Models with angular momentum transport

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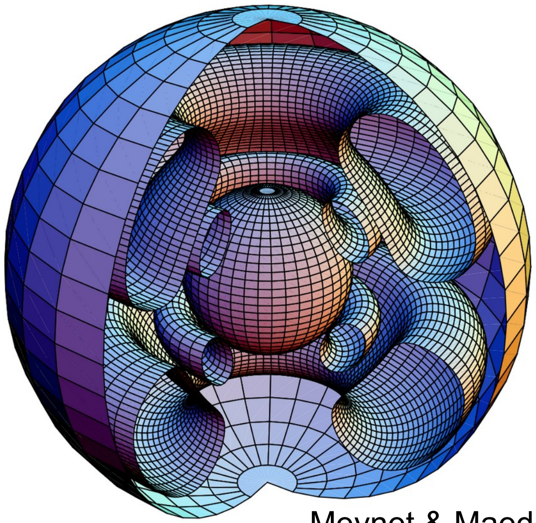
Transport of angular momentum

- Transport by meridional circulation and shear instability
  - Shellular rotation hypothesis (Zahn 1992) :
    turbulence induced by rotation is much stronger in the horizontal (along isobars) than in the vertical direction
    → *approximately constant Ω on the isobars*

$$f(P,\theta) = \overline{f}(P) + \tilde{f}(P)P_2(\cos\theta)$$



Meynet & Maeder 2002

  - Advective transport of AM
    by meridional currents :

$$u(r,\theta) = U(r)P_2(\cos\theta)$$

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right)_{M_r} = \frac{1}{5r^2}\frac{\partial}{\partial r}\left(\rho r^4\Omega U(r)\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\rho D r^4\frac{\partial\Omega}{\partial r}\right)$$

Transport of angular momentum

- Transport by meridional circulation and shear
  - Shear instability (Talon & Zahn 1997) :

$$D_{\rm shear} = 2\mathcal{R}i_{\rm crit} \frac{(dV/dz)^2}{N^2_{T,\rm ad}/(K + D_h) + N^2_{\mu}/D_h}$$

No free parameter f_μ to arbitrary reduce the inhibiting effects of chemical gradients (≠ diffusive scheme (e.g. MESA*) : $f_\mu \cong 0.05$ - 0.01)*

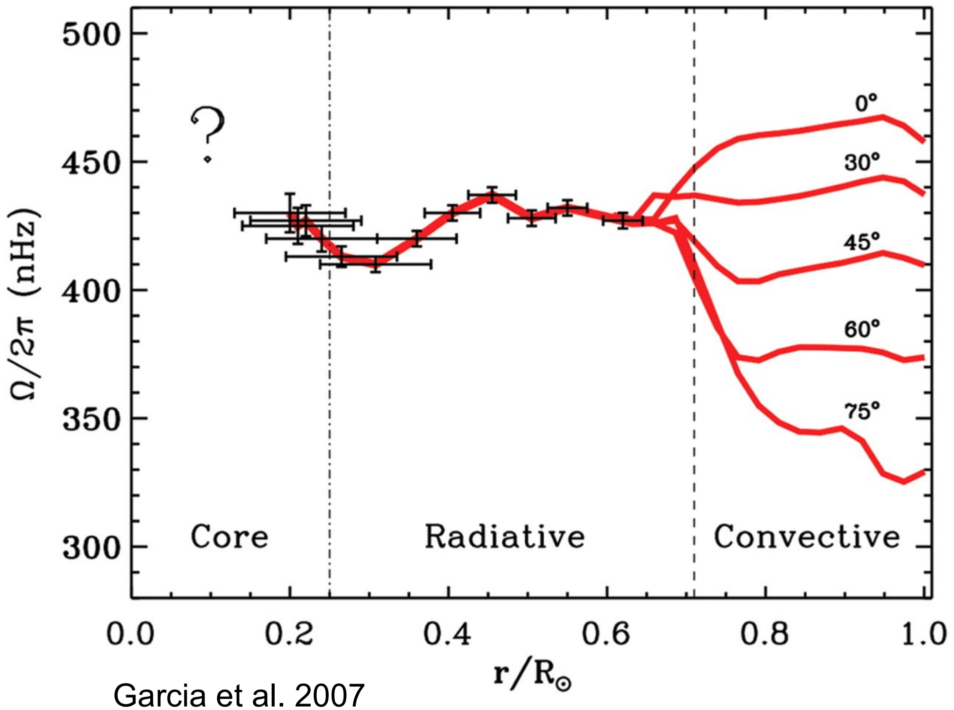
  - Only one free parameter in this formalism : the amplitude A of the horizontal turbulence $D_{\rm h}$ (Maeder 2003)

$$D_{\rm h} = Ar\left(r\bar{\Omega}(r)V[2V - \alpha U]\right)^{\frac{1}{3}}$$

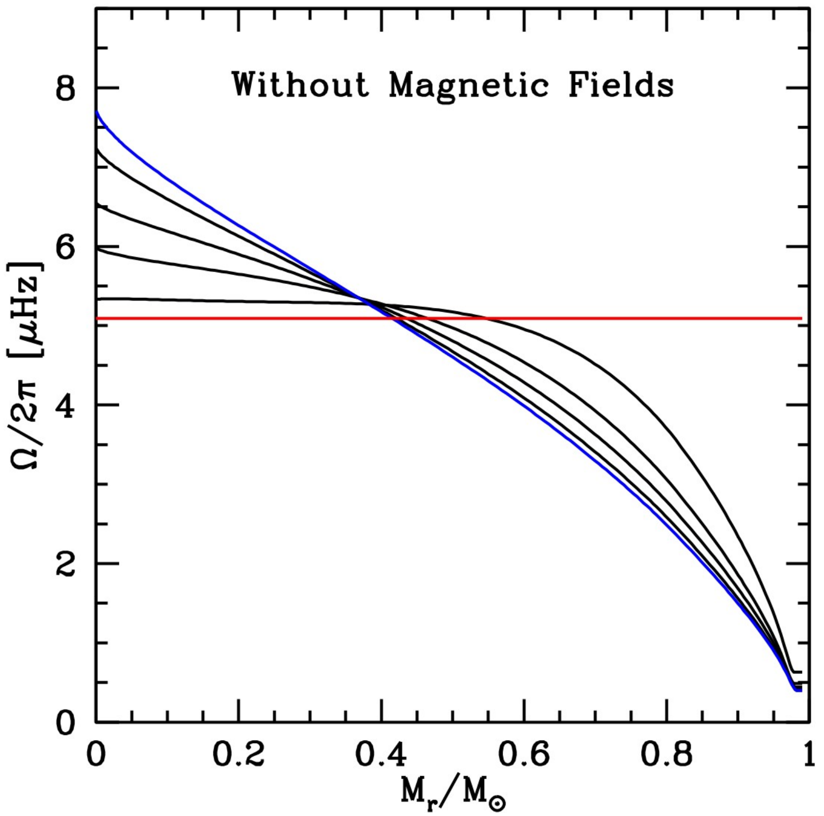
$$A = \left(\frac{3}{400n\pi}\right)^{\frac{1}{3}}$$

Transport of angular momentum

- The solar rotation profile
  - Helioseismic measurements



Garcia et al. 2007



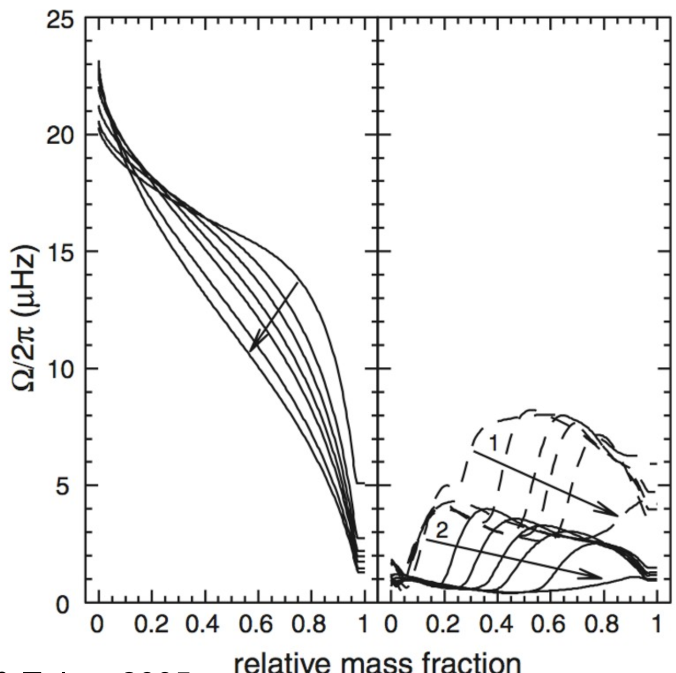
  - *Inefficient transport by hydrodynamic processes*
  - Steady internal fossil magnetic field in radiative zones ?

