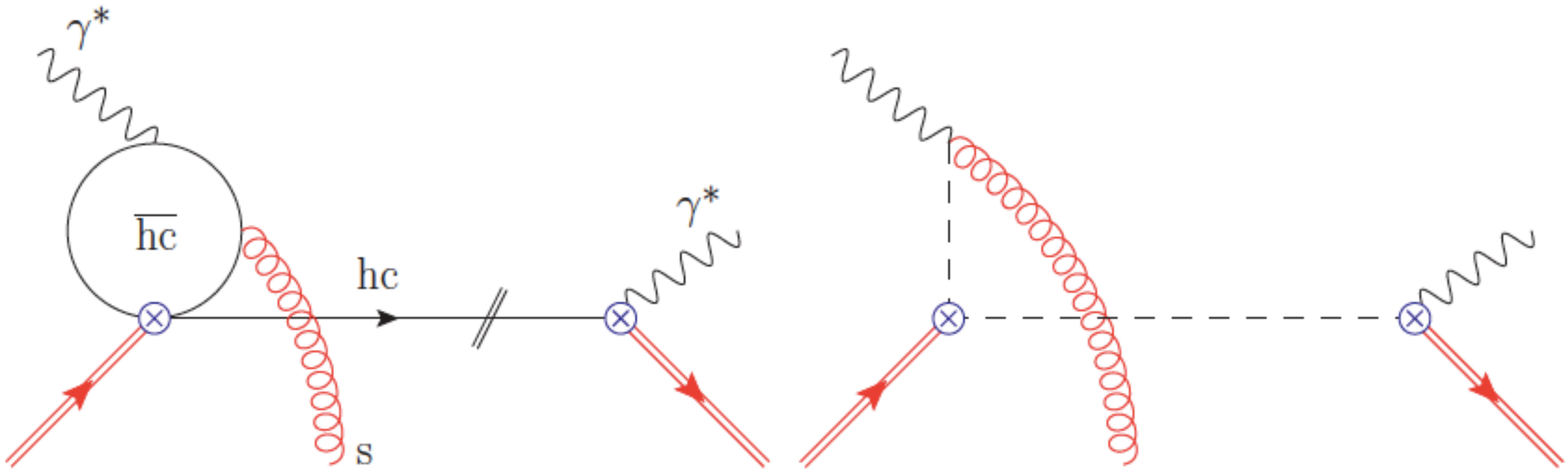
$\rightarrow \frac{\Lambda}{m_b}$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_1 and Q_7

$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J}$$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{1}{m_b}\int d\omega\, \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\varepsilon}$$

$$\frac{1}{\omega_1}\left[\bar{n}\cdot q\left(F\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - 1\right) - (\bar{n}\cdot q + \omega_1)\left(F\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q + \omega_1)}\right) - 1\right)\right.$$

$$\left. + \bar{n}\cdot q\left(G\left(\frac{m_c^2}{n\cdot q\bar{n}\cdot q}\right) - G\left(\frac{m_c^2}{n\cdot q(\bar{n}\cdot q + \omega_1)}\right)\right)\right] g_{17}(\omega, \omega_1)$$

$$g_{17}(\omega,\omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B}|\bar{h}(t\mathbf{n}) \ldots G_s^{\alpha\beta}(r\bar{\mathbf{n}}) \ldots h(0)|\bar{B}\rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \to 1$ limit (irreducible uncertainty)

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
  - PT invariance implies that soft functions are real
  - Moments of shape functions are related to HQET parameters
  - Soft functions have no significant structure outside the hadronic range
  - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

Numerical evaluation of the resolved contributions

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  - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^0 \, h_{17}(\omega_1, \mu) = 0.237 \pm 0.040 \, \mathrm{GeV}^2$$

New input:

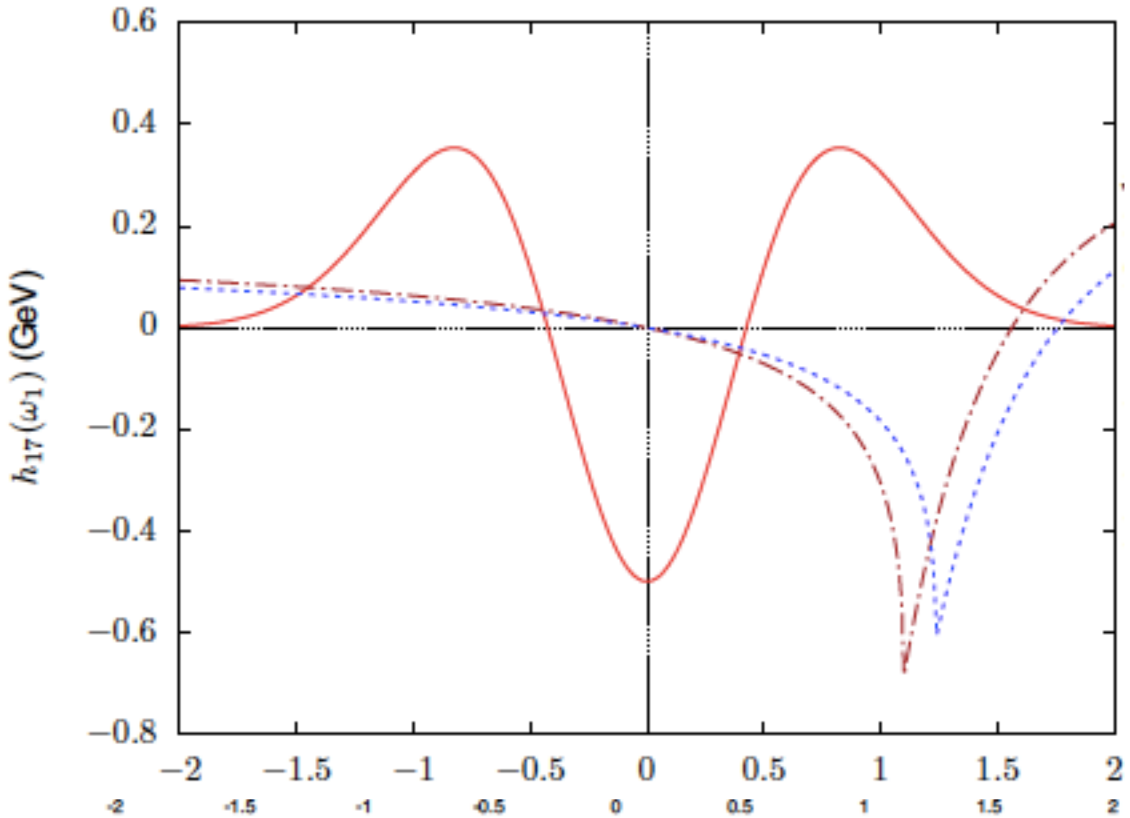
$$\int_{-\infty}^{\infty} d\omega_1 \, {\omega_1}^2 \, h_{17}(\omega_1, \mu) = 0.15 \pm 0.12 \ \mathrm{GeV}^4$$

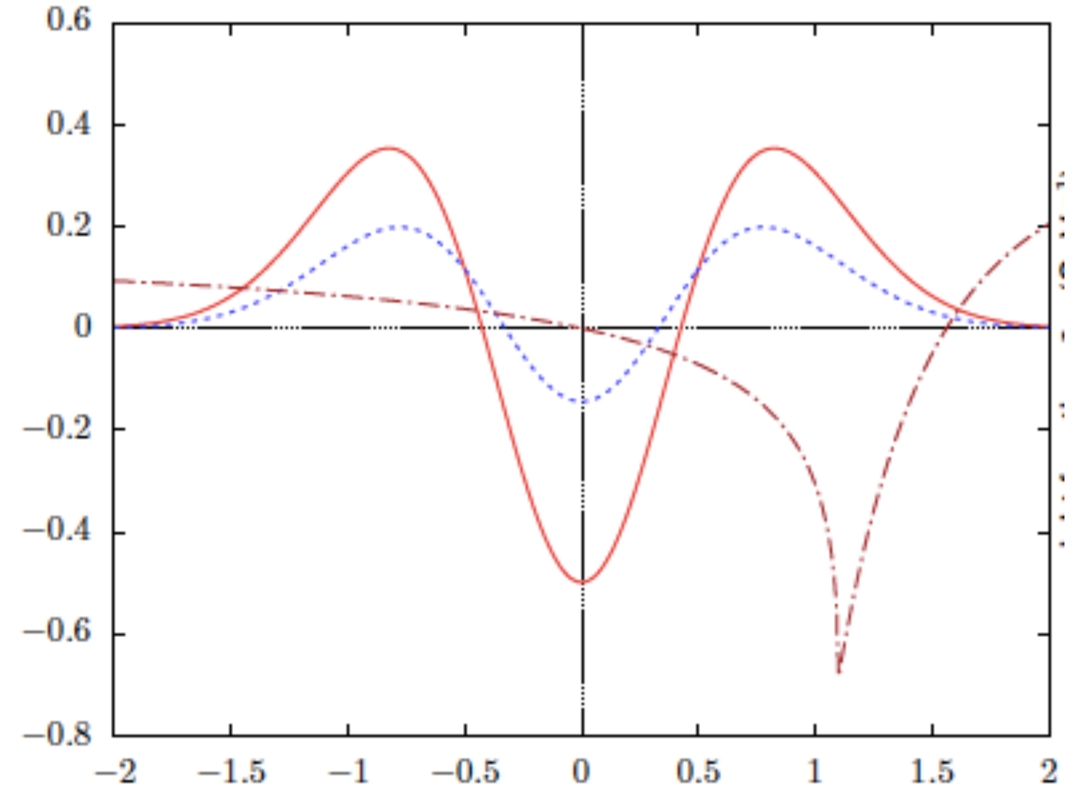
Paz et al. arXiv:1908.02812

Updated result for $\bar{B} \to X_s\gamma$

Benzke,Hurth,arXiv:2006.00624

Charm dependence of jet function: Constraint on shape function:





