ABUNDANCES MEASUREMENTS FROM HELIOSISMOLOGY

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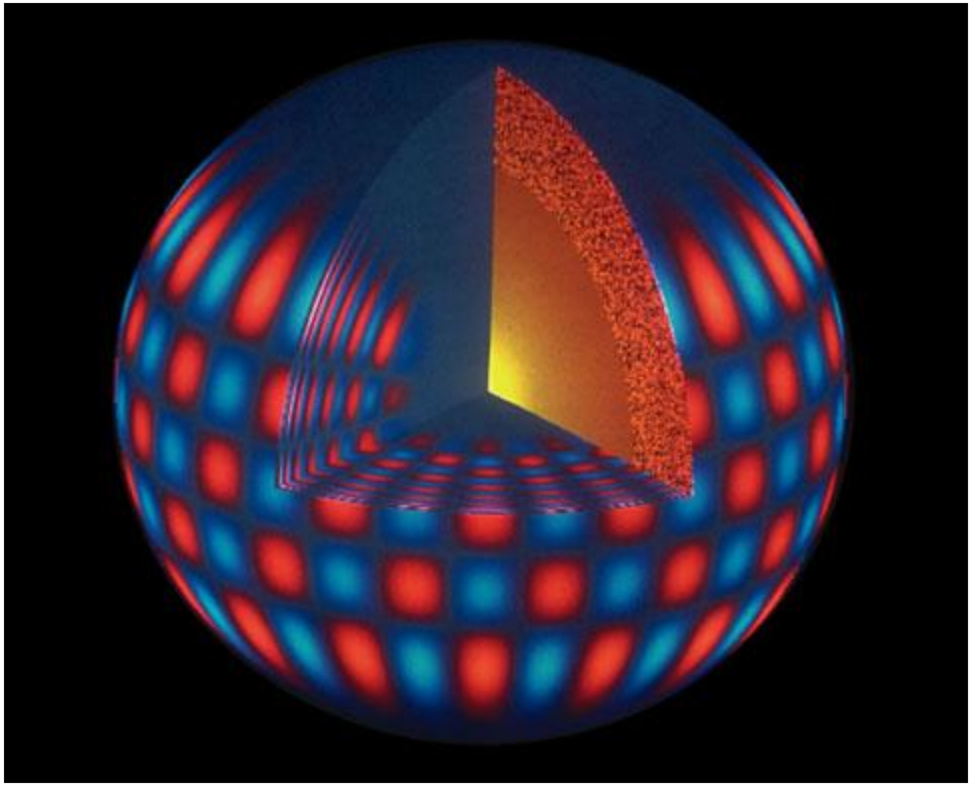
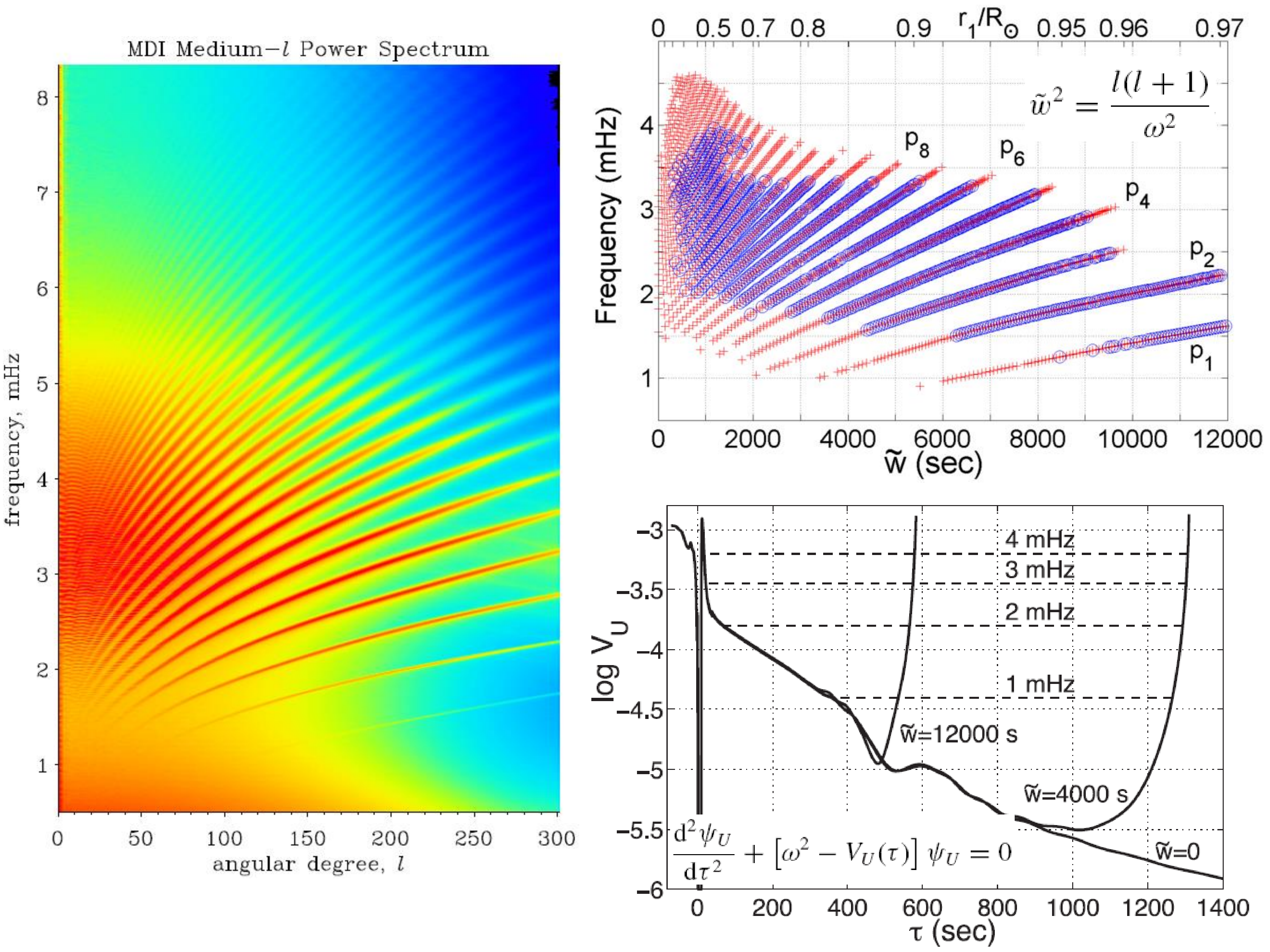
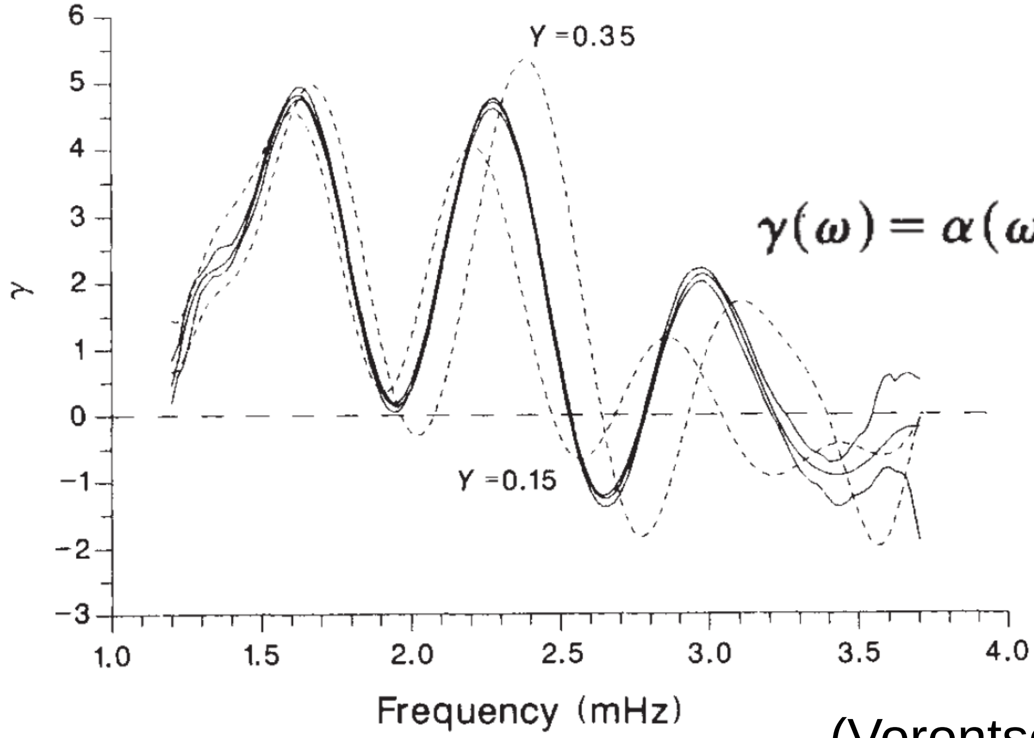