    issue: mechanical coupling to the convective zone

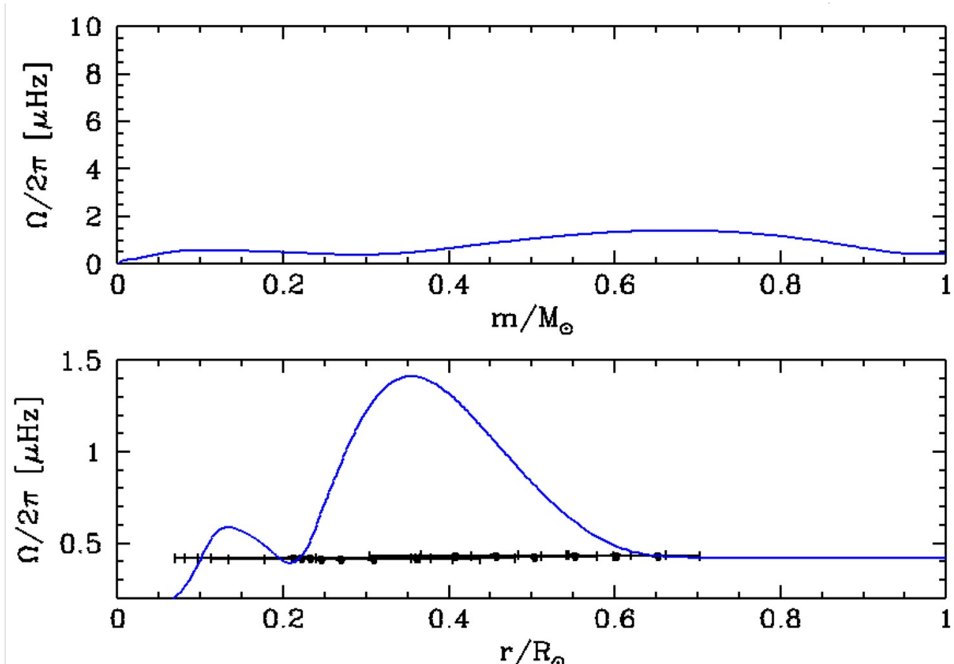
Transport of angular momentum

- The solar rotation profile : internal gravity waves ?

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left[r^2\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega U\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho (D_{\mathrm{shear}} + \nu_{\mathrm{waves}}) r^4 \frac{\partial \Omega}{\partial r}\right] - \frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathcal{L}_J(r)$$



Charbonnel & Talon. 2005



Difficult to reproduce the flat rotation profile of the Sun (Denissenkov et al. 2008)

Transport by IGW generated by penetrative convection ?

Transport of angular momentum

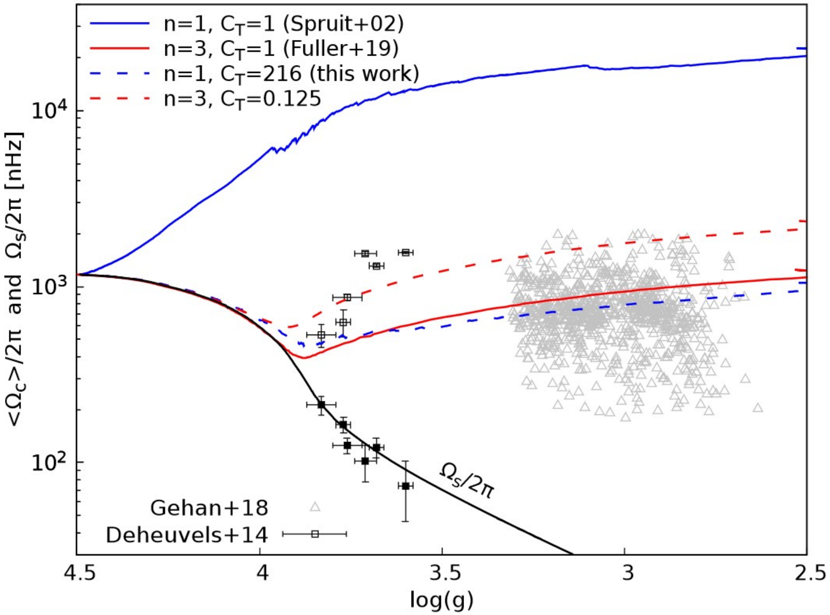
- Transport by the magnetic Tayler instability (Spruit 2002)
  - New general theoretical prescription (Eggenberger et al. 2022b)

$$\tau_{\rm damp} = C_{\rm T} \frac{1}{\omega_{\rm A}} \left(\frac{\Omega}{\omega_{\rm A}} \right)^n$$

$n=1$: Spruit (2002)
$n=3$: Fuller et al. (2019)

$$q_{\rm min,T} = C_{\rm T}^{-1} \left(\frac{N_{\rm eff}}{\Omega} \right)^{(n+2)/2} \left(\frac{\eta}{r^2 \Omega} \right)^{n/4}$$

$$\nu_{\rm T} = \frac{\Omega r^2}{q} \left(C_{\rm T}\, q \frac{\Omega}{N_{\rm eff}} \right)^{3/n} \left(\frac{\Omega}{N_{\rm eff}} \right)$$



Additional efficient magnetic AM transport

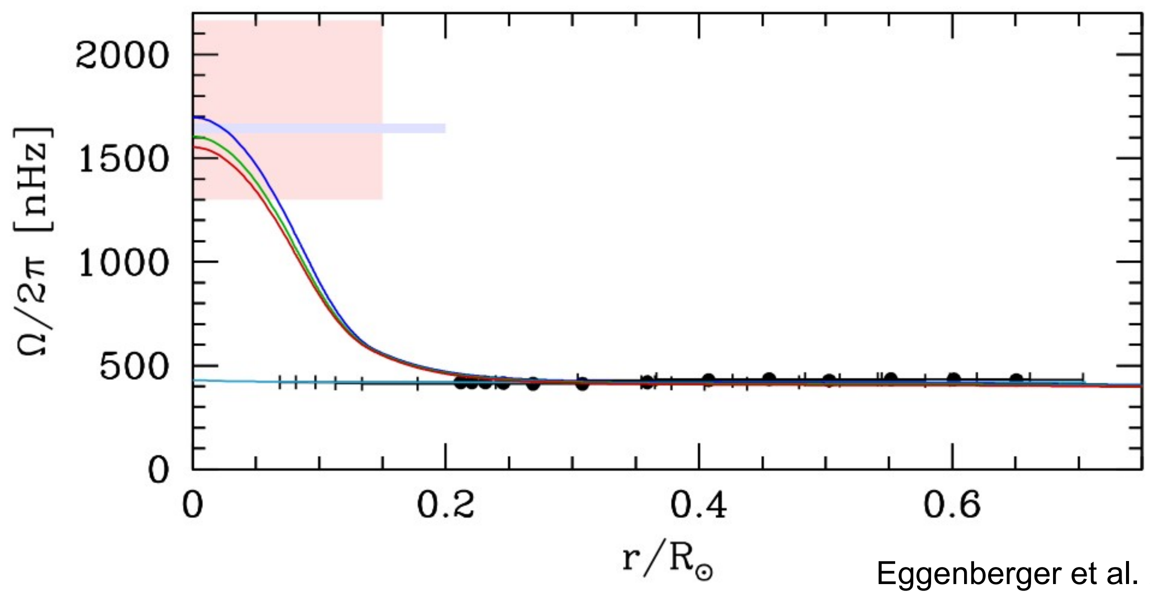
when $q \equiv |d\ln(\Omega)/d\ln(r)| \geq q_{\rm min}$

$$\rho \frac{\rm d}{{\rm d}t} \left(r^2 \Omega \right)_{M_r} = \frac{1}{5r^2} \frac{\partial}{\partial r} \left(\rho r^4 \Omega U(r) \right) + \frac{1}{r^2} \frac{\partial}{\partial r} \left(\rho (D_{\rm shear} + \nu_{\rm T}) r^4 \frac{\partial \Omega}{\partial r} \right)$$