Benzke,Hurth,arXiv:2006.00624

Neubert et al., arXiv: 1003.5012

$$\mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-0.4\%,\, 4.7\%] \qquad \mathcal{F}^{17}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-1.9\%,\, 4.7\%]$$

$$\mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-3.7\%,\, 6.5\%] \qquad \mathcal{F}^{\mathrm{total}}_{\mathrm{b}\to\mathrm{s}\gamma} \in [-5.2\%,\, 6.5\%]$$

(In addition: large scale dependence)

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \to X_s\gamma$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

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- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

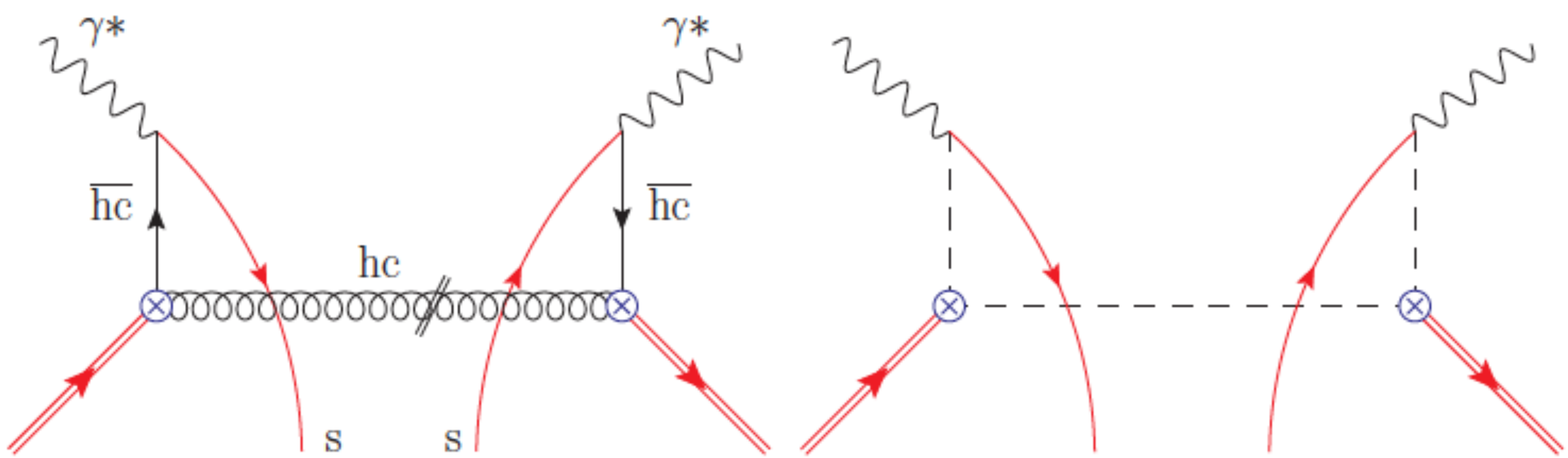
- Task 1 For NLL analysis we have to establish a factorisation theorem.

$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

- Task 2 Various steps of the NLL analysis   Bartocci,Böer,Hurth

- Task 1 For **NLL analysis** we have to establish a factorisation theorem.

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\rm res}}{dn\cdot q\, d\bar{n}\cdot q} \sim \frac{e_s^2\alpha_s}{m_b}\int d\omega\, \delta(\omega+p_+)\int\frac{d\omega_1}{\omega_1+\bar{n}\cdot q+i\varepsilon}\int\frac{d\omega_2}{\omega_2+\bar{n}\cdot q-i\varepsilon} g_{88}(\omega,\omega_1,\omega_2)$$

$$g_{88}(\omega,\omega_1,\omega_2) = \frac{1}{M_B}\langle\bar{B}|\bar{h}(\mathbf{tn})\ldots s(\mathbf{tn}+\mathbf{u\bar{n}})\bar{s}(\mathbf{r\bar{n}})\ldots h(\mathbf{0})|\bar{B}\rangle_{\rm F.T.}$$

- Subtlety in the Q_8-Q_8 contribution: convolution integral is UV divergent
  - This implies that there is no complete proof of the factorization formula yet.
  - Nevertheless one shows that scale dependence of direct and resolved contribution cancel. Benzke,Lee,Neubert,Paz,arXiv:1003.5012

- Task 1 For **NLL analysis** we have to establish a factorisation theorem.

Refactorisation in subleading $\bar{B} \to X_s\gamma$ Hurth,Szafron,arXiv:2301.01739

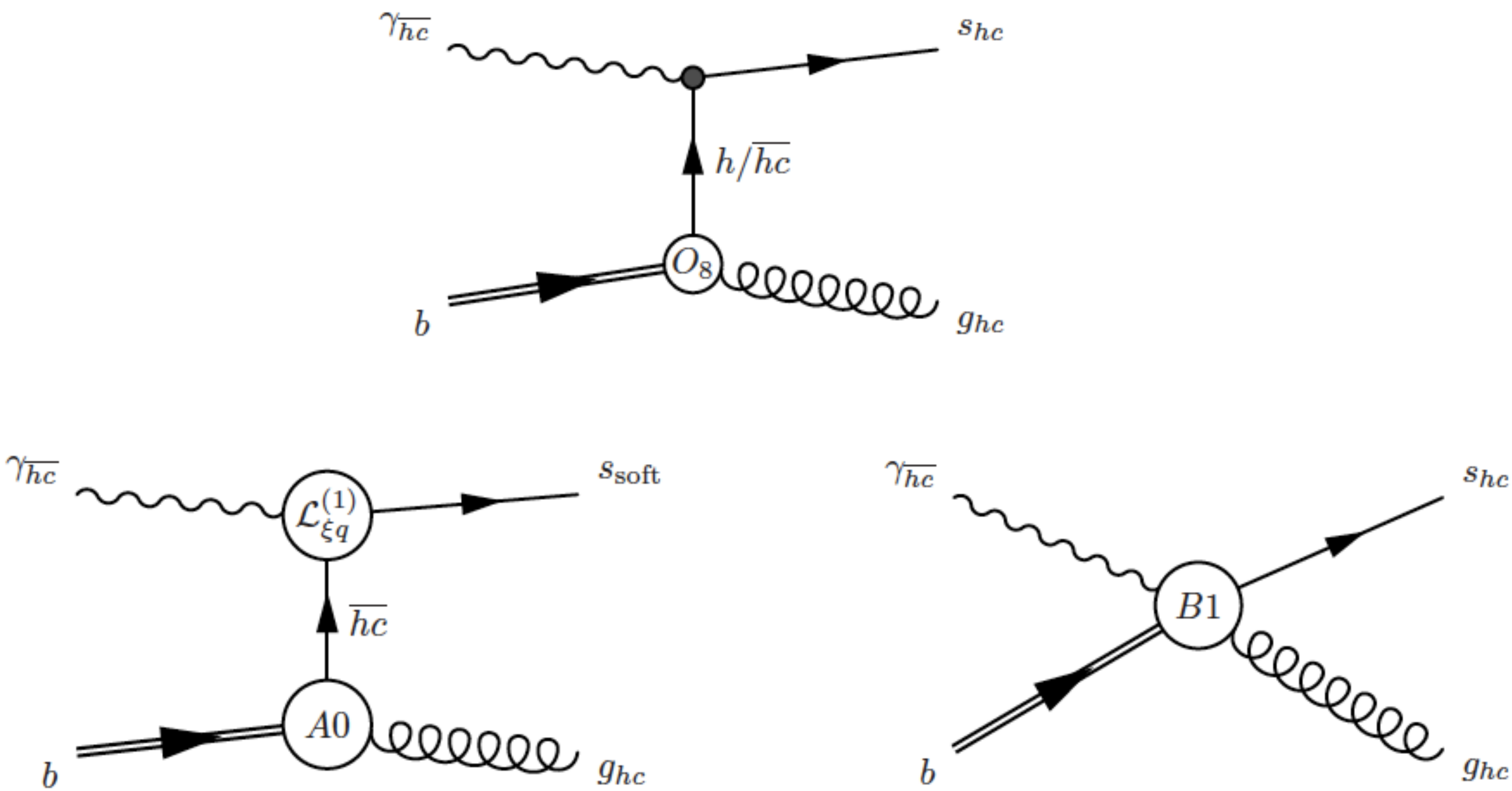
- Naive factorisation theorem with anti-hardcollinear Jet functions $\overline{J}$

$$d\Gamma(\bar{B} \to X_s\gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)}$$
$$+ \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

- Contribution of the gluon dipole operator does not factorise
- One can identify divergences in resolved *and* direct contribution in SCET-I as endpoint-divergences
- One can use refactorisation techniques developed in collider examples
  Neubert et al.,arXiv:2009.06779
- First QCD application with nonperturbative objects in flavour physics

Degeneracy in EFT leads to endpoint divergences

Hurth,Szafron,arXiv:2301.01739



$$\mathcal{O}^{A0}_{8g}(0) = \overline{\chi}_{\overline{hc}}(0)\,\frac{\not{n}}{2}\gamma_{\mu\perp}\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

$$\mathcal{O}^{B1}_{8g}(u) = \int \frac{dt}{2\pi} e^{-ium_b t}\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\,Q_s\,\mathcal{B}^{\nu}{}_{\overline{hc}\perp}(0)\,\gamma_{\mu\perp}\,\mathcal{A}^{\mu}{}_{hc\perp}(0)\,(1+\gamma_5)\,h(0)$$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO. **Not included in error above !**

- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.

- Task 1 For NLL analysis we have to establish a factorisation theorem.

