Gravity waves in the Sun:
generation, detection, and transport

Charly Pinçon

Workshop “Future of solar modeling”, Sierre 2023

Gravity waves in general

- **Subsonic (incompressible) waves:** buoyancy as the restoring force

$$k_r^2 = k_h^2\left(\frac{N^2}{\omega^2} - 1\right)$$

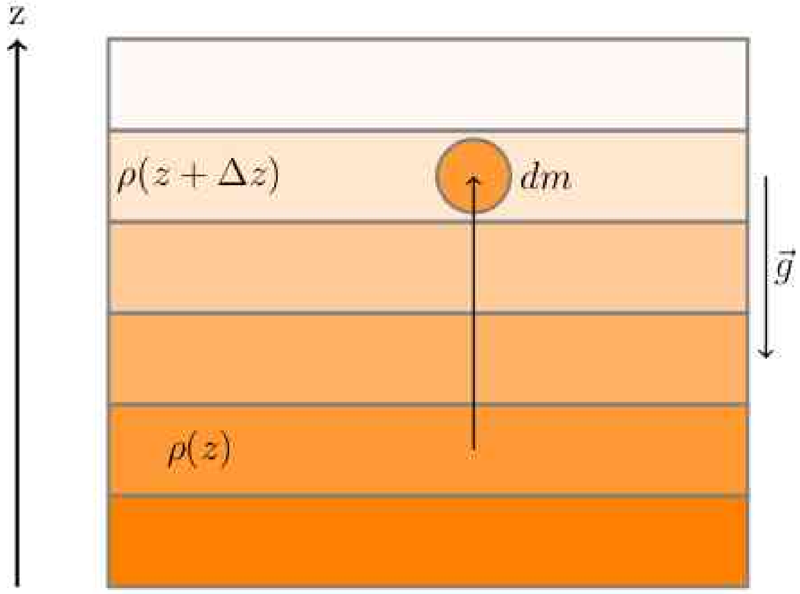
If $k_r^2 < 0$ ⇒ Evanescent

If $k_r^2 > 0$ ⇒ Propagation

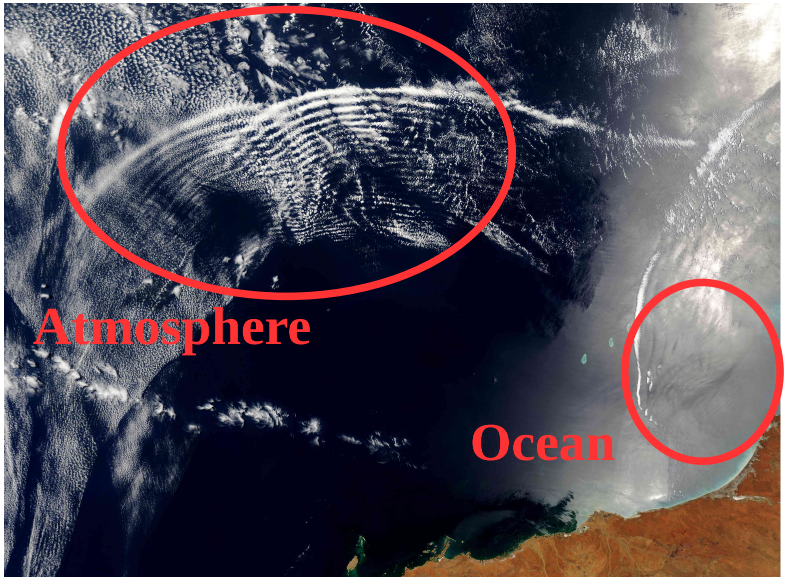
$$N^2 = \frac{g\,\delta}{C_P}\frac{ds}{dr}$$

Brunt-Väisälä

= stratification level



Credit: Pierrick Verwilghen



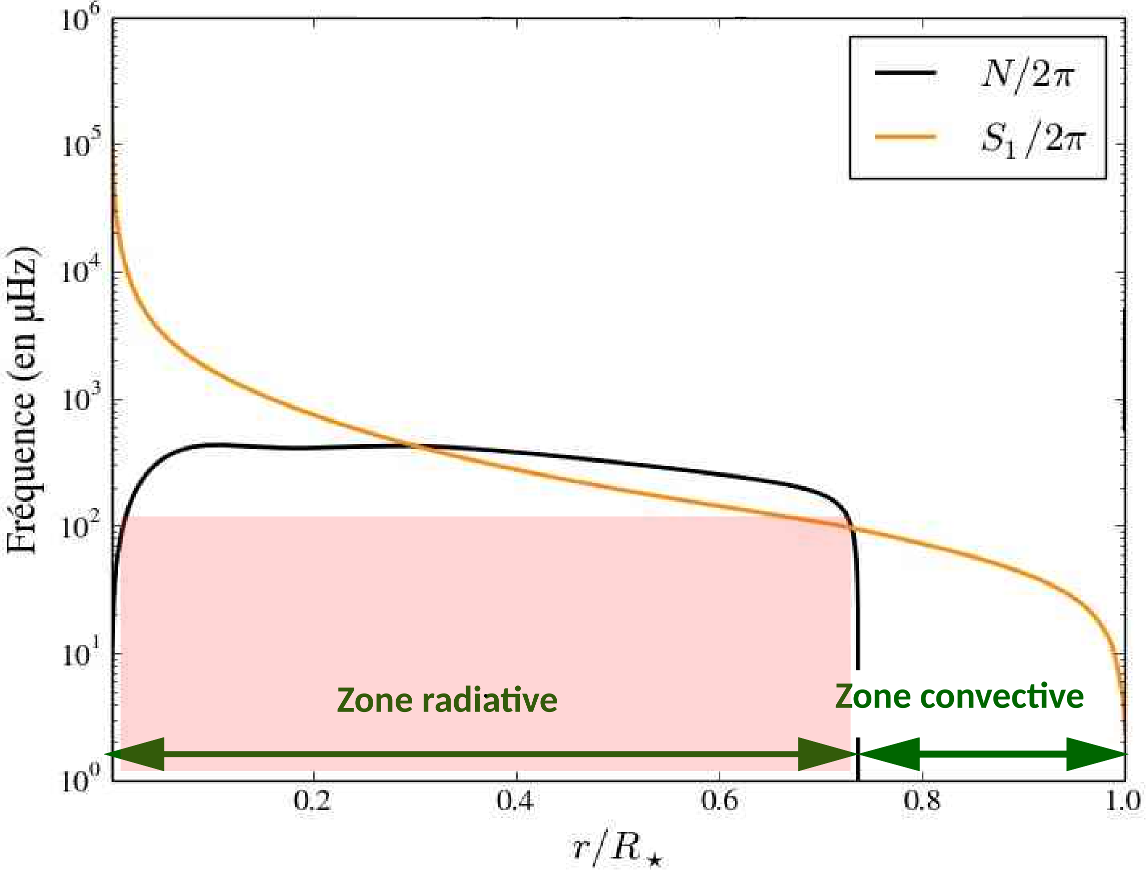
Credit : Jacques Descloitres, MODIS Rapid Response Team, NASA/GSFC

Internal gravity waves (IGW) in the Sun

$$k_r^2 = k_h^2 \left(\frac{N^2}{\omega^2} - 1 \right)$$

- **Propagation regions:**
  - between $0 < r < 0.7\ R_{sun}$ for $0 < \nu < 400\ \mu$Hz
  - $0 < \nu < 200\ \mu$Hz: evanesecent in the surface convective zone
  - $200 < \nu < 400\ \mu$Hz : coupling with surface acoustic modes = "Mixed modes"

Propagation diagram



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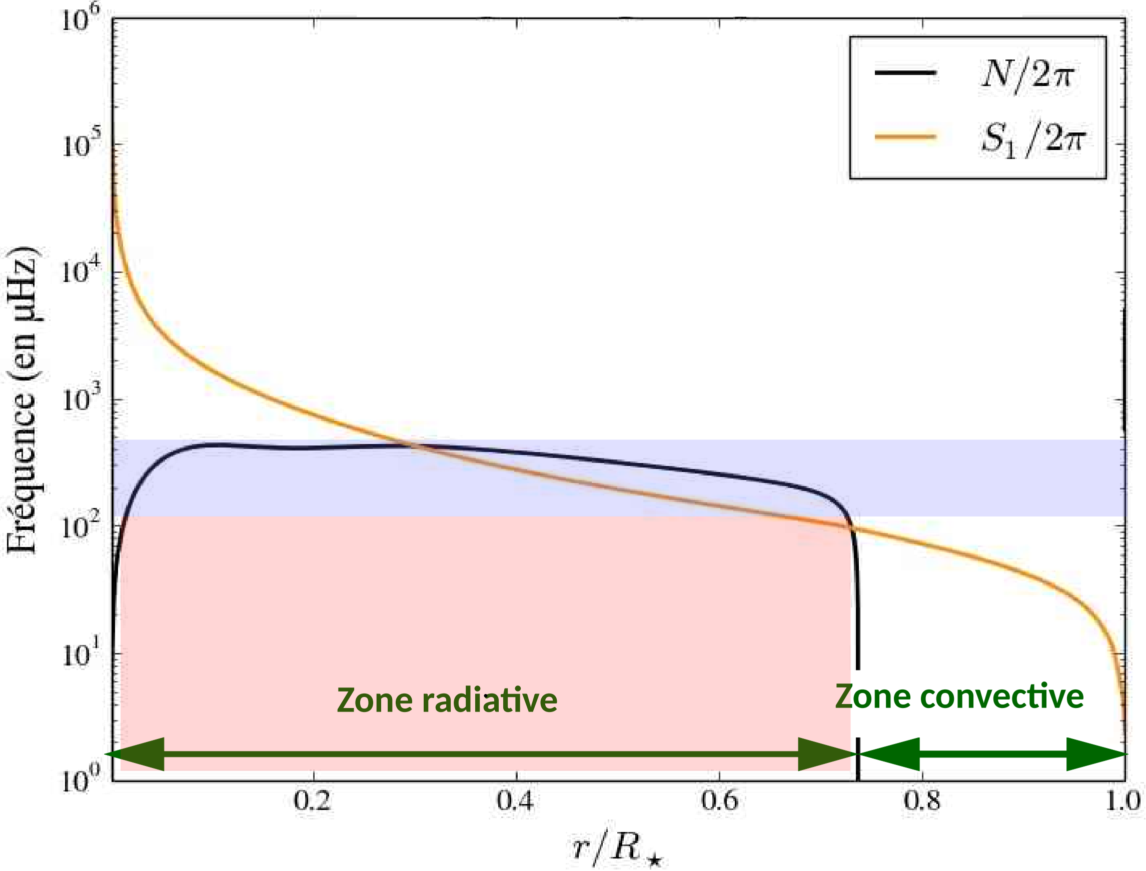
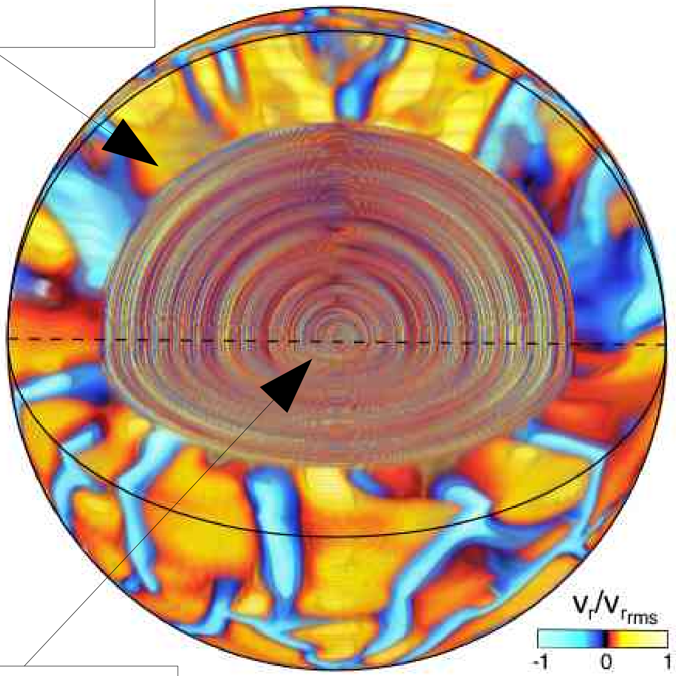


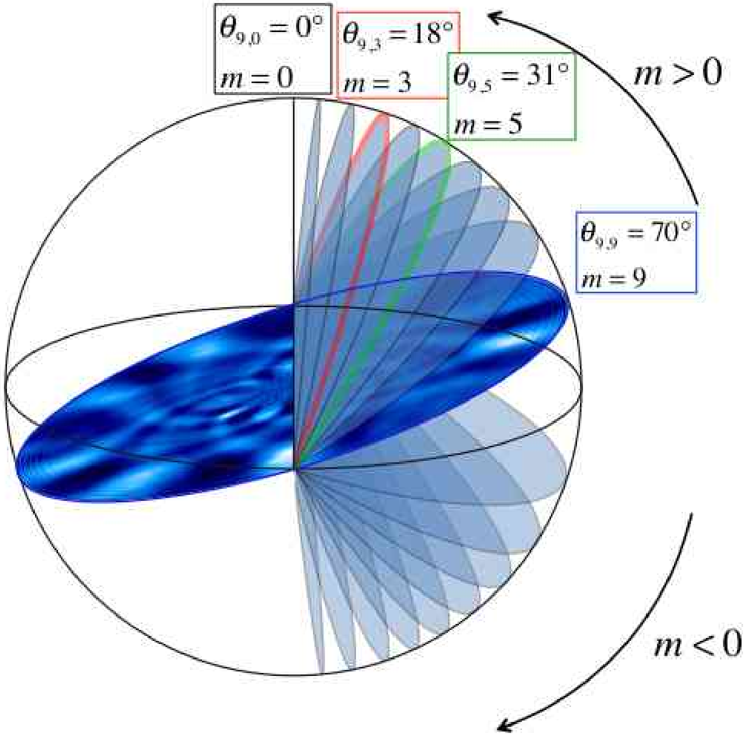
Energy ray paths in 3D

- Generated by the surface convection motions
- Energy propagating in inclined planes for given *l* and *m*

Convective Zone



Radiative Zone
⇒ IGW

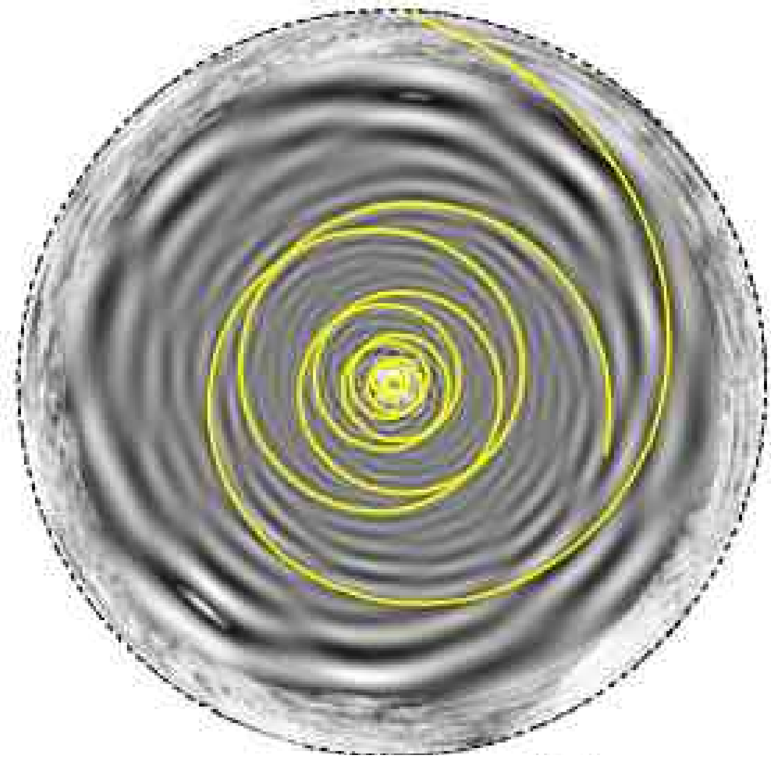


Alvan+2014,2015

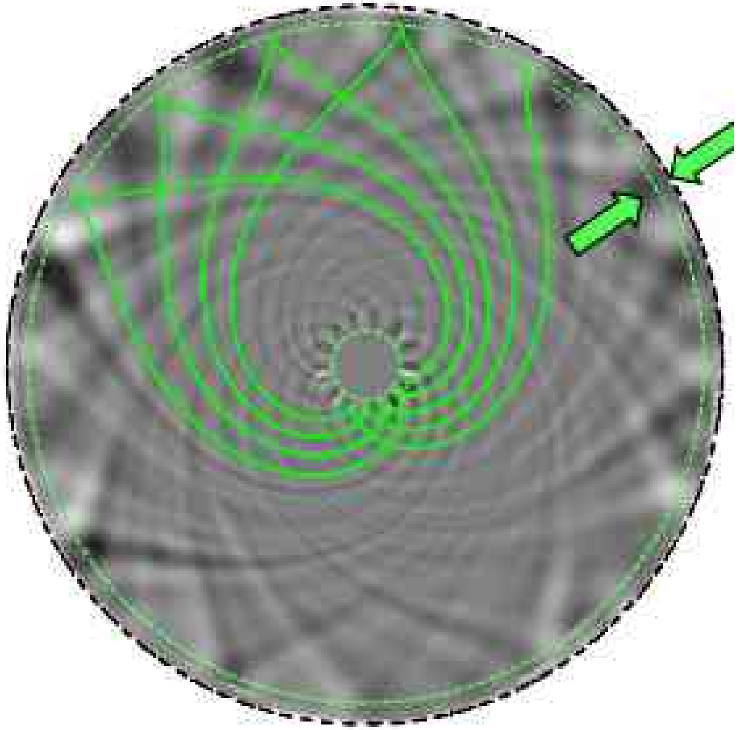
Energy ray paths in 3D

- ✓ Spiraling around the stellar center → pattern depends on the wave frequency

Equatorial plane
in the radiative zone



ν = 10 μHz

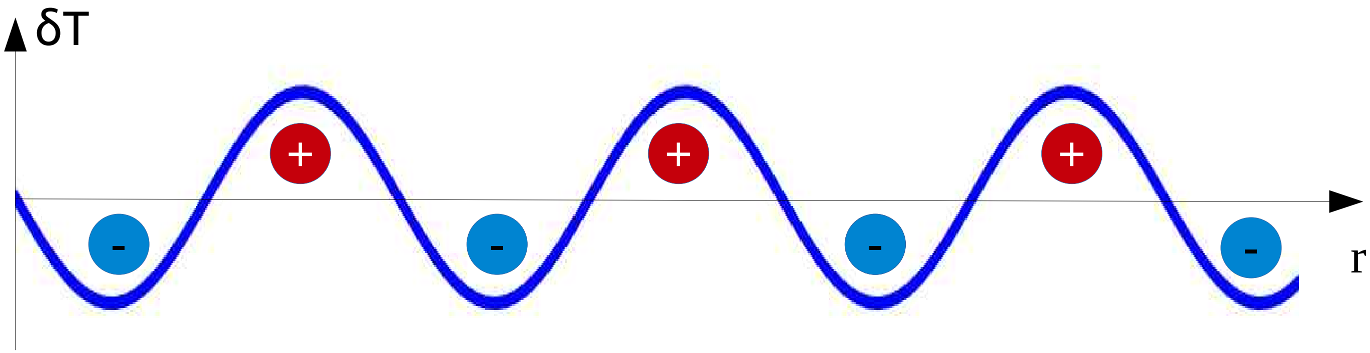


ν = 40 μHz

Alvan+2014,2015

Damping in the radiative region (only...)

- ✓ IGW induces local excess/deficiency of thermal internal energy
  - Radiative diffusion tends to smooth out these thermal gradients.



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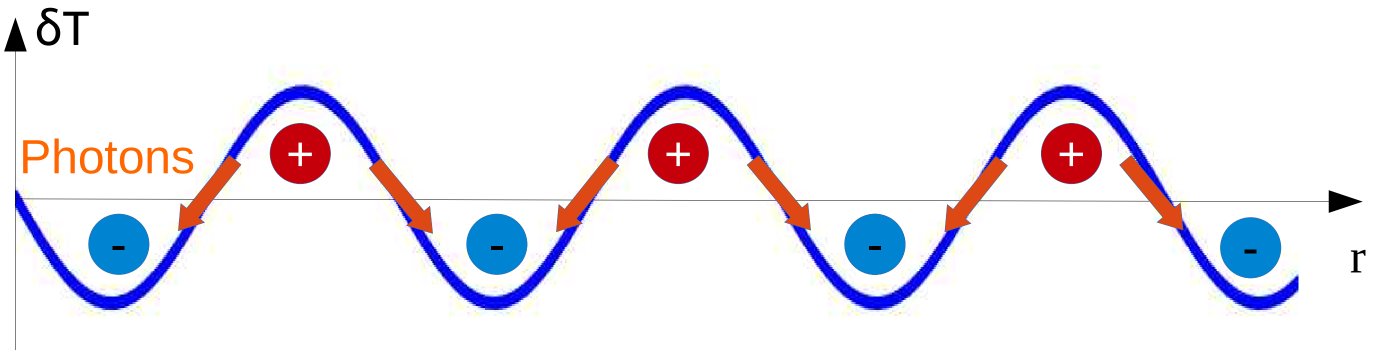

$$\frac{\partial \delta T}{\partial t} = -\frac{\partial}{\partial r}\left(\boxed{K_{rad}}\frac{\partial \delta T}{\partial r}\right)$$

Radiative diffusivity

$$t_{damping} \sim \frac{\boxed{\lambda_r^2}}{K_{rad}}$$

Radial wavelength

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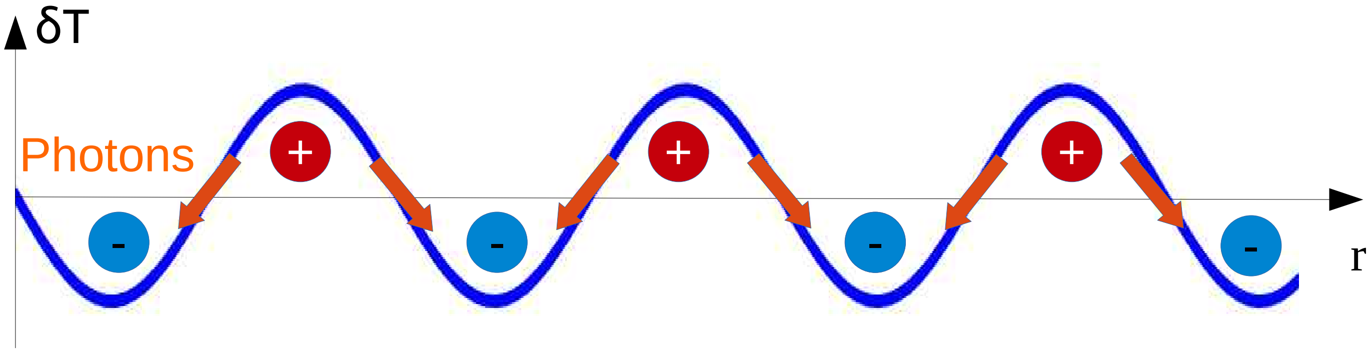

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Radial wavelength

- Two questions:
  - Deposit of energy in the medium → transport efficiency ?
  - Finite amplitude → detectability (question of g-modes) ?
  - → Need to know the wave amplitude ...