Transport of angular momentum

- The solar rotation profile: magnetic fields ?
  - Magnetic instabilities in radiative zones ?
    - Tayler instability and the Tayler-Spruit dynamo (Spruit 2002)
      - Analytical approach : ✓and ✗ Zahn et al. (2007) ; ✓ Fuller et al. (2019)
      - Numerical simulations : ✓ Braithwaite (2006) ; ✗ Zahn et al. (2007)

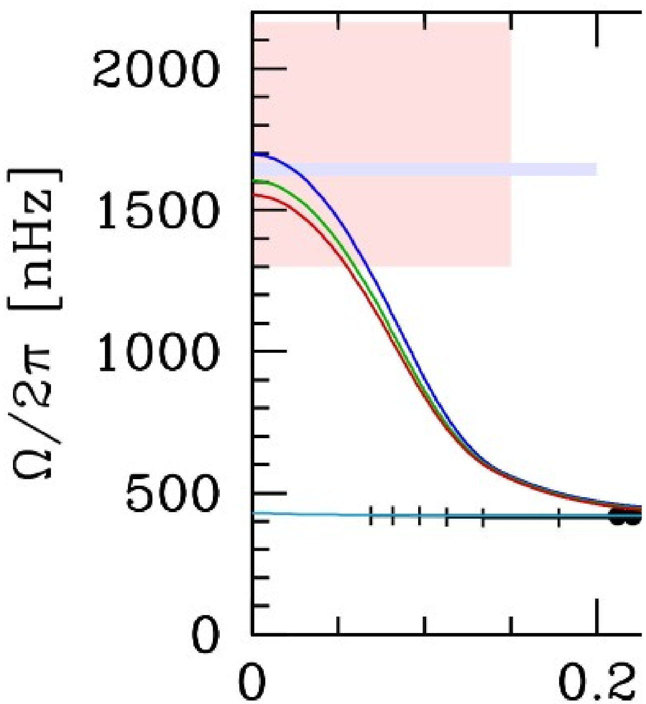
      recent results : ✓Petitdemange et al. (2023ab) ; Barrère et al. (2023) ; Ji et al. (2023)
    - MRI (strong shears) : Arlt et al. (2003) ; Rüdiger et al. (2014, 2015) ; Jouve et al. (2015)

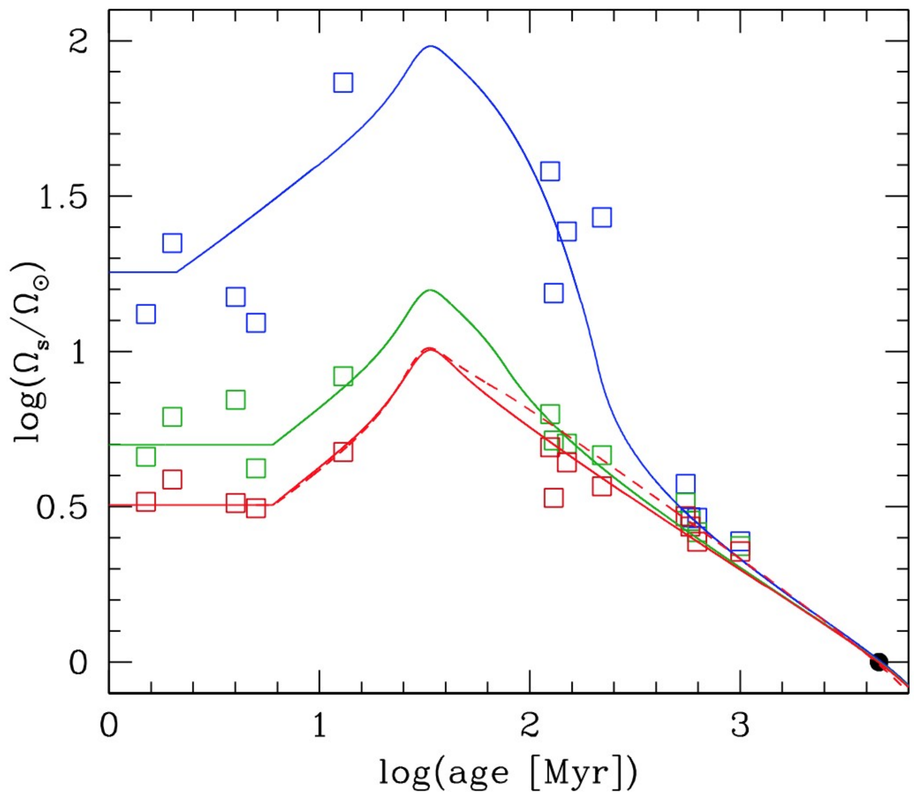


Eggenberger et al. 2019

Transport of angular momentum

- The solar rotation profile: magnetic fields ?
  - Rotation rate in the solar core : key constraint to the modelling of AM transport in layers with strong chemical gradients.





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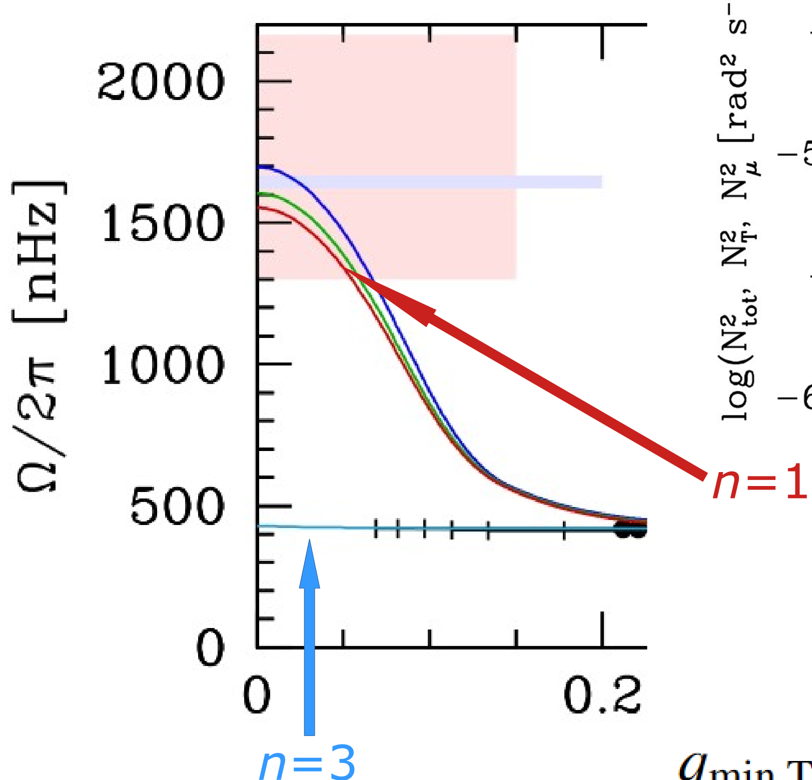
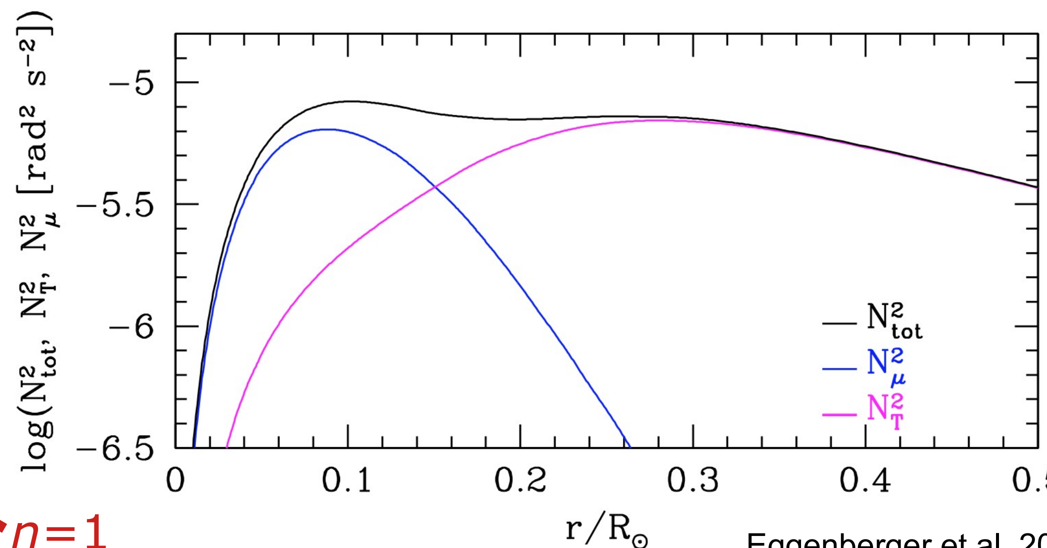