$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

- Task 2 Various steps of the NLL analysis   Bartocci,Böer,Hurth

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  - analysis of renormalisation properties of the soft function
  - α_s (two-loop) corrections to anti-jet function
  - hard matching at order α_s
  - α_s corrections to quark jet function
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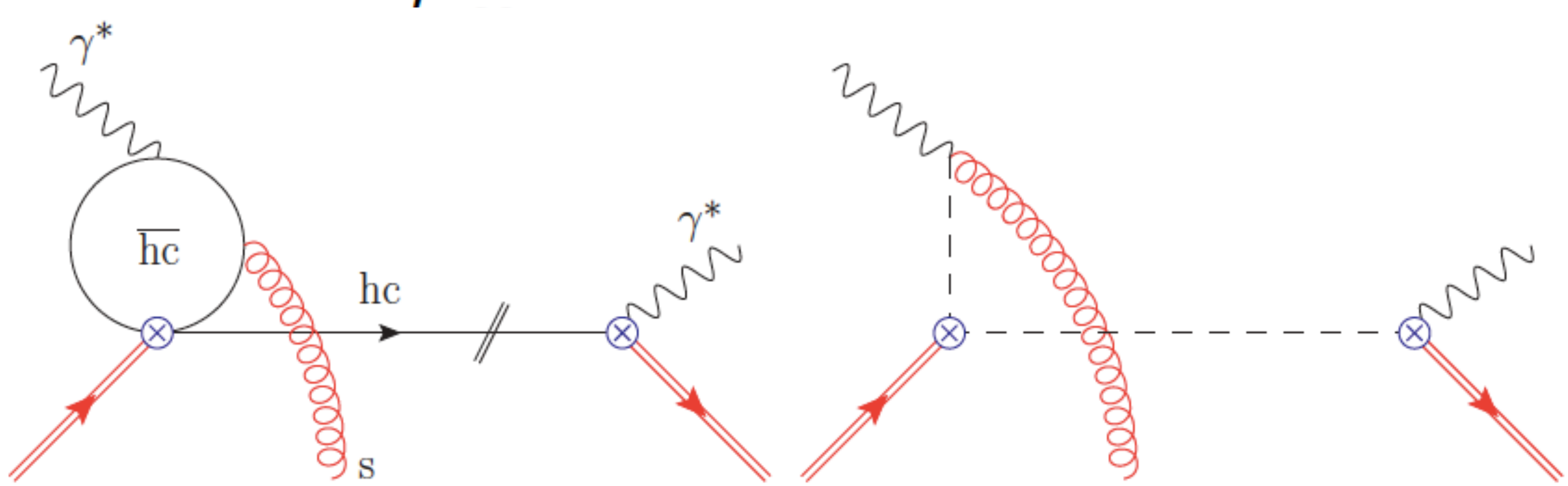
- Task 2 Various steps of the **NLL analysis** Bartocci,Böer,Hurth

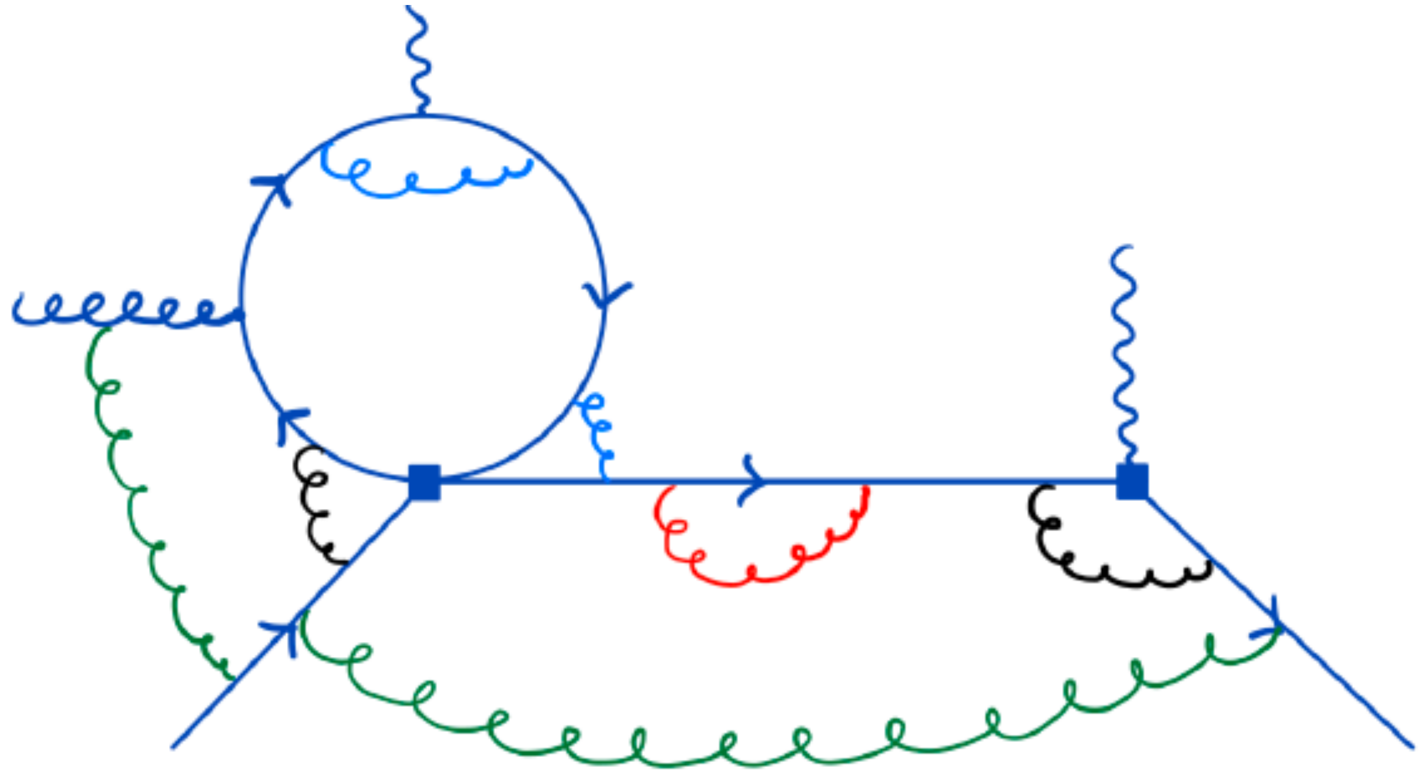
  – analysis of renormalisation properties of the soft function

$$\mathcal{S}_{ren}(\omega,\omega_1) = \int^{\bar{\Lambda}} d\omega' Z_{\mathcal{S}}(\omega,\omega',\omega_1,\omega_1')\, \mathcal{S}_{bare}(\omega',\omega_1').$$



Bartocci,Böer,Hurth, to appear next week

- Task 2 Various steps of the **NLL analysis** Bartocci,Böer,Hurth



$$d\Gamma \propto H \times J \otimes S \otimes \bar{J}$$

In SCET, we can compute gauge invariant pieces separately.

- analysis of renormalisation properties of **the soft function** ✓
- α_s (two-loop) corrections to **anti-jet function**

  We already calculated all diagrams for $m_c \to m_u = 0$ ✓
- α_s corrections to **quark jet function** known ✓
- **hard** matching at order α_s known ✓
- use RG techniques to run various functions to a common scale.

  We already checked the pole cancellation for $m_c \to m_u = 0$ ✓

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$$1.17\,\mathrm{GeV} \leq m_c \leq 1.23\,\mathrm{GeV}$$

We use scale variation of the hard-collinear scale

$$\mu_{\mathrm{hc}} \sim \sqrt{m_b\,\Lambda_{\mathrm{QCD}}} \quad \text{from} \quad 1.3\,\mathrm{GeV} \text{ to } 1.7\,\mathrm{GeV}$$ and get

$$1.14\,\mathrm{GeV} \leq m_c \leq 1.26\,\mathrm{GeV}$$

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This kinematic $1/m_b^2$ term has a $1/m_b$ shape function, all other $1/m_b^2$ contributions have a shape function of order $1/m_b^2$. So no cancellation expected. Benzke,Hurth,arXiv:2303.06447

The large kinematic $1/m_b^2$ term can be used as conservative estimate of all $1/m_b^2$ contributions to resolved $\mathcal{O}_{7\gamma} - \mathcal{O}_1$.

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Underestimation of the uncertainty due to the resolved contribution.

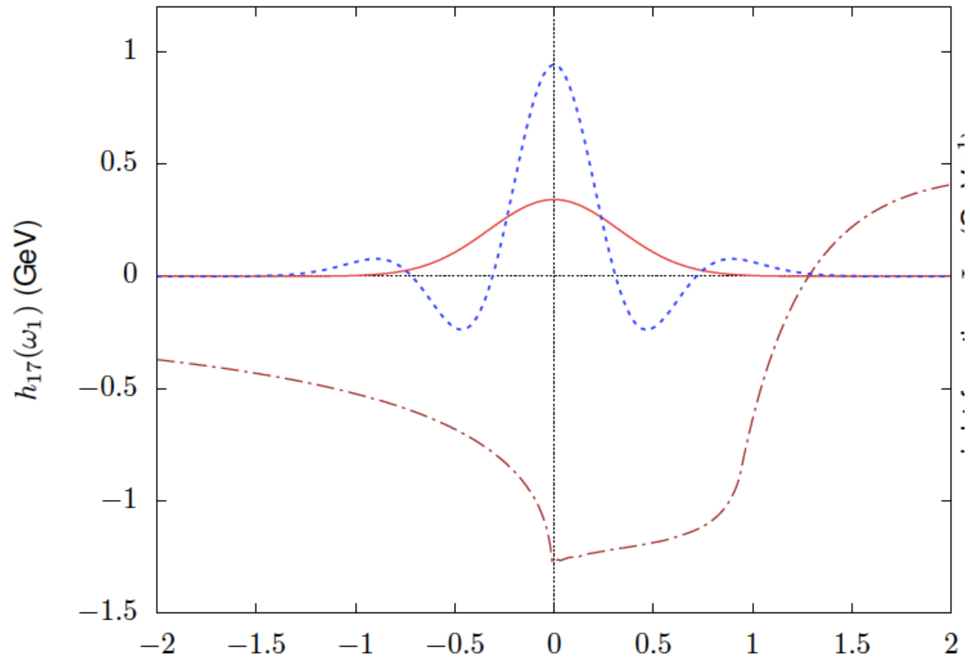
But used in recent $b \to s\gamma$ analysis. Misiak, Rehman, Steinhauser, arXiv:2002.01548v2