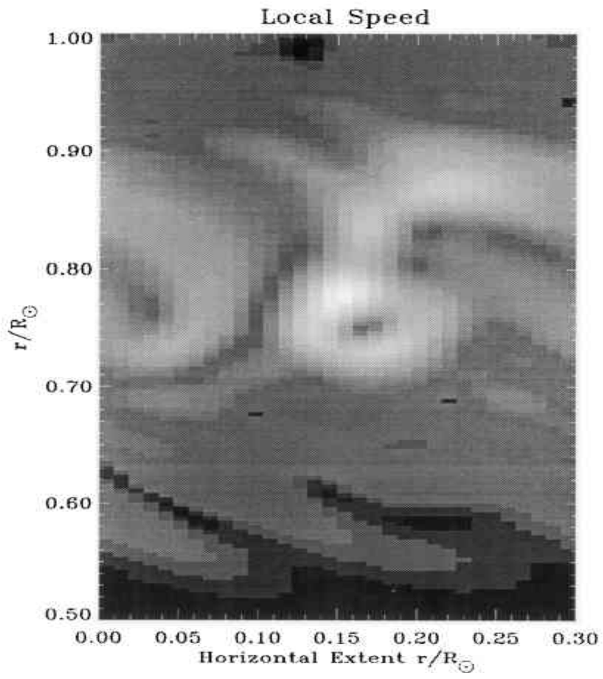
IGW generation in numerical simulations of the Sun

Andersen 1996

→ 2D Cartesian geometry

→ Open boundaries

→ Efficiency < 0.1% = progressive IGW

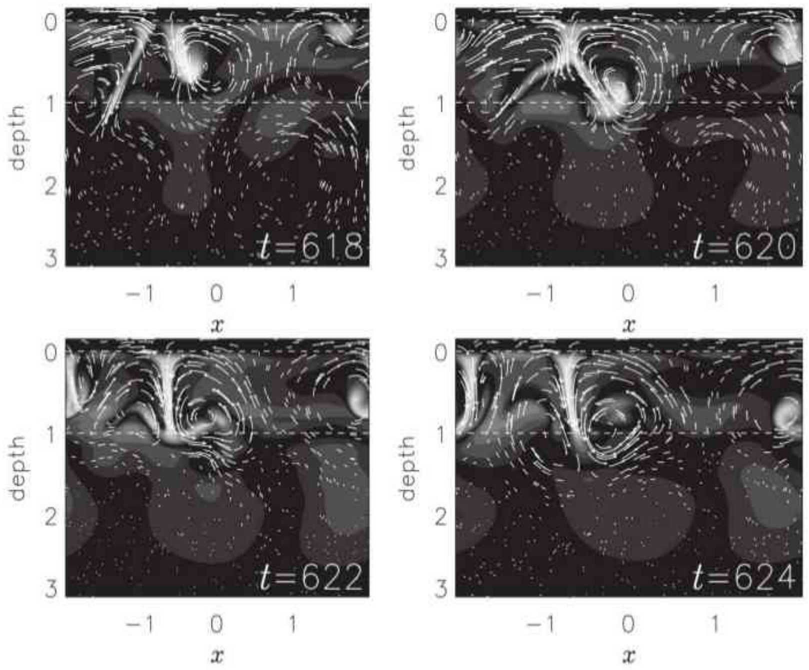


Dintrans+2005

→ 2D Cartesian geometry

→ Closed boundaries

→ Efficiency up to 40% = eigenmodes



→ Discrepancies: toward realistic setup and spectral studies.

IGW generation in numerical simulations of the Sun

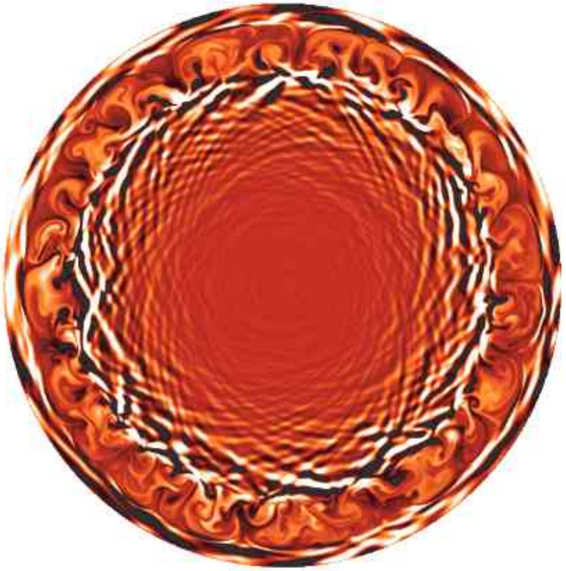
Rogers+2005,2006

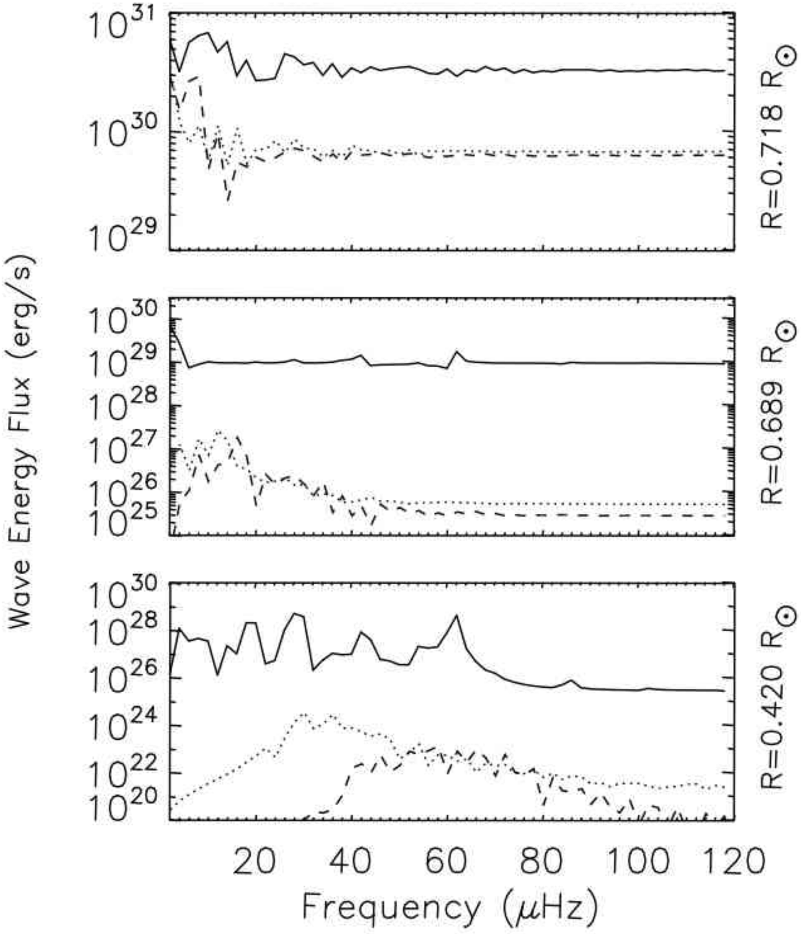
→ Polar geometry, realistic thermal structure

→ Flat wave spectrum

→ But enhanced thermal diffusivity

= overestimated flux / generation

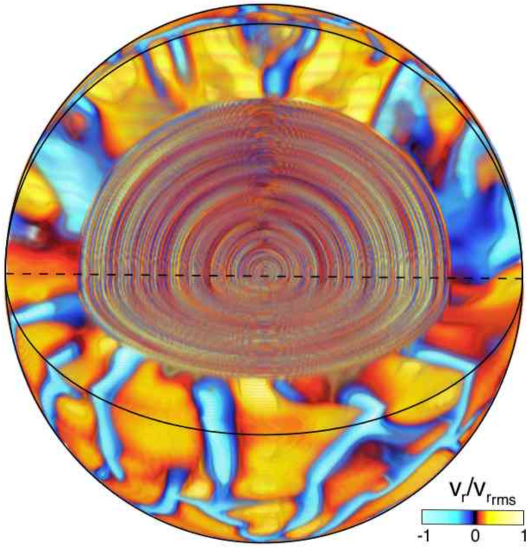


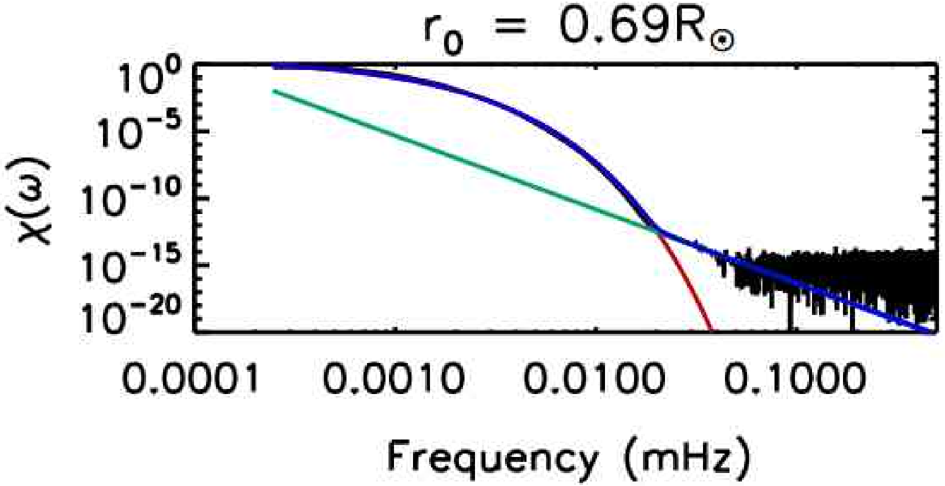


IGW generation in numerical simulations of the Sun

Alvan+2014,2015

- → Spherical geometry geometry, realistic thermal structure
- → Gaussian at low frequency / Power law at high frequency
- → Still enhanced thermal diffusivity, question of the thermal relaxation...





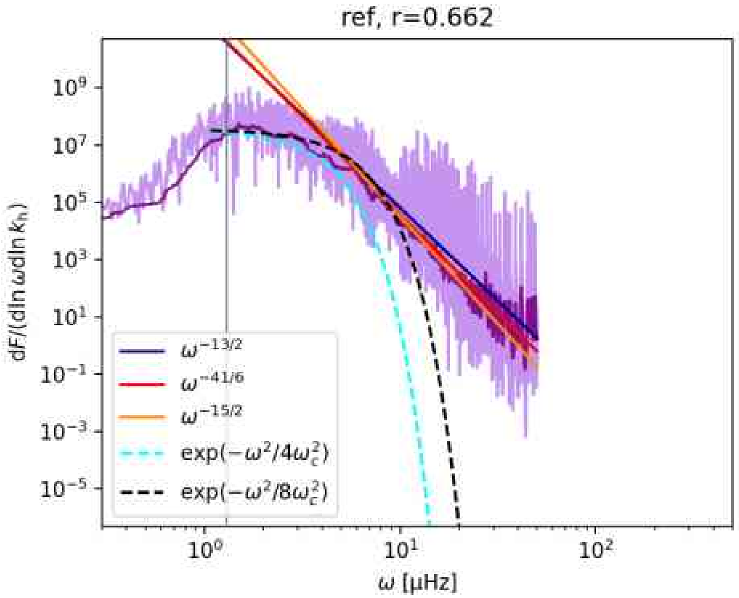
IGW generation in numerical simulations of the Sun

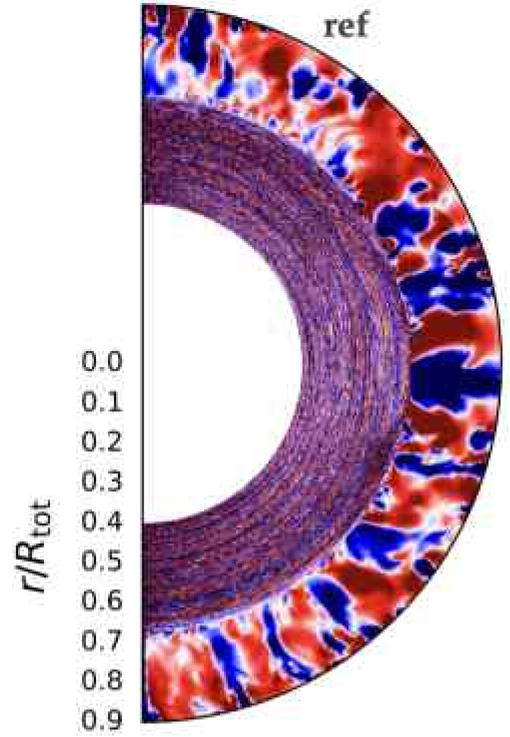
Le Saux+2022

→ ~ Same spectral shape as Alvan+2014

→ Study of the impact of the enhanced thermal diffusivity

→ Conclude that simulations must be considered with caution ...

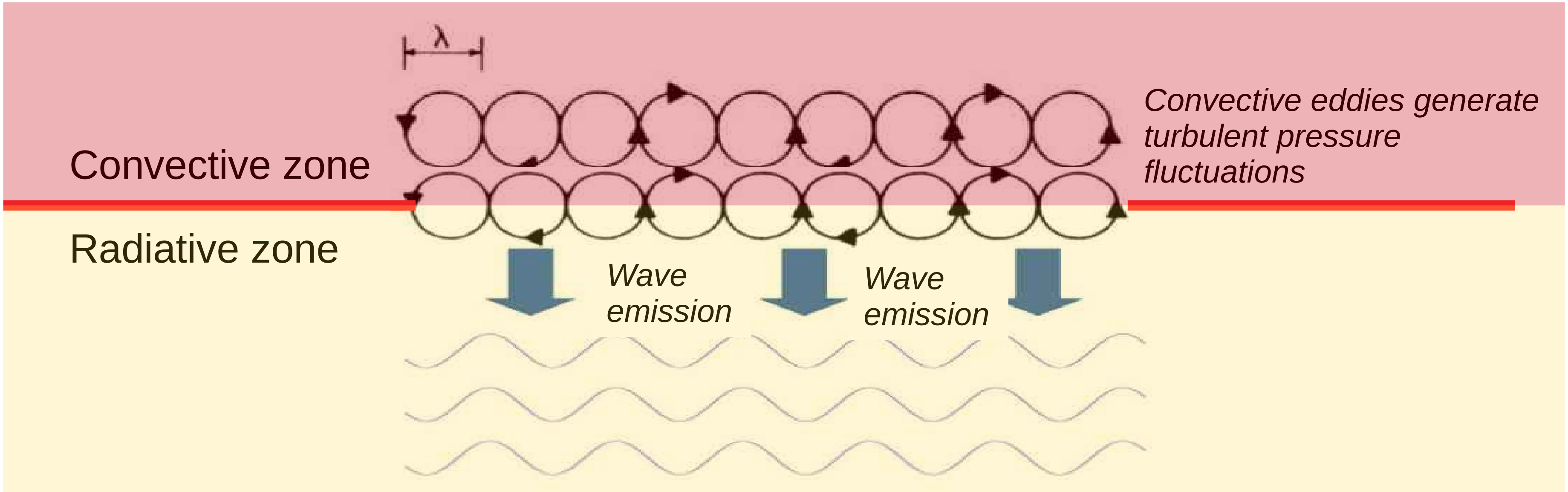




Overall, numerical simulations are good guides, but still not realistic enough (cf. simu Re ~ 10^5 / stars Re ~10^{12})

→ Need for complementary semianalytical estimates !

1- Excitation by turbulent Reynold stress



Several excitation models, but a general mechanism/expression

- Pressure matching at the rad/conv interface (Press 1981, Garcia-Lopez+1991, Zahn+1997)
- Reynold forcing through the convective bulk (Kumar 1999, Lecoanet 2013)

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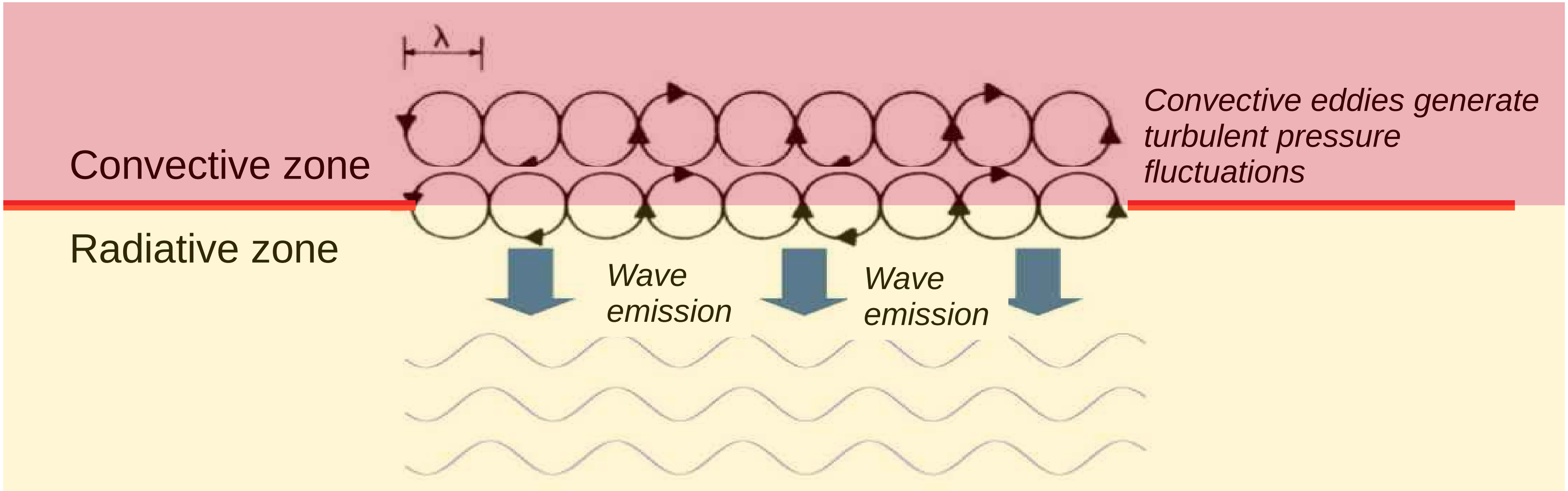

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Turnover timescale of the biggest eddies

Wave energy flux

$$F_{E,w} = F_c \left(\frac{\omega_c}{\omega}\right)^a \left(\frac{l_c}{l}\right)^b M_c$$

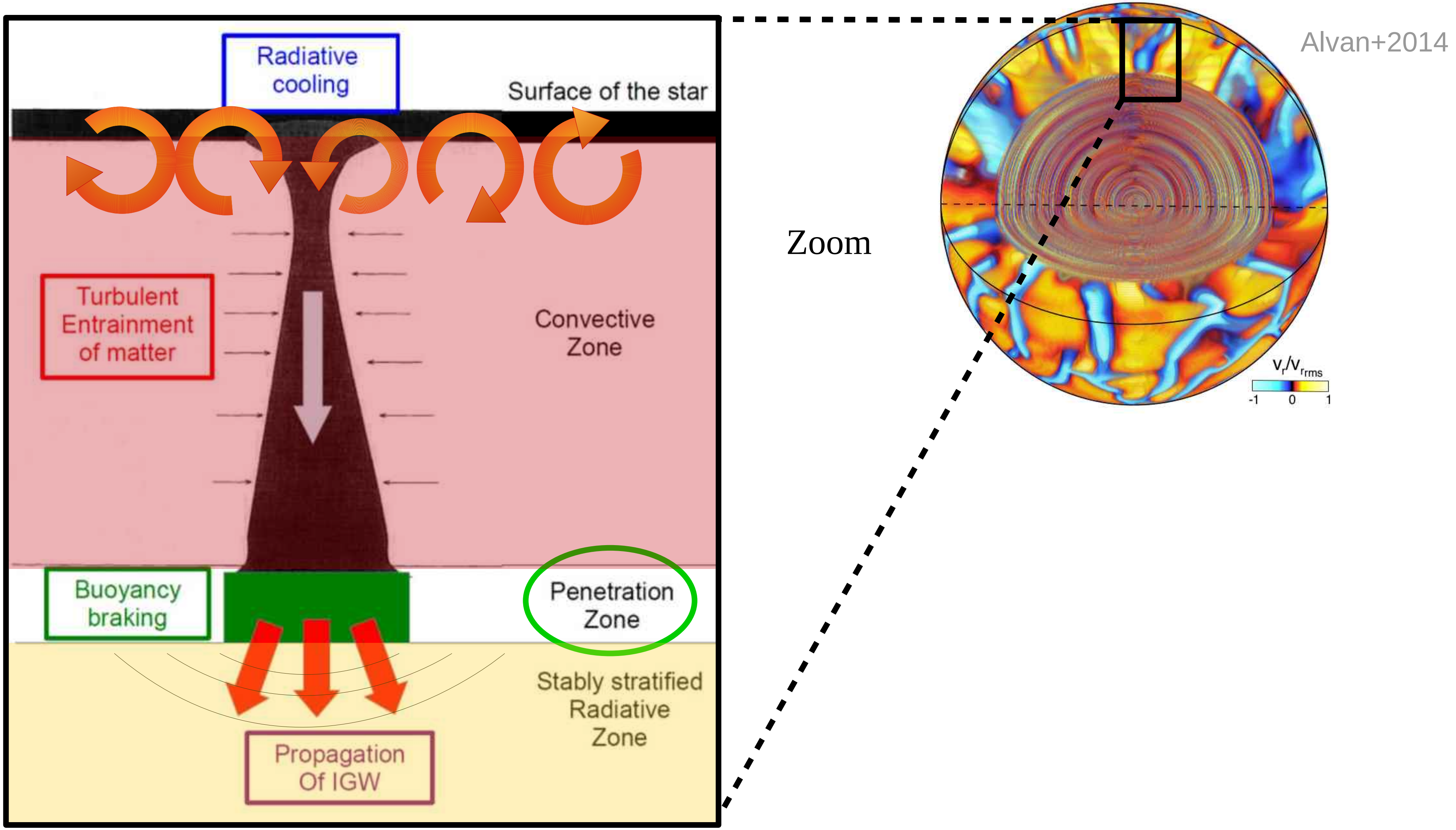
Horizontal « size » of the biggest eddies

