Dynamo action in stellar radiative layers

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Context

Angular momentum in the radiative zone of stars

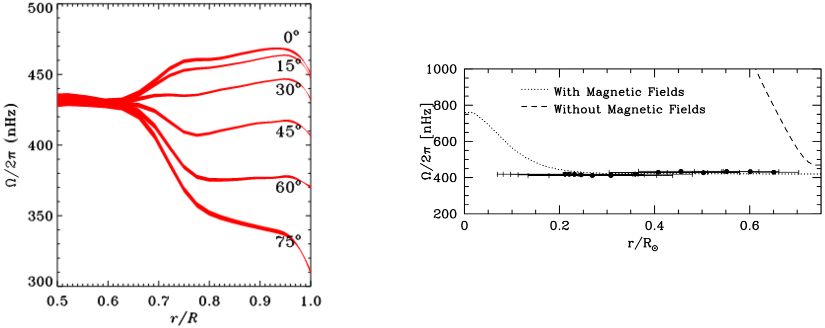


Figure: Left: Rotation profile at constant latitude in the sun [*Brown et al. 1989*]. Right: Numerical simulation for the radiative zone of the sun, with and without magnetic field, compared to observations [*Eggenberger et al. 2005*]

Seismic constraints on the radial dependence of the internal rotation profiles of six *Kepler* subgiants and young red giants (Deheuvels 2014)

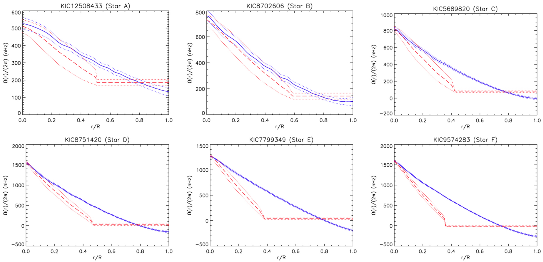
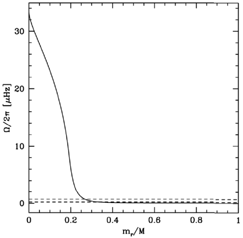


Fig. 10. Best rotation profiles obtained by applying the RLS method with a smoothness condition on the rotation profile on the entire star (solid blue lines) or only in the radiative interior while the convective envelope is assumed to rotate as a solid body (long-dashed red lines). The dotted lines indicate the 1σ error bars for both types of inversions.

Deheuvels *et al* 2014



Ceillier *et al* 2013

The core rotation rate is 200 times lower than predicted by 1D models of stellar evolution (Zahn 1992 formalism).

Observations correspond qualitatively to what is expected by the conservation of AM: spin-up of the core results from its contraction. . . i.e. emergence of differential rotation in radiative zones.

Conclusion: need for an additional efficient physical mechanism (a missing transport process) or better understanding turbulence in stably-stratified rotating spherical shells (radiative zones).

Evolution of the rotation profile for late evolutionary phases

Rotation profile in subgiants:

Need for an efficient transport of angular momentum

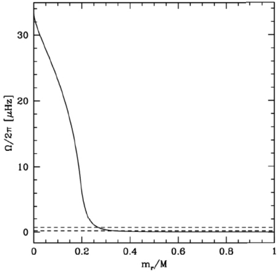
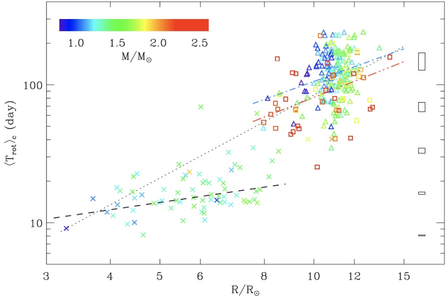


Fig. 3. Rotation profile of the selected model at the end of the evolutionary track (solid line). The two dashed lines correspond to the core and the surface rotation rates derived by Deheuvels et al. (2012).

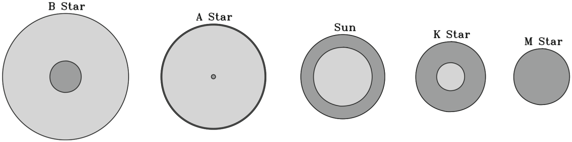
Ceillier *et al* 2013.