Updated result for $\bar{B} \to X_s \ell\ell$

Benzke,Hurth,arXiv:2006.00624

Rather symmetric jet function $\to$

Various shape functions lead to very similar values of the convolution



arXiv:2006.00624

$$\mathcal{F}^{17}_{\mathrm{b}\to \mathrm{s}\ell\ell} \in [+0.2\%, +2.6\%]$$

arXiv:1705.10366

$$\mathcal{F}^{17}_{\mathrm{b}\to \mathrm{s}\ell\ell}|_{1/m_b} \in [-0.5\%, +3.4\%]$$

We find large scale dependence of the results in both penguins

$\Rightarrow$ α_s corrections desirable

Numerical relevant contributions to $O(1/m_b^2)$

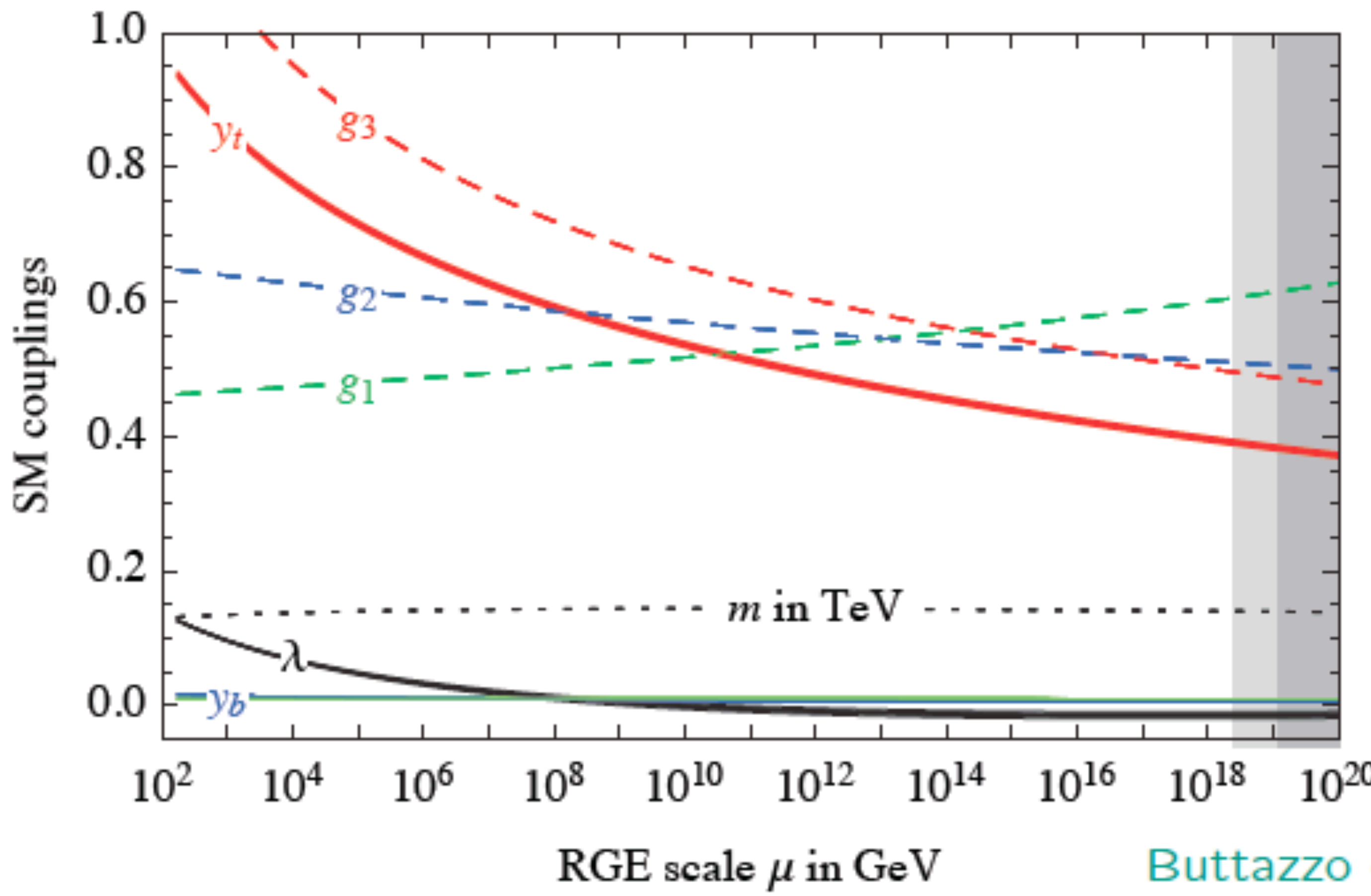
$\mathcal{F}_{19}$: $O(1/m_b^2)$ but $|C_{9/10}| \sim 13|C_{7\gamma}|$

Epilogue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



Buttazzo et al. arXiv:1307.3536

High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- Dark matter (visible matter accounts for only 4% of the Universe)
- Neutrino masses (Dirac or Majorana masses ?)
- Baryon asymmetry of the Universe (new sources of CP violation needed)

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Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

- The days of "guaranteed" discoveries or no-lose theorems in particle physics are over, at least for the time being .....

- but the big questions of our field remain open (hierarchy problem. flavour, neutrinos, dark matter, baryogenesis,...)

- This simply implies that, more than for the past 30 years, future HEP's progress is to be driven by experimental exploration, possibly renouncing/reviewing deeply rooted theoretical bias.

Spares 1

Refactorisation

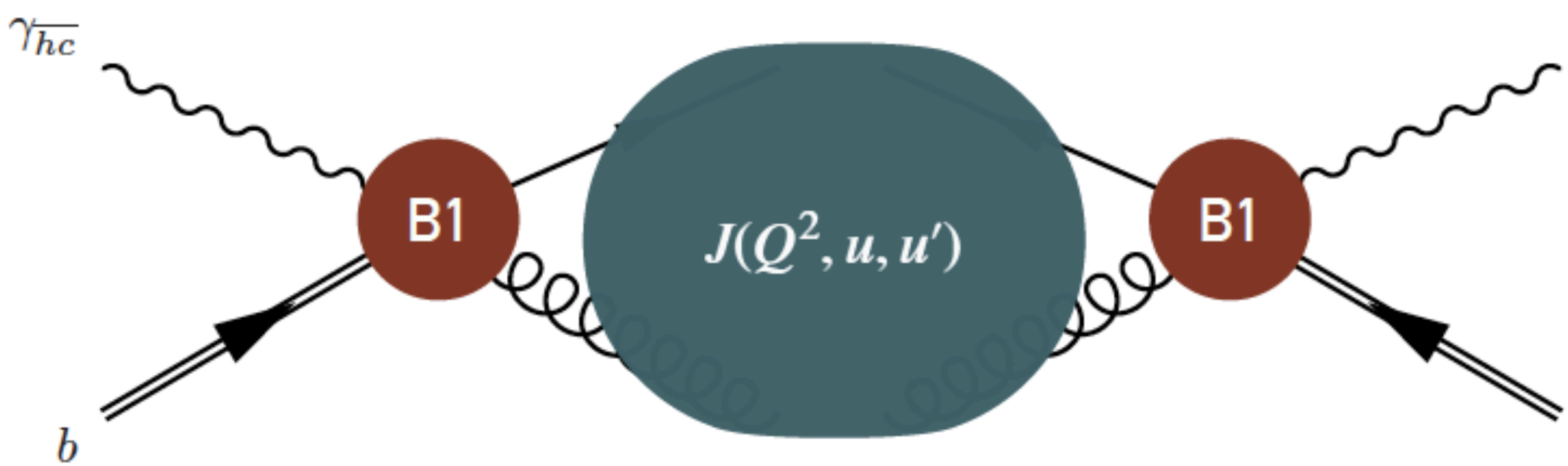
Hurth, Szafron, arXiv:2301.01739

Factorisation of direct contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du\, \mathbf{C^{B1}}\left(\mathbf{m_b}, \mathbf{u}\right) \int_0^1 du'\, \mathbf{C^{B1*}}\left(\mathbf{m_b}, \mathbf{u'}\right) \int_{-p_+}^{\overline{\Lambda}} d\omega\, \mathbf{J}\left(\mathbf{M_B}\left(\mathbf{p_+} + \omega\right), \mathbf{u}, \mathbf{u'}\right) \mathcal{S}\left(\omega\right)$$

$$\mathbf{J}\left(\mathbf{p^2}, \mathbf{u}, \mathbf{u'}\right) = \frac{(-1)}{2N_c} \frac{1}{2\pi} \int \frac{dt dt'}{(2\pi)^2}\, d^4x\; e^{-im_b(ut-u't')+ipx}$$

$$\text{Disc}\left[\langle 0|\, tr\left[\frac{1+\not v}{2}(1-\gamma_5)\mathcal{A}_{hc\perp}(x)\,\gamma_\perp^\nu \chi_{hc}(t'\bar{n}+x)\,\overline{\chi}_{hc}(t\bar{n})\,\gamma_{\nu\perp}\mathcal{A}_{hc\perp}(0)\,(1+\gamma_5)\right]|0]\rangle\right]$$