MDI Medium−l Power Spectrum
frequency, mHz
angular degree, l
$r_1/R_\odot$
0 0.5 0.7 0.8 0.9 0.95 0.96 0.97
Frequency (mHz)
$\tilde{w}^2 = \frac{l(l+1)}{\omega^2}$
p_8
p_6
p_4
p_2
p_1
$\tilde{w}$ (sec)
log V_U
4 mHz
3 mHz
2 mHz
1 mHz
$\tilde{w}$=12000 s
$\tilde{w}$=4000 s
$\tilde{w}$=0
$\frac{d^2\psi_U}{d\tau^2} + \left[\omega^2 - V_U(\tau)\right]\psi_U = 0$
τ (sec)

High-frequency asymptotic description of solar p modes

$$F(\tilde{w}) + \frac{1}{\omega^2}\Phi(\tilde{w}) \simeq \pi \frac{n + \alpha_0(\omega) + \tilde{w}^2 \alpha_2(\omega)}{\omega}$$

$$F(\tilde{w}) = \int_{r_1}^{R} \tilde{s}\, \mathrm{d}r, \qquad \tilde{s} = \left(\frac{1}{c^2} - \frac{\tilde{w}^2}{r^2}\right)^{1/2}, \qquad \tilde{w} = \frac{l + 1/2}{\omega}$$



$$\gamma(\omega) = \alpha(\omega) - \omega \frac{\mathrm{d}\alpha}{\mathrm{d}\omega} - \omega^2 \frac{\mathrm{d}^2\alpha}{\mathrm{d}\omega^2}$$

Result: Y=0.25±0.01
(Vorontsov, Baturin and Pamyatnykh 1991)

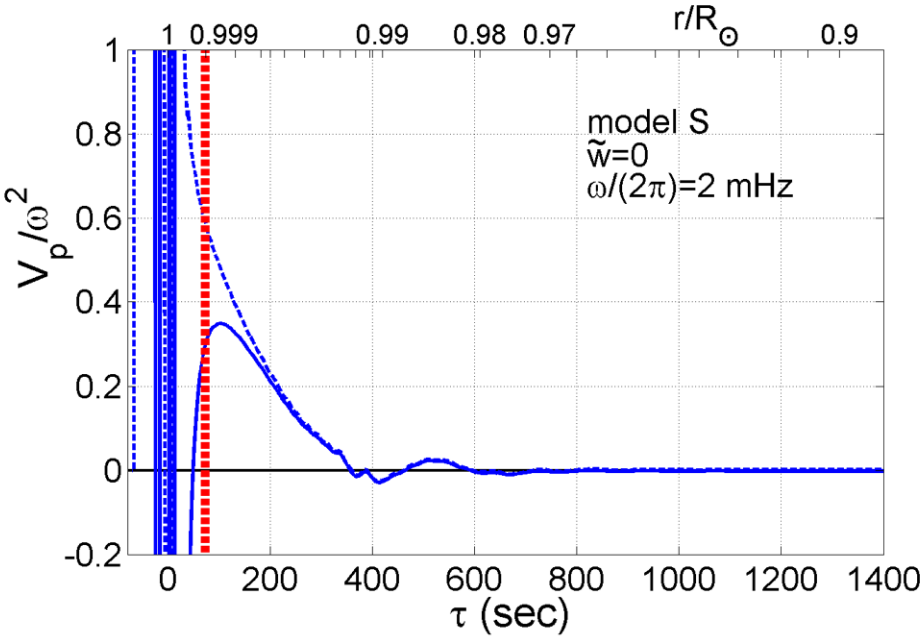
Non-asymptotic analysis

Inversion of frequency differences (Sun – model)

$$\frac{\delta\nu}{\nu} = \int_0^R K_{\Gamma_1,\rho} \frac{\delta\Gamma_1}{\Gamma_1}\, dr + \int_0^R K_{\rho,\Gamma_1} \frac{\delta\rho}{\rho}\, dr + \frac{f(\nu)}{I}$$