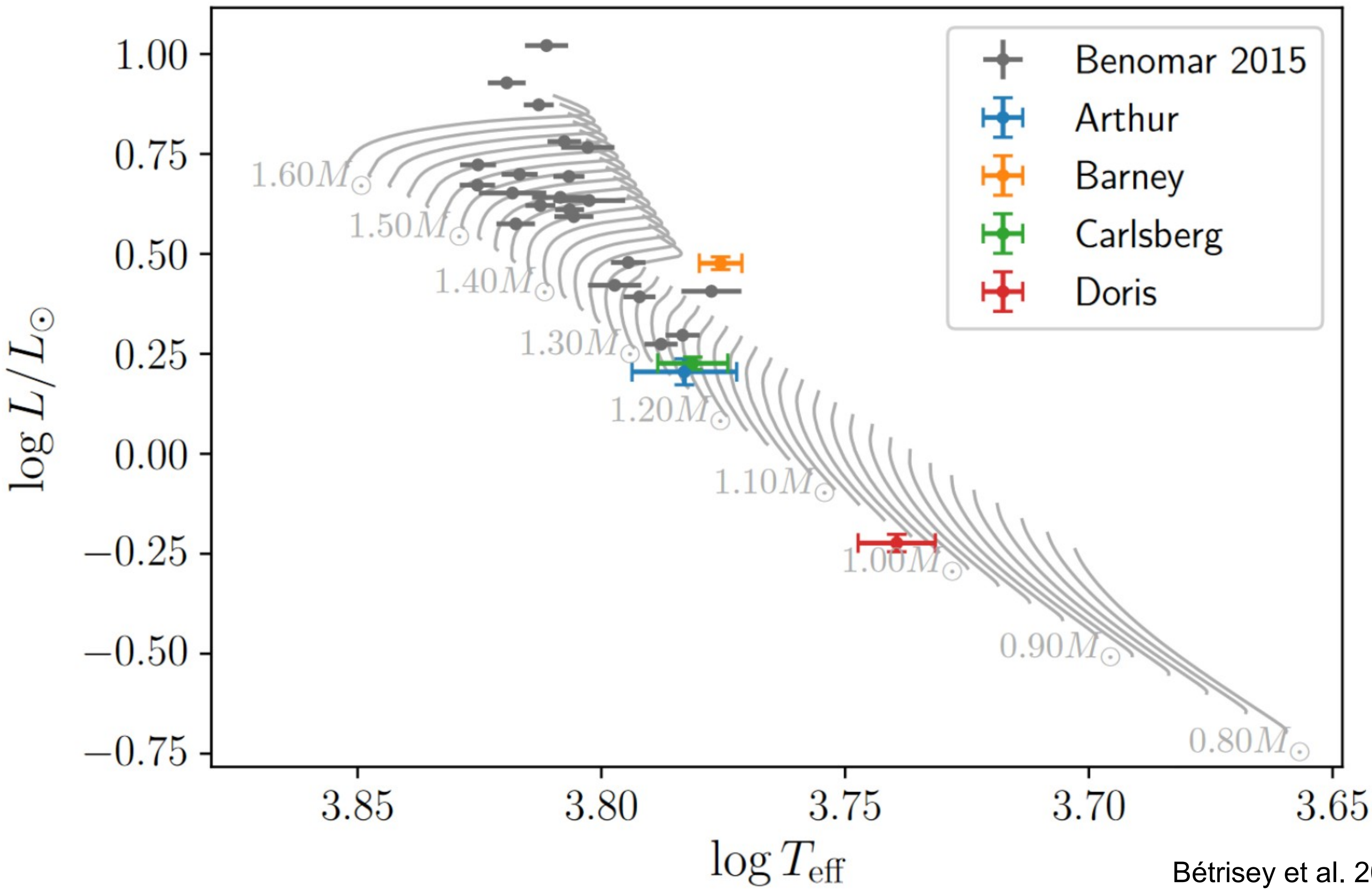
Eggenberger et al. 2019

$$N^2_{\mathrm{eff}} = \frac{\eta}{K} N^2_T + N^2_\mu$$

$$q_{\mathrm{min,T}} = C_{\mathrm{T}}^{-1} \left(\frac{N_{\mathrm{eff}}}{\Omega}\right)^{(n+2)/2} \left(\frac{\eta}{r^2\Omega}\right)^{n/4}$$

Transport of angular momentum

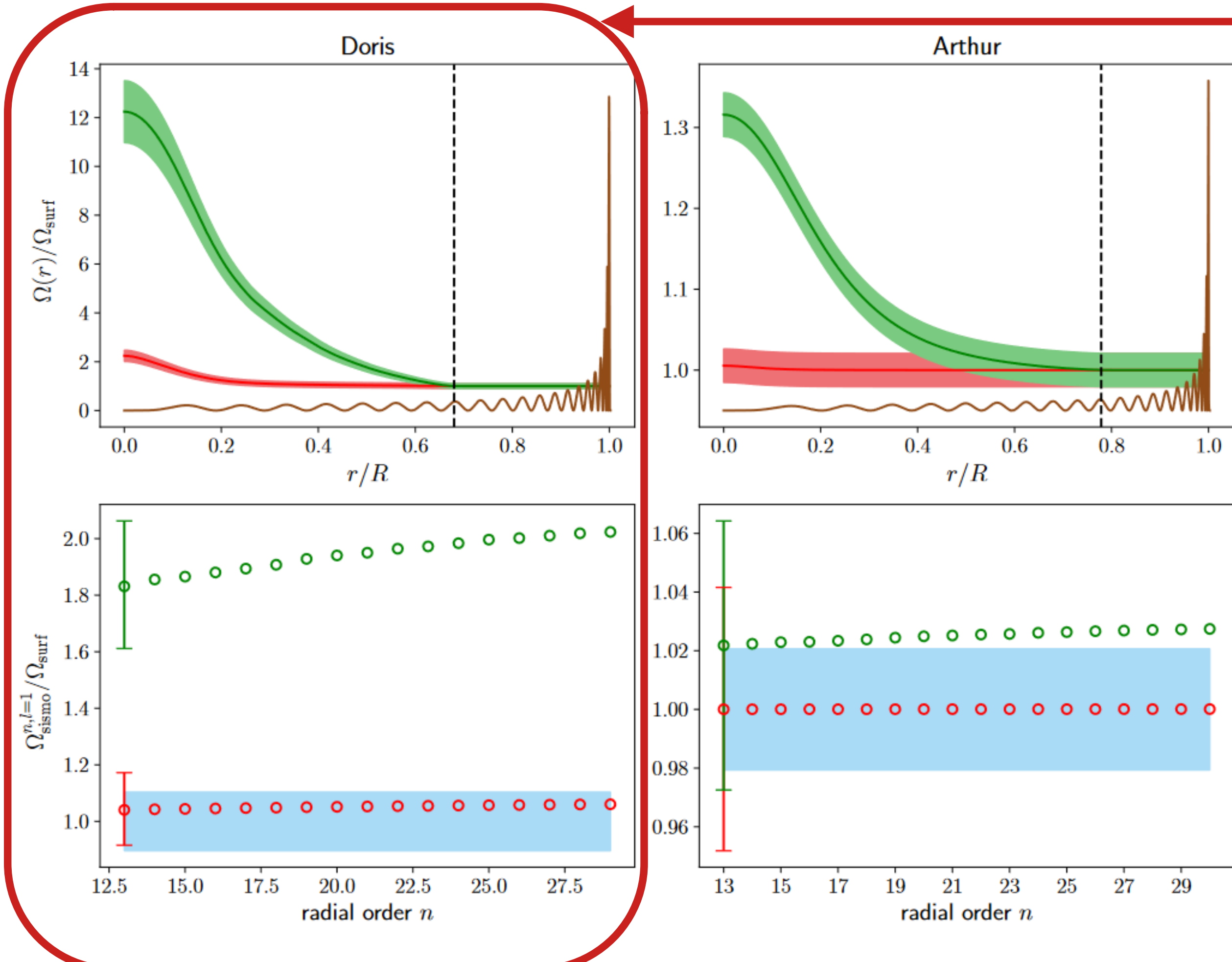
- Tayler instability in main-sequence stars