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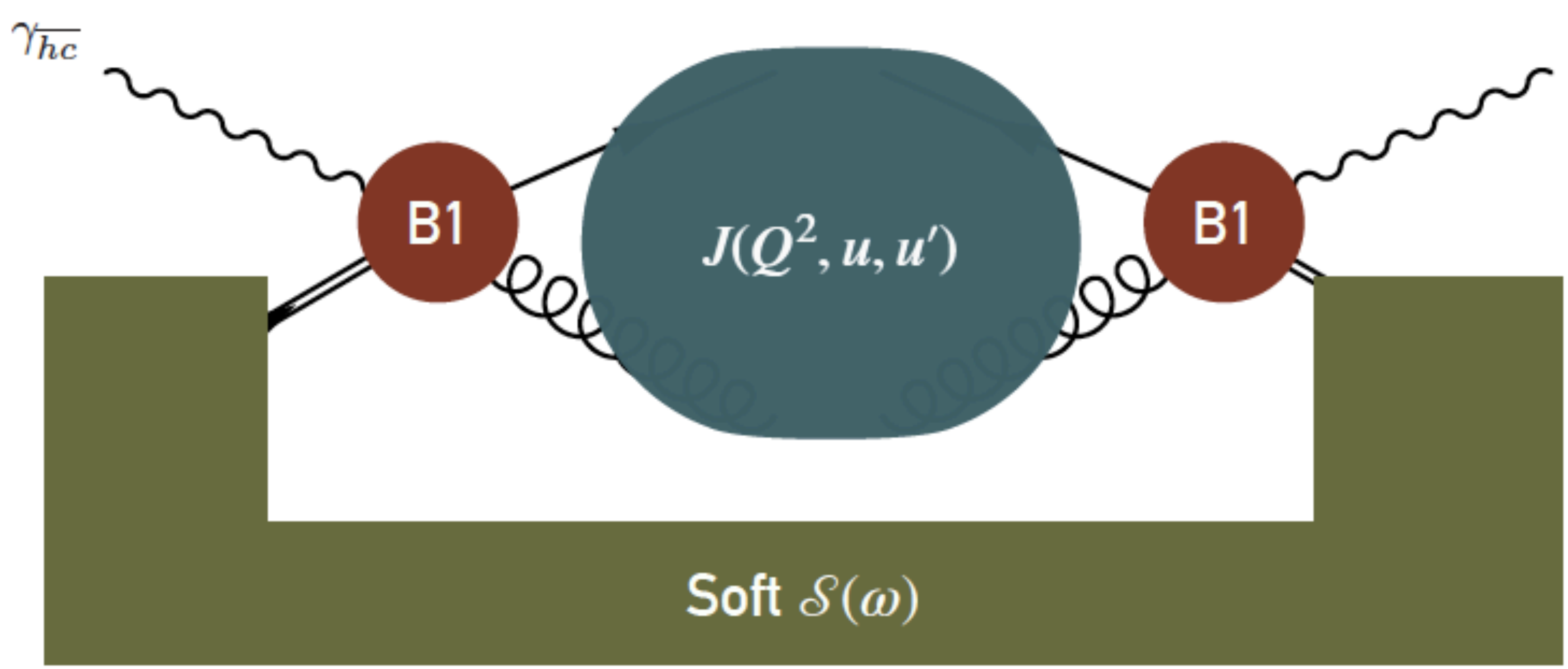
$$\mathcal{S}\left(\omega\right) = \frac{1}{2m_B} \int \frac{dt}{2\pi} e^{-i\omega t} \left\langle B \right| h\left(tn\right) S_n\left(tn\right) S_n^\dagger\left(0\right) h\left(0\right) \left| B \right\rangle$$



Endpoint divergence in direct contribution at leading order

Hard matching coefficients

$$\mathbf{C^{B1}_{LO}(m_b, u)} = (-1)\frac{\overline{u}}{u}\frac{m_b^2}{4\pi^2}\frac{G_F}{\sqrt{2}}\lambda_t C_{8g} = (-1)\frac{\overline{u}}{u}C^{A0}_{LO}(m_b)$$

convoluted with jet function

$$\mathbf{J\left(p^2, u, u'\right)} = C_F\frac{\alpha_s}{4\pi\, m_b}\theta(p^2)\, A(\epsilon)\, \delta(u-u')u^{1-\epsilon}(1-u)^{-\epsilon}\left(\frac{p^2}{\mu^2}\right)^{-\epsilon}$$

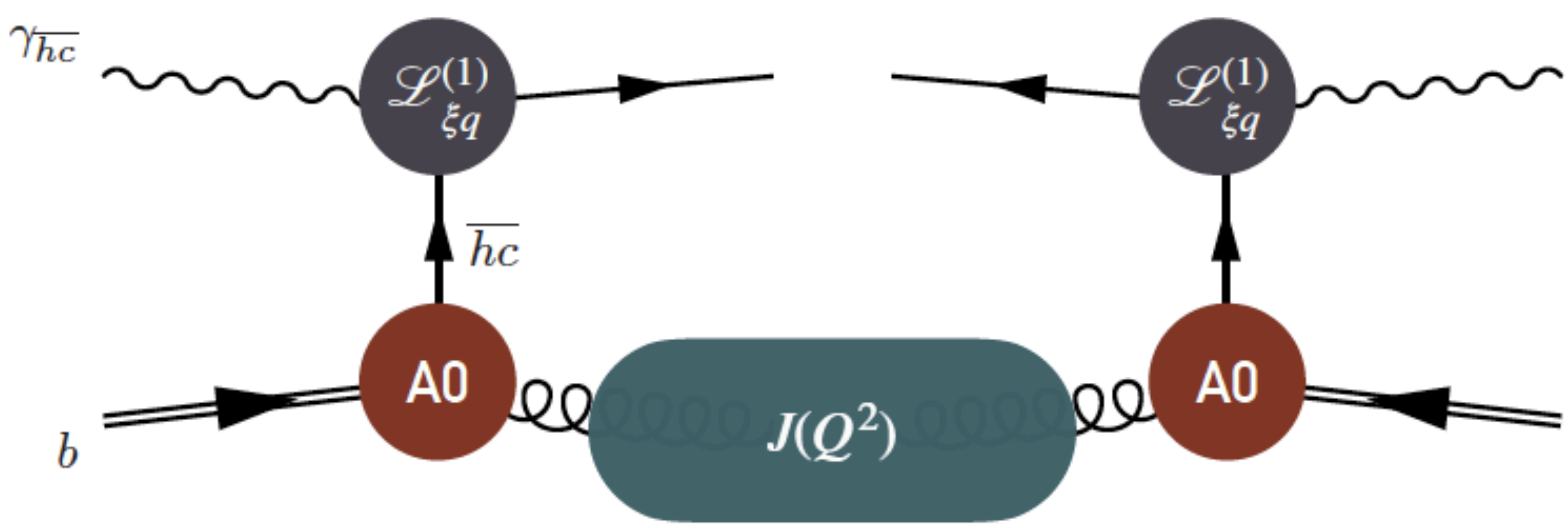
lead to endpoint divergence in the $u \to 0$ limit

$$\int_0^1 du\frac{1}{u}\int_u^1 du'\frac{1}{u'}u^{1-\epsilon}\delta(u-u') \sim \int_0^1 du\frac{1}{u^1+\epsilon}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|\mathbf{C^{A0}}\left(\mathbf{m_b}\right)\right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega\, \mathbf{J_g}\left(\mathbf{m_b}\left(\mathbf{p_+} + \omega\right)\right) \int d\omega_1 \int d\omega_2\, \bar{\mathbf{J}}\left(\omega_1\right) \bar{\mathbf{J}}^*\left(\omega_2\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

$$-g_s^2 \delta_{ab} g_\perp^{\mu\nu}\ \mathbf{J_g}(\mathbf{p^2}) = \frac{1}{2\pi i} \mathrm{Disc}\left[i \int d^4x e^{ipx} \langle 0| T\left[\mathcal{A}^{a\mu}_{hc\perp}(x), \mathcal{A}^{b\nu}_{hc\perp}(0)\right] |0\rangle\right]$$



Factorisation of resolved contribution

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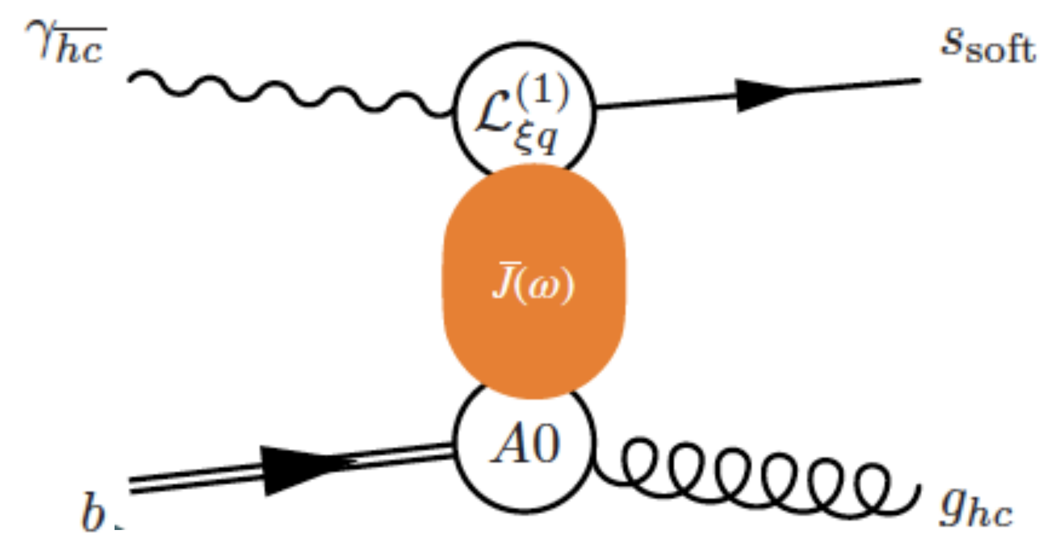
Anti-hardcollinear jet function $\overline{J}(\omega)$ is defined on the amplitude level.