Convective energy flux at the BCZ

Convective Mach number at the BCZ = Efficiency

Ex : Sun Mc ~ 10^{-4}

2- Excitation by penetrative convection



Radiative cooling

Surface of the star

Turbulent Entrainment of matter

Convective Zone

Buoyancy braking

Penetration Zone

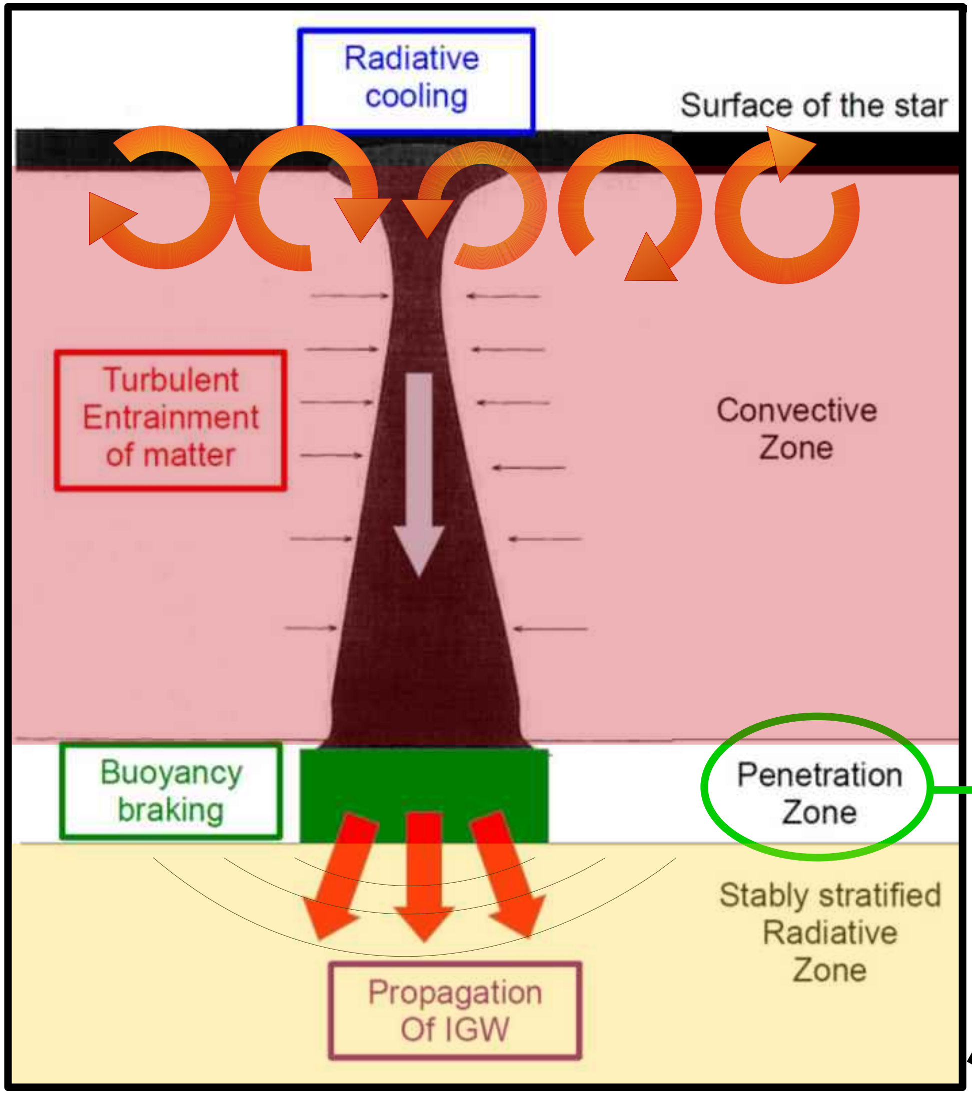
Stably stratified Radiative Zone

Propagation Of IGW

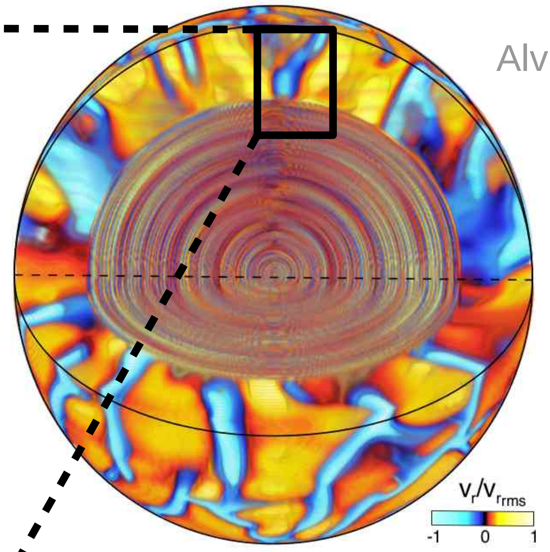
Zoom

Alvan+2014

2- Excitation by penetrative convection



Zoom



Penetration zone in the Sun:

Péclet number: Pe~ 10^5 -10^7

⇒ the plume keeps its thermal identity

⇒ Efficient buoyancy braking!

To the contrary, in simulations :

Pe ~ 1-10^2⇒ the plume is more rapidly mixed

⇒ Weak buoyancy braking, lack of realism ...

2- Excitation by penetrative convection

- **Driving = Ram pressure exerted by an ensemble of incoherent plumes** (Pinçon+2016)

  - Momentum equation: $\rho\, \partial_t^2 \boldsymbol{\xi} + \boldsymbol{\nabla} p' - \rho' \boldsymbol{g} + \rho \boldsymbol{\nabla} \psi' = -\boldsymbol{\nabla} \cdot (\rho \boldsymbol{\mathcal{V}}_{\rm p} \otimes \boldsymbol{\mathcal{V}}_{\rm p})$
  - Radial plume velocity $\boldsymbol{\mathcal{V}}_{\rm p}\ (\boldsymbol{r}, t) = f\left(\frac{t}{\tau_{\rm p}}\right) \mathcal{V}_r(r)\, e^{-S_h^2/2b^2}\, \boldsymbol{e}_r$

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Gaussian plume time profile

$$f_{\rm G}\left(\frac{t}{\tau_{\rm p}}\right) \equiv e^{-t^2/\tau_p^2}$$

Radial profile

Rieutord=1995 in CZ

Zahn 1991 in penetration zone

Gaussian horizontal profile

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Gaussian horizontal profile

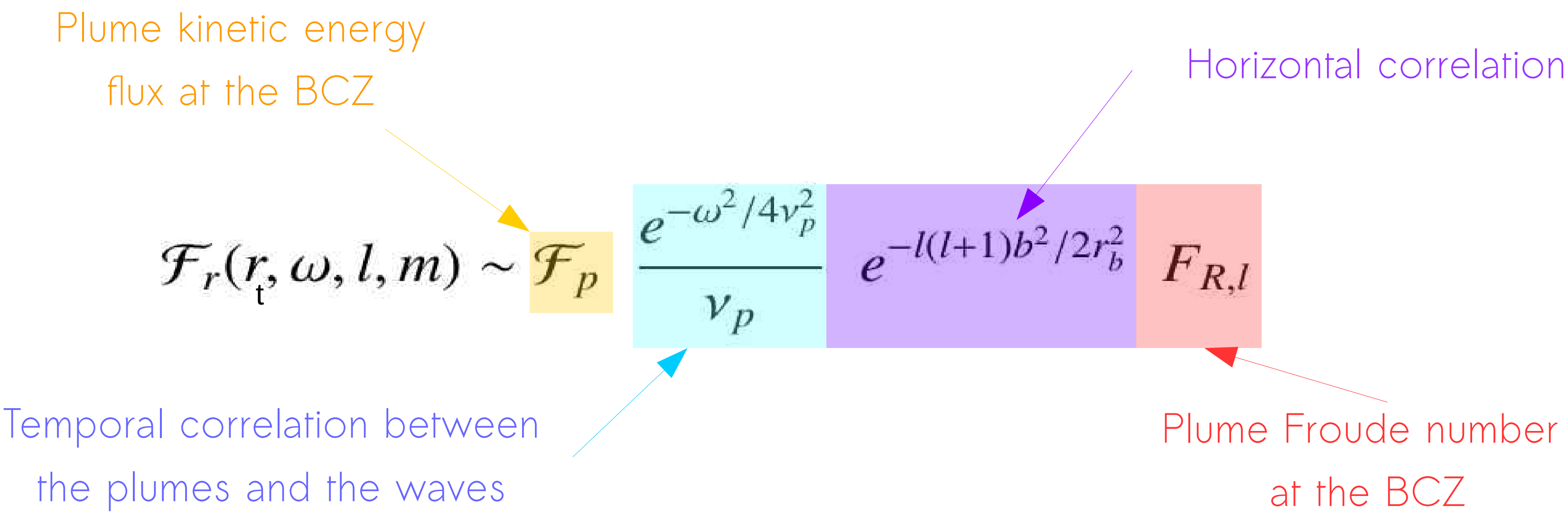
- ✓ **No heat exchange during the excitation (adiabatic approximation)**
  → Ram pressure dominates on the plume lifetime.

- ✓ **Excitation assumed stationary, ergodic and uniform horizontally**
  → Statistical approach: semi-analytical approximation of the wave energy flux.

2- Excitation by penetrative convection

- Outgoing flux at the BCZ as a function of pulsation **ω** and angular degree *l*

(Pinçon 2016)



Plume kinetic energy flux at the BCZ

Horizontal correlation

$$\mathcal{F}_r(r_t, \omega, l, m) \sim \mathcal{F}_p \frac{e^{-\omega^2/4\nu_p^2}}{\nu_p} e^{-l(l+1)b^2/2r_b^2} F_{R,l}$$

Temporal correlation between the plumes and the waves

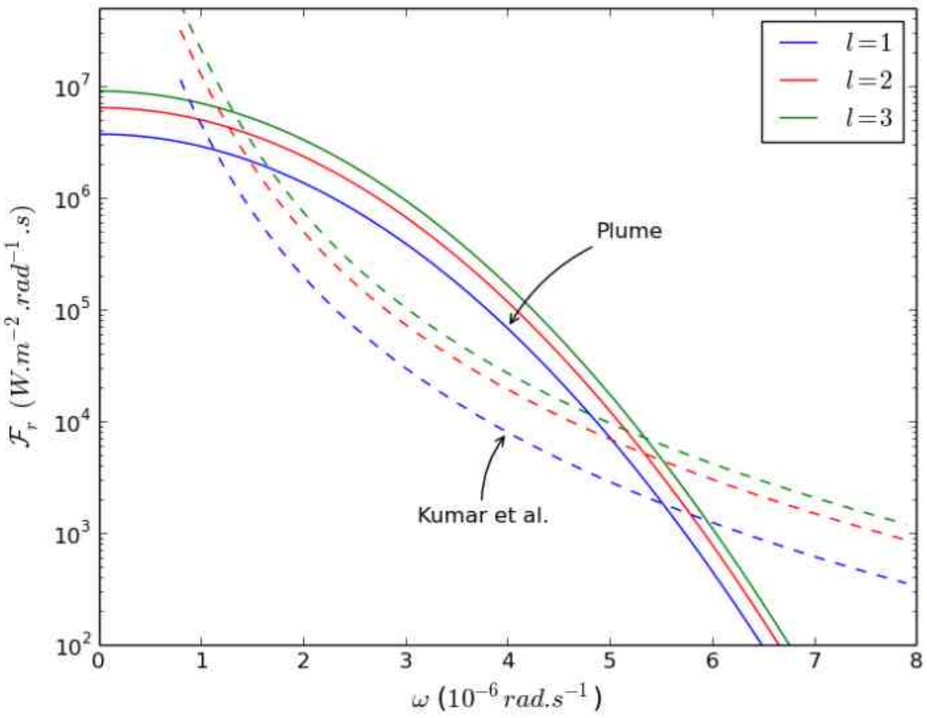
Plume Froude number at the BCZ

- Excitation efficiency

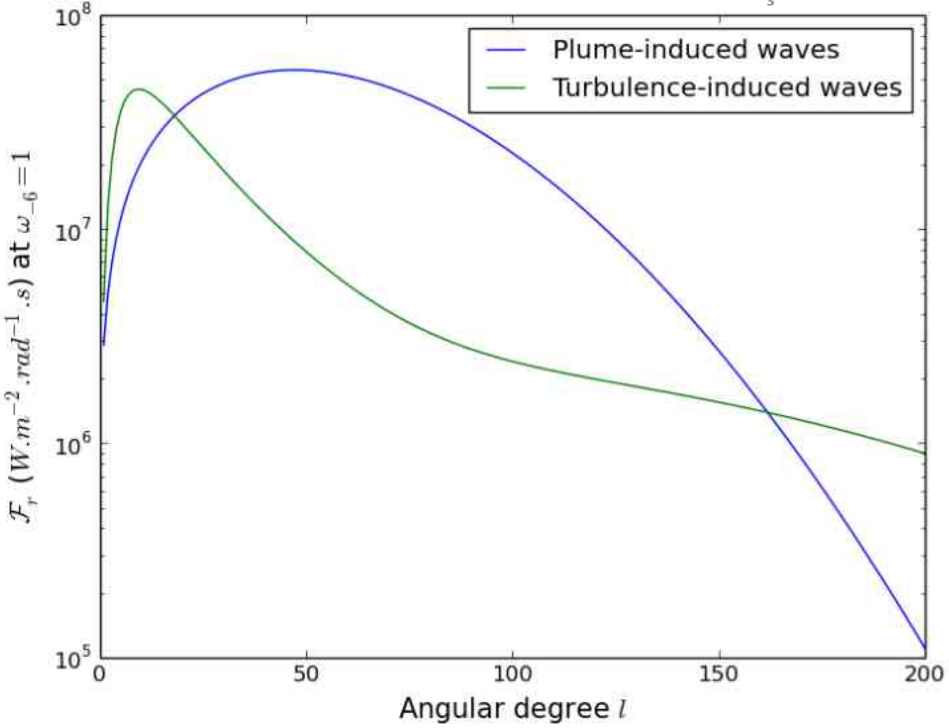
→ Froude number ~ reaction of the medium to the plume penetration.

→ In the Sun, $F_R \sim 10^{-3}$

Penetrative convection vs Reynold stress



Pinçon+2016

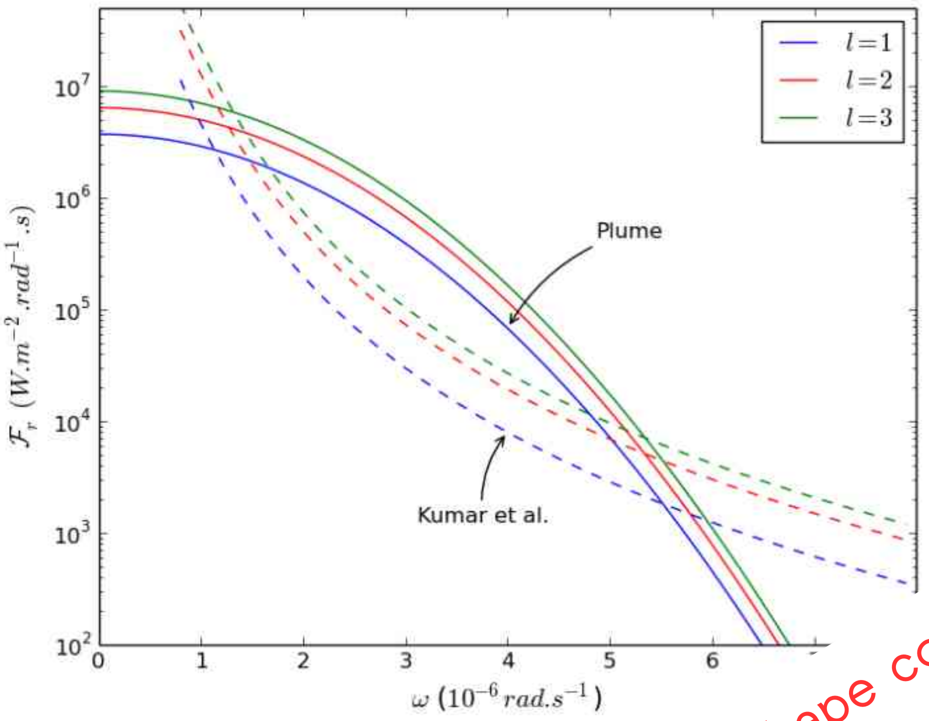


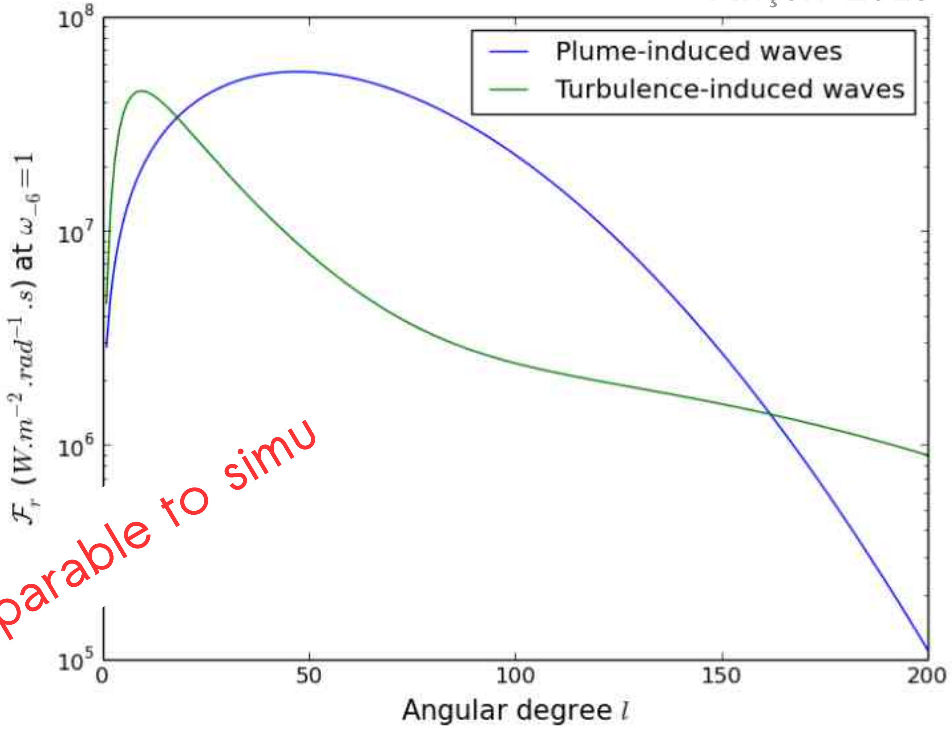
Turbulent pressure vs Penetrative convection :

Shape : decreasing power laws vs Gaussian
Degree at maximum : size of convective eddies > size of plumes at the BCZ
Total flux : ~ 0.1 % vs ~ 0.6 % of the solar energy flux

(Note : the total spectra is a weighted sum of both contributions)

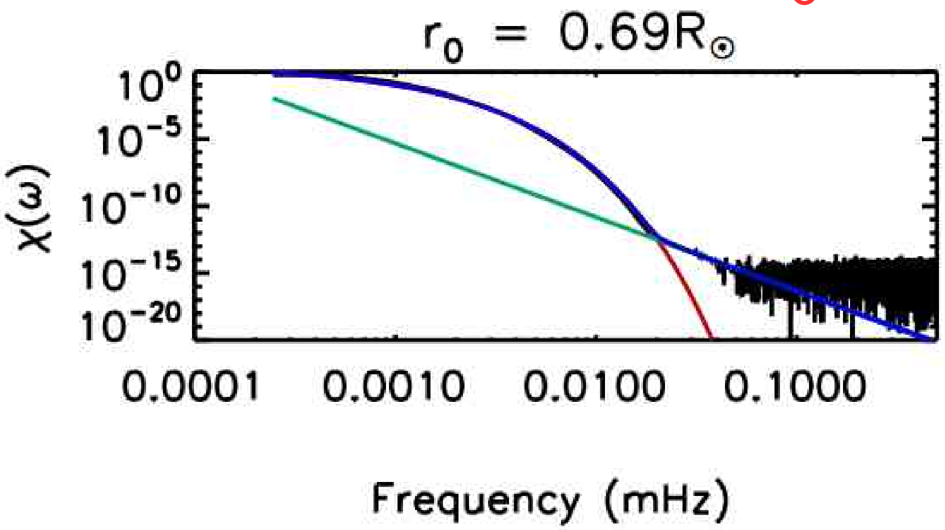
Penetrative convection vs Reynold stress