Localized Averaging
Tikhonov Regularization (RLS)

Differential-response technique



Direct model calibration
Constrained structural inversion

Helium abundance measurements

Vorontsov, Baturin and Pamyatnykh (1991)
EOS: Saha (rejected), Saha+electrostatic corrections, MHD
Method: Asymptotic, calibration with phase shifts
Result: **$Y=0.25\pm0.01$**

Christensen-Dalsgaard and Perez Hernandez (1991)
EOS: EFF, MHD
Method: Differential asymptotic, calibration with phase shifts
Result: **$Y\approx0.25$**

Dappen, Gough, Kosovichev and Thompson (1991)
EOS: MHD with Z=O+Fe ions in ground states
Method: Localized Averaging
Result: **$Y=0.268\pm0.002$**

Dziembowsky, Pamyatnikh and Sienkiewisz (1991)
EOS: MHD with Z=O ions in all excited states
Method: Tikhonov regularization (RLS)
Result: **$Y=0.235\pm0.005$**

Perez Hernandez and Christensen-Dalsgaard (1994)
EOS: MHD
Method: Differential asymptotic, calibration with phase shifts
Result: **Y=0.242±0.003**

Antia and Basu (1994)
EOS: MHD
Method: Differential asymptotic, calibration with sound speed and phase shifts
Result: **Y=0.252±0.003**

Kosovichev (1996)
EOS: OPAL
Method: Localized Averaging
Result: **Y=0.248±0.006**

Basu (1998)
EOS: OPAL
Method: Localized Averaging
Result: **Y=0.248±0.001**

Richard, Dziembowski, Sienkiewicz and Goode (1998)
EOS: OPAL, MHD
Method: Tikhonov regularization (RLS)
Result**: Y=0.248±0.002 (OPAL)**
Y≈0.242 (MHD)

Di Mauro, Christensen-Dalsgaard, Rabello-Soares and Basu (2002)

EOS: MHD, OPAL

Method: Localized Averaging

Result**: Y=0.2457±0.0005 (MHD)**

Y=0.2539±0.0005 (OPAL)

Metallicity measurements

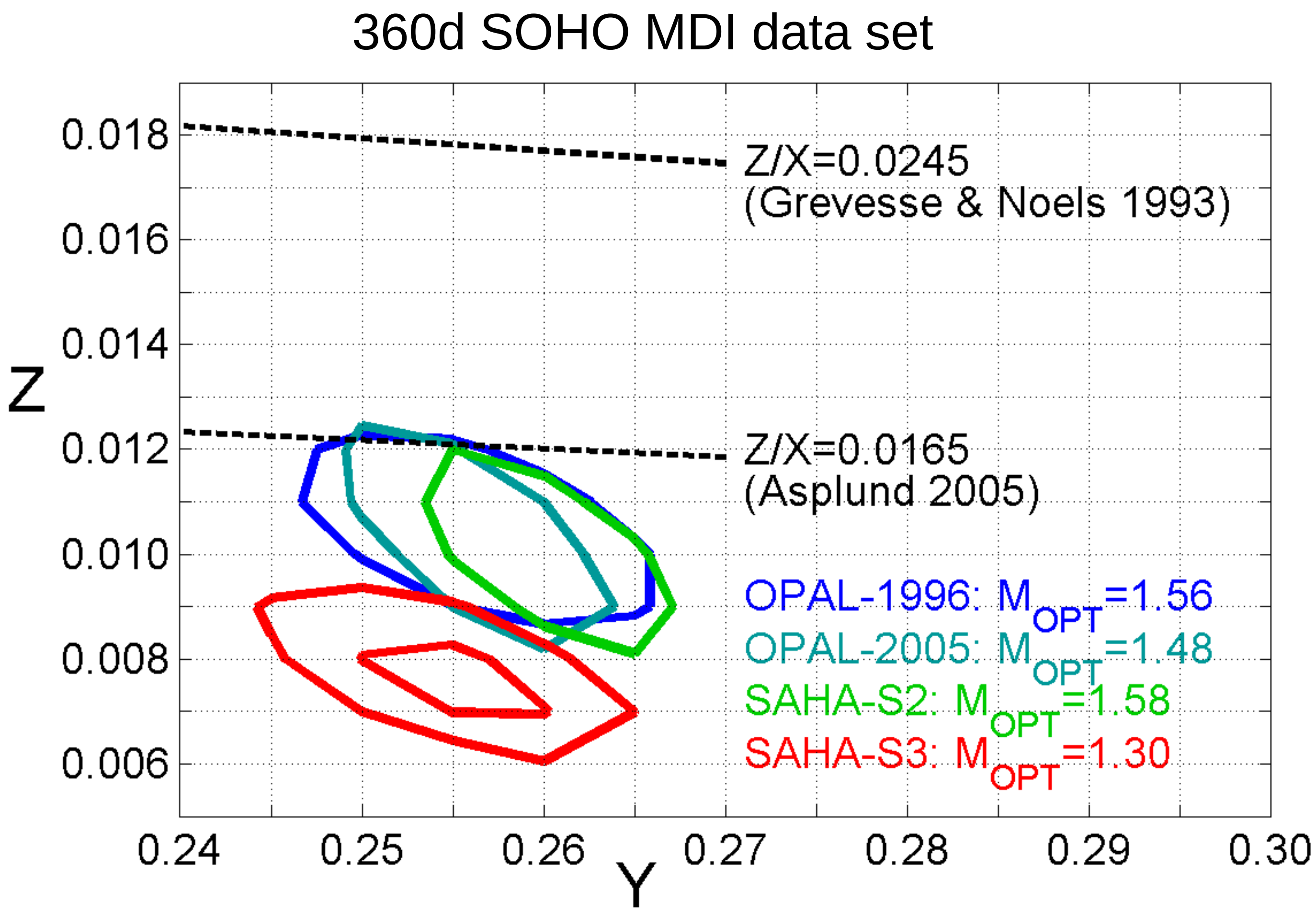
Antia and Basu (2006)
EOS: OPAL, CEFF
Method: Tikhonov regularization (RLS), sound-speed gradient
Result: **Z=0.0172±0.002**