$$O_{T\xi q} = i\int d^d x T\left[\mathcal{L}_{\xi q}(x), O_{8g}^{A0}(0)\right]$$

$$= \int d\omega \int \frac{dt}{2\pi} e^{-it\omega} [\overline{q_s}]_\alpha (tn) \left[\overline{\mathbf{J}}(\omega)\right]^{\mathbf{a}\,\nu\mu}_{\alpha\beta} Q_s \mathcal{B}^\nu_{\overline{hc}\perp}(0)\, \mathcal{A}^{\mu\, a}_{hc\perp}(0)\, [h(0)]_\beta$$

Decomposition to all orders: $\left[\overline{\mathbf{J}}(\omega)\right]^{\mathbf{a}\,\nu\mu}_{\alpha\beta} = \overline{J}(\omega)\, t^a \left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{\bar n}\not{n}}{4}\right]_{\alpha\beta}$

Factorisation of resolved contribution

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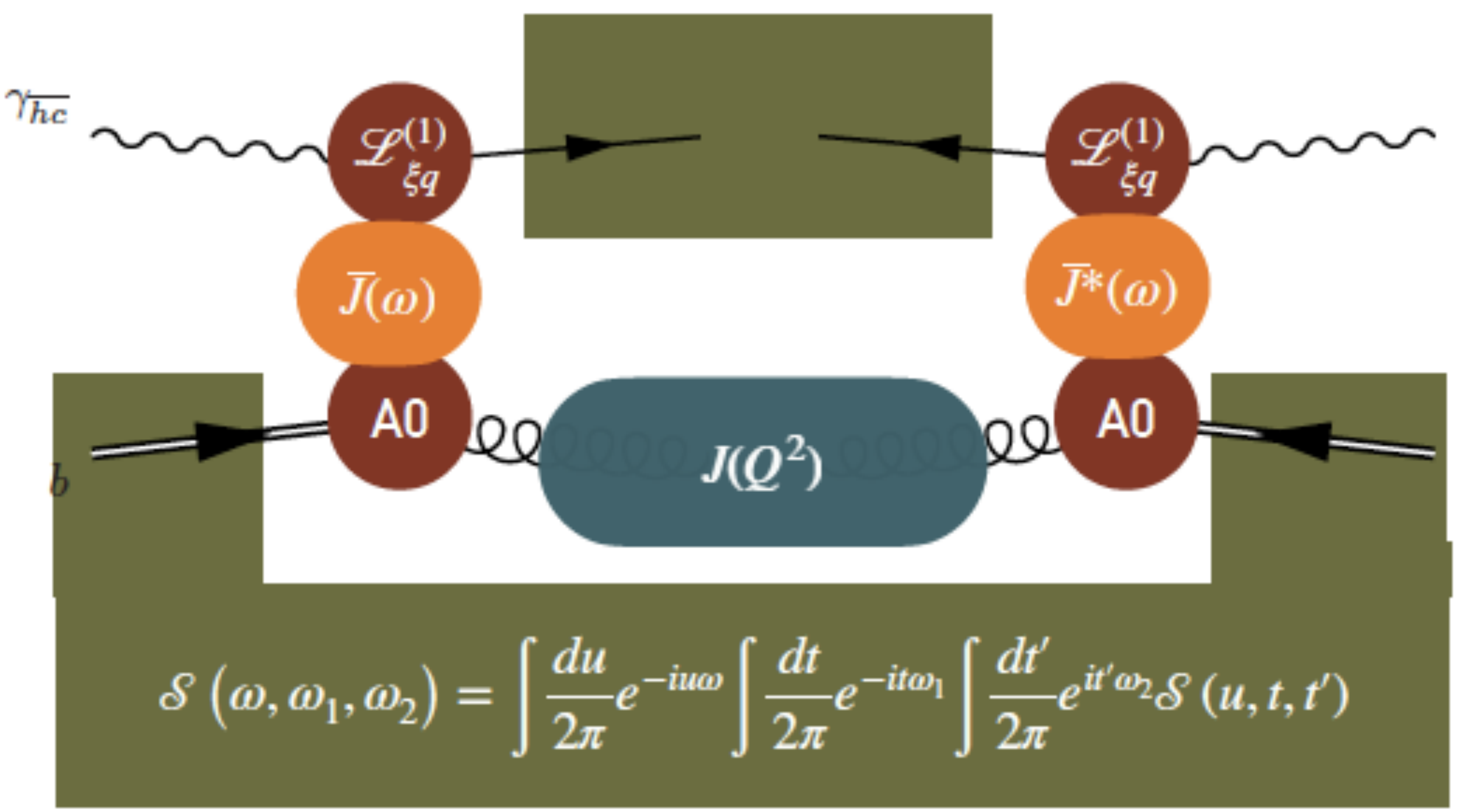
Operatorial definition of the soft function in position space $\mathcal{S}\left(\mathbf{u}, \mathbf{t}, \mathbf{t}'\right)$

$$\mathcal{S}\left(\mathbf{u}, \mathbf{t}, \mathbf{t}'\right) = (d-2)^2 g_s^2 \langle B| \bar{h}\left(un\right)\left(1-\gamma_5\right)\left[S_n\left(un\right) t^a S_n^\dagger\left(un\right)\right] S_{\bar{n}}\left(un\right) S_{\bar{n}}^\dagger\left(t'\bar{n}+un\right)$$
$$\frac{\not{n}\not{\bar{n}}}{4} q_s\left(t'\bar{n}+un\right) \bar{q}_s\left(t\bar{n}\right) \frac{\not{\bar{n}}\not{n}}{4} S_{\bar{n}}\left(t\bar{n}\right) S_{\bar{n}}^\dagger\left(0\right)\left[S_n\left(0\right) t^a S_n^\dagger\left(0\right)\right]\left(1+\gamma_5\right) h\left(0\right)|B\rangle \,/\, \left(2m_B\right)$$



Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\bar\Lambda} d\omega\, J_g(m_b(p_+ + \omega)) \int d\omega_1 \int d\omega_2\, \bar{J}(\omega_1)\, \bar{J}^*(\omega_2)\, \mathcal{S}(\omega, \omega_1, \omega_2)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

$\omega_{1,2} \to \infty$ corresponds to $t, t' \to 0$ and $q_s(un) \to S_n(un) q_{hc}(un)$, $\bar{q}_s(0) \to q_{hc} S_n^+(0)$

$$\mathcal{S}(u,t,t') = (d-2)^2 g_s^2 \langle B|\bar{h}(un)(1-\gamma_5)\left[S_n(un)\, t^a S_n^\dagger(un)\right] S_{\bar n}(un) S_{\bar n}^\dagger(t'\bar n + un) \frac{\not{\bar n}\not{n}}{4} q_s(t'\bar n + un)\, \bar{q}_s(t\bar n) \frac{\not{n}\not{\bar n}}{4} S_{\bar n}(t\bar n) S_{\bar n}^\dagger(0) \left[S_n(0)\, t^a S_n^\dagger(0)\right](1+\gamma_5) h(0)|B\rangle \,/\, (2m_B)$$

$$\mathcal{S}(u) = \langle B|\bar{h}(un)\, S_n(un)\, S_n^\dagger(0)\, h(0)|B\rangle \,/\, (2m_B)$$

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More general:
Asymptotic ($\omega_1 \sim \omega_2 \leq \omega$) soft function $\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)$ is a convolution of a perturbabtive kernel K and the leading power soft function.

$$\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2) = \int d\omega' K(\omega,\omega',\omega_1,\omega_2)\mathcal{S}(\omega')$$

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Asymptotic $(\omega_1 \sim \omega_2 \leq \omega)$ soft function $\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)$ is a convolution of a perturbabtive kernel K and the leading power soft function.

$$\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2) = \int d\omega' K(\omega,\omega',\omega_1,\omega_2)\mathcal{S}(\omega')$$

Leading order in α_s:

$$\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2) = C_F A(\epsilon)\frac{\alpha_s}{(4\pi)}\, \omega_1^{1-\epsilon}\delta(\omega_1-\omega_2) \int_\omega^{\bar\Lambda} d\omega'\, \mathcal{S}(\omega') \left(\frac{(\omega'-\omega)}{\mu^2}\right)^{-\epsilon}$$

Refactorisation at leading order

$$\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0} = -\mathcal{N}\,\left|C_{LO}^{A0}\,(m_b)\right|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}\,A(\epsilon)\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega)\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}|_A^{\mathrm{asy}} = \mathcal{N}|C_{LO}^{A0}\,(m_b)\,|^2\,\frac{\alpha_s C_F}{(4\pi)\,m_b}\,\frac{1}{\epsilon}A(\epsilon)\,\int_{-p_+}^{\overline{\Lambda}} d\omega\,\mathcal{S}_{LO}(\omega')\left(\frac{m_b(\omega+p_+)}{\mu^2}\right)^{-\epsilon}$$

One verifies that

$$\frac{d\Gamma}{dE_\gamma}|_A^{\mathrm{asy}} = (-1)\,\frac{d\Gamma}{dE_\gamma}|_B^{u,u'\to 0}$$

Refactorisation conditions can be formulated on the operator level

Express the fact that in the limits $u \sim u' \ll 1$ and $\omega_1 \sim \omega_2 \gg \omega$ the two terms of the subleading $\mathcal{O}_8 - \mathcal{O}_8$ contribution have the same structure.