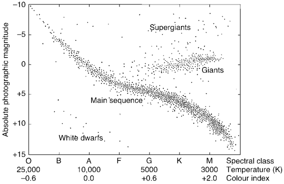


Mosser *et al* 2012

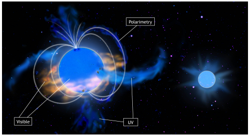
Stellar evolution models predict strong shear layers at different life phases that are not observed by asteroseismology.

Massive and intermediate-mass stars





Magnetic properties



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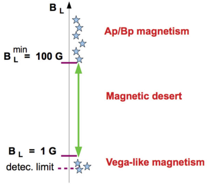
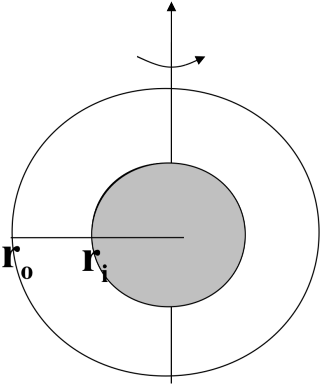


Lignières *et al* 2014

- Strong dipolar fields would result from initial conditions (Fossil fields).
- A MHD instability (MRI?) could explain the observed magnetic desert.
- Vega-like fields could result from dynamo action ? in which layers ? In this case, what is the dynamo mechanism ?
- . . .

A simple numerical model for radiative zones with the main physical ingredients



Set up

- A Boussinesq fluid in a rotating spherical shell with
  - ⋆ constant kinematic viscosity ν
  - ⋆ constant thermal diffusivity κ
  - ⋆ constant magnetic diffusivity η
- The rotation rates of the spheres are maintained (Spherical Couette Flow).

Boundary conditions

- fixed temperature ΔT
- no-slip for the flow
- conducting inner core and insulating outer sphere

Systematic parameter studies (anelastic models)

Six control parameters

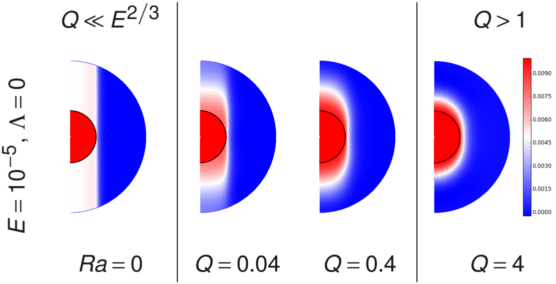
Number	Symbol			Sun
Rayleigh number	Ra	$\frac{\alpha g \Delta T L^3}{\nu\kappa}$	$\leq 10^{13}$	10^{30}
mag Prandtl	Pm	ν/η	$0.2 \leq Pm \leq 25$	10^{-4}
Prandtl	Pr	ν/κ	0.1	10^{-6}
Ekman	E	$\nu/(\Omega L^2)$	$10^{-4} \geq E \geq 10^{-7}$	10^{-15}
aspect ratio	χ	r_i/r_o	0.35	

The relevant parameter in rotating stably-stratified spherical shells seems to be $Q = Pr\left(\frac{N}{\Omega_0}\right)^2 = E^2 Ra$

Results obtained by using the *PaRoDy* code.

Stably-Stratification affects the geometry of the angular velocity

Parameter study with $Pr = 0.05$



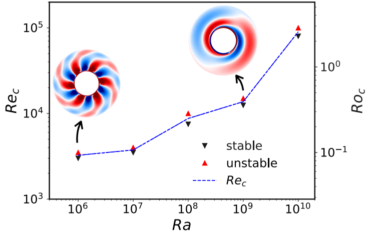
$Q = Pr\left(\frac{N}{\Omega_0}\right)^2 = E^2 Ra$ where N: Buoyancy frequency