Vorontsov, Baturin, Ayukov and Gryaznov (1913)
EOS: OPAL, SAHA-S
Method: Differential Responce, direct calibration
Result: Z=0.008-0.013, Y=0.240-0.255

Buldgen, Salmon, Noels, Scuflaire, Dupret and Rees (2017)
EOS: CEFF, FreeEOS
Method: Localized Averaging
Result**: Z=0.008-0.014, Y=0.242-0.255**

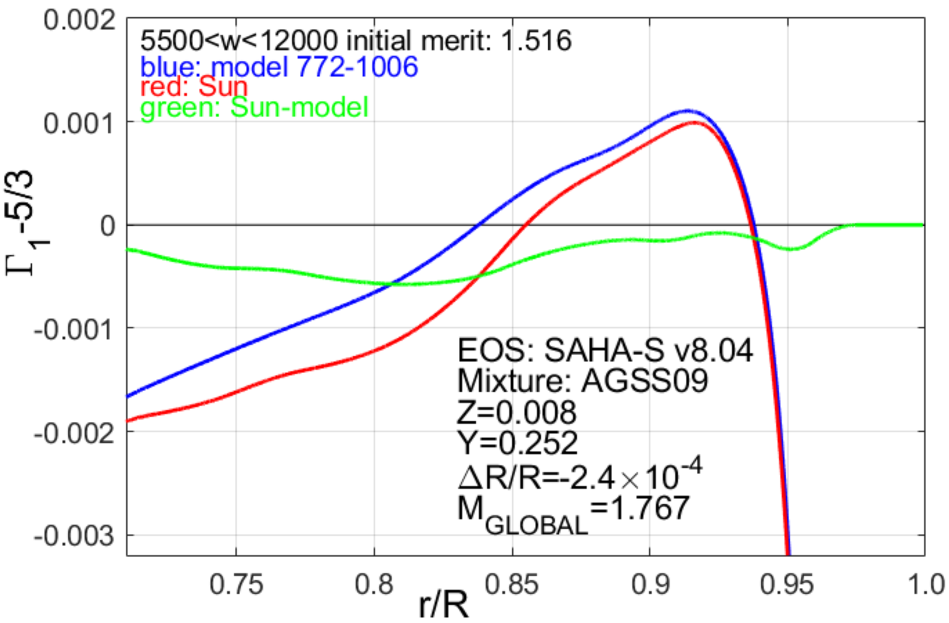
Buldgen, Noels, Baturin, Oreshina, Ayukov, Scuflaire, Amarsi and Grevesse (in press)
EOS: SAHA-S, FreeEOS
Method: Localized Averaging
Result: **Z=0.0120-0.0151, Y=0.255-0.273**

