CURRENT STATE OF SOLAR (& stellar) ROTATION MEASUREMENTS

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THE FUTURE OF SOLAR MODELLING
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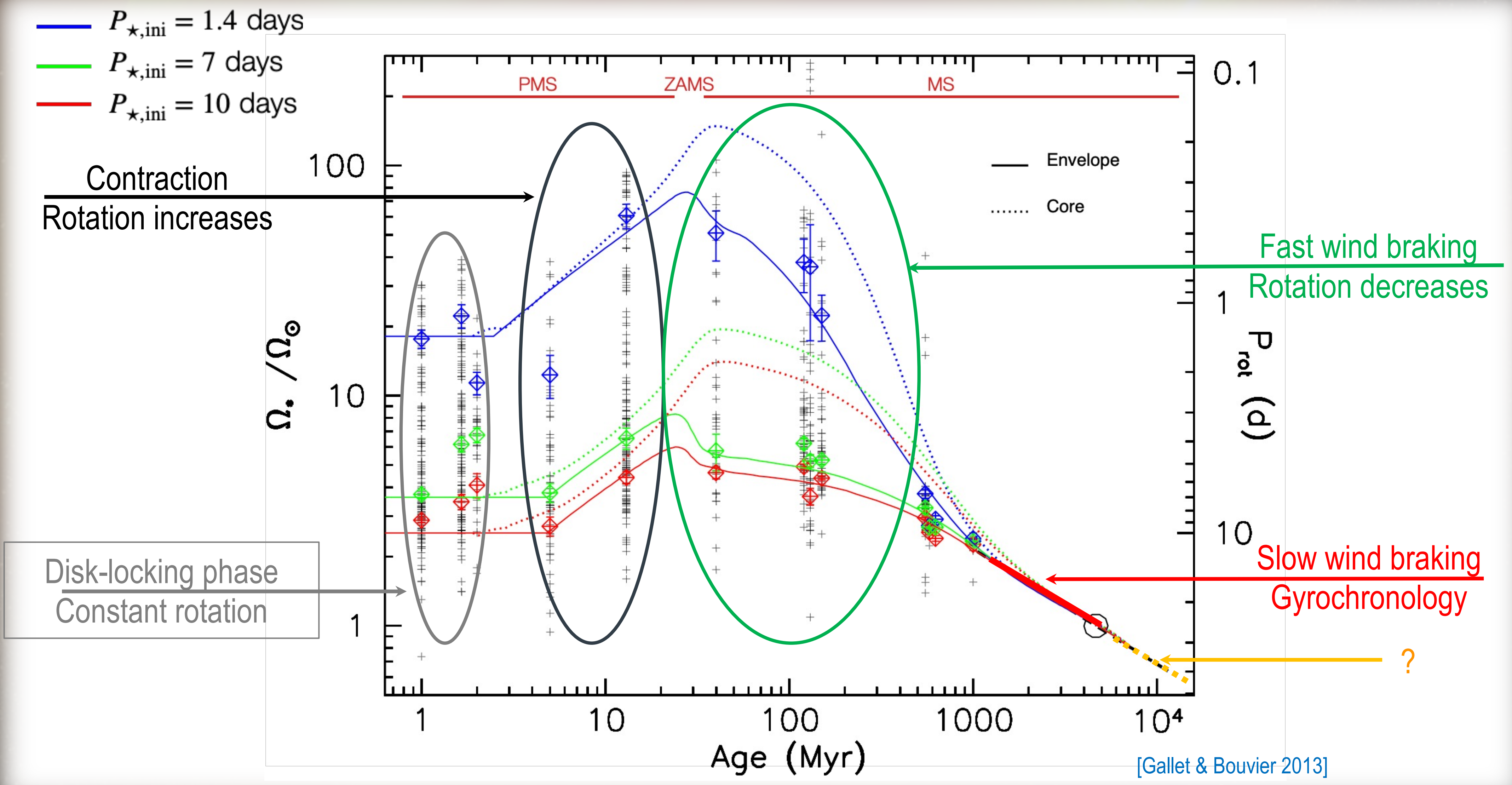
CONTENT

- I-Introduction
  - Why Rotation?
  - Importance in the context of Solar (stellar) modelling?
  - Some open questions still need to be solved
- II- Measuring Surface and Internal rotation
  - Methods and some historical context
  - Methods to infer internal rotation
- III-Results
  - Convective Zone, Tachocline, Radiative interior
  - Current limits
    - High latitudes & Core
  - Temporal (cyclic) variations
    - Torsional oscillations
- IV-Stellar rotation
  - Is the Sun rotation profile representative of other MS stars?
- V-Conclusions & Perspectives

I- Introduction

I-WHY ARE WE INTERESTED IN ROTATION?