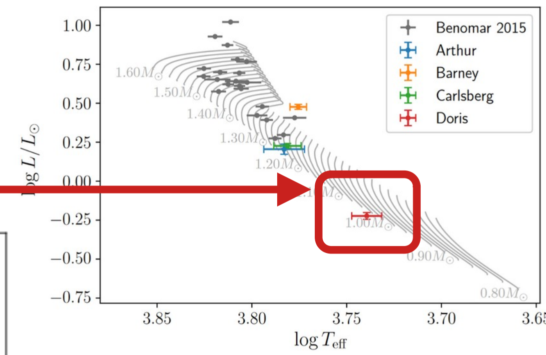


Bétrisey et al. 2023

Transport of angular momentum

- Tayler instability in main-sequence stars
  - Solar-type stars





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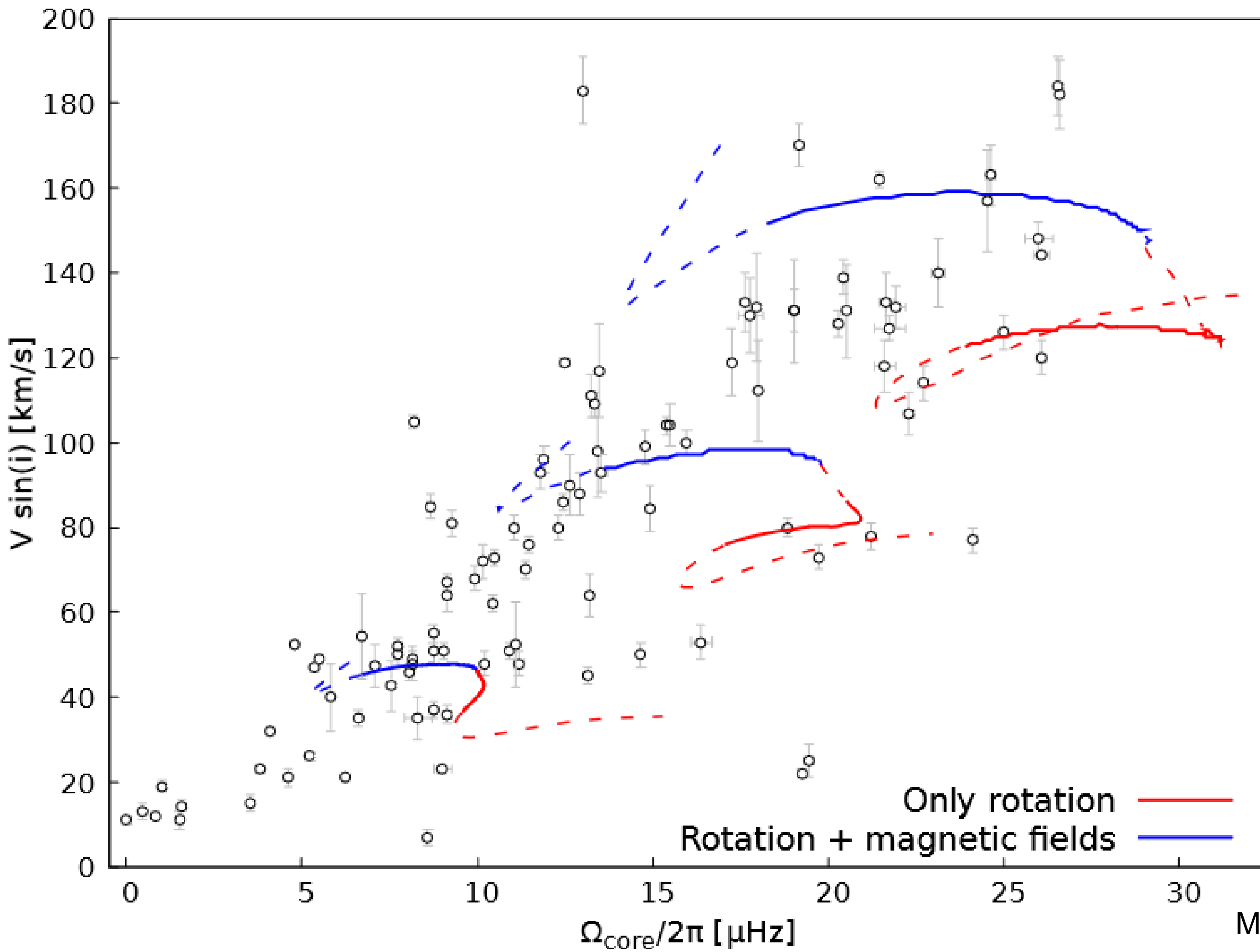
Transport of angular momentum

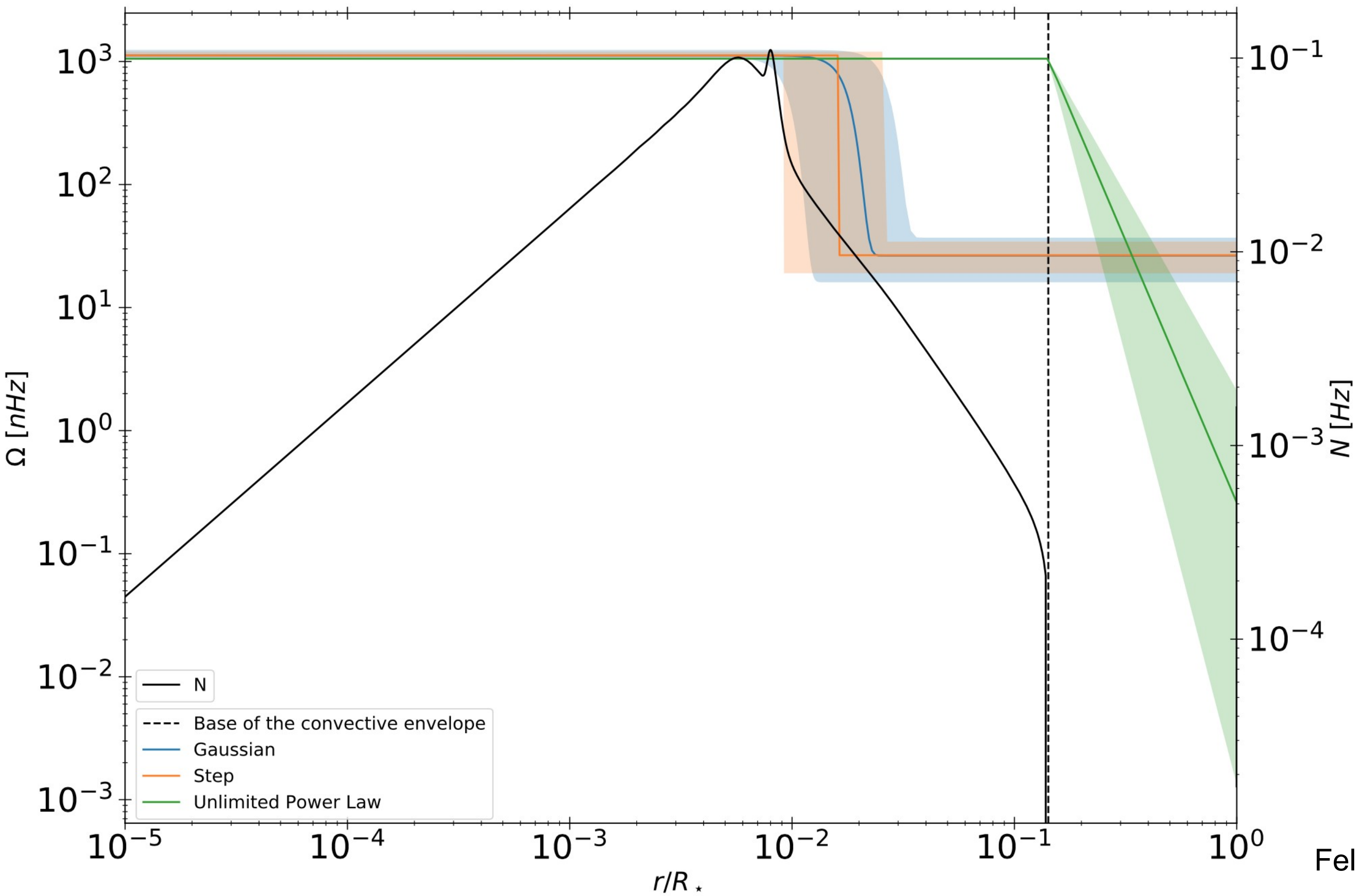
- Tayler instability in main-sequence stars
  - Gamma Doradus pulsators



Moyano et al. 2023

Transport of angular momentum

- Sharp discontinuity in the rotation profiles of evolved stars



Fellay et al. 2021

transport process with reduced efficiency when $\nabla_\mu \nearrow$?