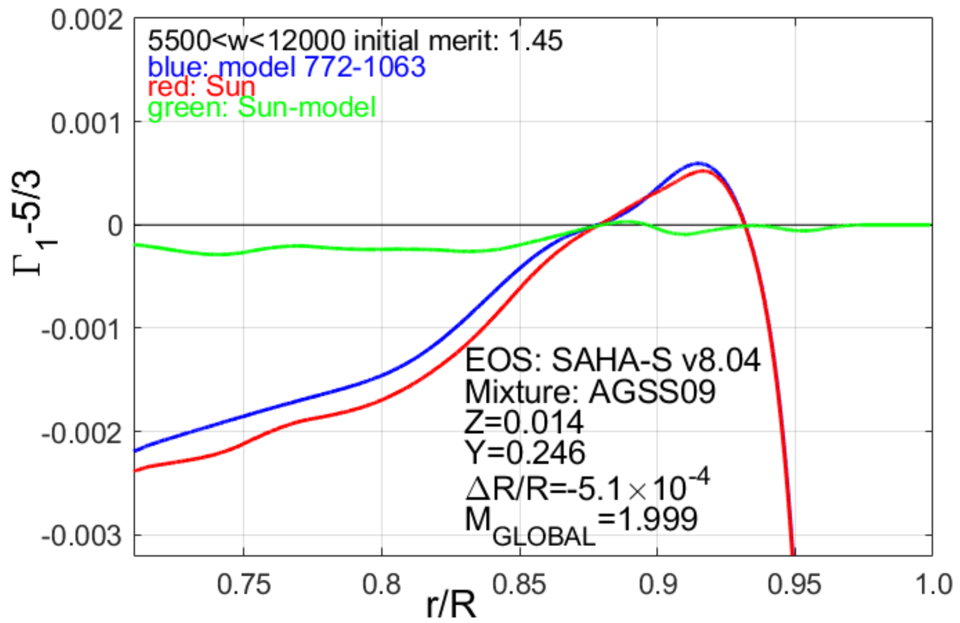
Calibration with differential response



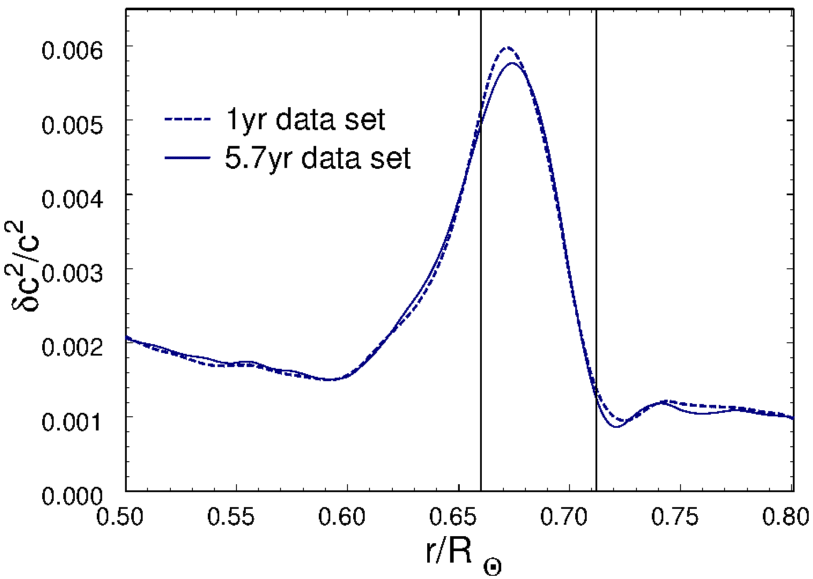
Some more recent results

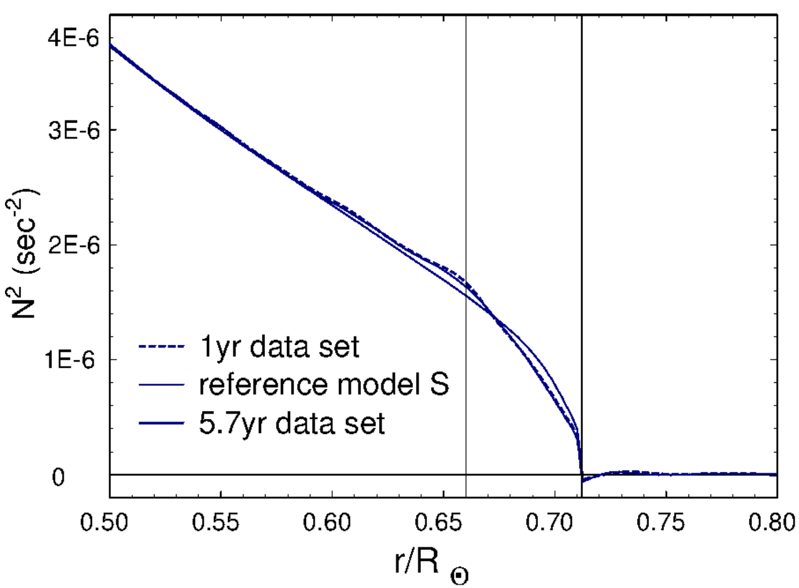
Updated versions of the SAHA-S EOS
Updated data set (2304d SDO HMI)