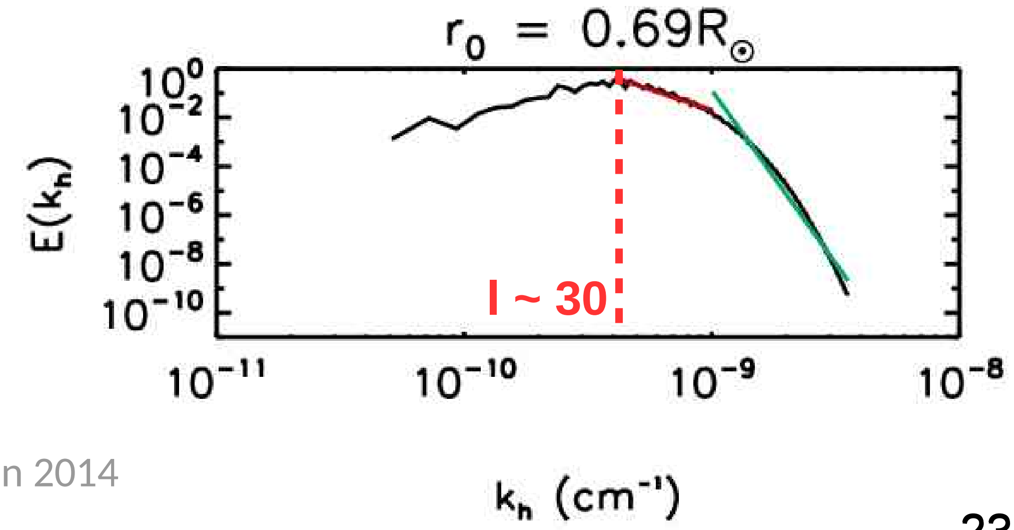




Pinçon+2016

Shape comparable to simu

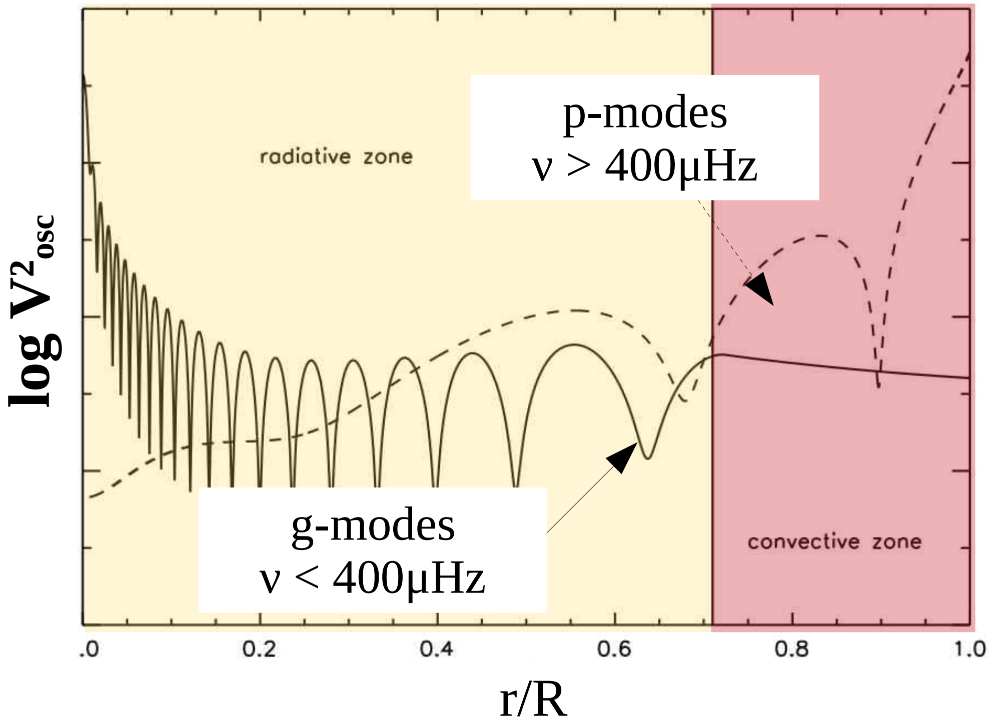




Alvan 2014

Detectability: the quest of the g-modes.

Belkacem 2011



- Resonant gravity modes = potential diagnostic of the solar core:

  - Core stratification ⇒ complementarity with neutrinos (e.g., Salmon+2021)

    ⇒ constraints on metallicity, nuclear reactions, electron screening

  - Constraint on the angular momentum redistribution (e.g., Eggenberger 2019)

Still no robust detection

- ✓ Evanescent, very small amplitudes at the solar surface
    - Several claims of detection, but no confirmation (e.g., Brookes+1976, Severnyi+1976, Delache+1983, Thomson+1995, Turck-Chièze+2004, Garcia+2007)

- ✓ Most recent claim by Fossat+2017,2018
    - Search for the signature of g-modes in the p-mode spectrum
    - Not reproduced (Schunker2018, Appourchaux 2019)
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  - p- and g-mode coupling too small (Scherrer 2019, Böning 2019)
- ✓ Theoretical estimates of the mode amplitudes are thus useful to
  - Guide observational strategies and future instrument design
  - When detected, diagnostics of the excitation and damping mechanisms

Semianalytical estimates

- Mean g-mode amplitude = balance driving/damping

$$\frac{\mathrm{d}E_{\mathrm{osc}}}{\mathrm{d}t}(t) = \underset{\underset{\text{Driving}}{\uparrow}}{\mathcal{P}} - 2\,\underset{\underset{\text{Damping rate}}{\uparrow}}{\eta}\,E_{\mathrm{osc}}(t)$$

$$\underbrace{\overline{\frac{\mathrm{d}E_{\mathrm{osc}}}{\mathrm{d}t}(t)} = 0}_{\text{Stationary over long timescale}} \Longrightarrow \overline{E}_{\mathrm{osc}} = \frac{\overline{\mathcal{P}}}{2\eta}$$

- Most recent estimates (see also Goldreich+1977, Gough+1985)

1) ν < 100 μHz: radiative damping, analytical

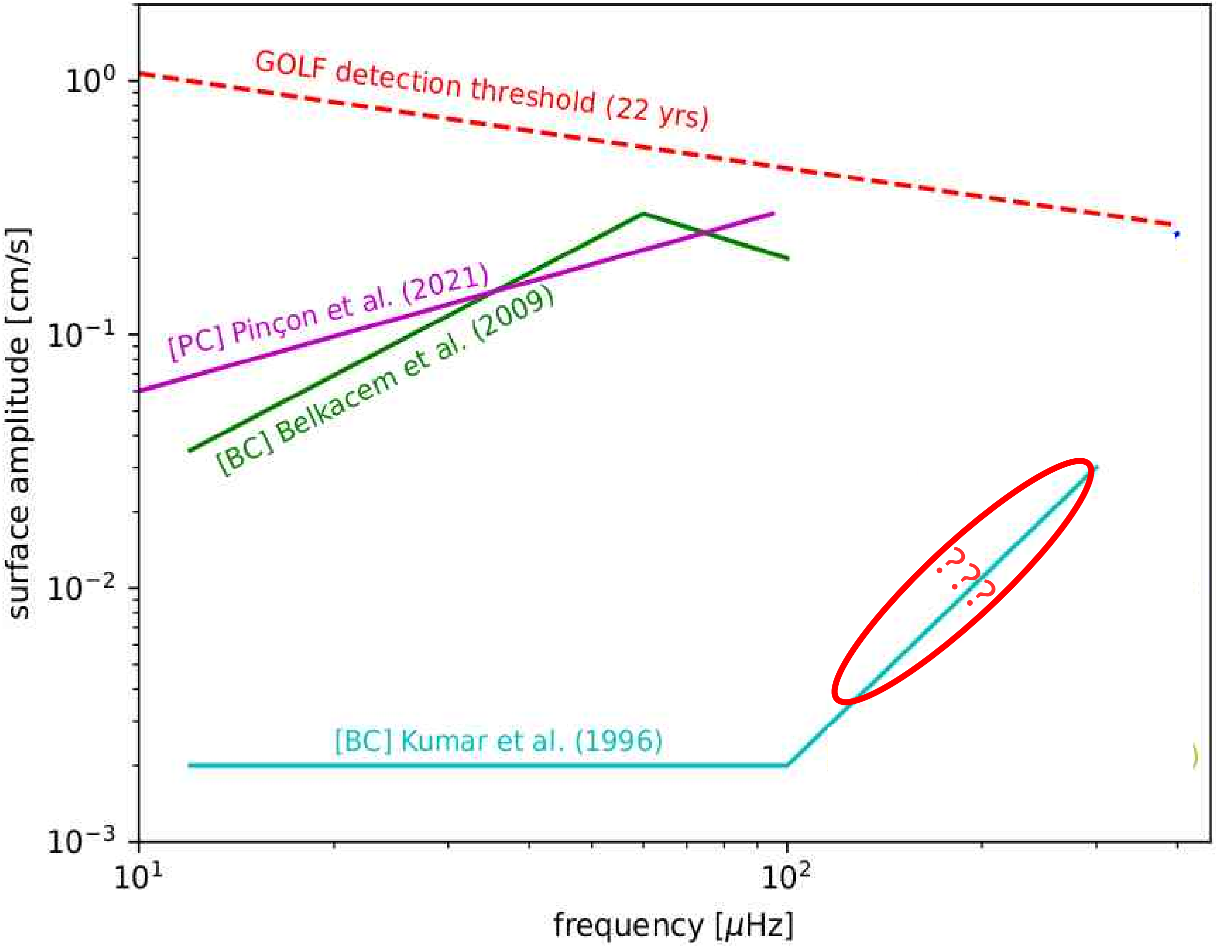
  - Kumar+1996: Reynold stress, Gaussian eddy-time correlation.
  - Belkacem+2011: Reynold stress, Lorentzian eddy-time correlation (simus 3D).
  - Pinçon+2021: Penetrative convection, exponential plume time profile

$$f_{\mathrm{E}}\left(\frac{t}{\tau_{\mathrm{p}}}\right) \equiv e^{-|t|/\tau_p}$$

2) ν > 100 μHz: mode-convection interaction dominates damping, uncertain ...

Semianalytical estimates

Belkacem 2022



A new estimate for the solar mixed modes

- **ν > 200 μHz:** Same properties as low-order acoustic radial modes in the envelope
  - → Assumption: similar driving and damping for close frequencies
  - → Their amplitudes are related

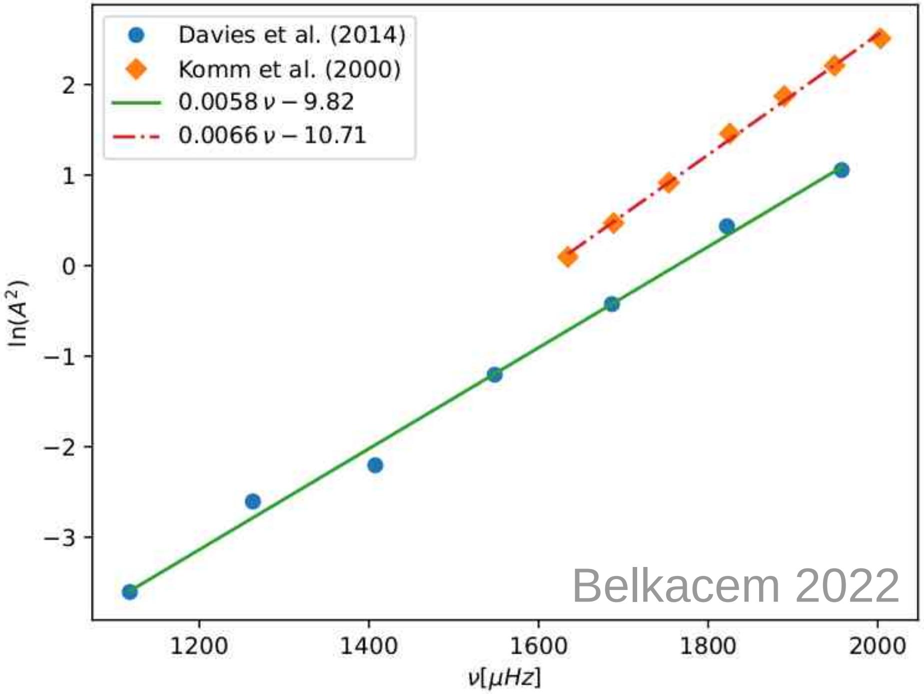
Mixed

Radial

$$\frac{A_{\rm m}^2}{A_{\rm ref}^2} = \frac{\mathcal{M}_{\rm ref}}{\mathcal{M}_{\rm m}},$$

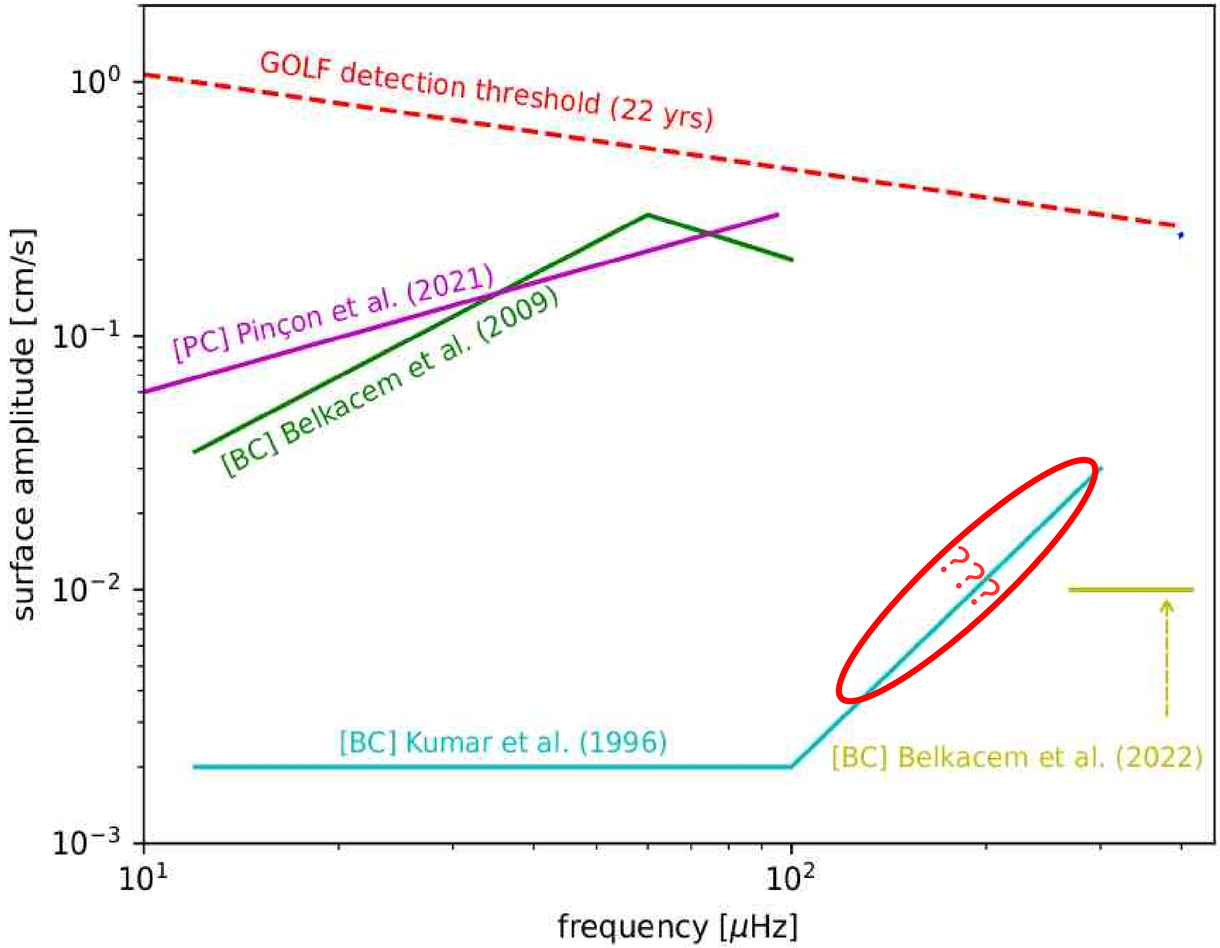
- Estimate of the radial mode amplitude in 400 **μ**Hz > **ν** > 200 **μ**Hz

  → Extrapolation from the observed low-frequency radial modes (1mHz > **ν** > 2mHz)



g-modes amplitudes: current status

Belkacem 2022



- ✓ Low frequency domain (10 **μ**Hz < **ν** < 100 **μ**Hz) more suited to detect g-modes
- ✓ Theoretical uncertainties ~ gap with the detection threshold (e.g., GOLF)

→ Need for improvement in the description of convection.

IGW transport in current stellar models

✓ A quite “simple” formalism in a first step

- **Internal structure:** - Spherical (1D), shellular rotation $\Omega(r,\theta) \sim \Omega(r)$ (Zahn 1992)

- **Propagation:** - Horizontally-averaged radial wave flux
  - Effect of frequency Doppler-shift
  - No Coriolis force/rotation gradient effect

- **Excitation/damping:** - Available Reynold stress / penetrative convection models
  - Quasi-adiabatic damping (Press 1981)

IGW transport in current stellar models

- Angular momentum transport equation

$$\varrho \frac{\partial}{\partial t}(r^2\overline{\Omega})_{M_r} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}(\varrho r^4 \overline{\Omega} U_2(r))}_{\text{Meridional circulation}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\varrho D_{\mathrm{v}} r^4 \frac{\partial\overline{\Omega}}{\partial r}\right)}_{\text{Shear turbulence}} + \underbrace{\frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathcal{F}_{J,\mathrm{w}}(r)}_{\text{IGW}}$$

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$$\mathcal{F}_{J,\mathrm{w}}(r) = \sum_{l}\sum_{m=-l}^{m=+l}\int_{-\infty}^{+\infty}\frac{m}{\omega}\frac{r_{\mathrm{t}}^2}{r^2}\mathcal{F}_{E,\mathrm{w}}(r_{\mathrm{t}},\omega,l,m)\,\mathrm{e}^{-\tau(r,\hat{\omega},l)}\,\mathrm{d}\omega,$$

Flux at the top of the radiative zone

Radiative damping

with

$$\tau(r,\hat{\omega},l) = [l(l+1)]^{3/2}\int_r^{r_{\mathrm{t}}} K\frac{N N_{\mathrm{T}}^2}{\hat{\omega}^4}\left(\frac{N^2}{N^2-\hat{\omega}^2}\right)^{1/2}\frac{\mathrm{d}r}{r^3},$$

Radiative diffusivity **Wave intrinsic frequency**

$\hat{\omega}(r,\omega,m) = \omega - m\delta\Omega(r)$ and $\delta\Omega = \Omega(r) - \Omega_{BCZ}$

IGW transport in current stellar models

- Angular momentum transport equation

$$\varrho \frac{\partial}{\partial t}(r^2\overline{\Omega})_{M_r} = \underbrace{\frac{1}{5r^2}\frac{\partial}{\partial r}(\varrho r^4 \overline{\Omega} U_2(r))}_{\text{Meridional circulation}} + \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(\varrho D_v r^4 \frac{\partial \overline{\Omega}}{\partial r}\right)}_{\text{Shear turbulence}} + \underbrace{\frac{3}{8\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\mathcal{F}_{J,\mathrm{w}}(r)}_{\text{IGW}}$$

$$\mathcal{F}_{J,\mathrm{w}}(r) = \sum_{l}\sum_{m=-l}^{m=+l}\int_{-\infty}^{+\infty}\frac{m}{\omega}\frac{r_t^2}{r^2}\mathcal{F}_{E,\mathrm{w}}(r_t,\omega,l,m)\,\mathrm{e}^{-\tau(r,\hat{\omega},l)}\,\mathrm{d}\omega,$$

Flux at the top of the radiative zone

Radiative damping

with

$$\tau(r,\hat{\omega},l) = [l(l+1)]^{3/2}\int_{r}^{r_t} K\frac{N N_T^2}{\hat{\omega}^4}\left(\frac{N^2}{N^2-\hat{\omega}^2}\right)^{1/2}\frac{\mathrm{d}r}{r^3},$$

Radiative diffusivity

Wave intrinsic frequency

$$\hat{\omega}(r,\omega,m) = \omega - m\delta\Omega(r) \quad \text{and} \quad \delta\Omega = \Omega(r) - \Omega_{BCZ}$$

- Transport results from 3 ingredients: Damping + Excitation + Rotation contrast

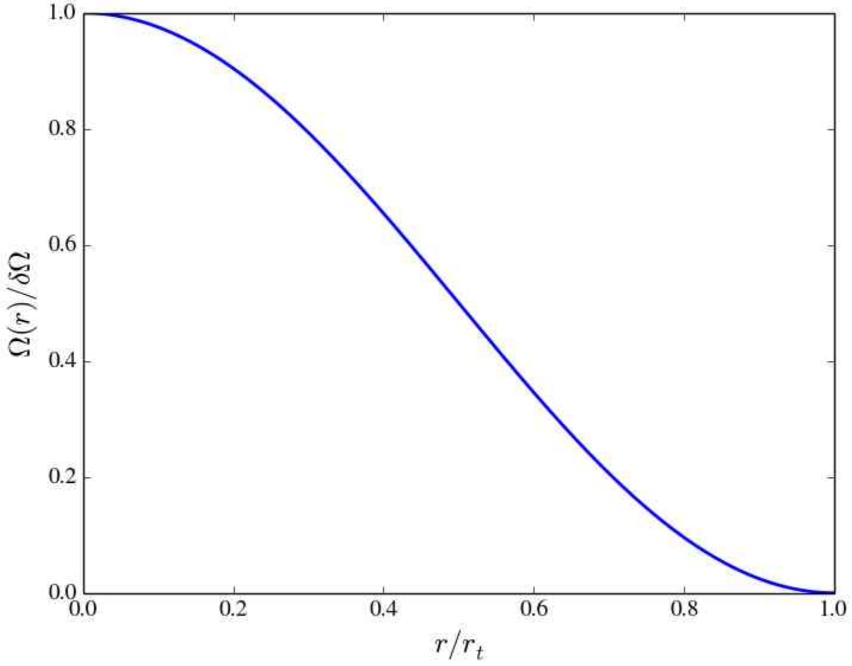
Transport efficiency : a simple estimate