- Rotation is a key parameter of stellar evolution



Contraction

Rotation increases

Disk-locking phase

Constant rotation

Fast wind braking

Rotation decreases

Slow wind braking

Gyrochronology

?

[Gallet & Bouvier 2013]

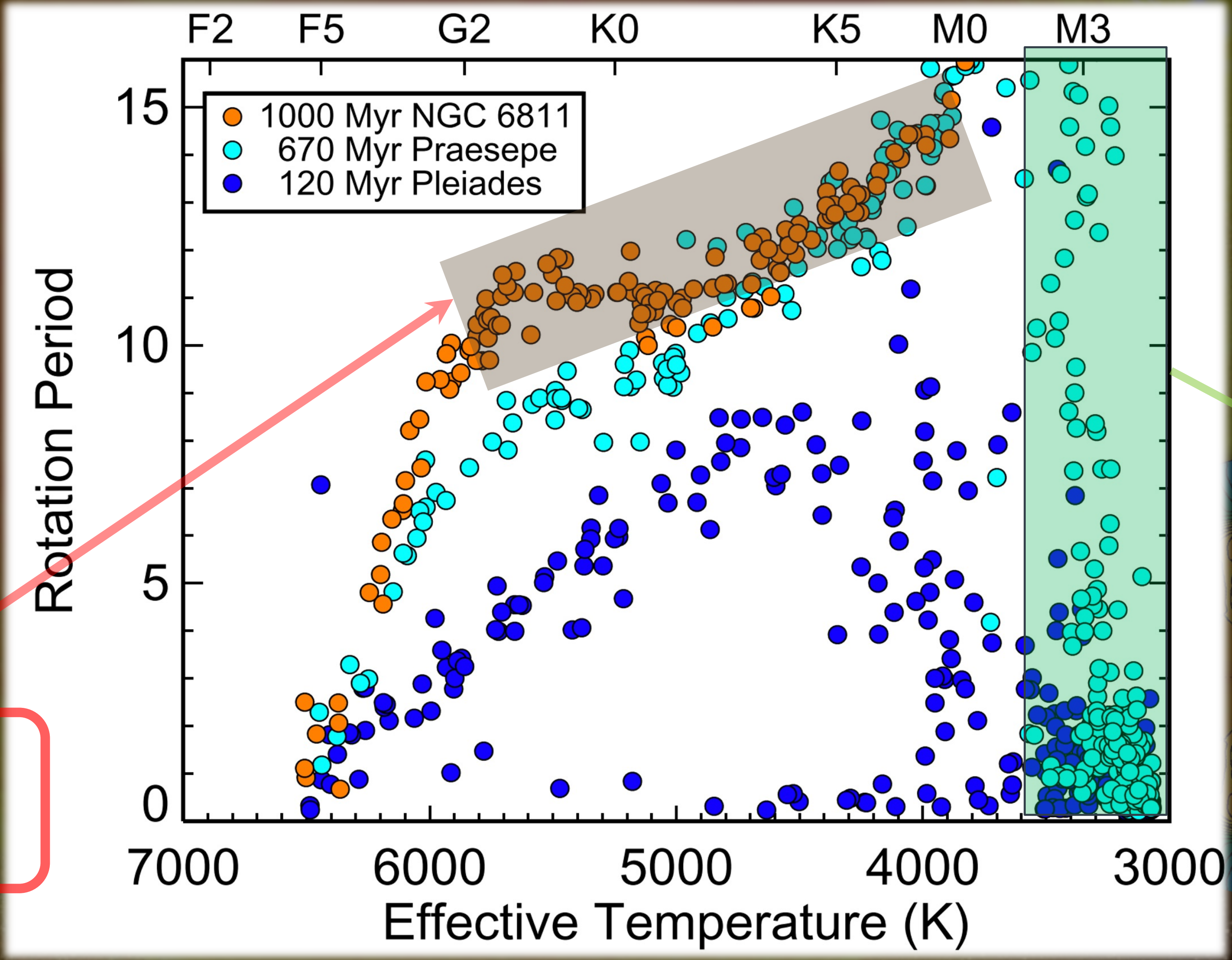
[Skumanich 1972]