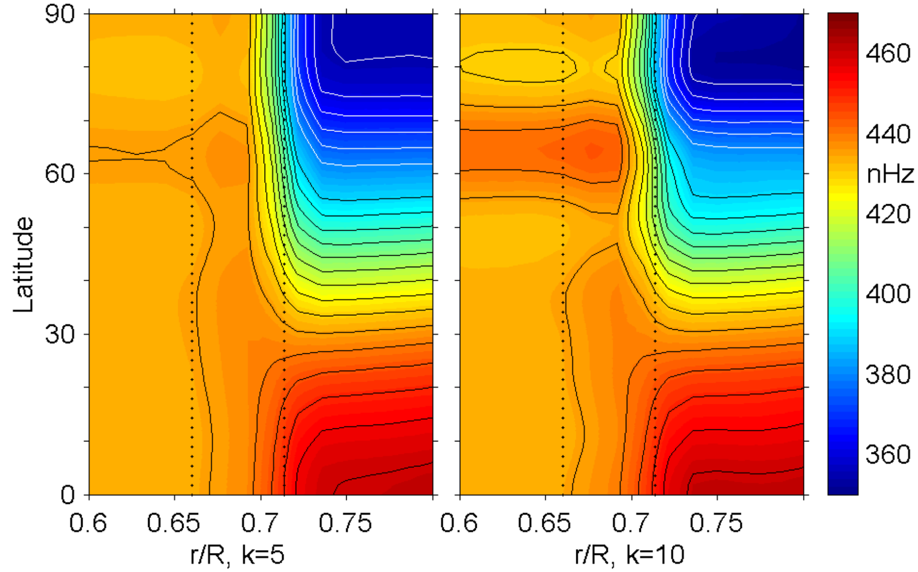


The solar tachocline

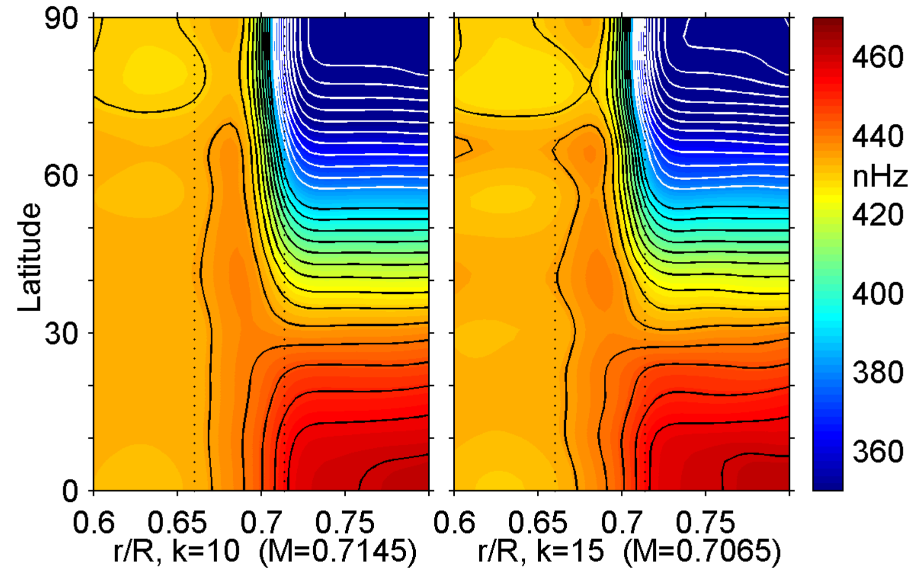




31×72d SOHO MDI

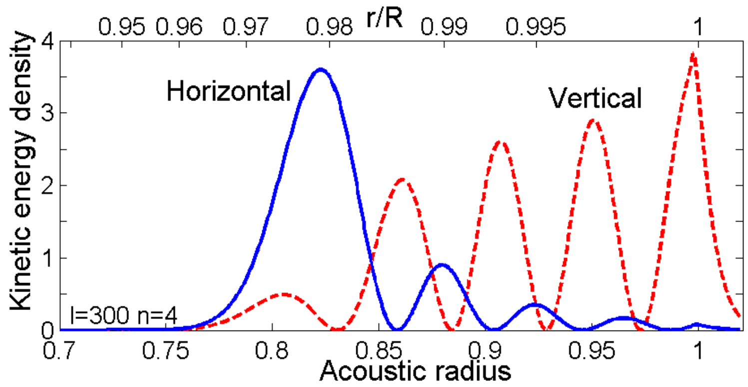


2304d SDO HMI



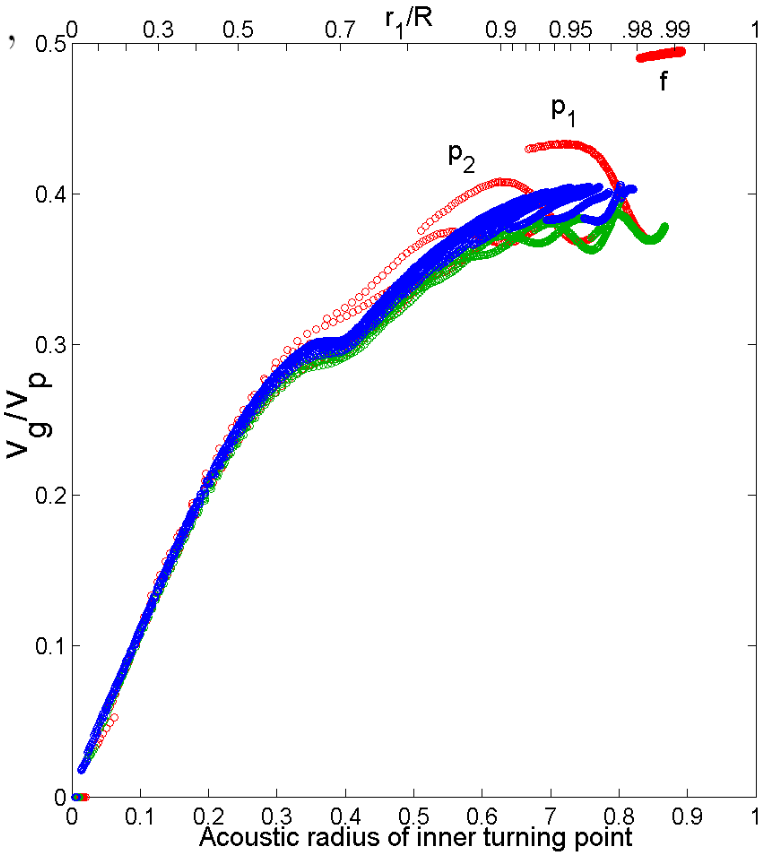
Group velocity of surface waves

$$v_p = \frac{\omega}{L} R_\odot, \qquad v_g = \left(\frac{\partial \omega}{\partial L}\right)_n R_\odot, \qquad L^2 = l(l+1).$$



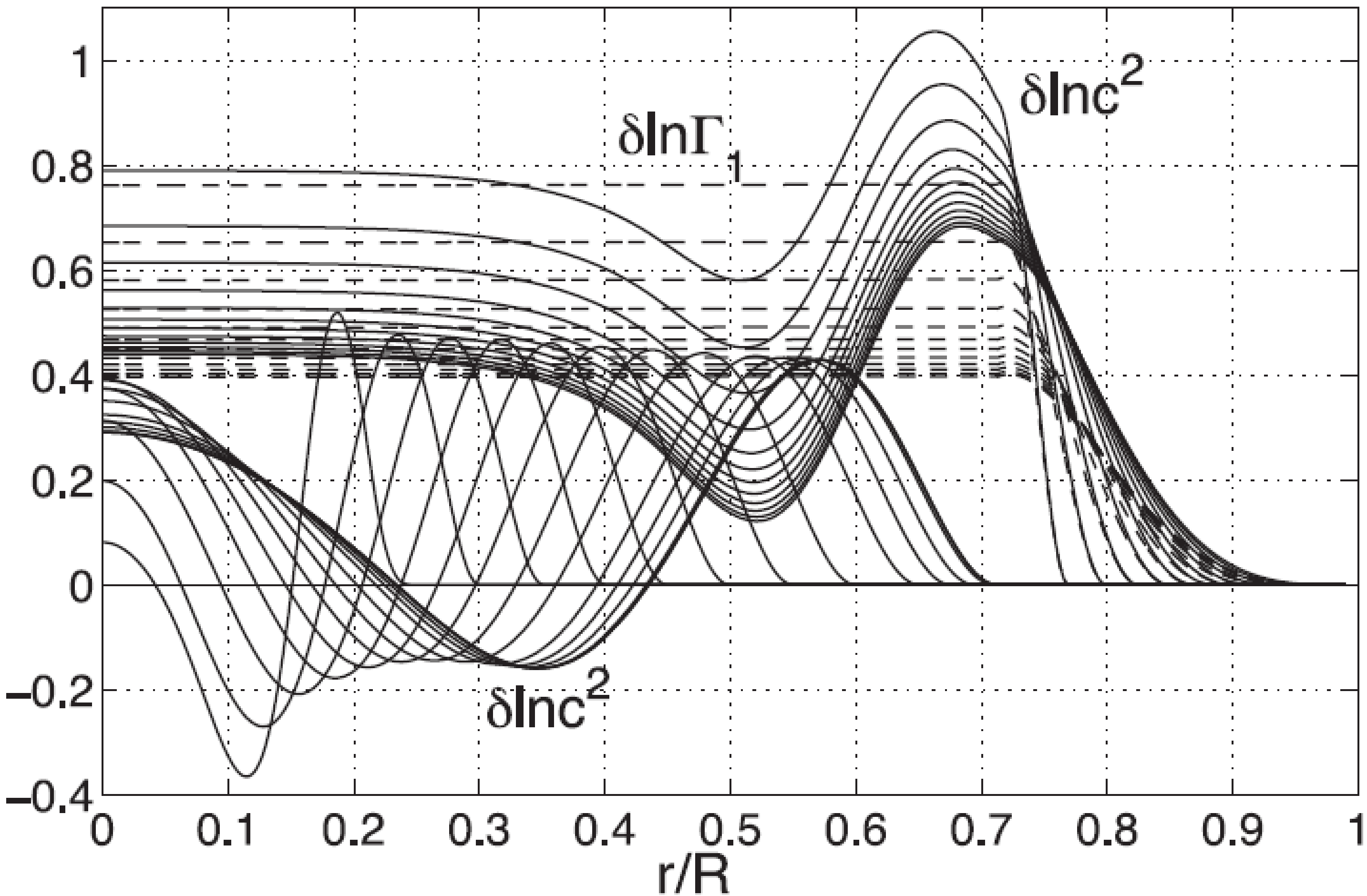
$$\frac{v_g}{v_p} \equiv \left(\frac{\partial \ln \omega}{\partial \ln L}\right)_n = \frac{\int_0^R \rho_0 r^2 L^2 V^2 dr + \frac{L^2}{4\pi G \omega^2} \int_0^\infty P^2 dr}{\int_0^R \rho_0 r^2 \left(U^2 + L^2 V^2\right) dr},$$