Get a stellar model…

Assume a given rotation profile…

- assumed smooth here.
- varying amplitude of the rotation contrast :

$$\delta\Omega = \Omega_{core} - \Omega_{BCZ}$$



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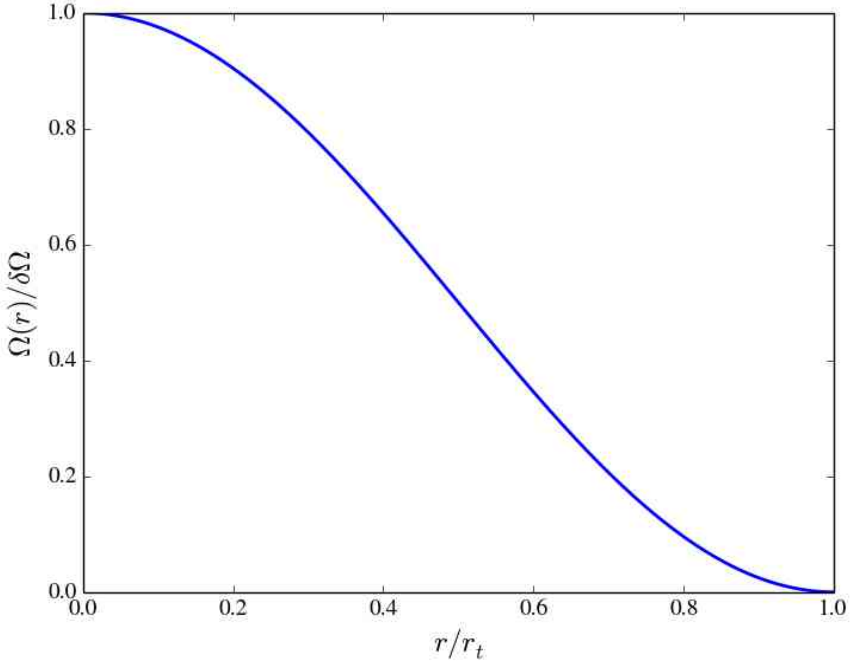

... and compute (at a given time)

$$T_L \sim \frac{\rho r^2 \Omega}{-\nabla.(F_{J,w})}$$

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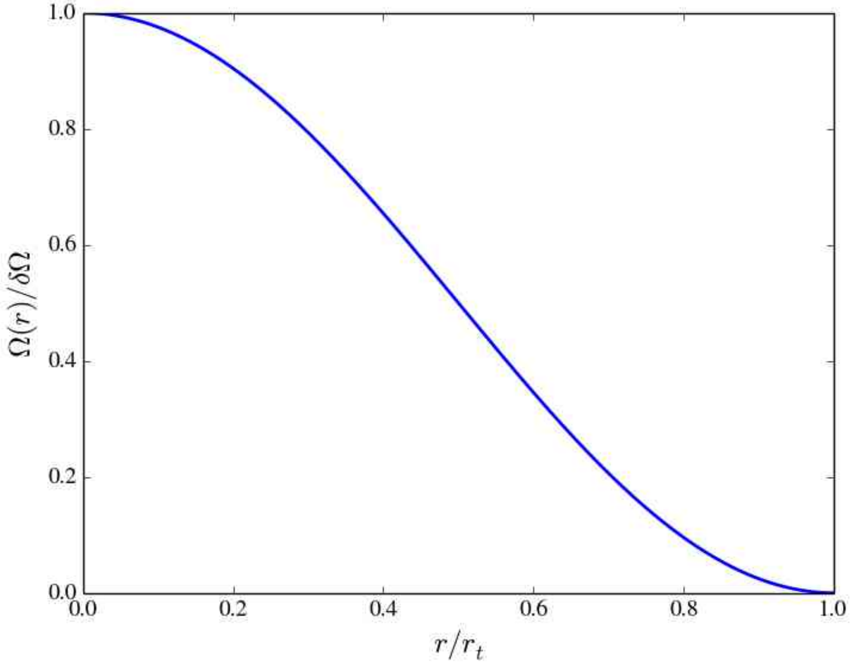


... and compute (at a given time)

Local timescale on which IGW can modify the rotation

$$T_L \sim \frac{\rho r^2 \Omega}{-\nabla.(F_{J,w})}$$

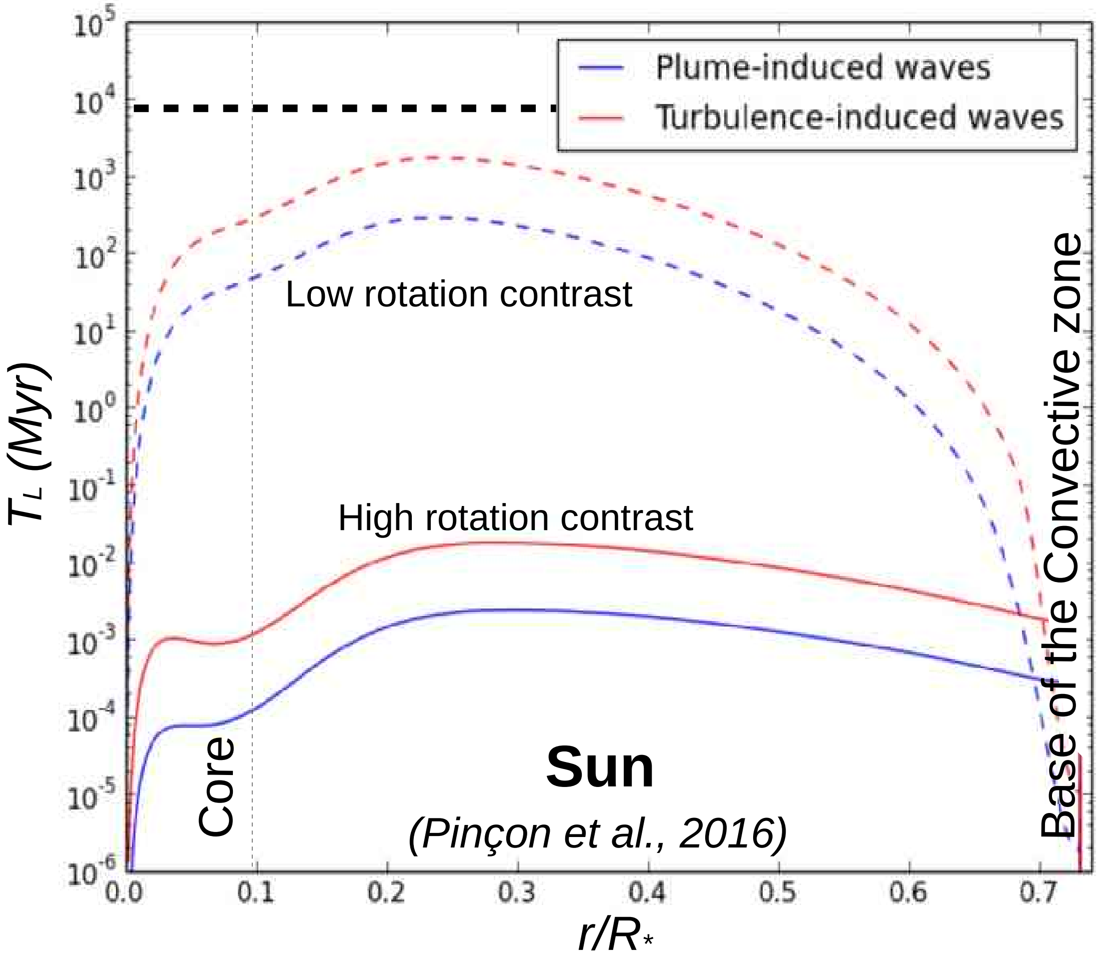
Angular momentum density

Divergence of the wave flux

➨ Criterion : IGW are efficient if **T_L** < **Evolution/contraction timescale.**

Transport efficiency in the solar case

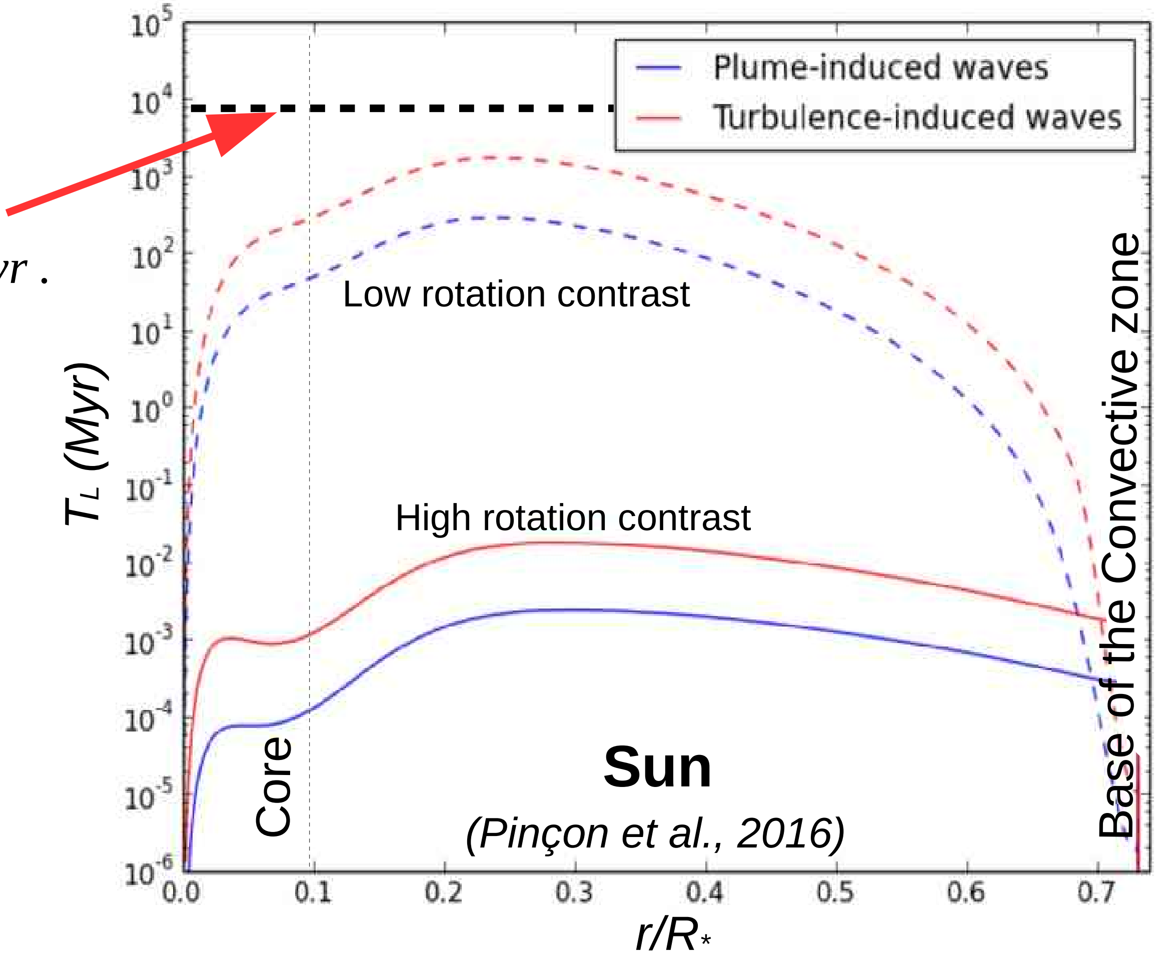
In the Sun,



Transport efficiency in the solar case

In the Sun,

- *...IGW can modify rotation on* $T_L < 10\text{Gyr}$ *.*
- *...plume-induced IGW are more efficient than turbulence-induced IGW.*
- *The higher the rotation contrast, the more efficient the transport by IGW*



(Pinçon et al., 2016)

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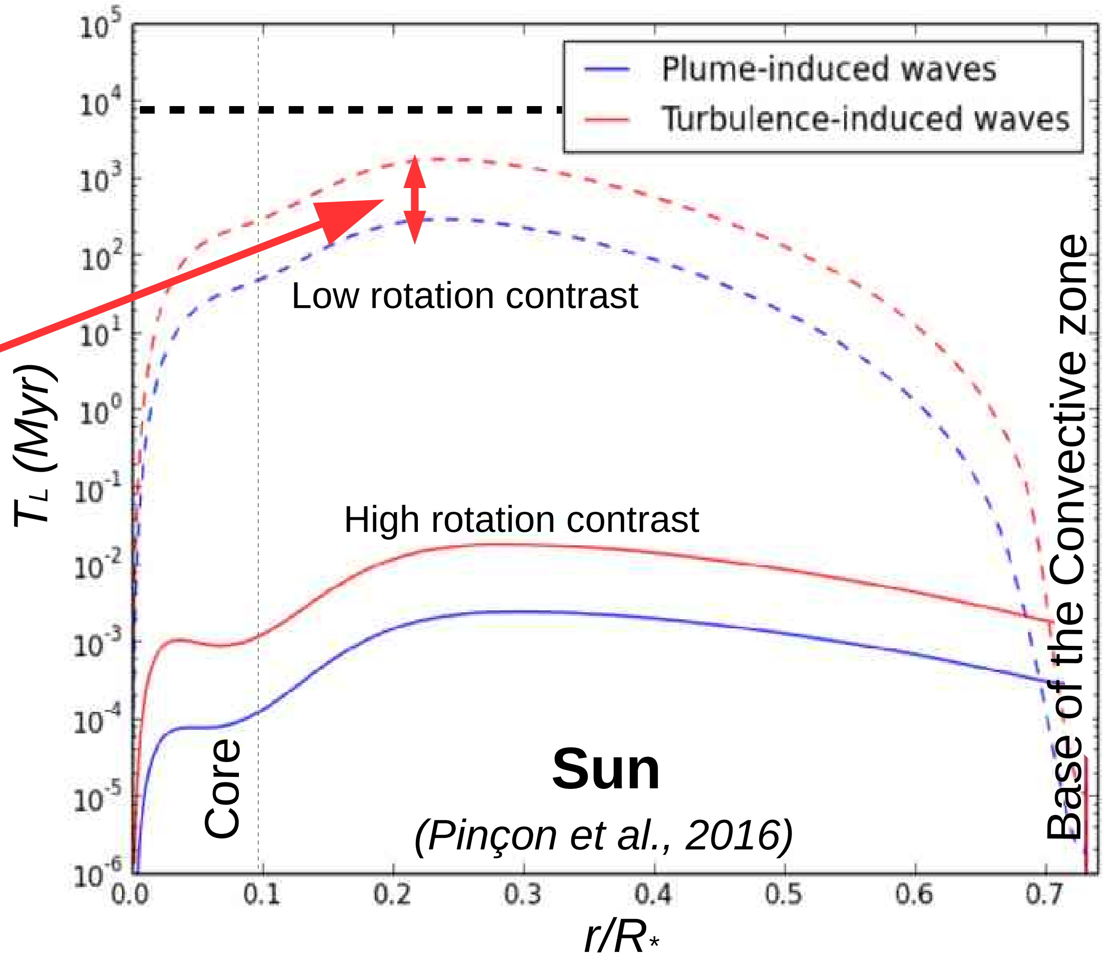

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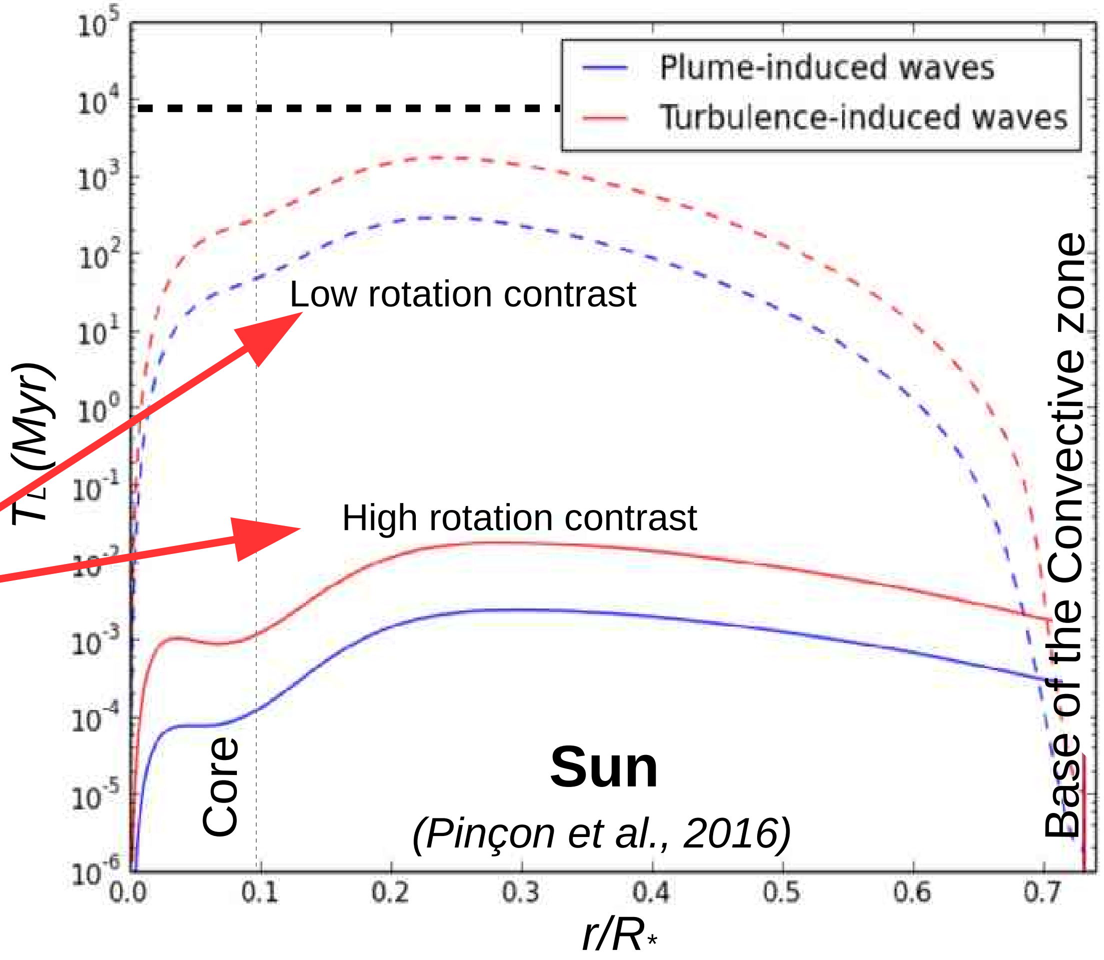

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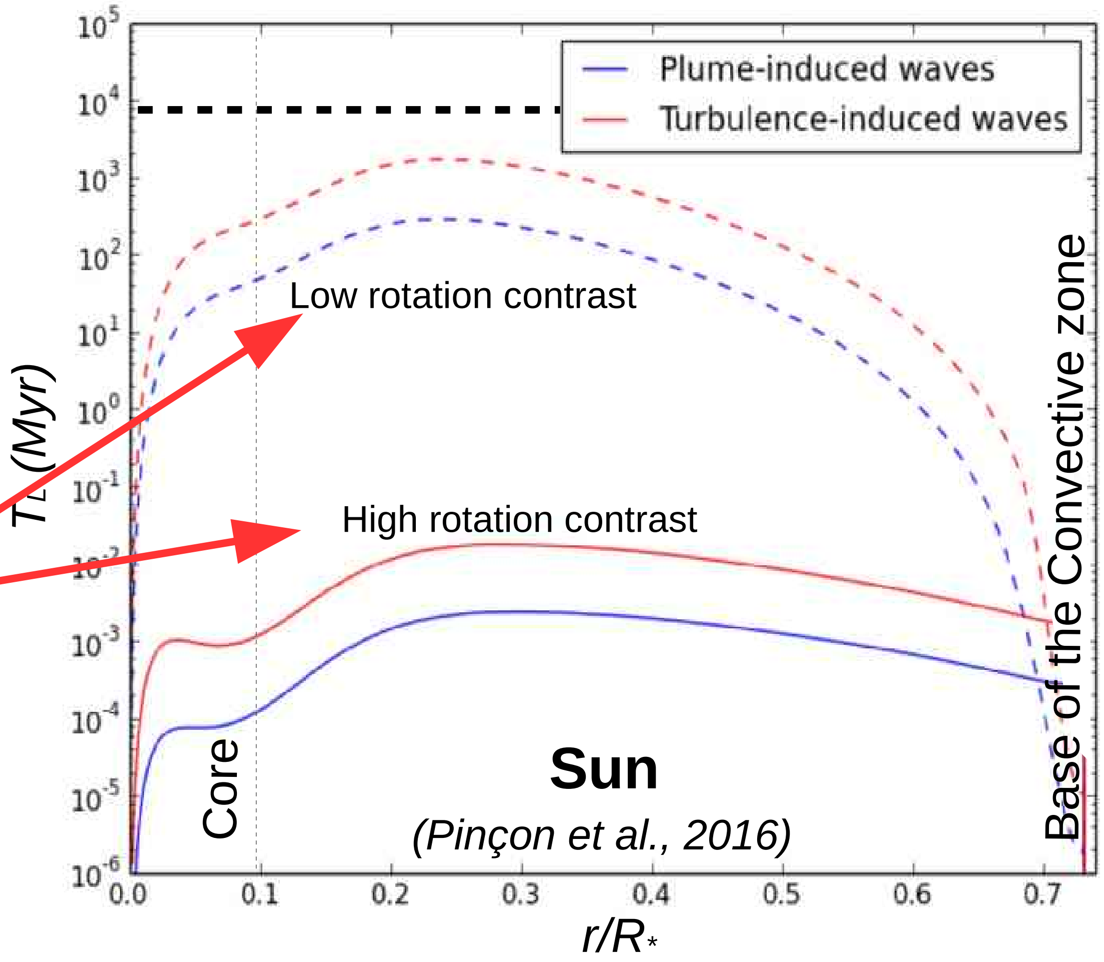
$$\gamma_{damp} \sim (\omega + m\,\delta\Omega)^{-4} \searrow$$



⇒ IGW can efficiently slow down the core if $\delta\Omega > \delta\Omega_{th}$ (~ 0.1μHz in the Sun)

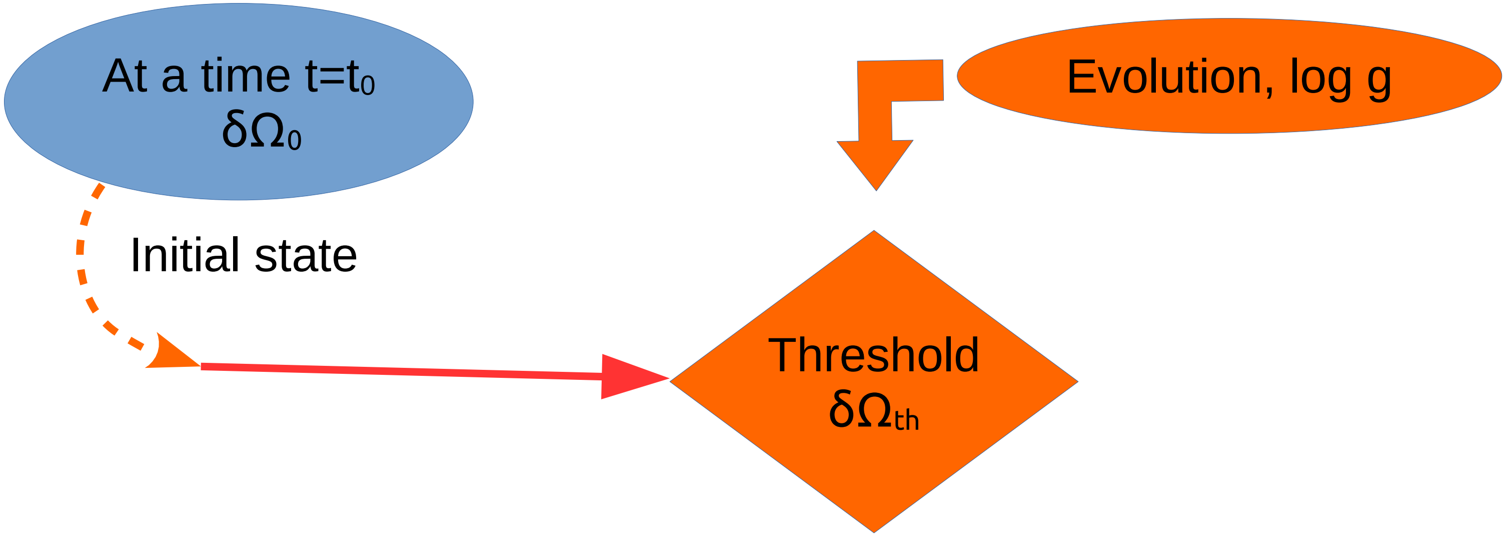
Regulation mechanism driven by IGW ?