[Barnes 2003]

[Van Saders+ 2016]

I-WHY ARE WE INTERESTED IN ROTATION?

- Temporal evolution of stellar rotation
  - Shows complex behaviour pointing out to different physical processes at work as a function of time



Stalling of the K dwarfs

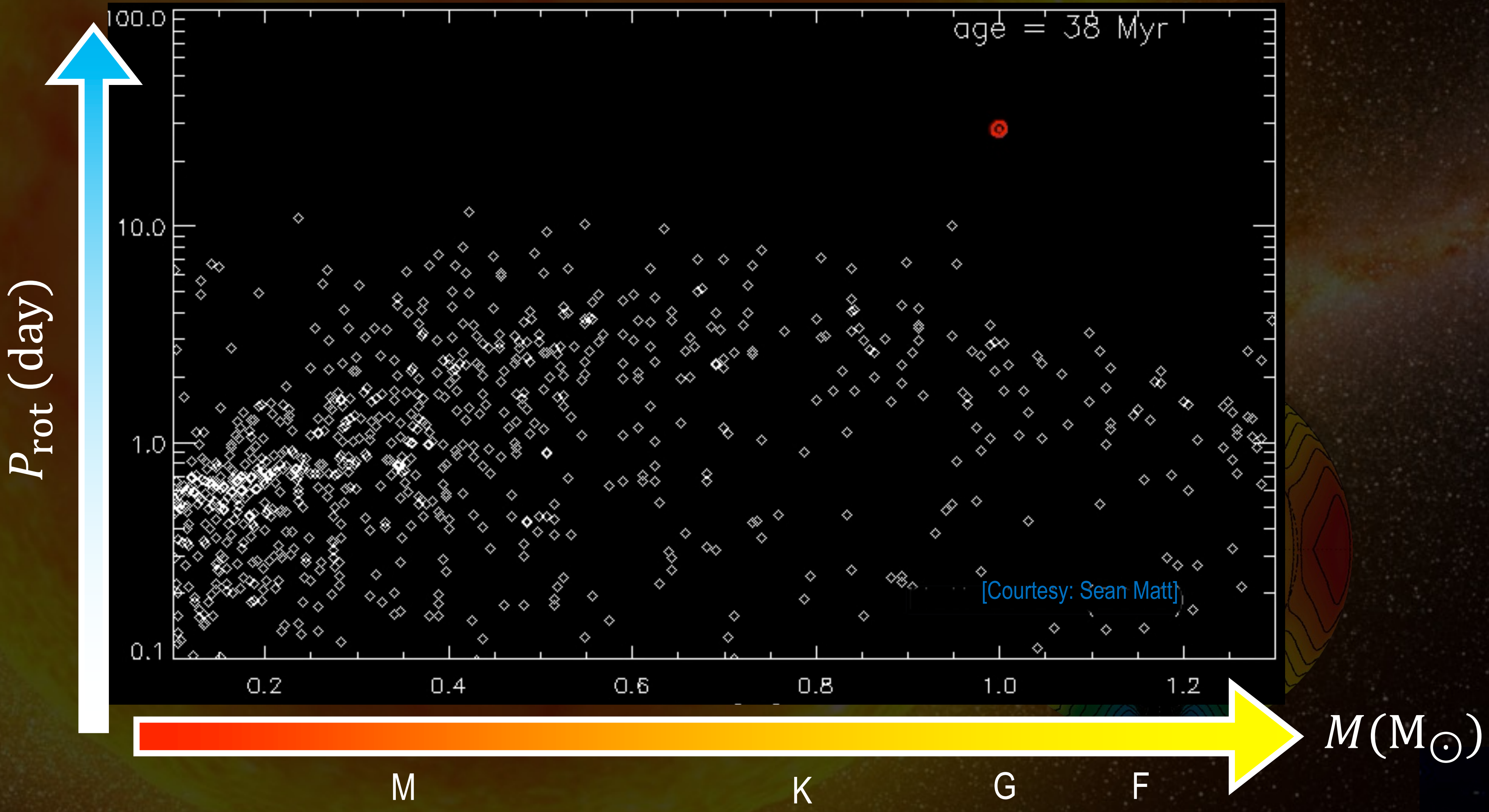
Many M dwarfs from clusters not converged into the gyrochronology sequence… yet!!