- $[\![C^{B1}(m_b, u)]\!] = (-1)C^{A0}(m_b)\, m_b \overline{J}(u m_b)$

  ($[\![g(u)]\!]$ only denotes the leading term of a function $g(u)$ in the limit $u \to 0$)

- $\widetilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ corresponds to $\mathcal{S}(\omega, \omega_1, \omega_2)$ in the limit $\omega_1 \sim \omega_2 \gg \omega$

  (In this limit: $q_s \to q_{sc}$ and higher power corrections in $\omega/\omega_{1,2}$ are neglected)

- $\int_{-p_+}^{\overline{\Lambda}} d\omega\, [\![J(m_b(p_+ + \omega), u, u')\, \mathcal{S}(\omega)]\!] = \int_{-p_+}^{\overline{\Lambda}} d\omega\, J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$

  (In this limit $\chi_{hc} \to q_{sc}$, brackets indicate again that the $u \to 0$ *and* $u' \to 0$ limits)

The refactorisation relations are operatorial relations that guarantee the cancellation of endpoint divergences between the two terms to all orders in α_s.

Finally we show that refactorisation and renormalisation commute.

Refactorised (endpoint finite) factorisation theorem

We subtract the two asymptotic terms

$$0 = 2\mathcal{N}\left|C^{A0}(m_b)\right|^2 \int_{-p_+}^{\Lambda} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \int_{m_b}^{\infty} d\omega_1 \overline{J}\left(\omega_1\right) \int_0^{\omega_1} d\omega_2 \overline{J}^*\left(\omega_2\right) \widetilde{\mathcal{S}}\left(\omega, \omega_1, \omega_2\right)$$

$$+ 2\mathcal{N} \int_0^1 du \left[\!\left[C^{B1}\left(m_b, u\right)\right]\!\right] \int_u^1 du' \left[\!\left[C^{B1*}\left(m_b, u'\right)\right]\!\right] \int_{-p_+}^{\overline{\Lambda}} d\omega \left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)\right]\!\right]$$

with

$$\left[\!\left[J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)\right]\!\right] = J_g(m_b(p_+ + \omega))\widetilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

$$\left[\!\left[C^{B1}\left(m_b, u'\right)\right]\!\right] = (-1) C^{A0}\left(m_b\right) m_b \overline{J}\left(u m_b\right)$$

from the all-order factorisation theorems we derived

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}\left|C^{A0}(m_b)\right|^2 \int_{-\infty}^{\infty} d\omega_1 \overline{J}\left(\omega_1\right) \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}^*\left(\omega_2\right) \int_{-p_+}^{\overline{\Lambda}} d\omega J_g\left(m_b\left(p_+ + \omega\right)\right) \mathcal{S}\left(\omega, \omega_1, \omega_2\right)$$

$$+ 2\mathcal{N} \int_0^1 du C^{B1}\left(m_b, u\right) \int_u^1 du' C^{B1*}\left(m_b, u'\right) \int_{-p_+}^{\overline{\Lambda}} d\omega J\left(m_b\left(p_+ + \omega\right), u, u'\right) \mathcal{S}(\omega)$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$
\begin{aligned}
\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N}\int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ & J_g(m_b(p_+ + \omega))\left|C^{A0}\left(m_b\right)\right|^2 \\
& \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\,\overline{J}^*(\omega_2)\left[\mathcal{S}\left(\omega,\omega_1,\omega_2\right) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] \\
& + \int_0^1 du \int_u^1 du' \Big[C_{LO}^{B1}\left(m_b,u\right)\, C^{B1*}\left(m_b,u'\right)\, J\left(m_b\left(p_+ + \omega\right),u,u'\right)\mathcal{S}\left(\omega\right) \\
& - \llbracket C^{B1}\left(m_b,u\right)\rrbracket\, \llbracket C^{B1*}\left(m_b,u'\right)\rrbracket\, \llbracket J\left(m_b\left(p_+ + \omega\right),u,u'\right)\mathcal{S}(\omega)\rrbracket \Big] \Big\} ,
\end{aligned}
$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$\frac{d\Gamma}{dE_\gamma}|_{A+B} = 2\mathcal{N} \int_{-p_+}^{\overline{\Lambda}} d\omega \Big\{ J_g(m_b(p_+ + \omega)) \left|C^{A0}\left(m_b\right)\right|^2 \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \overline{J}(\omega_1)\, \overline{J}^*(\omega_2) \left[\mathcal{S}\left(\omega,\omega_1,\omega_2\right) - \theta(\omega_1 - m_b)\theta(\omega_2)\widetilde{\mathcal{S}}(\omega,\omega_1,\omega_2)\right] + \int_0^1 du \int_u^1 du' \Big[C^{B1}_{LO}\left(m_b,u\right) \, C^{B1*}\left(m_b,u'\right) \, J\left(m_b\left(p_+ + \omega\right),u,u'\right) \mathcal{S}\left(\omega\right) - [\![C^{B1}\left(m_b,u\right)]\!] \, [\![C^{B1*}\left(m_b,u'\right)]\!] \, [\![J\left(m_b\left(p_+ + \omega\right),u,u'\right) \mathcal{S}(\omega)]\!] \Big] \Big\},$$

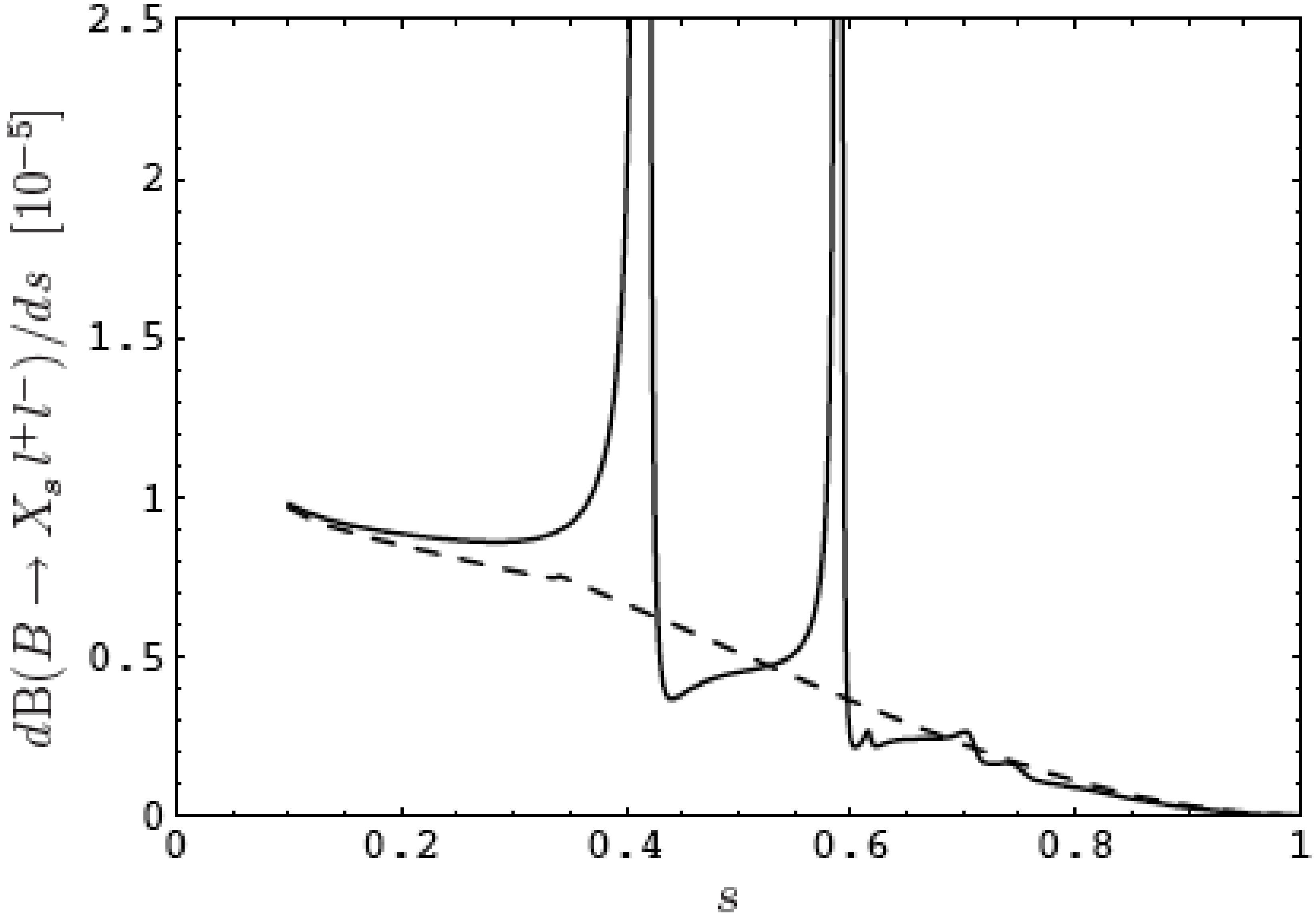
Finally we show that refactorisation and renormalisation commute.

Spares II

Quark-hadron duality violated in $\bar{B} \to X_s \ell^+ \ell^-$?

BBNS, arXiv:0902.4446

Within integrated branching ratio the resonances J/ψ and ψ' exceed the perturbative contributions by two orders of magnitude.