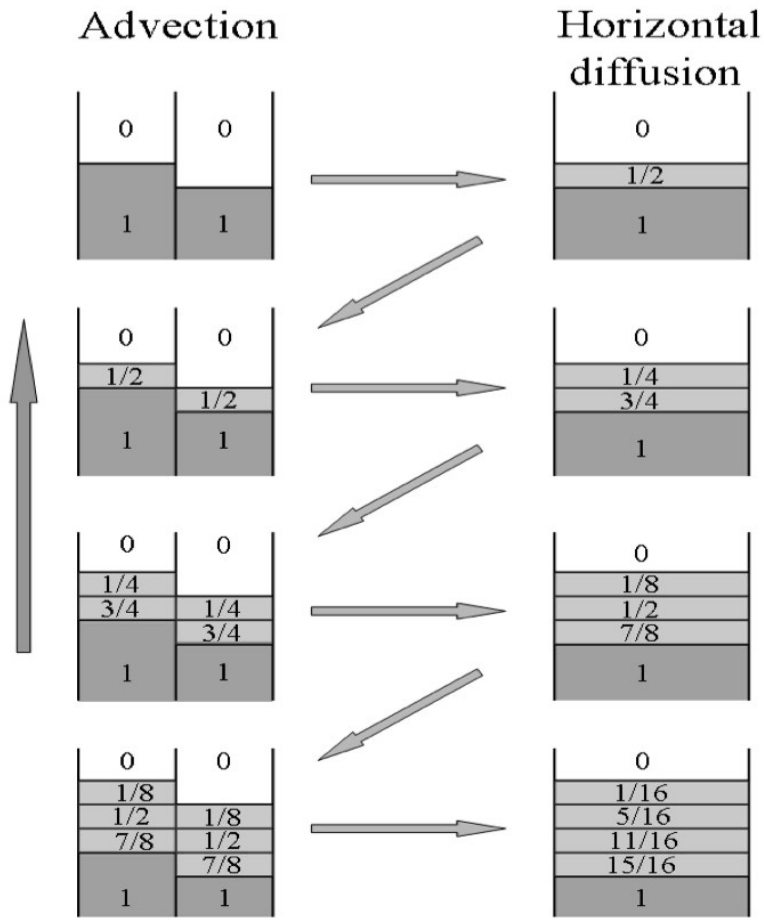
Transport of AM + chemicals

- Coherent transport of angular momentum and chemicals
  - Advective transport of chemical elements by meridional currents + impact of horizontal turbulence (Chaboyer & Zahn 1992)

$$D_{\rm eff} = \frac{|rU(r)|^2}{30 D_{\rm h}}$$

No free parameter f_c to arbitrary differentiate the efficiency of the transport of AM and chemicals

$$D_{\rm shear}(\text{chemicals}) = D_{\rm shear}(\text{AM})$$



Zahn 1992 ; Maeder 2009

Ok with simulations : $D_t \cong (0.8 - 1)\, \nu_t$ (e.g. Prat et al. 2016)

≠ *diffusive scheme (e.g.* MESA*) :* $D_t = f_c\, \nu_t$ with $f_c \cong 0.02 - 0.04$

Transport of AM + chemicals

- Coherent transport of angular momentum and chemicals
  - Direct transport of chemicals by the Tayler instability :
    Equation to determine the magnetic transport of chemicals

$$\frac{r^2\Omega}{q^2K}\left(N_{\rm T}^2 + N_\mu^2\right)x^4 - \frac{r^2\Omega^3}{K}x^3 + 2N_\mu^2\,x - 2\Omega^2q^2 = 0$$

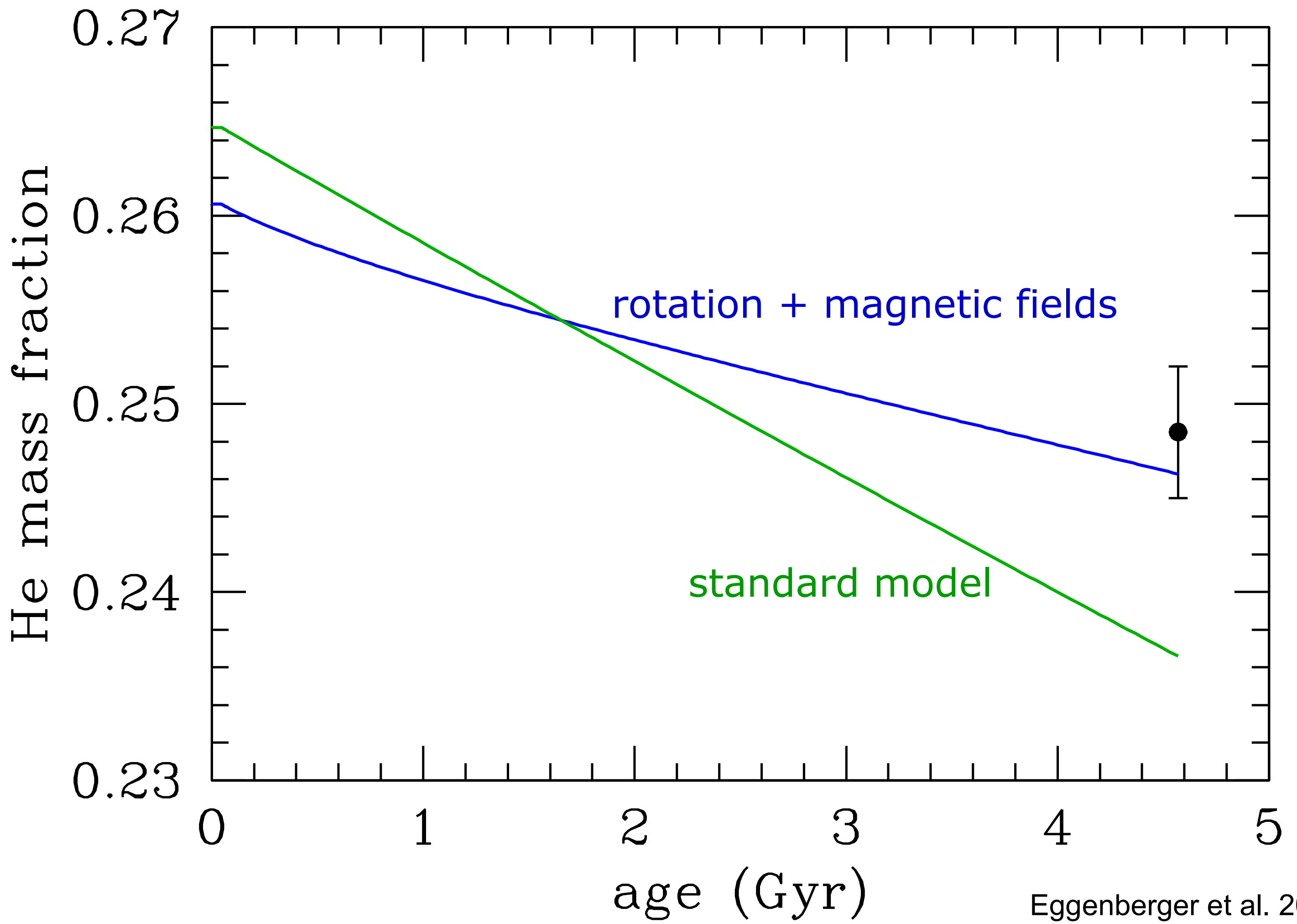
$$x = \left(\frac{\omega_{\rm A}}{\Omega}\right)^2 \rightarrow D_{\rm Tayler} = \frac{r^2\,\Omega}{q^2}\left(\frac{\omega_{\rm A}}{\Omega}\right)^6$$

  - Equation for the evolution of chemicals :

$$\rho\frac{\partial X_i}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho(D_{\rm eff} + D_{\rm shear} + D_{\rm Tayler})\frac{\partial X_i}{\partial r}\right] - \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\rho v_i\right] + \rho\dot{X}_i^{\rm nucl}$$