At a time $t=t_0$

$\delta\Omega_0$

Initial state

Regulation mechanism driven by IGW ?



At a time $t=t_0$

$\delta\Omega_0$

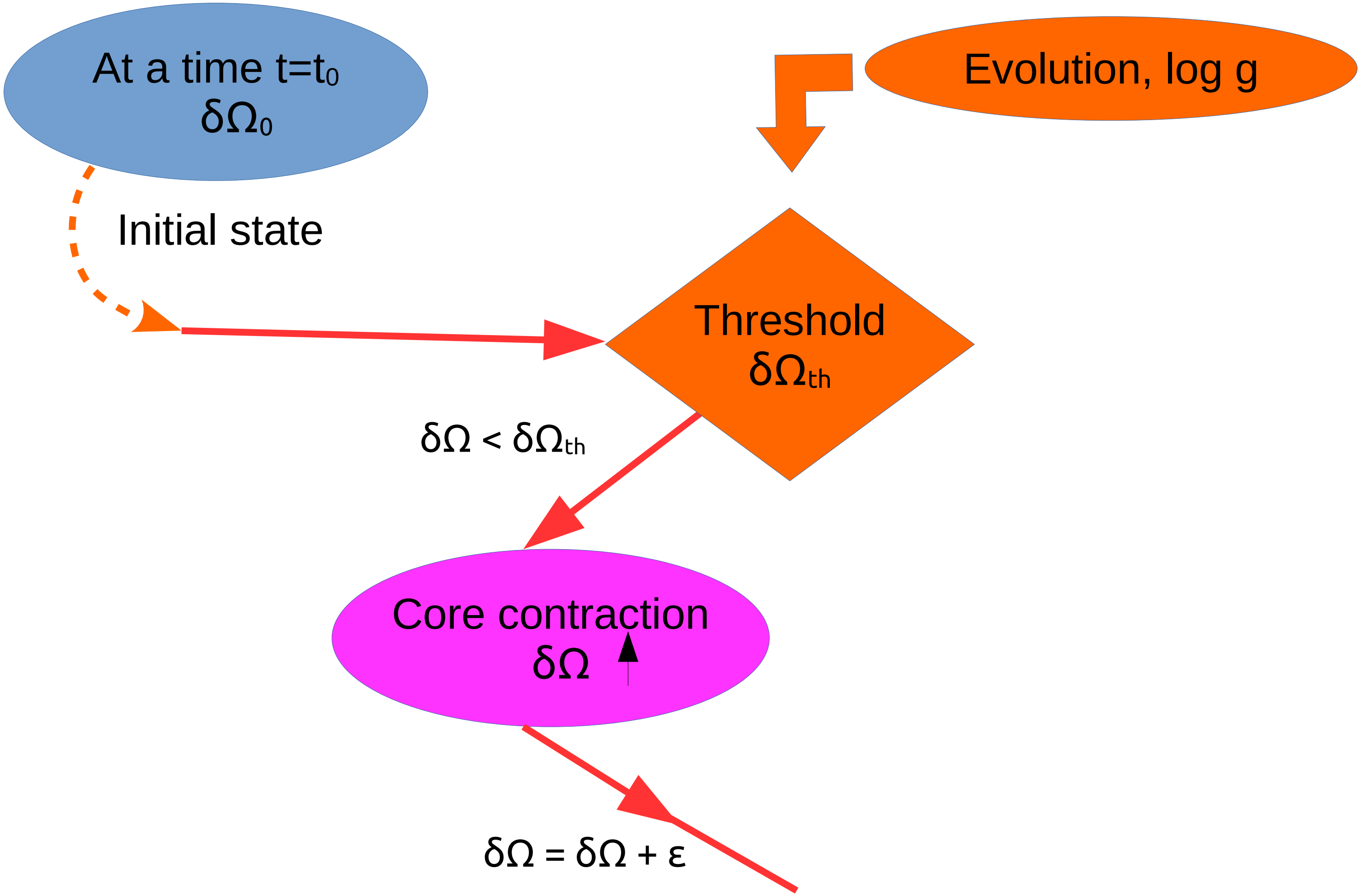
Initial state

Evolution, log g

Threshold

$\delta\Omega_{th}$

Regulation mechanism driven by IGW ?



At a time t=t_0
$\delta\Omega_0$

Evolution, log g

Initial state

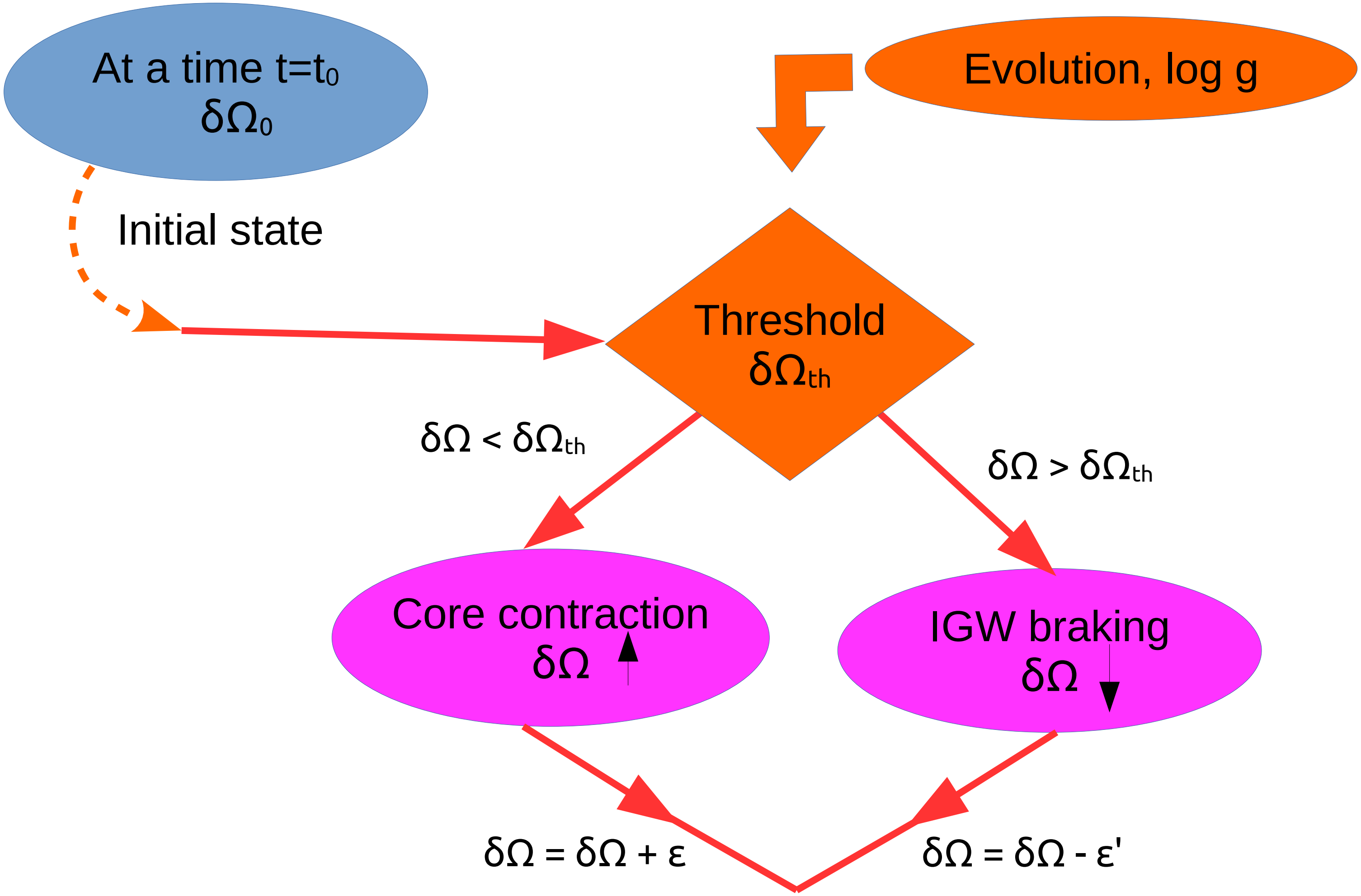
Threshold
$\delta\Omega_{th}$

$\delta\Omega < \delta\Omega_{th}$

Core contraction
$\delta\Omega \uparrow$

$\delta\Omega = \delta\Omega + \varepsilon$

Regulation mechanism driven by IGW ?



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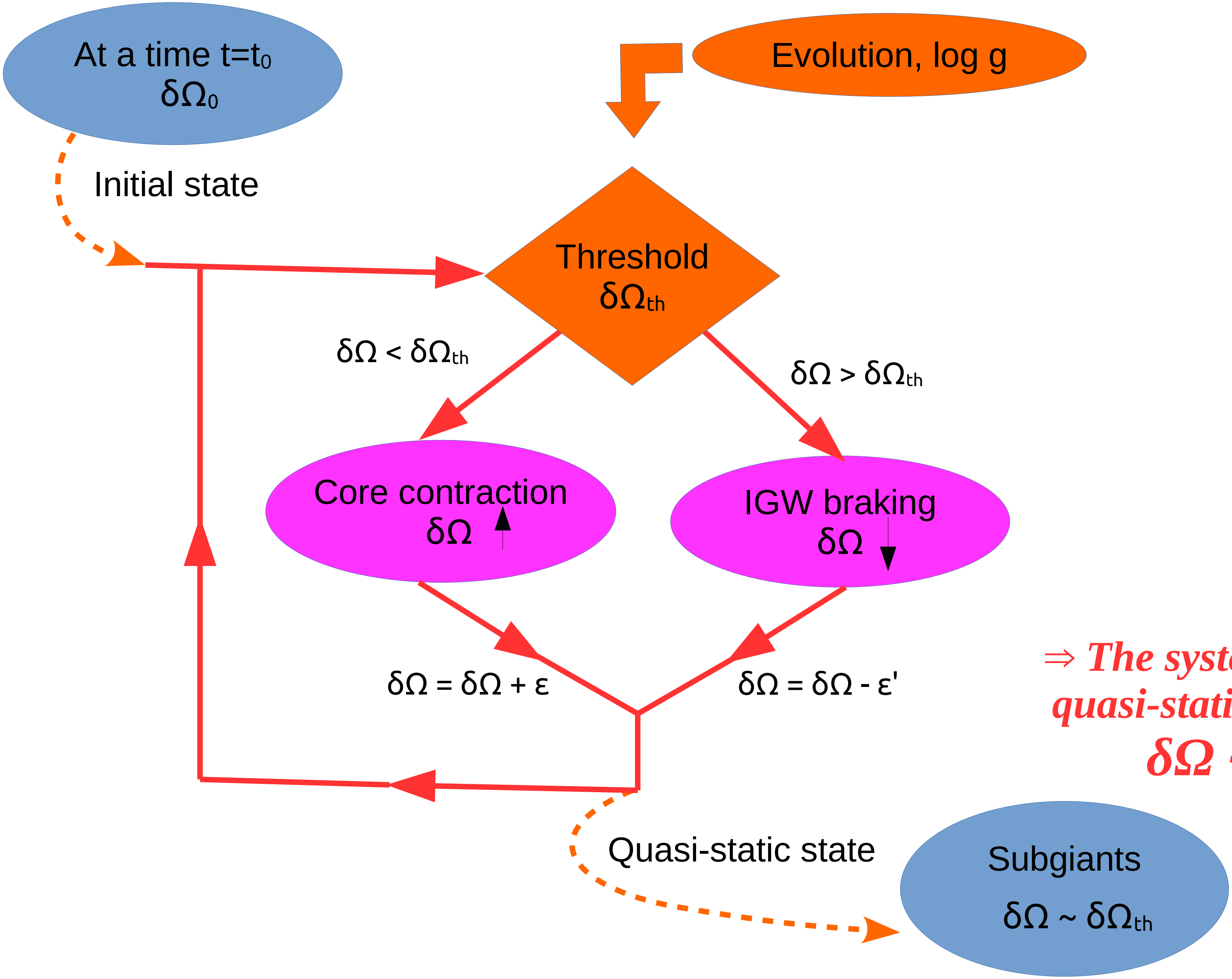


⇒ The system reaches a quasi-static state where $\delta\Omega \sim \delta\Omega_{th}$

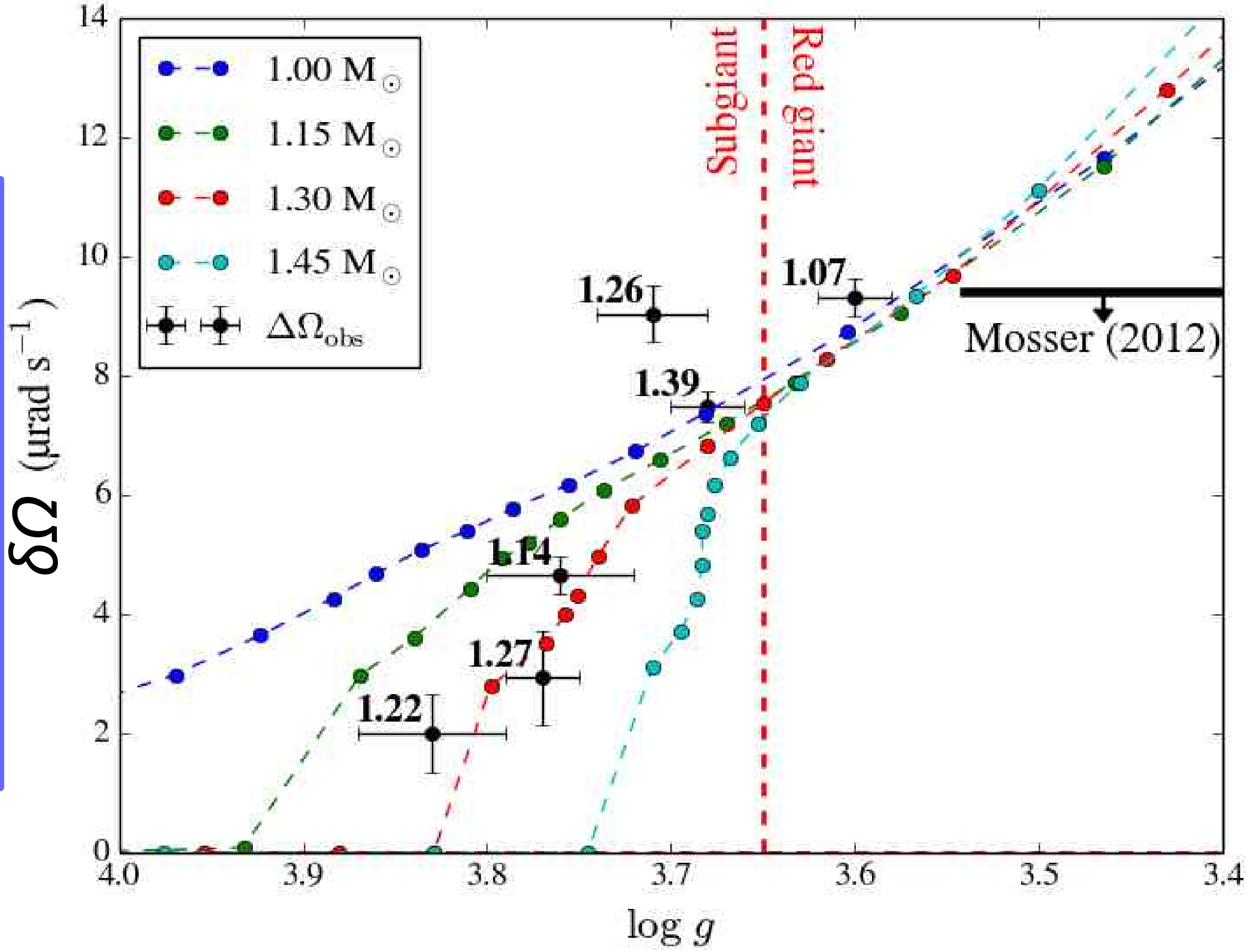
IGW-driven regulation in subgiant stars ?

- Good agreement!

$$\boldsymbol{\delta\Omega_{obs} \sim \delta\Omega_{th}}$$

- $\delta\Omega_{th}$ increases to counterbalance the increase of the radiative damping

$$\gamma_{damp} \propto N^2$$

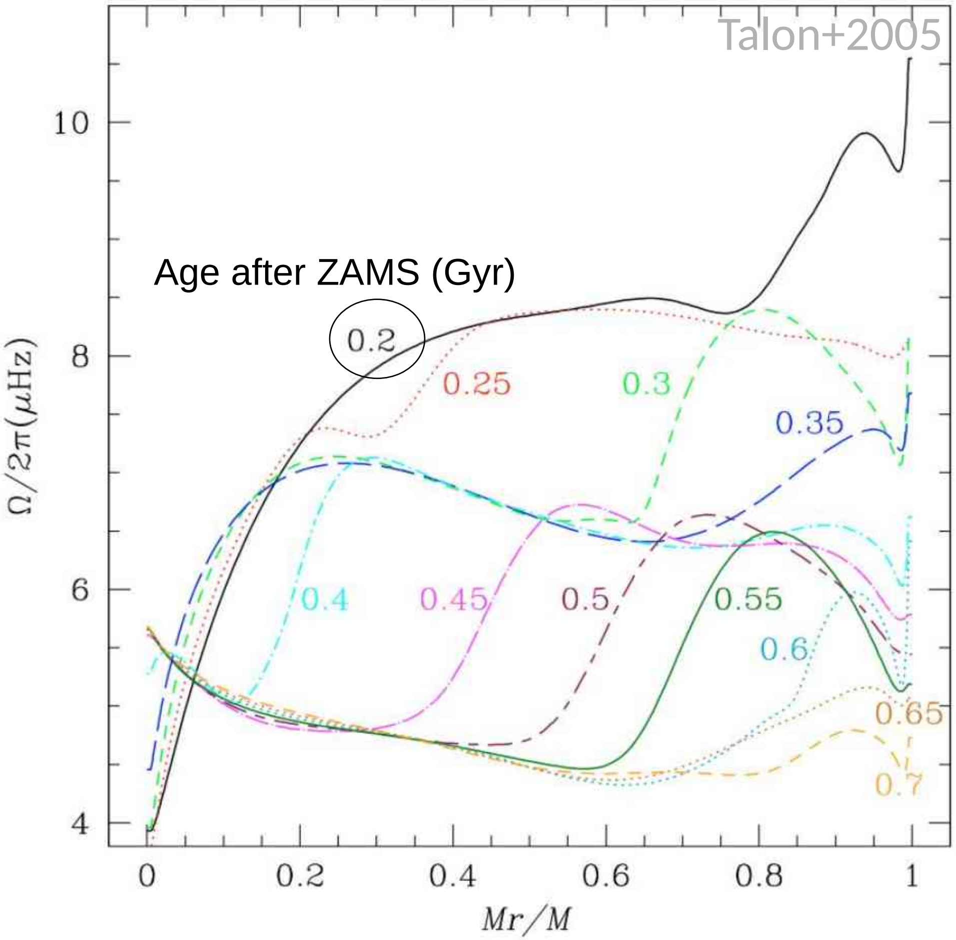


→ Low-frequency IGW can slow down subgiants…

…but still insufficient for Red Giants.

Evolutionary models including transport by IGW

- ✓ **Talon+2005, Charbonnel+2005, Mathis+2013**
  - → Account for shear turbulence + meridional circulation + IGW.
  - → Can somehow explain the "flat" solar rotation profile.