$$\mathbf{u} = \hat{\mathbf{r}} U(r) Y_{lm}(\theta, \phi) + V(r) \nabla_1 Y_{lm}(\theta, \phi),$$
$$\psi' = -P(r) Y_{lm}(\theta, \phi).$$



[Vorontsov et al. 2014]

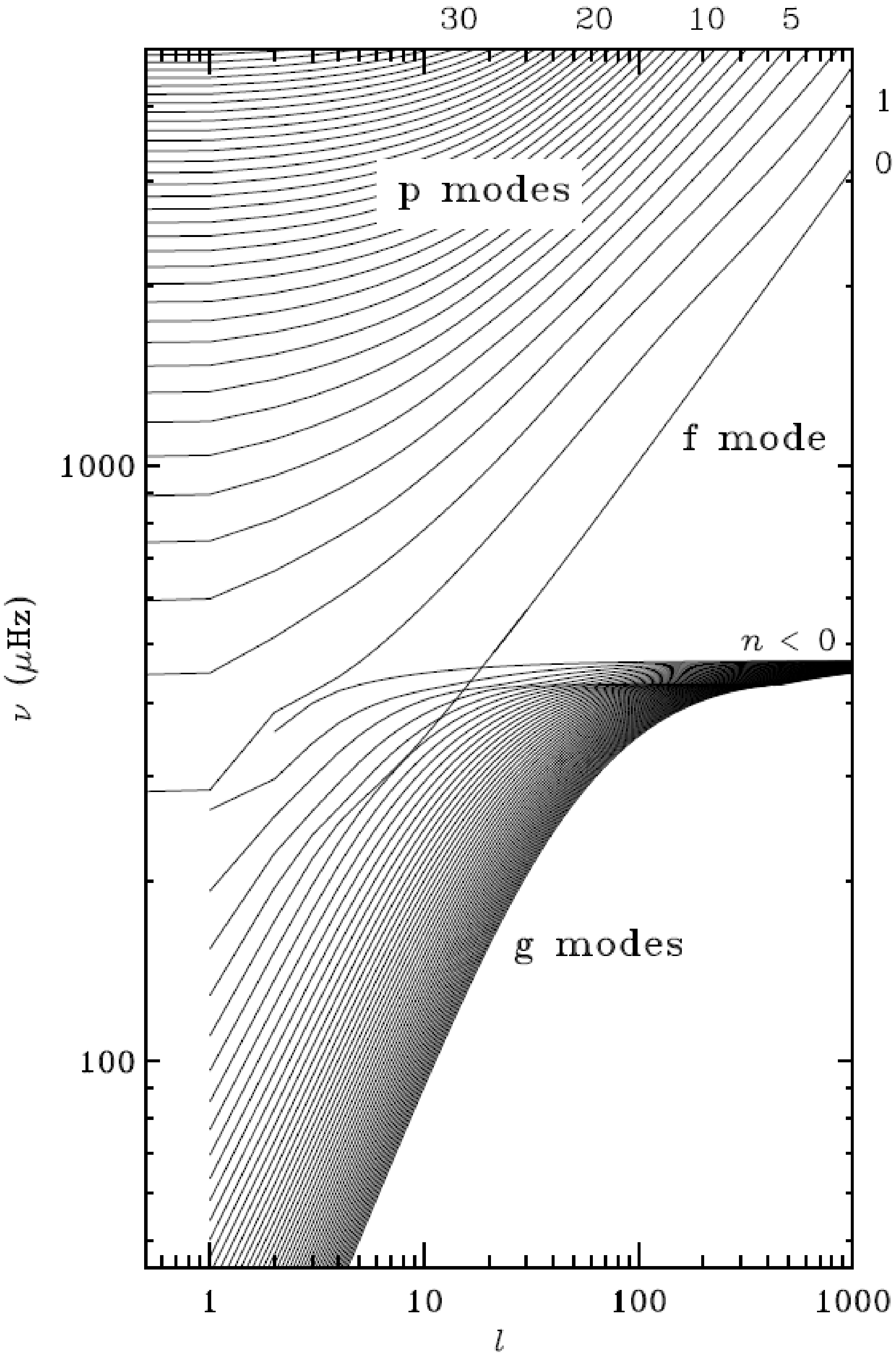
Constrained structural inversion with differential-response technique: set of elementary model variations



Homology rescaling: dimentionless variables

$$\tilde{\rho} = \frac{4\pi\,\mathrm{R}_{\odot}^3}{\mathrm{M}_{\odot}}\rho_0, \quad \tilde{p} = \frac{4\pi\,\mathrm{R}_{\odot}^4}{G\mathrm{M}_{\odot}^2}p_0, \quad \tilde{g} = \frac{\mathrm{R}_{\odot}^2}{G\mathrm{M}_{\odot}}g_0,$$