Philidet *et al* 2019 *GAFD*

Stabilizing effects of stratification on the shear instability Ro_c

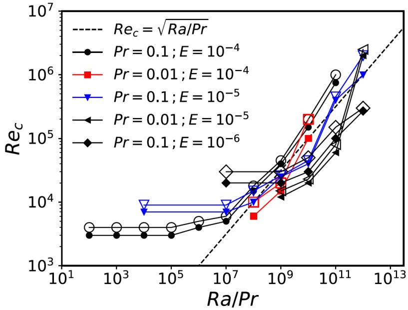
Reynolds number $Re = \frac{\Delta\Omega r_i r_e}{\nu}$;
$E = 10^{-5}$, $Pr = 0.1$

$$Q = Pr\left(\frac{N}{\Omega_0}\right)^2 = E^2 Ra$$



Evolution of the critical Reynolds number Re_C (or Rossby number Ro_C) as a function of stratification (the Rayleigh number Ra). When Re exceeds Re_C (red upwards triangles), non-axisymmetric modes are maintained over time.

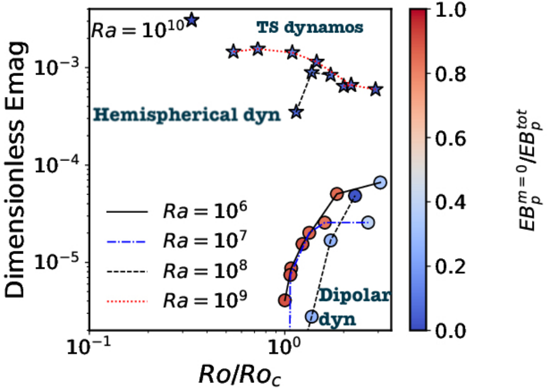
Petitdemange *et al* submitted to A& A



This behaviour corresponds to $Ri < 1/4$ (Richardson number).

Magnetic field topology depends on the level of stratification (Ra)

stars: $E_{mag,toro} > 0.8 E_{mag}$; circles: dipolar dynamos

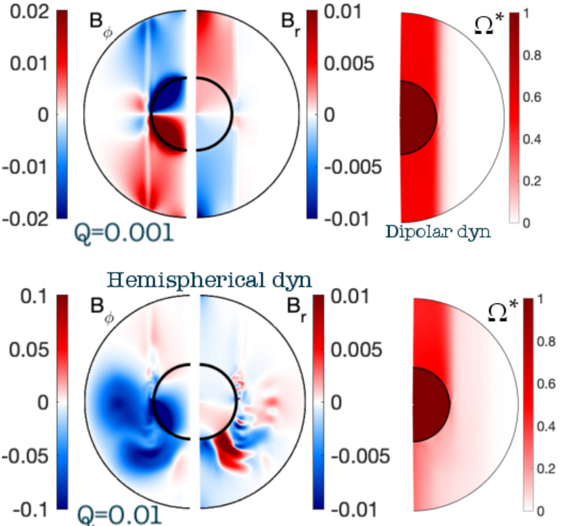


$E = 10^{-5}$, $Pm = 1$

When stratification is low, results are similar to Unstratified dipolar dynamos (Guervilly & Cardin 2010).

Different magnetic topologies in simulated dynamos with $E = 10^{-5}$

Dipolar fields: regardless the initial magnetic field with $Q = 0.001$ ($Ra = 10^7$) ; a bistable regime exists with $Q = 0.01$ ($Ra = 10^8$).



Typical axisymmetric sections for dynamo solutions obtained with different levels of stratification.

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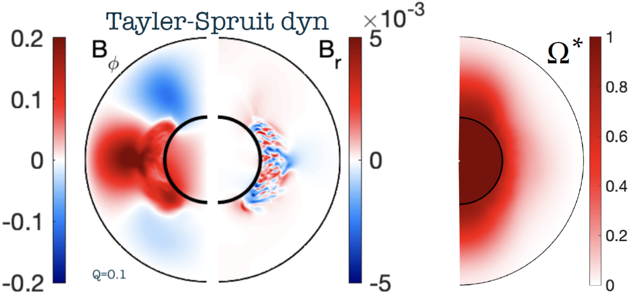
Different magnetic topologies in simulated dynamos with $E = 10^{-5}$

Strong toroidal fields generated when $Q = Pr(N/\Omega_0)^2 = E^2 Ra = 0.1$ $(Ra = 10^9)$