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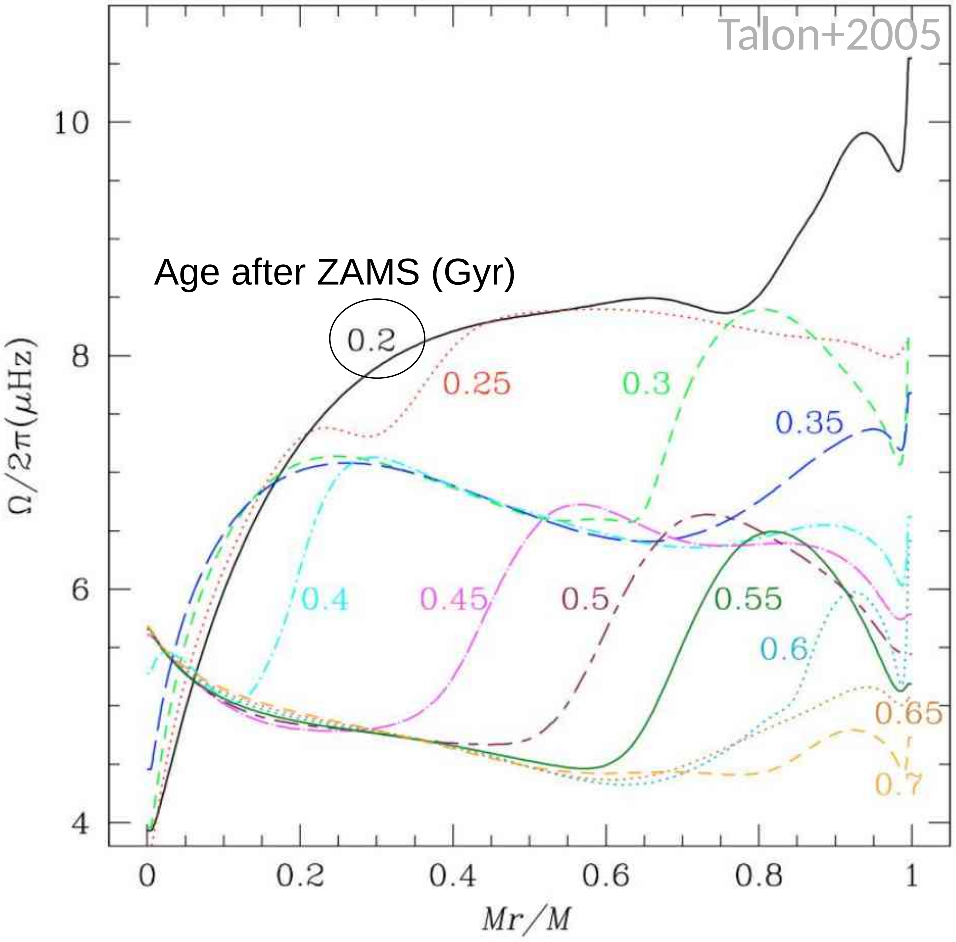

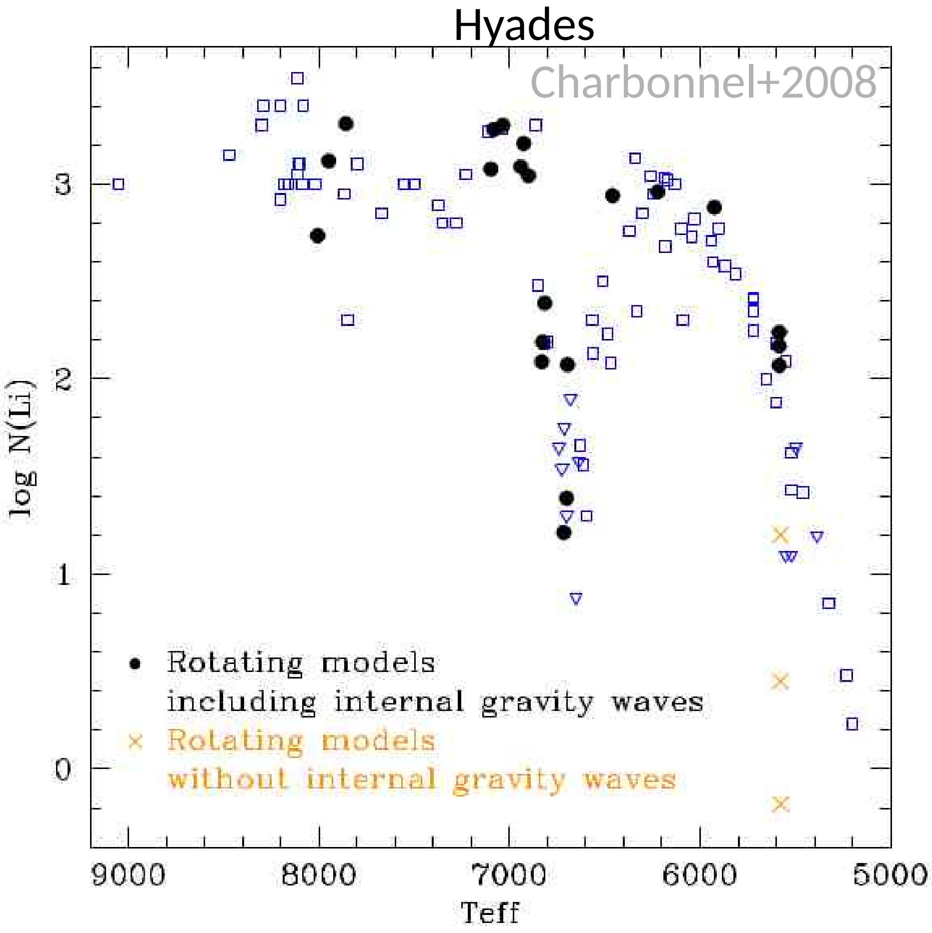

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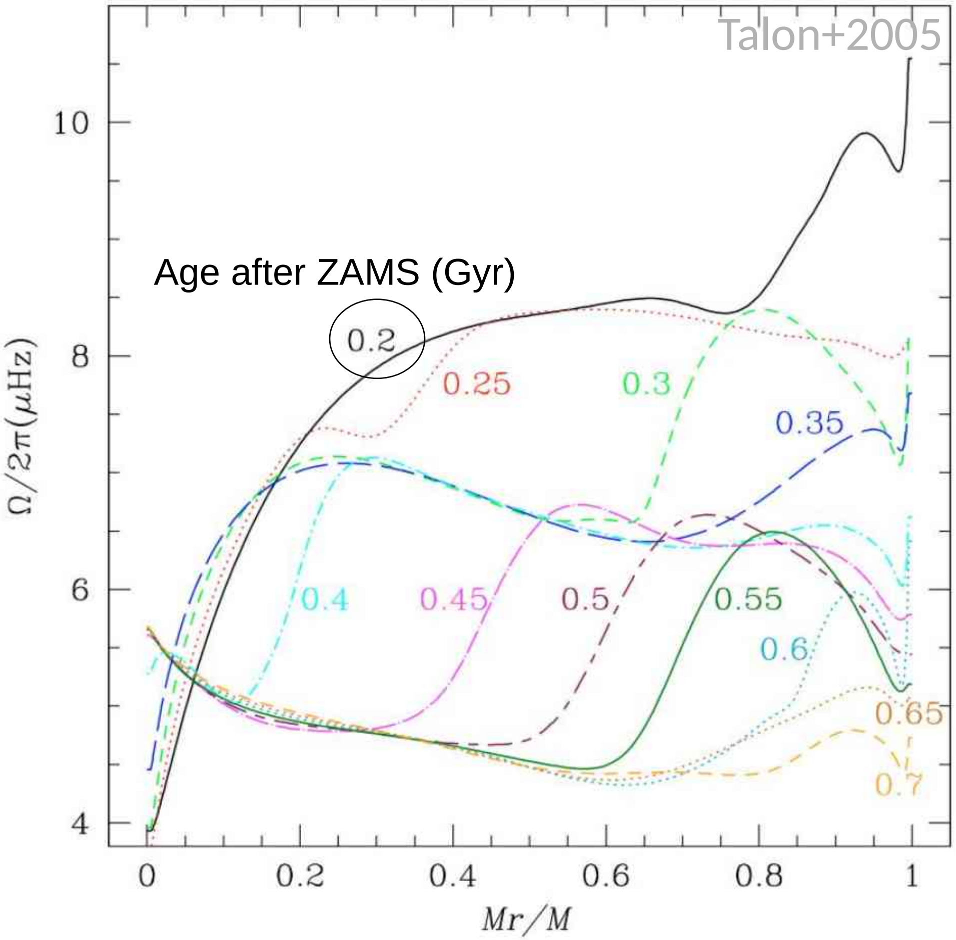

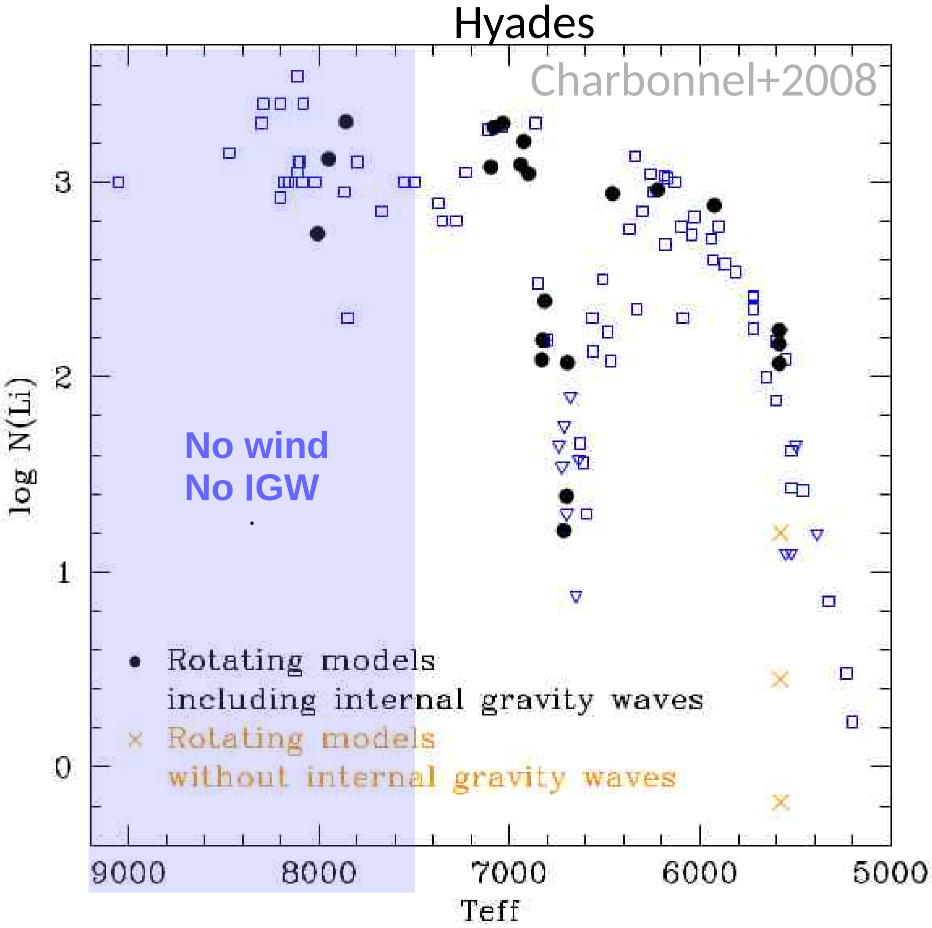

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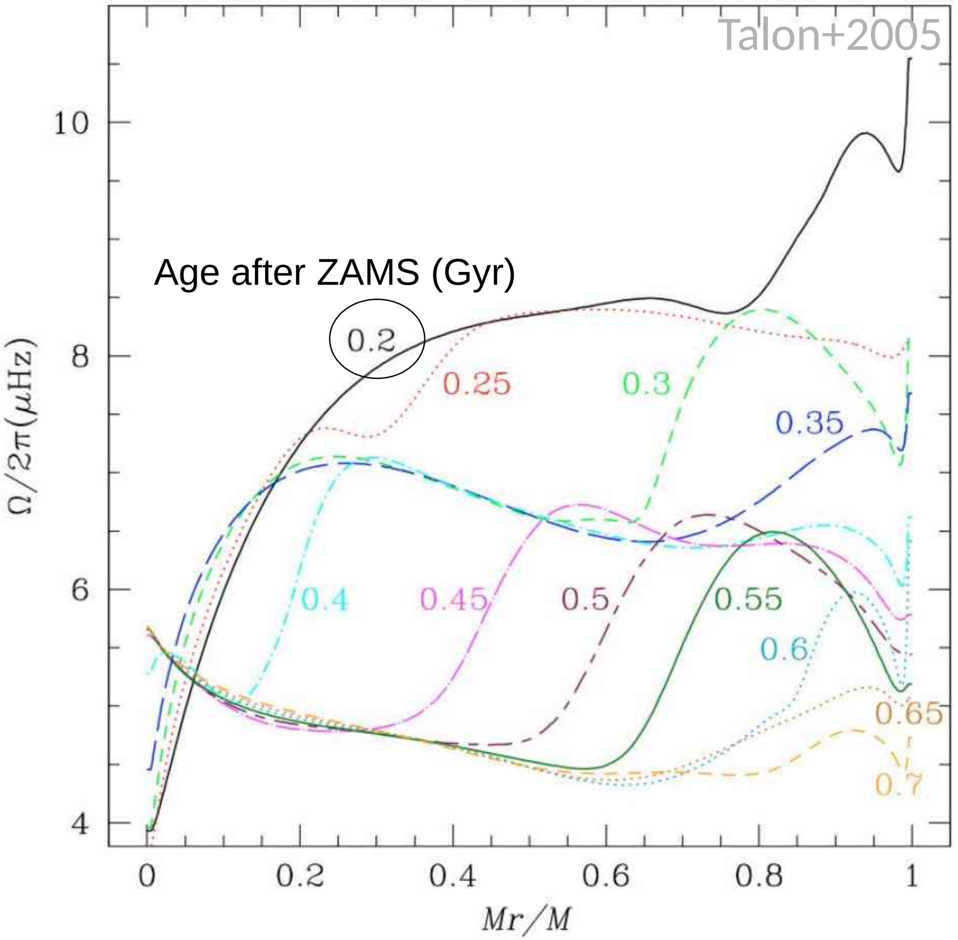

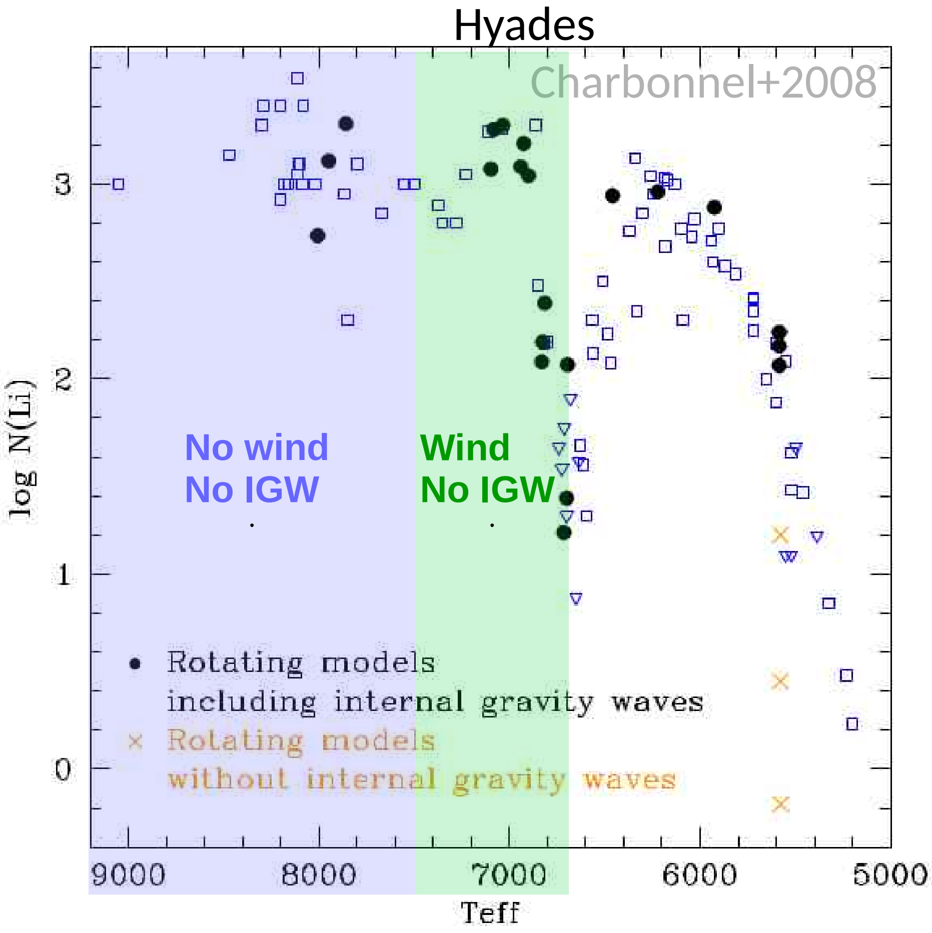

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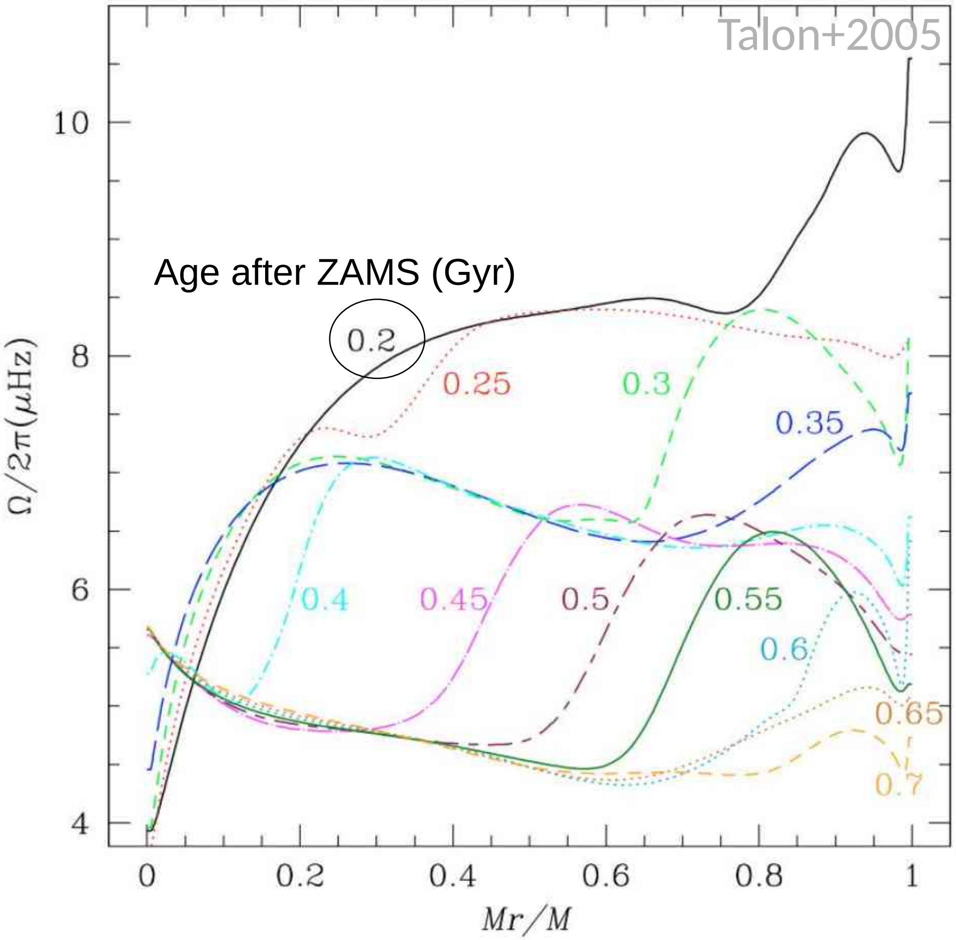
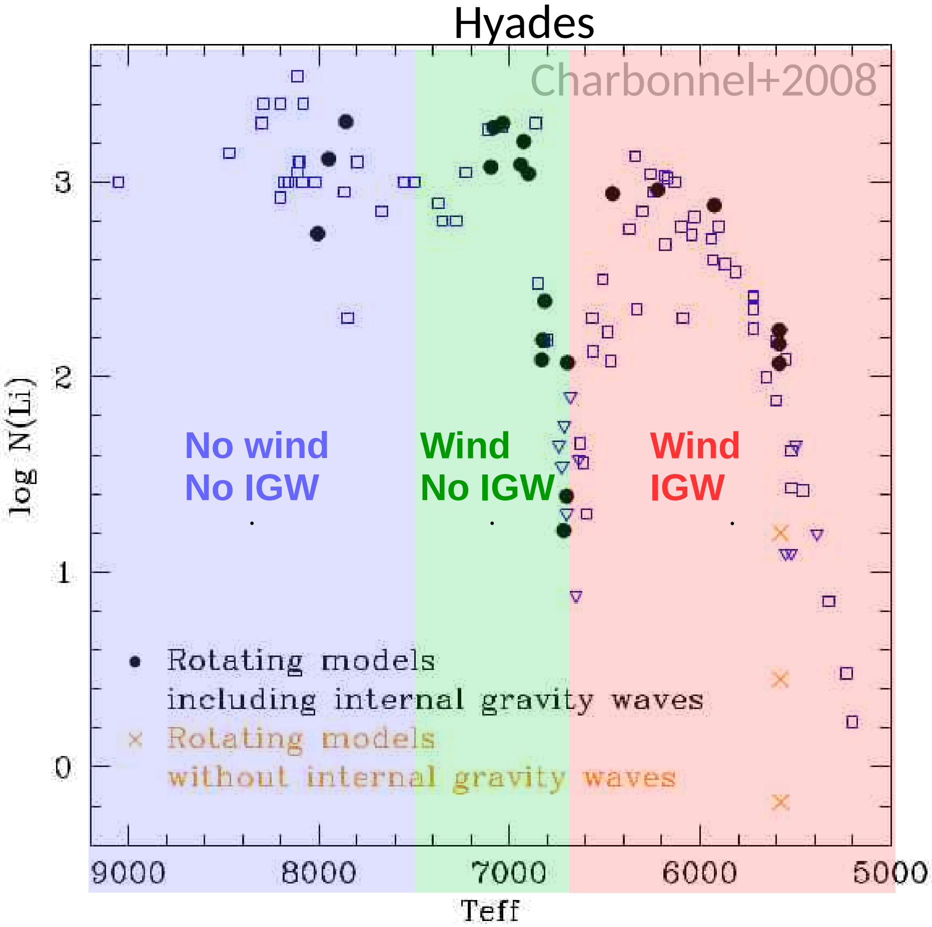


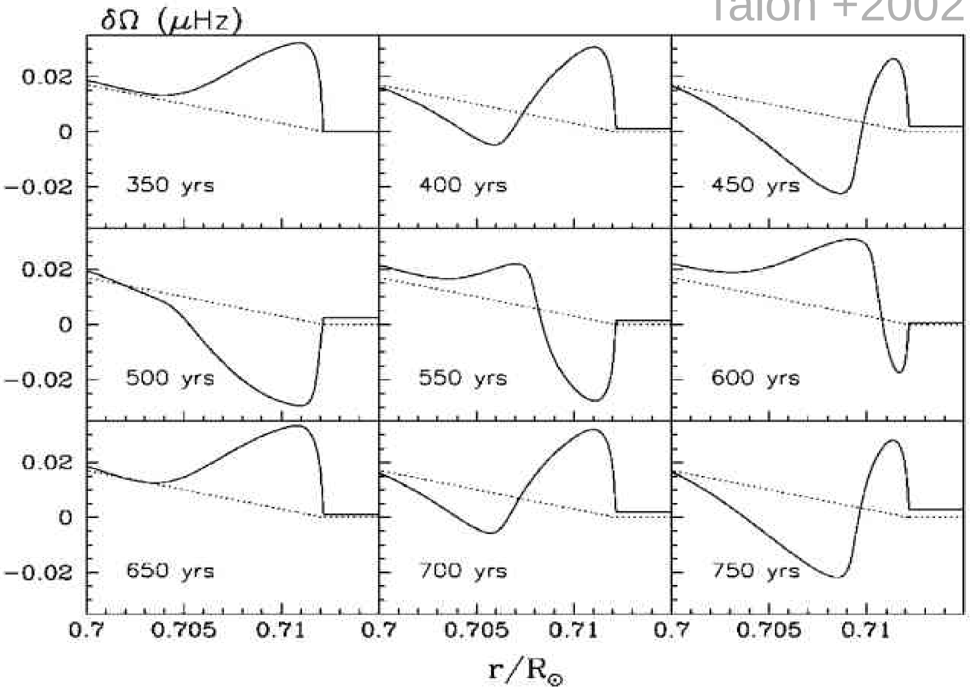


Going beyond this simple picture

- **Shear Layer Oscillation**

  - Wave/shear interaction
  - Very localized at the BCZ
  - Impact on global evolution ?