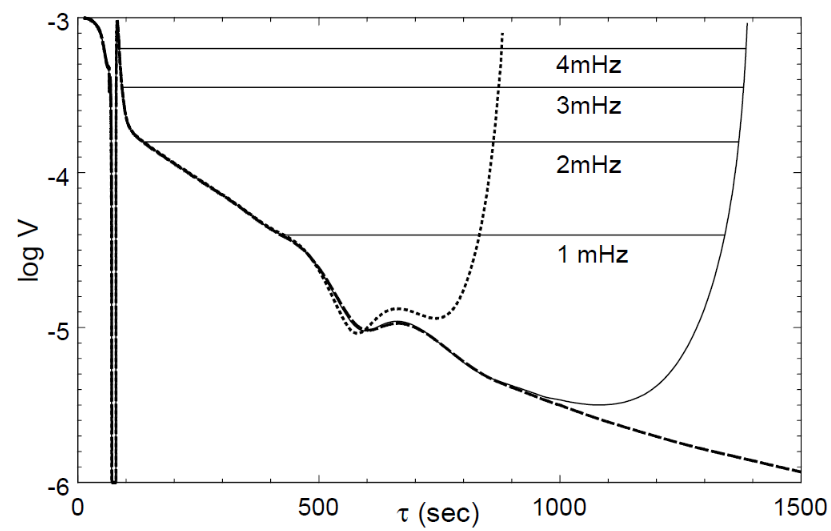
$$\tilde{c}^2 = \Gamma_1\frac{\tilde{p}}{\tilde{\rho}} = \frac{\mathrm{R}_{\odot}}{G\mathrm{M}_{\odot}}c^2, \quad \tilde{N}^2 = \frac{\mathrm{R}_{\odot}^3}{G\mathrm{M}_{\odot}}N^2, \quad x = \frac{r}{\mathrm{R}_{\odot}}.$$



$$\frac{d^2}{d\tau^2}\zeta + \left[\omega^2 - V(\tau)\right]\zeta = 0,$$

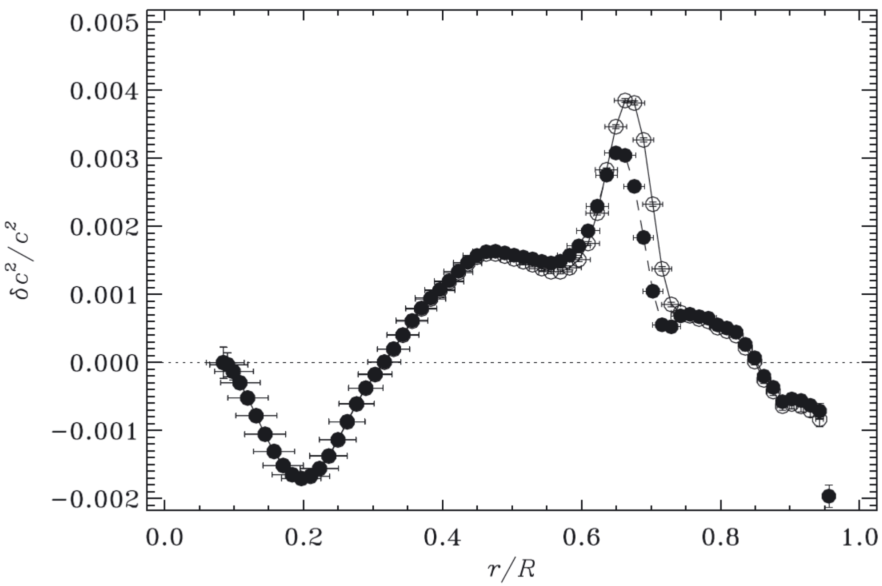
$$\tau = \mathrm{sgn}\left(s^2\right)\int_r^R |s|\, dr, \qquad s^2 = \frac{1}{c^2} - \frac{\tilde{w}^2}{r^2},$$

$$\tilde{w} = \frac{\ell + 1/2}{\omega}, \qquad \zeta = s^{-1/2} r \xi_r,$$

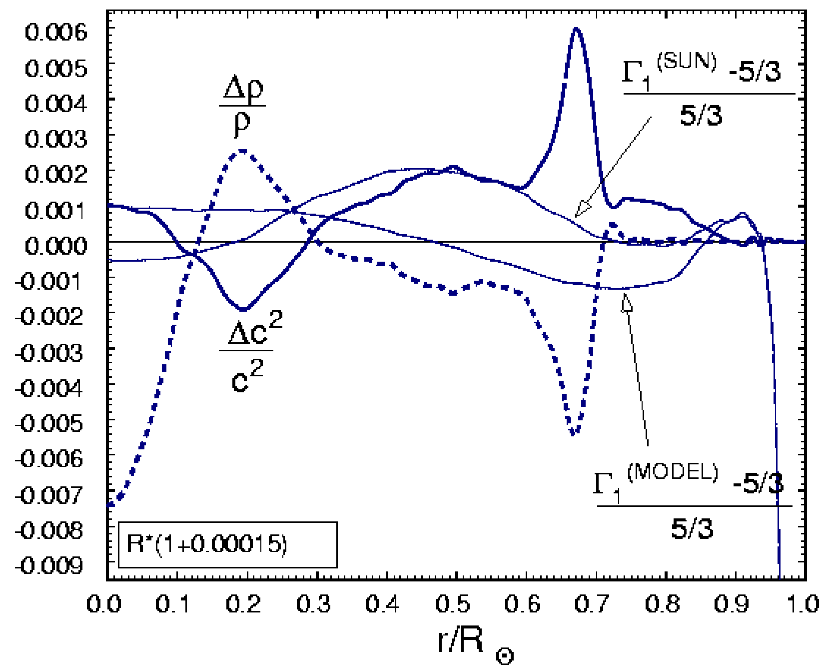


Acoustic potential

STRUCTURAL INVERSION



Christensen-Dalsgaard et al. 2011
Localized Averaging



Vorontsov 2001
Iterative regularization
(Strakhov & Vorontsov 2001)