[Curtis+ 2019]

I-WHY ARE WE INTERESTED IN ROTATION?

- Simulated time evolution of stars up to the solar age
    - Placing the Sun in a more stellar context

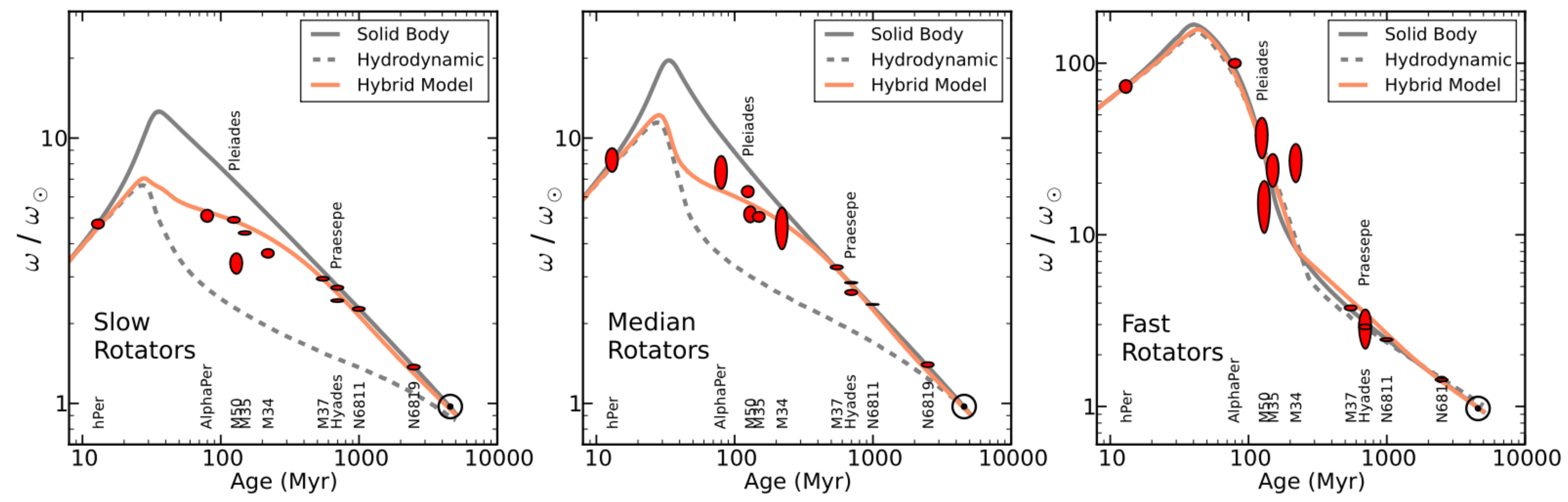
[Breimann+ 2021]



[Courtesy: Sean Matt]

I-WHY ARE WE INTERESTED IN ROTATION?

- Rotation is a key parameter of stellar evolution
  - Diagnostics of different physics in stellar interior models
  - The Sun is not always the best calibrator. We need to look at earlier and later ages



[Somers & Pinsonneault 2016]

[Data from Gallet & Bouvier 2015]

- YREC Models
  - Hydro and solid body models: the weakest and strongest possible coupling of AM
  - Hybride models: in addition to AM diffusion induced by hydrodynamics, drive additional parametrized constant diffusion.