  → Need for multi-timing method...



- **And more to tackle ...**
  - → Non-adiabatic effects (in progress)
  - → Non-linear waves, turbulence in radiative zones
  - → Effect of Coriolis acc.
  - → Transport of chemical elements (e.g., Stokes drift, wave breaking, ...)

Concluding remarks

- ✓ Gravity waves in the Sun and in stars:
  - - Invoked for more than 30 years (e.g., Schatzmann 1993).
  - - Available “simple” prescriptions, a few evolutionary computations.
- ✓ Potential probing of the solar core: the quest of g-modes
  - - No robust detection, low-frequency range more suited for a future detection.
  - - Need improvements in the description of convection to be more realistic.
- ✓ Transport of AM and chemical elements:
  - - Very promising (solar an subgiant rotation, Li abundance)
  - - But still need to go beyond the current approximations.
  - - Using complementarity between semianalytical developments and simulations.

Effect of the Coriolis force at very low frequencies

Previous excitation/propagation wave models neglect the Coriolis force

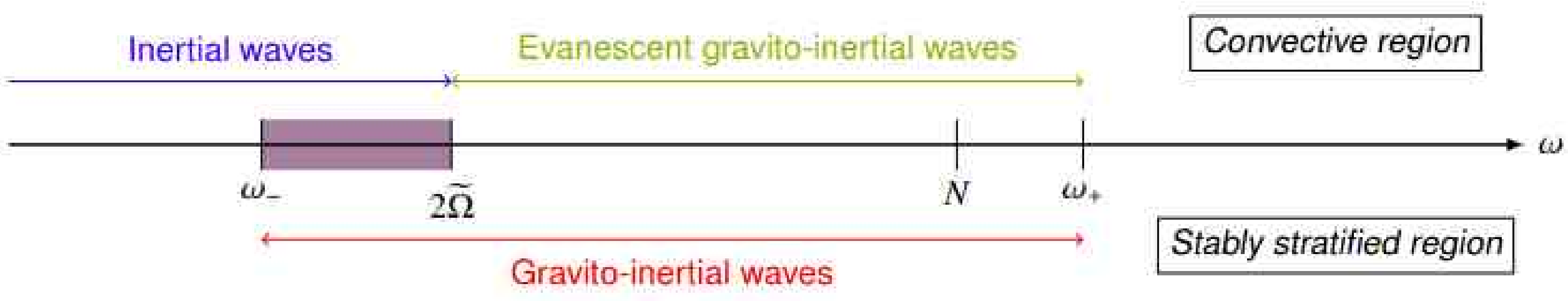
- $\omega \lesssim \Omega \Rightarrow$ this approximation fails (even for slow rotators)

Local Dispersion relation (e.g., Gerkema 2005)**:**

$$k_z^2 = k_\perp^2 \left[\frac{N^2 - \omega^2}{\omega^2 - f^2} + \left(\frac{\omega \tilde{f}_s}{\omega^2 - f^2} \right)^2 \right]$$

Coriolis parameters

New behaviors : - Propagative inertial waves in convective regions
- Evanescent sub-inertial waves in radiative regions



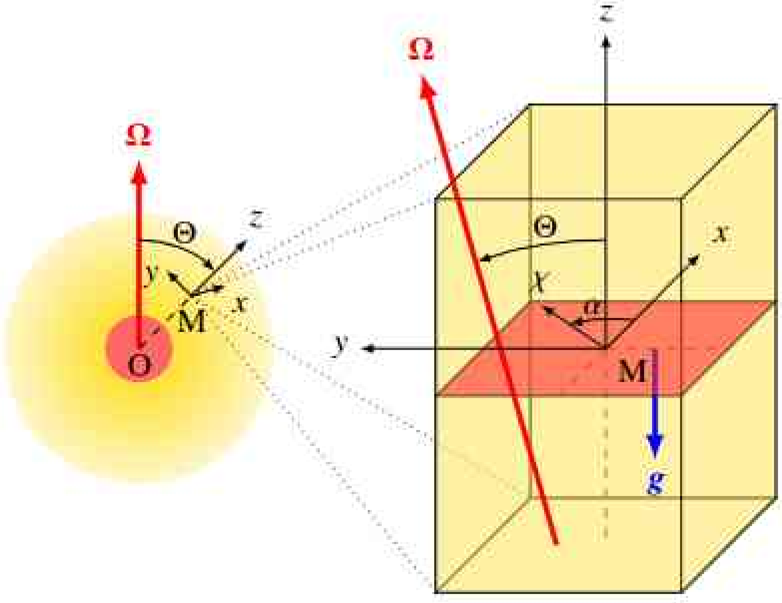
André 2017

Impact on the excitation ?

Traditional approximation fails at these frequencies (e.g., Eckart 1960, Mathis 2014)**...**

⇒ Complex GLOBAL 2D problem since no more separable in the radial and latitudinal coordinates

... BUT stil analytically tractable in the non-traditional LOCAL f-plane !
(e.g., Mathis 2014, Augustson 2020)

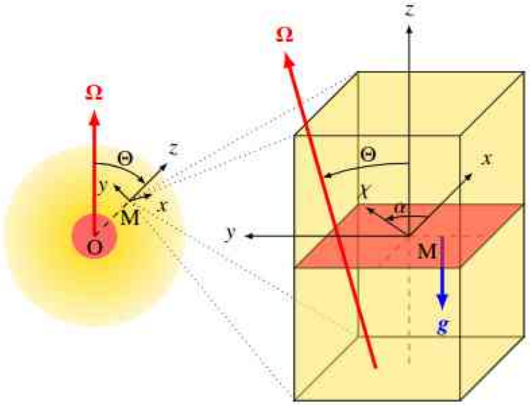


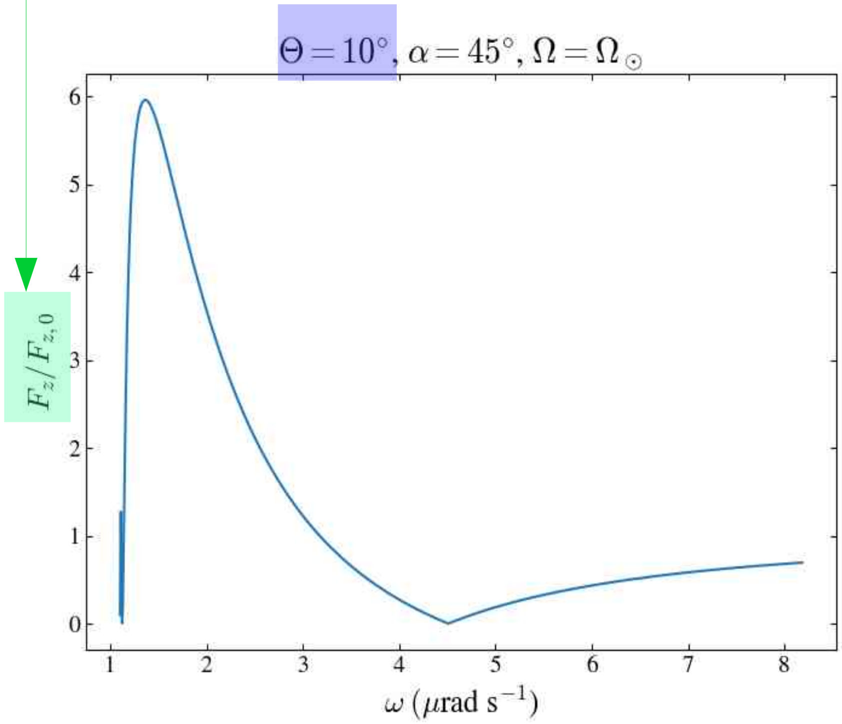
André 2017

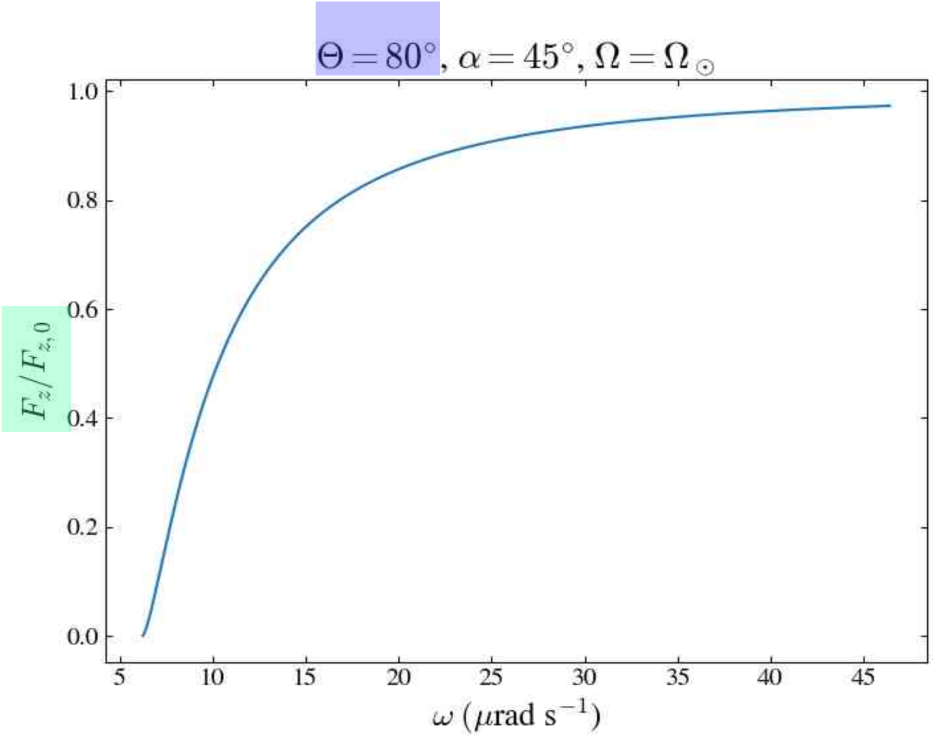
⇒ How the Coriolis force influence the wave excitation by penetrative plumes ?
(Note : Local approach valid only for horizontal wavelength much smaller than the radius)

Preliminary results for a Sun

$$\frac{\textit{Flux with rotation}}{\textit{Flux without rotation}}$$

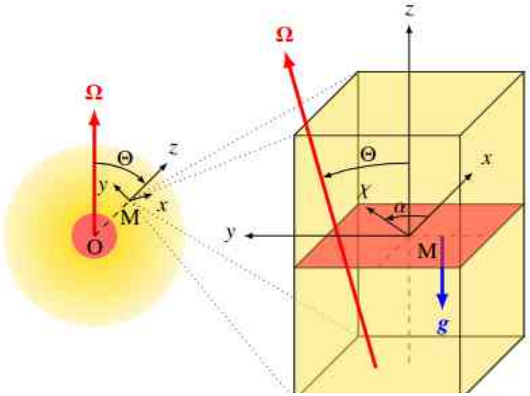






⇒Depending on the latitude and frequency, the wave flux is increased/decreased.

Preliminary results for a Sun



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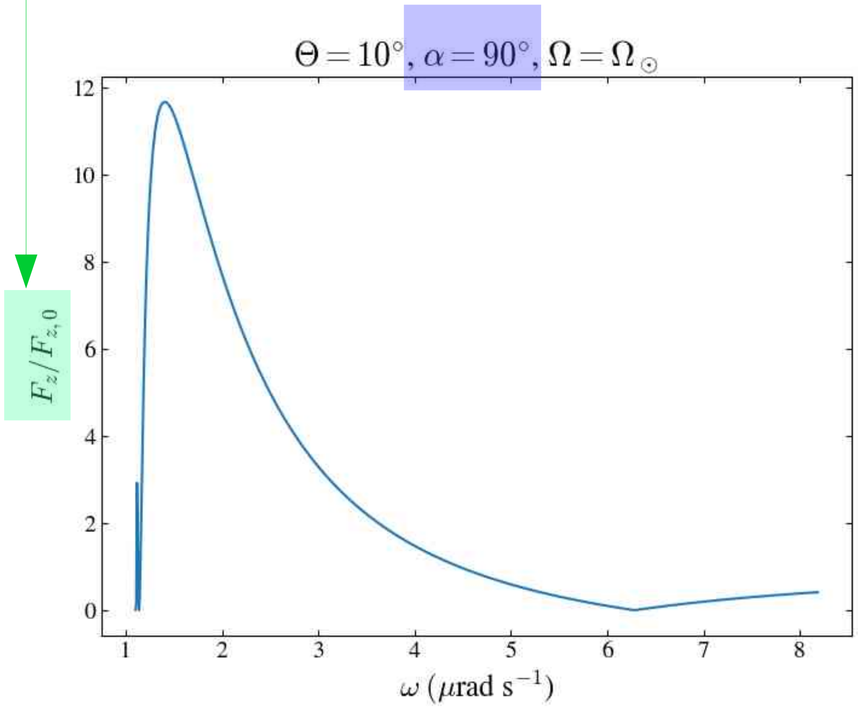

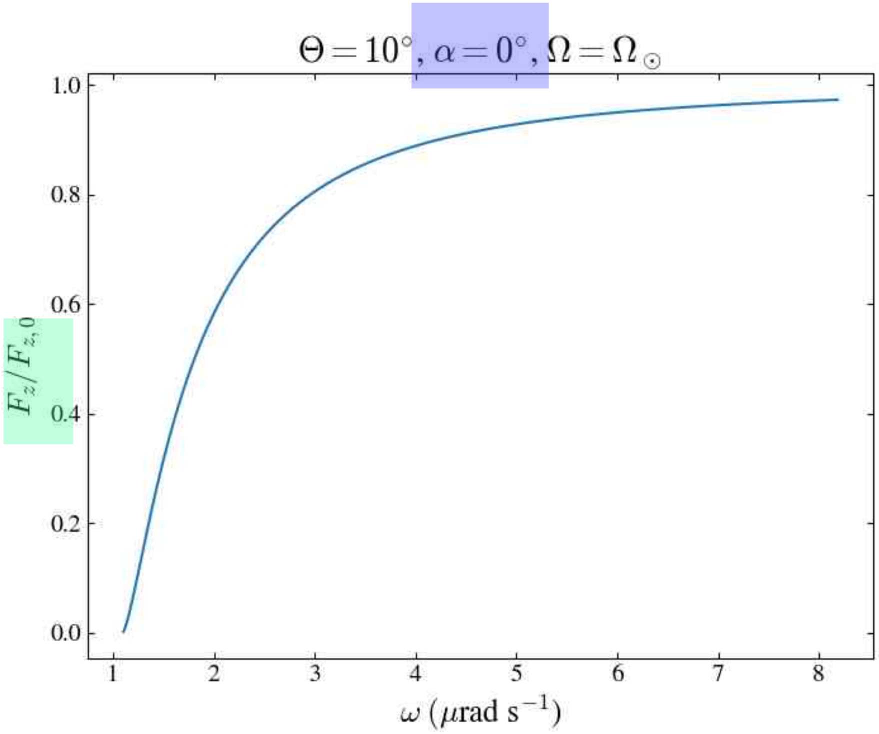

⇒ But it also depends on the horizontal direction of propagation !

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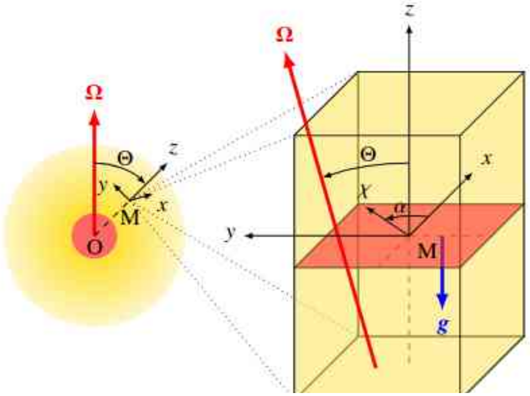

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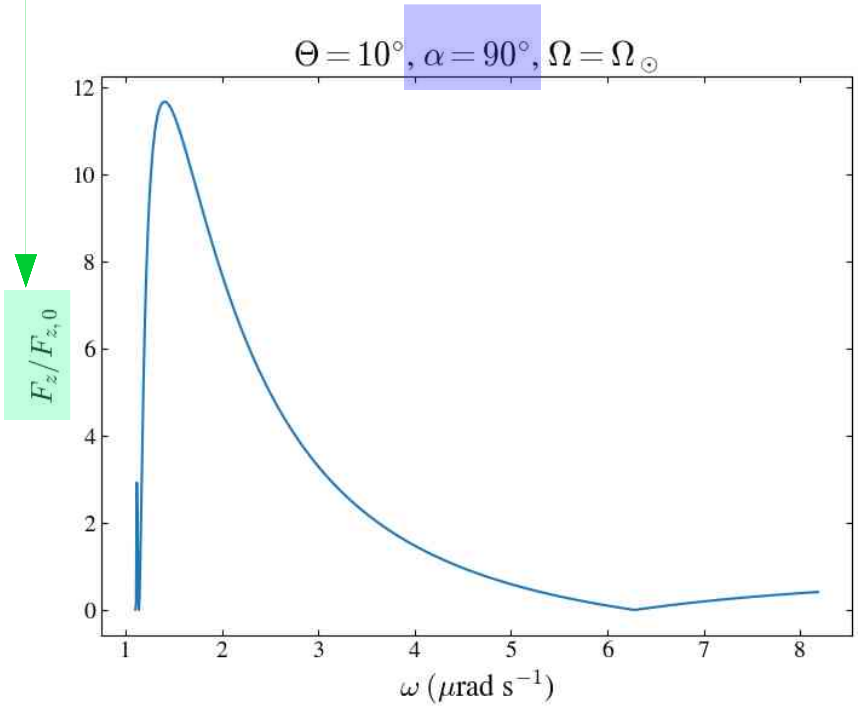
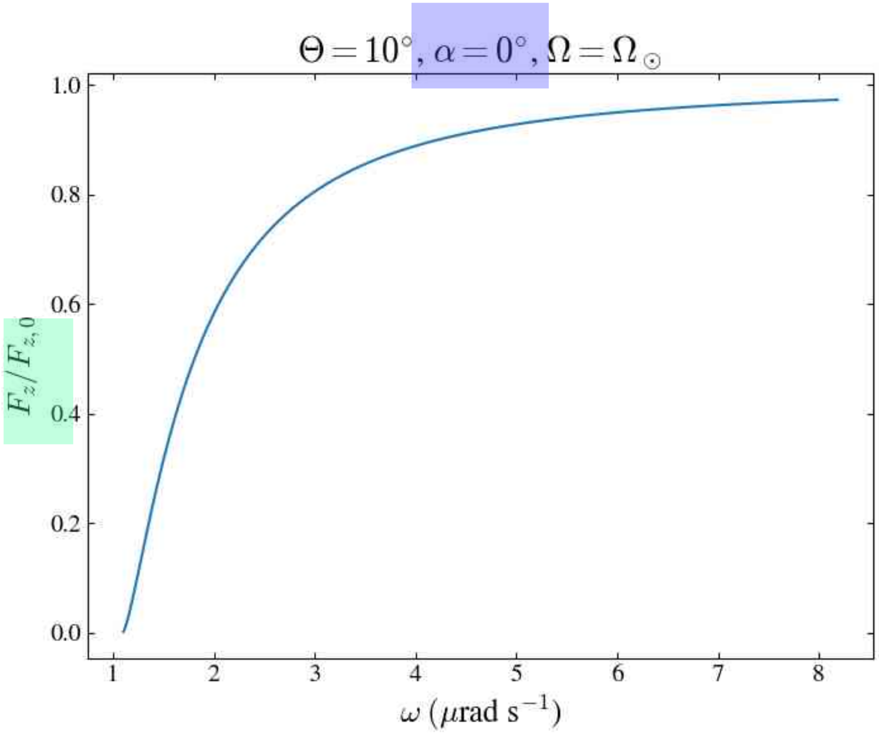

$\Theta = 10°, \alpha = 90°, \Omega = \Omega_\odot$


$\Theta = 10°, \alpha = 0°, \Omega = \Omega_\odot$

⇒ But it also depends on the horizontal direction of propagation !

⇒ Complex horizontal dependence : work in progress (with S. Mathis & K. Augustson)