Transport of AM + chemicals

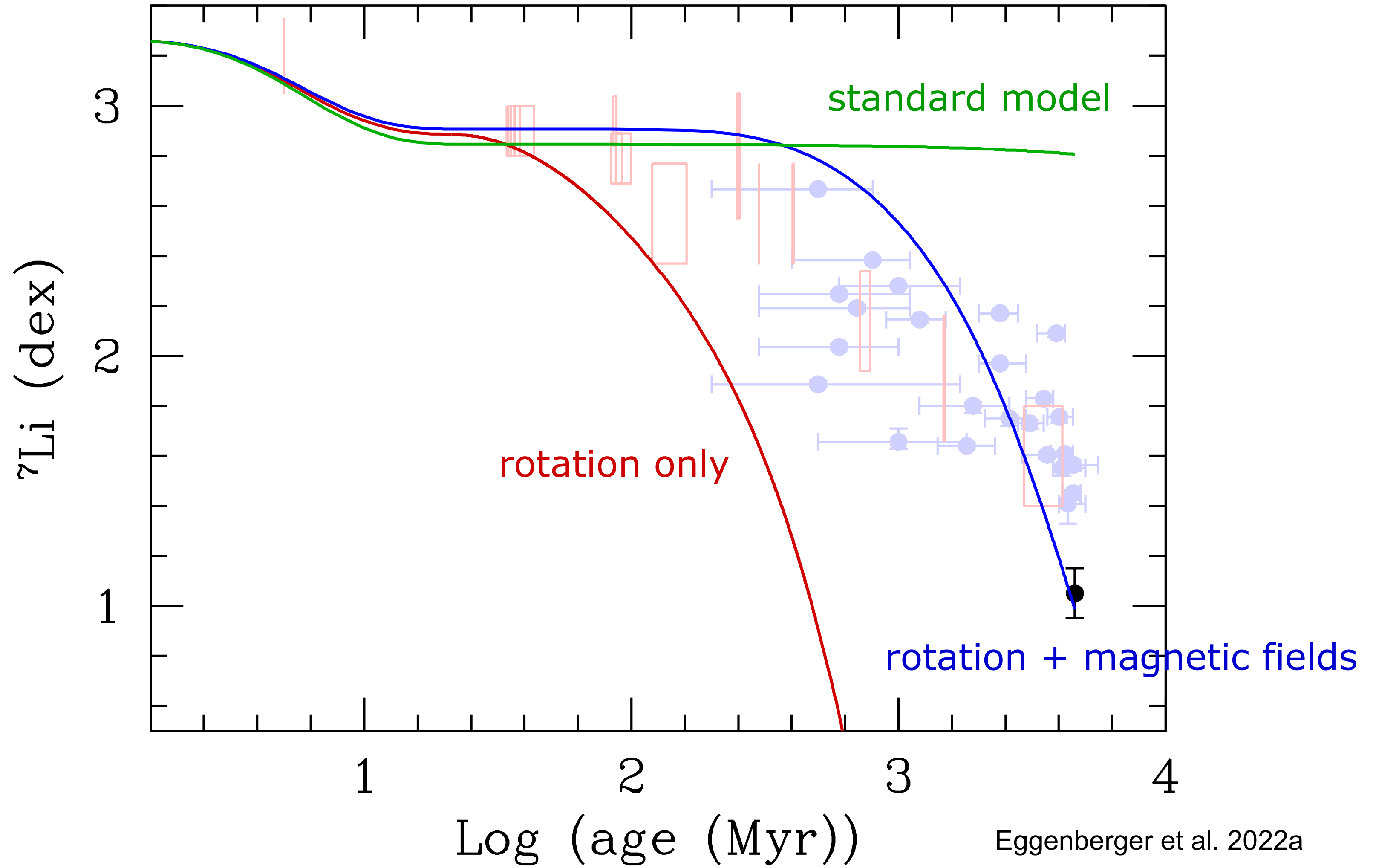
- The solar He abundance



Eggenberger et al. 2022a

Transport of AM + chemicals

- The solar Li abundance



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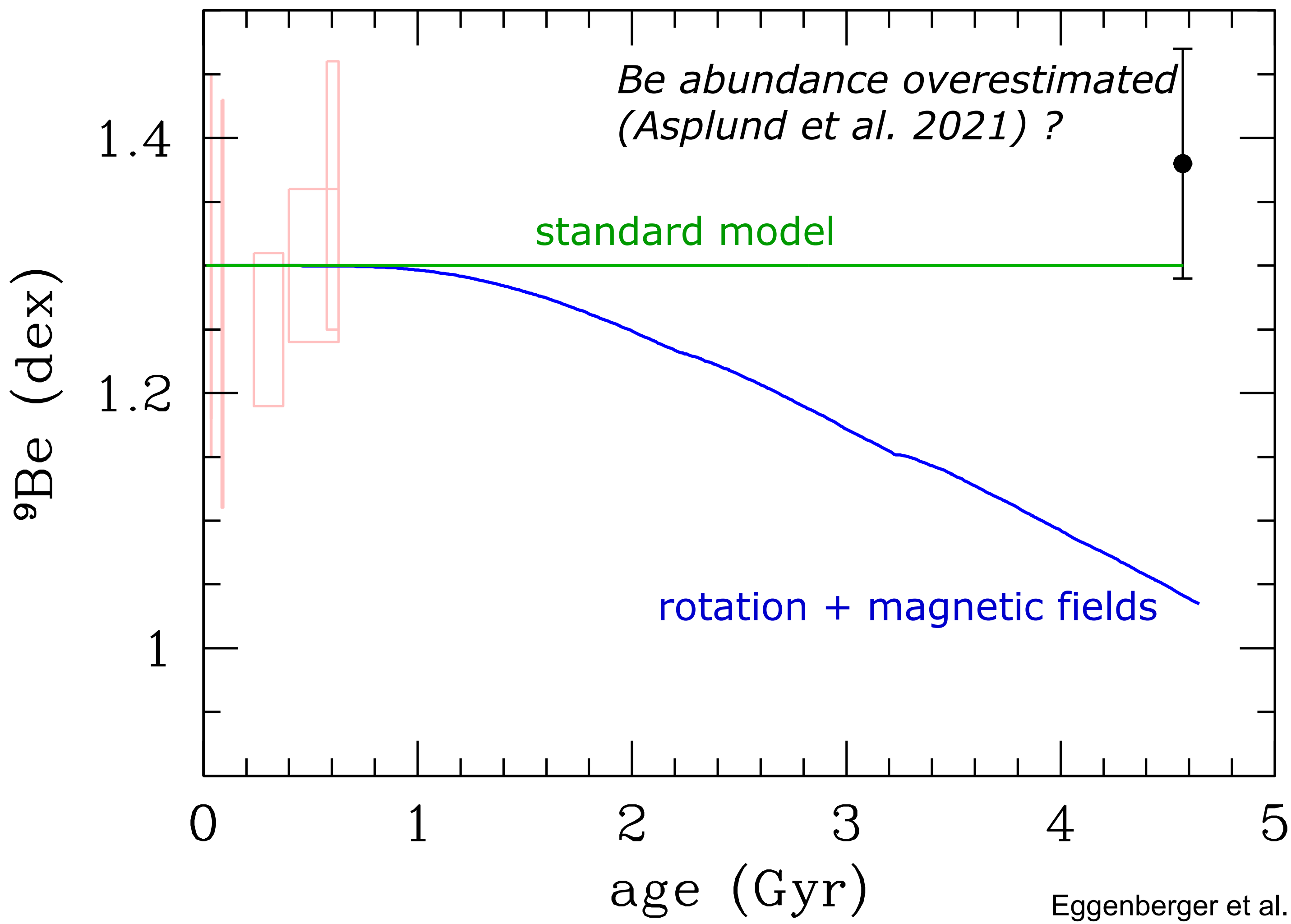
Transport of AM + chemicals