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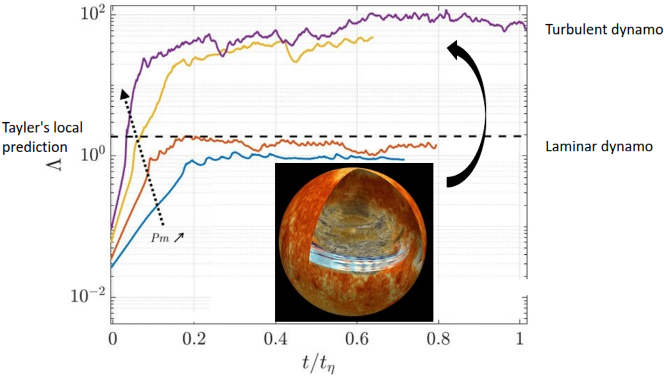
Tayler-Spruit dynamos: Secondary growth



Timeseries of the magnetic energy (measured by the Elsasser number $\Lambda = \frac{1}{V}\int B^2/\rho_0\mu_0\Omega\eta$) for $E = 10^{-5}$, $N/\Omega = 1.24$, $P_r = 0.1$, $Ro = 0.78$ and varying $Pm = [0.35; 0.42; 0.5; 1]$, in resistive timescales. [*Petitdemange et al. 2023*].

Tayler instability with cylindrical coordinates (s, ϕ, z)

Instability conditions (Spruit 1999)

$$s \frac{d}{ds}\left(\frac{V_A^2}{s^2}\right) > 0 \quad \text{for} \quad m = 0 \qquad (1)$$

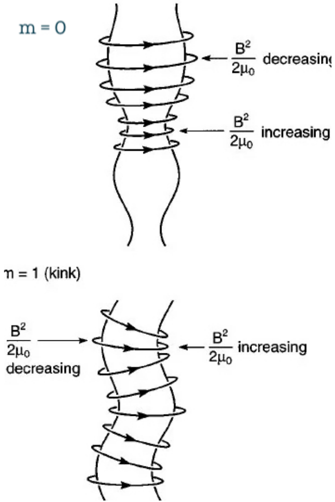
$$\frac{1}{s^3}\frac{d}{ds}\left(s^2 V_A^2\right) > \frac{m^2 V_A^2}{s^2} \quad \text{for} \quad m > 0 \qquad (2)$$

The $m = 1$ modes require less steep variations of $V_A = B_\phi / \sqrt{\mu\rho}$.
Stabilizing effects: rotation, diffusion and stratification. Using heuristic arguments:

$$\frac{\omega_{A0}}{\Omega} > \left(\frac{N}{\Omega}\right)^{1/2}\left(\frac{\eta}{r^2\Omega}\right)^{1/4} \qquad \kappa = 0 \ (3)$$

$$\frac{\omega_{A1}}{\Omega} > \left(\frac{N}{\Omega}\right)^{1/2}\left(\frac{\kappa}{r^2\Omega}\right)^{1/4}\left(\frac{\eta}{\kappa}\right)^{1/2} \qquad (4)$$

and spherical symmetry (shellular) $\omega_A = V_A / r$.



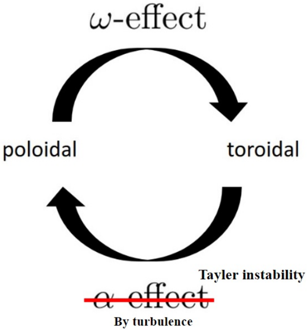
Tayler-Spruit dynamo: theory (in short), Spruit 2002

Spruit's predictions are based on analytical approximations. Their robustness have to be confirmed by numerical and/or experimental studies. One of them is the existence of an effective resistivity η_e resulting from the Tayler instability.

$$\eta_{e0} = r^2\Omega\left(\frac{\omega_A}{\Omega}\right)^4\left(\frac{\Omega}{N}\right)^2 \qquad (5)$$

and

$$\eta_{e1} = r^2\Omega\left(\frac{\omega_A}{\Omega}\right)^2\left(\frac{\Omega}{N}\right)^{1/2}\left(\frac{\kappa}{r^2 N}\right)^{1/2} \qquad (6)$$