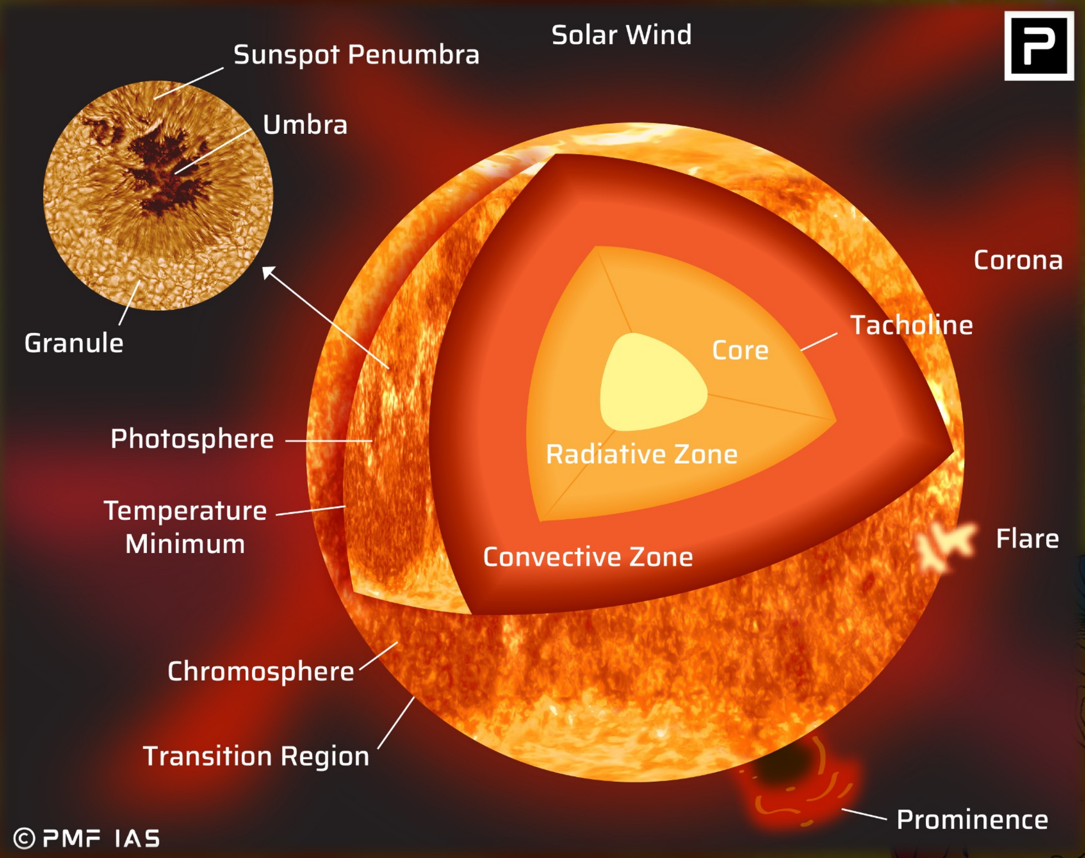
I-WHY ARE WE INTERESTED IN ROTATION?

- Surface (differential) Rotation
  - Main probe of the convective envelope
  - One of the main ingredients
    - Magnetic activity (& Cycles)

[e.g. Brun & Browning 2017]

- Role of the interactions with the environment
  - Solar wind [e.g. Finley & Brun 2023]
  - Space weather [e.g. Georgieva 2013]
  - Planets
    - Dynamics
    - Habitability
    - Are planet host stars rotating slower?

[e.g.Sibony, Helled, Feldmann 2022, Ferraz-Mello & Beaugé 2023]]