- The solar Be abundance



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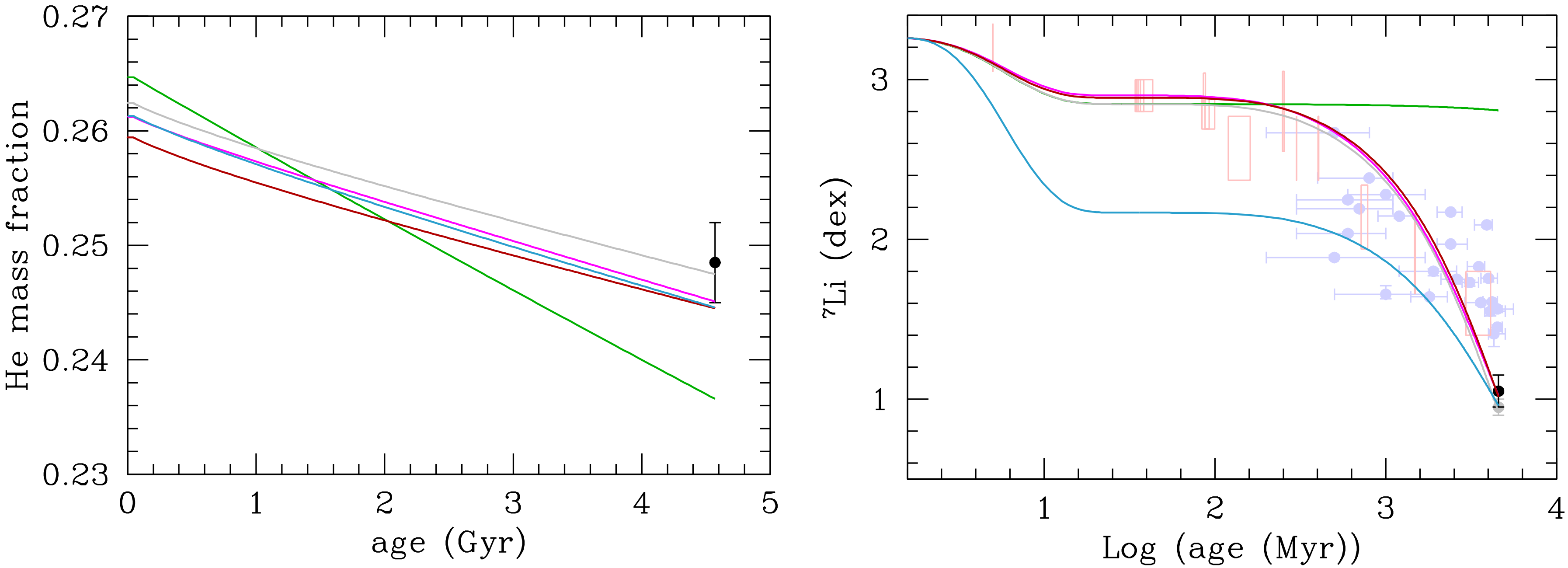
Transport of AM + chemicals

- The general link between solar He and Li abundances

  - Non-rotating models with an arbitrary parametric diffusion coefficient for the transport of chemicals (Proffitt & Michaud 1991) :

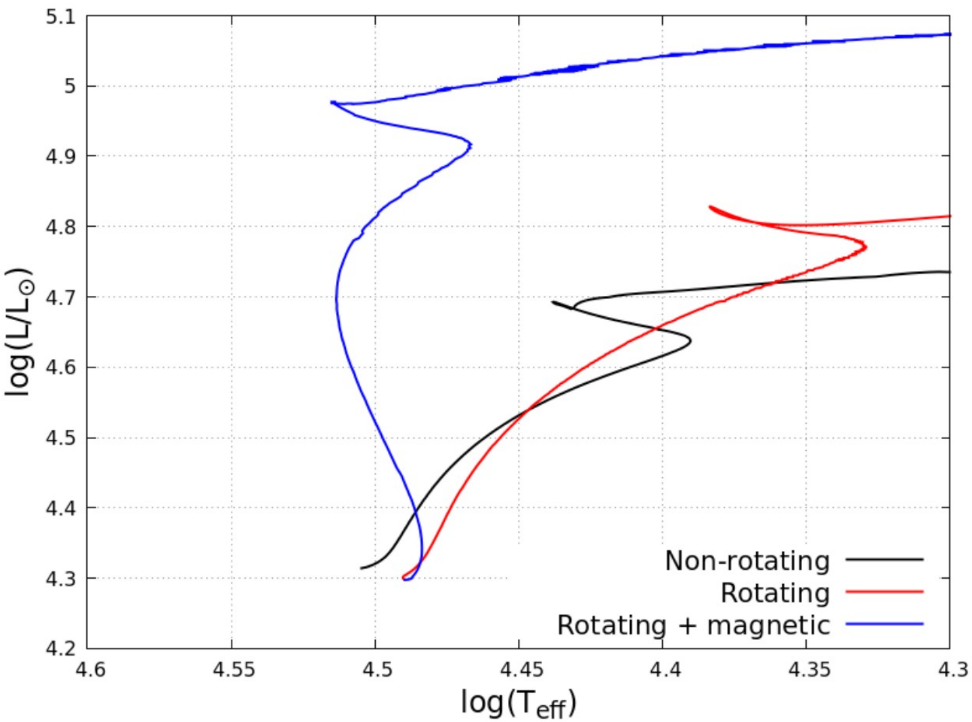
$$D_{\text{tot}} = C \left(\frac{\rho_{\text{BCZ}}}{\rho(r)} \right)^n$$

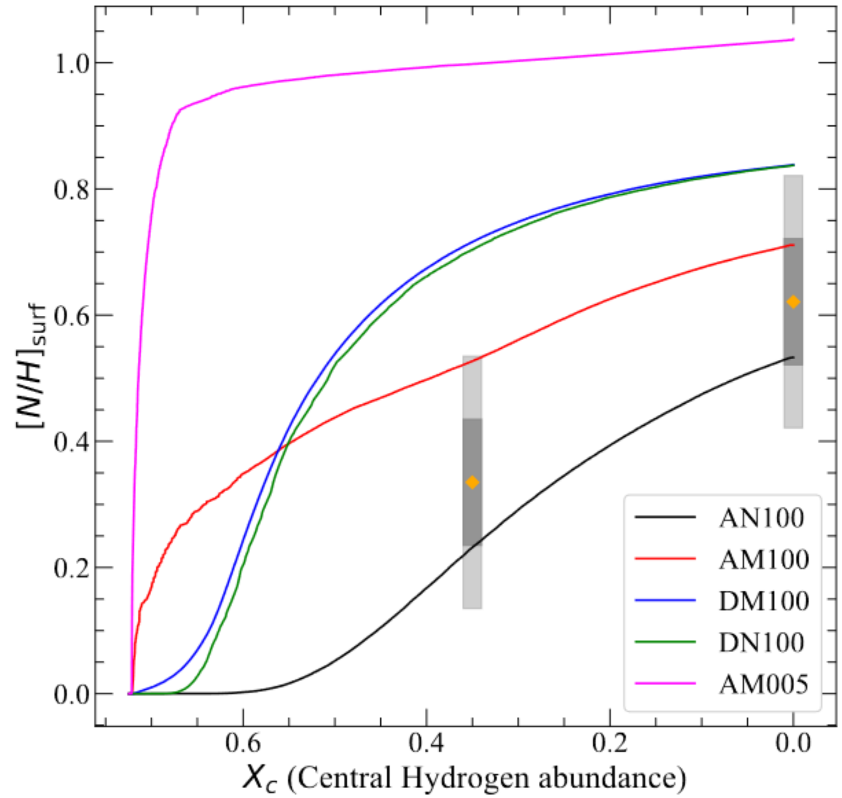


Eggenberger et al. 2022a

Transport of AM + chemicals

- Impact for massive stars
  - Transport by magnetic instabilities also important during the main-sequence evolution of massive stars





Griffiths et al. 2022

Summary : the Sun

- Solar models with rotation + magnetic instabilities :

  ✓ Evolution of surface rotation rates observed in open clusters
  ✓ Solar rotation profile
  ✓ Photospheric solar Li abundance
  ✓ Helioseismic He abundance

  Physical explanation to the solar internal rotation and He-Li abundances

  - **Be abundance ? core rotation of the Sun ?**

  ✗ Location of the base of the convective zone
  ✗ Sound speed, density profiles

- Solar modelling problem :

  - AGSS09 abundances compatible with helioseismic He
  - He-Li link independent of a specific transport process

Summary : Stars

- Stellar models with rotation + magnetic fields
  - Simultaneous and coherent theoretical description of both angular momentum and chemical elements transport
  - Key impact of the advective scheme for angular momentum and chemicals transport
  - Central role of horizontal turbulence as the only free parameter
  - Predictive power of these models: no free parameters to arbitrary differentiate angular momentum/chemicals transport efficiency or to arbitrary reduce the inhibiting effects of chemical gradients (f_c and f_μ in the diffusive scheme)