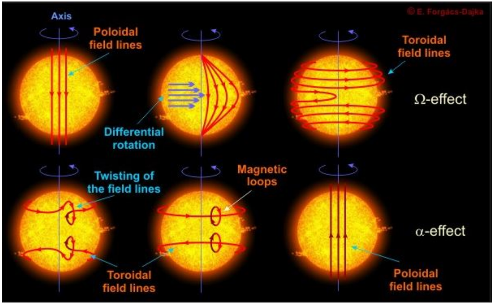
Saturated fields:

$$\frac{\omega_{A0}}{\Omega} = q\frac{\Omega}{N} \qquad (7)$$

$$\frac{\omega_{A1}}{\Omega} = q^{1/2}\left(\frac{\Omega}{N}\right)^{1/8}\left(\frac{\kappa}{r^2 N}\right)^{1/8} \qquad (8)$$

with $q = r\partial_r\Omega/\Omega$

Mean-field concept



Decomposition into a mean part and a fluctuating part:

$$\mathbf{v} = \overline{\mathbf{V}} + \mathbf{v}' \qquad \mathbf{B} = \overline{\mathbf{B}} + \mathbf{B}'\,.$$

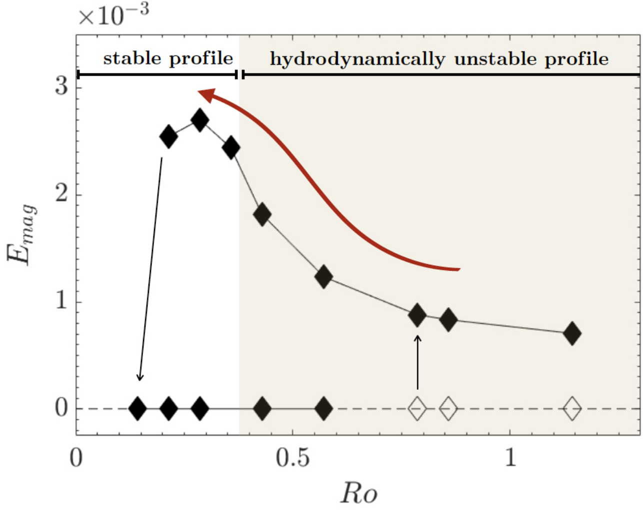
Under some assumptions:

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\xi + \overline{\mathbf{v}} \times \overline{\mathbf{B}} - \eta \nabla \times \overline{\mathbf{B}})$$

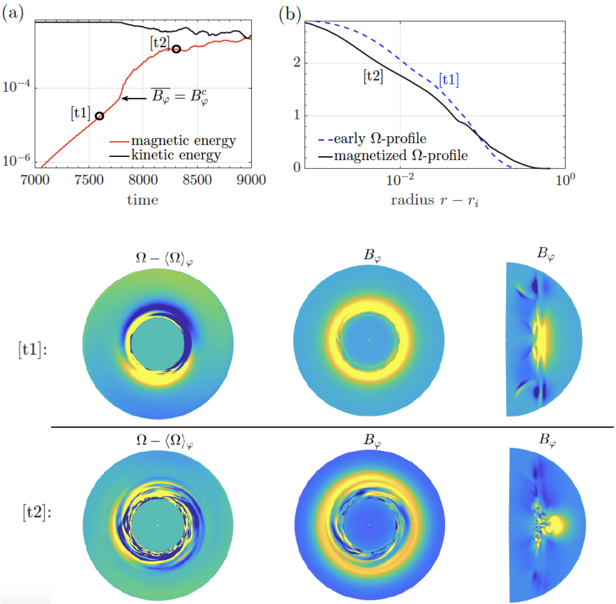
$$\nabla \times (\overline{\mathbf{v}} \times \overline{\mathbf{B}}) = (\overline{\mathbf{B}} \cdot \nabla)\overline{\mathbf{v}} - (\overline{\mathbf{v}} \cdot \nabla)\overline{\mathbf{B}}$$

ω-effec: stretching of field by shear.