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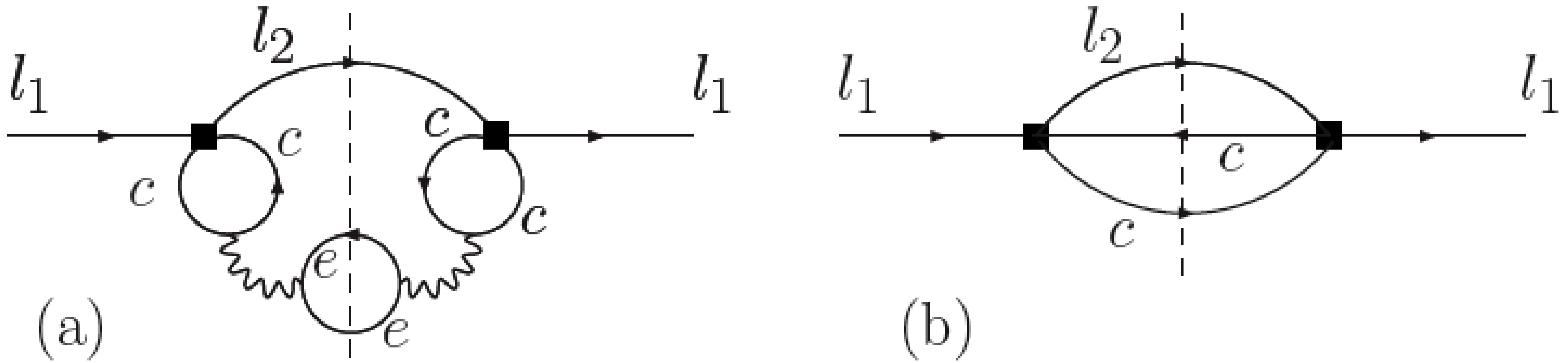


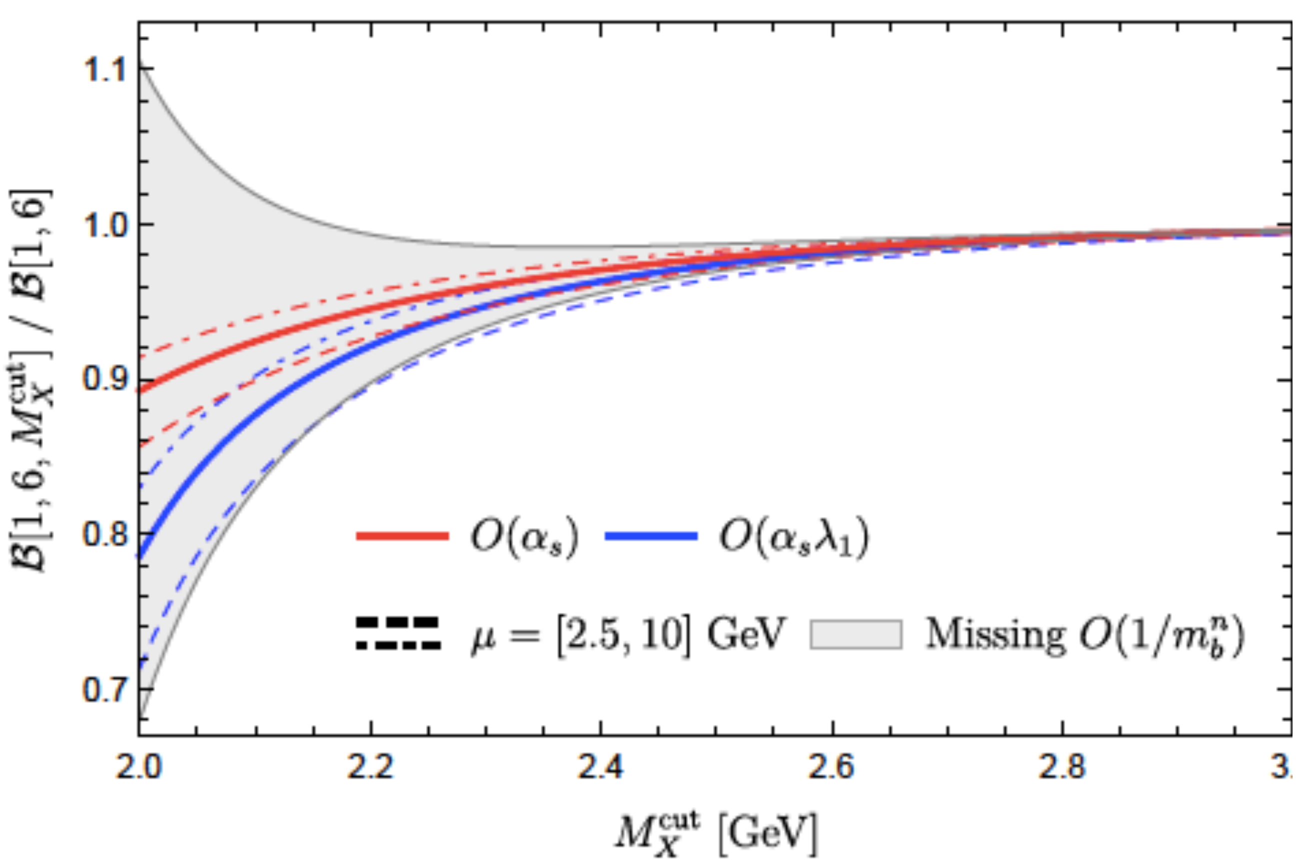
The rate $l_1 \to l_2 e^+ e^-$ (a) is connected to the integral over $|\Pi(q^2)|^2$ for which global duality is NOT expected to hold.

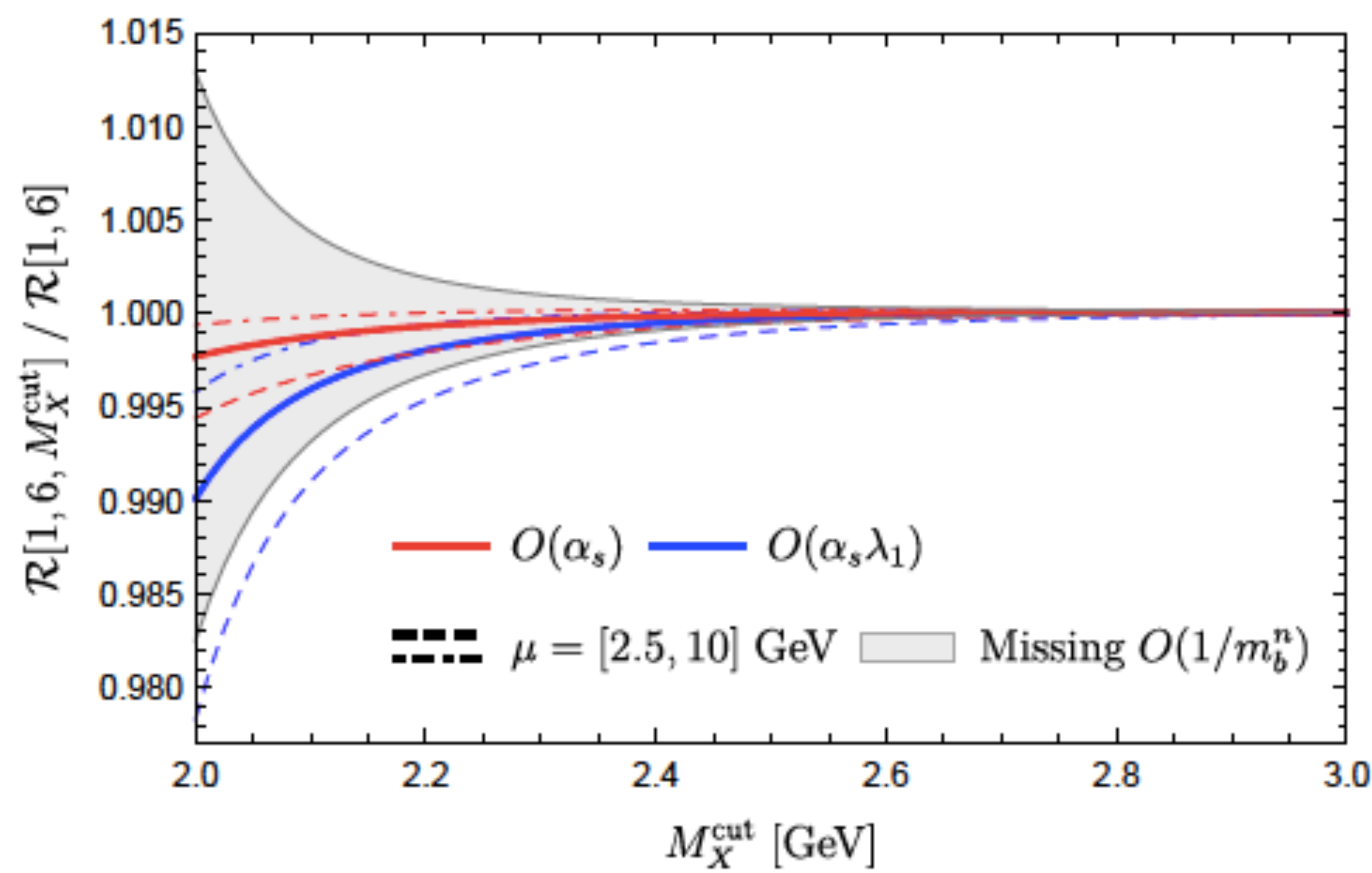
In contrast the inclusive hadronic rate $l_1 \to l_2 X$ (b) corresponds to the imaginary part of the correlator $\Pi(q^2)$.

Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

Huber,Hurth,Jenkins,Lunghi arXiv 2306.03134

- We computed the fully differential distribution of $\bar{B} \to X_s \ell^+ \ell^-$ at $O(\alpha_s)$ in the OPE
- Also the three $\bar{B} \to X_s \ell^+ \ell^-$ angular observables, together with the $\bar{B} \to X_u \ell^- \nu$ branching fraction, all with the same hadronic mass cut
- We find effective Independence of the hadronic mass cut





Hadronic cut dependence in $\bar{B} \to X_s \ell\ell$

- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \to c(\to s e^+ \nu) e^- \bar{\nu}$)

- Previous SCET calculation with some simplifications and certain problems with SCET scaling (q assumed to be hard)
  Uncertainty due to subleading shape functions estimated to 5 – 10%

  Lee,Ligeti,Stewart,Tackmann hep-ph/0512191

  Lee,Tackmann arXiv:0812.0001

- New Strategy to minimise uncertainty

  Huber,Hurth,Jenkins,Lunghi arXiv 2306.03134

  – Calculation of cut dependence using OPE for mild hadronic cuts

  – Analyse breakdown of OPE via λ_1 power corrections

  – Try to interpolate betweeen SCET and OPE calculation

  – Use cut-independent ratios in OPE and SCET to analyse interpolation