Bifurcation diagram with $Q = 0.1$, $E = 10^{-5}$, $Pm = 1$, $Pr = 0.1$



Development and saturation of TS-dynamo in our DNS

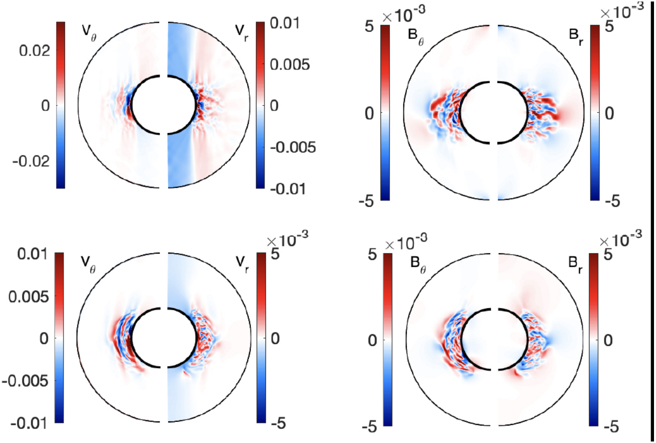


Typical axisymmetric sections at a given time

Varying the level of stratification (*Ra* or *Q*) with $E = 10^{-5}$.

$Q = 0.1$

$Q = 1$



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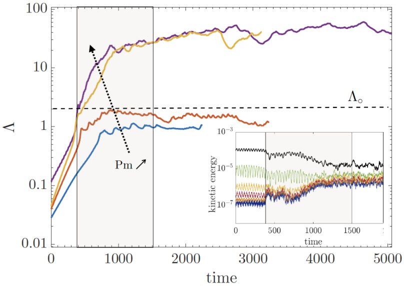
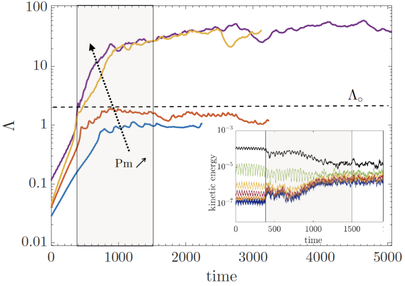
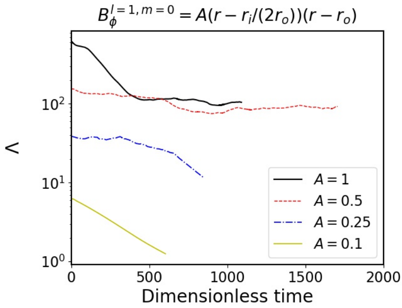
Routes to the Tayler-Spruit dynamo



Routes to the Tayler-Spruit dynamo



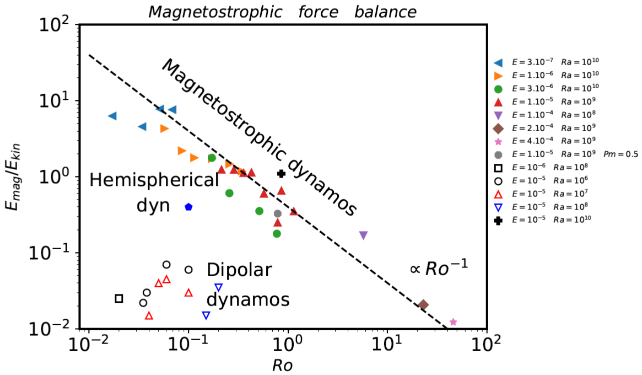
$Ro = 0.79$



$Ro = 0.57$: no primary instability

Tayler-Spruit dynamos can be obtained when the toroidal field strength is sufficiently high. The primary instability is not necessary!

Field strength



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Angular momentum transport

The magnetic torque G_{mag} as a function of the spherical radius r is

$$G_{mag} = \iint r^3 \sin^2\theta\, V_{Ar} V_{A\phi} d\theta d\phi = \langle 4\pi r^3 \sin^2\theta\, V_{Ar} V_{A\phi} \rangle \tag{9}$$

where $\mathbf{V}_A$ is the Alfvèn speed. The angular momentum conservation is

$$\frac{\partial \langle u_\phi r \sin\theta \rangle}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \left\langle (u_r u_\phi - V_{Ar} V_{A\phi}) \sin\theta\, r + \nu r^2 \sin^2\theta \frac{\partial \Omega}{\partial r} + \Omega_o r^2 \sin^2\theta\, u_r \right\rangle = 0 \tag{10}$$

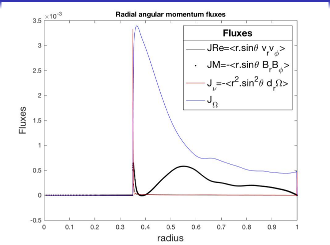
$$\frac{\partial \langle u_\phi r \sin\theta \rangle}{\partial t} + \nabla \cdot \mathbf{F} = 0 \tag{11}$$

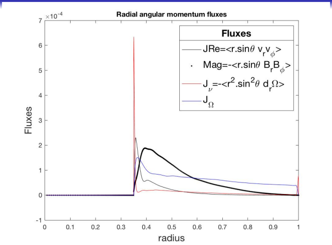
From Spruit's theory:

$$S_0 \approx \frac{B_{r0} B_{\phi 0}}{4\pi} = \rho \Omega^2 r^2 q^3 \left(\frac{\Omega}{N}\right)^4 \quad \kappa = 0 \tag{12}$$

$$S_1 \approx \frac{B_{r1} B_{\phi 1}}{4\pi} = \rho \Omega^2 r^2 q \left(\frac{\Omega}{N}\right)^{1/2} \left(\frac{\kappa}{r^2 N}\right)^{1/2} \quad \kappa \neq 0 \tag{13}$$

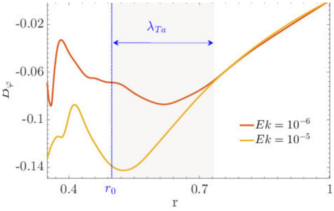
Radial distribution of anular momentum fluxes

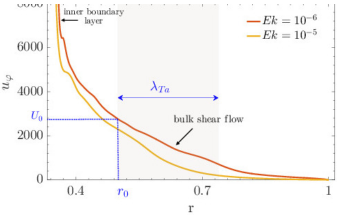




$Pm = 0.35$

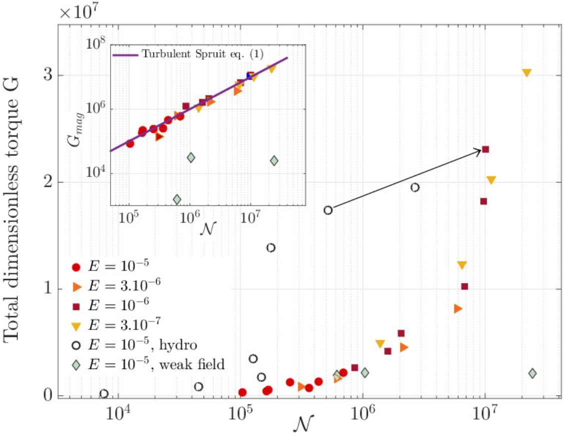
$Pm = 1$





The measured shear rate $q = kU_0/\Omega$ where $k = N\sqrt{\mu\rho}/B_\phi = 2\pi/\lambda_{Ta}$ is the radial wavenumber for the Tayler instability.

Spruit's prediction for the magnetic torque G_{mag}



Spruit's prediction: $B_r B_\phi/\mu = \rho r^2 \Omega^2 q^3 \left(\frac{\Omega}{N}\right)^4$ corresponds to our results when $q \sim kU_0/\Omega \Rightarrow B_r B_\phi/\mu \propto \rho (U_0\Omega)^{3/2}/N$.

$$G_{mag} = \mathcal{N} = \beta r_i^{5/2} \frac{(U_0\Omega)^{3/2}}{N\nu^2} \quad \text{Dimensionless torque} \qquad (14)$$

$\beta \sim 0.1$ is an adjustable parameter.

Conclusion

- Tayler-Spruit (TS) dynamo seems to be now supported by Direct Numerical Simulations.
- Subcritical behaviour: hidden magnetic fields trigger MHD turbulence that transport angular momentum.
- Stratification enables the development of TS dynamos in DNS.
- In progress:
  - Exploring different parameter regimes.
  - Determining the transport coefficients and the dynamo properties.
  - Considering a more realistic model for the radiative zone.
  - Link with observations…
- The Tayler-Spruit dynamo at different evolutionary phases…