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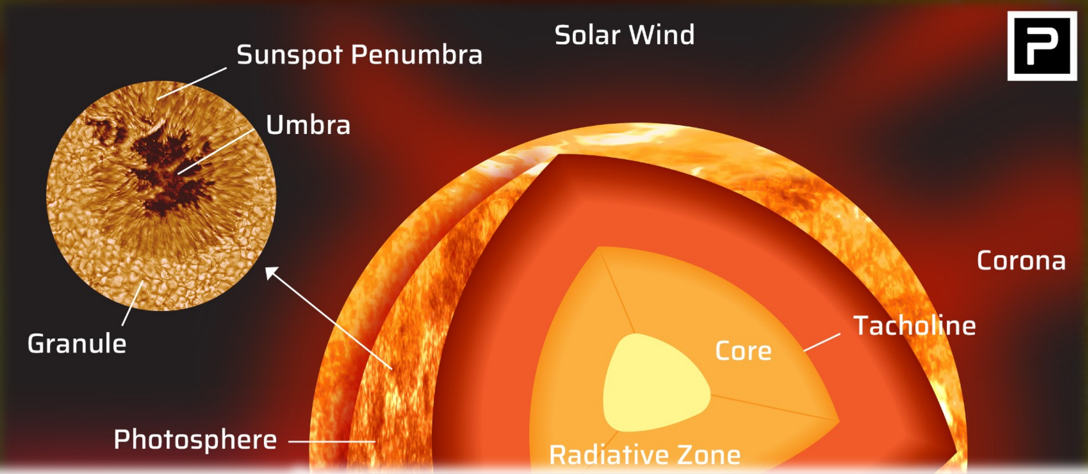
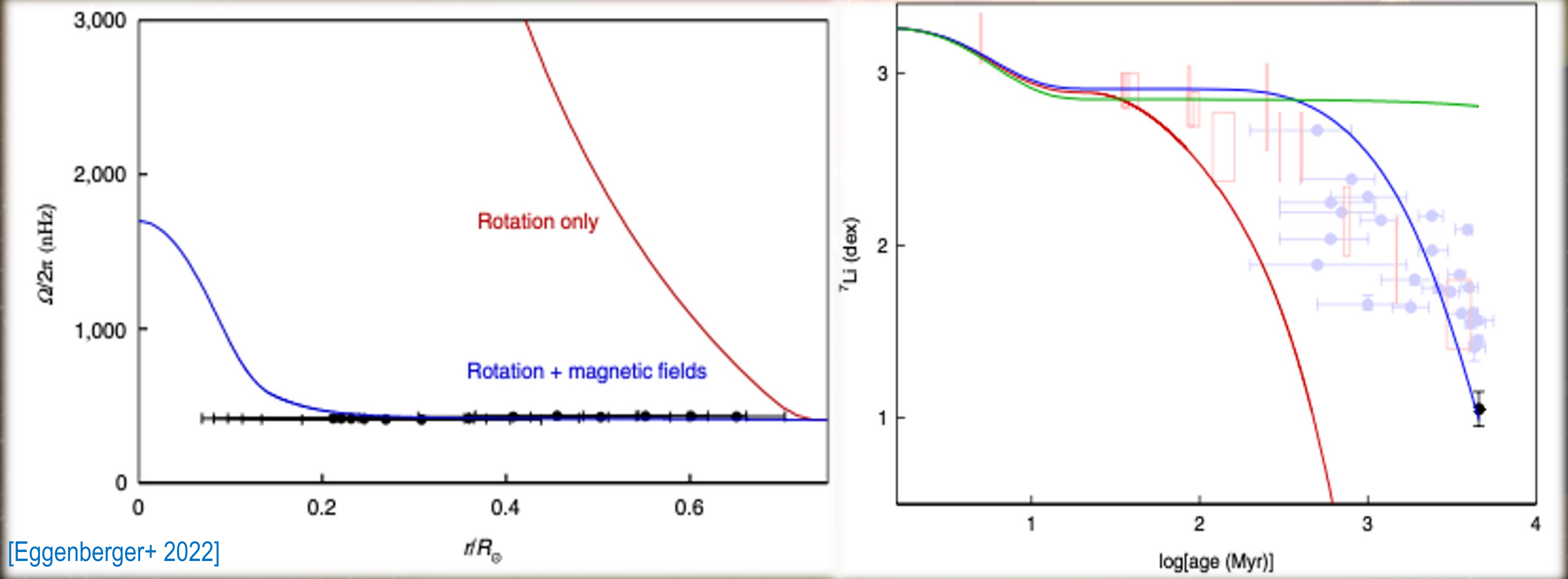
[e.g. Brun & Browning 2017]



- Internal (differential) Rotation
  - Angular momentum transport
    - Internal mixing
  - Connexion with surface abundances (e.g. Li, He)

[Eggenberger+ 2022]

- Role of the interactions with the

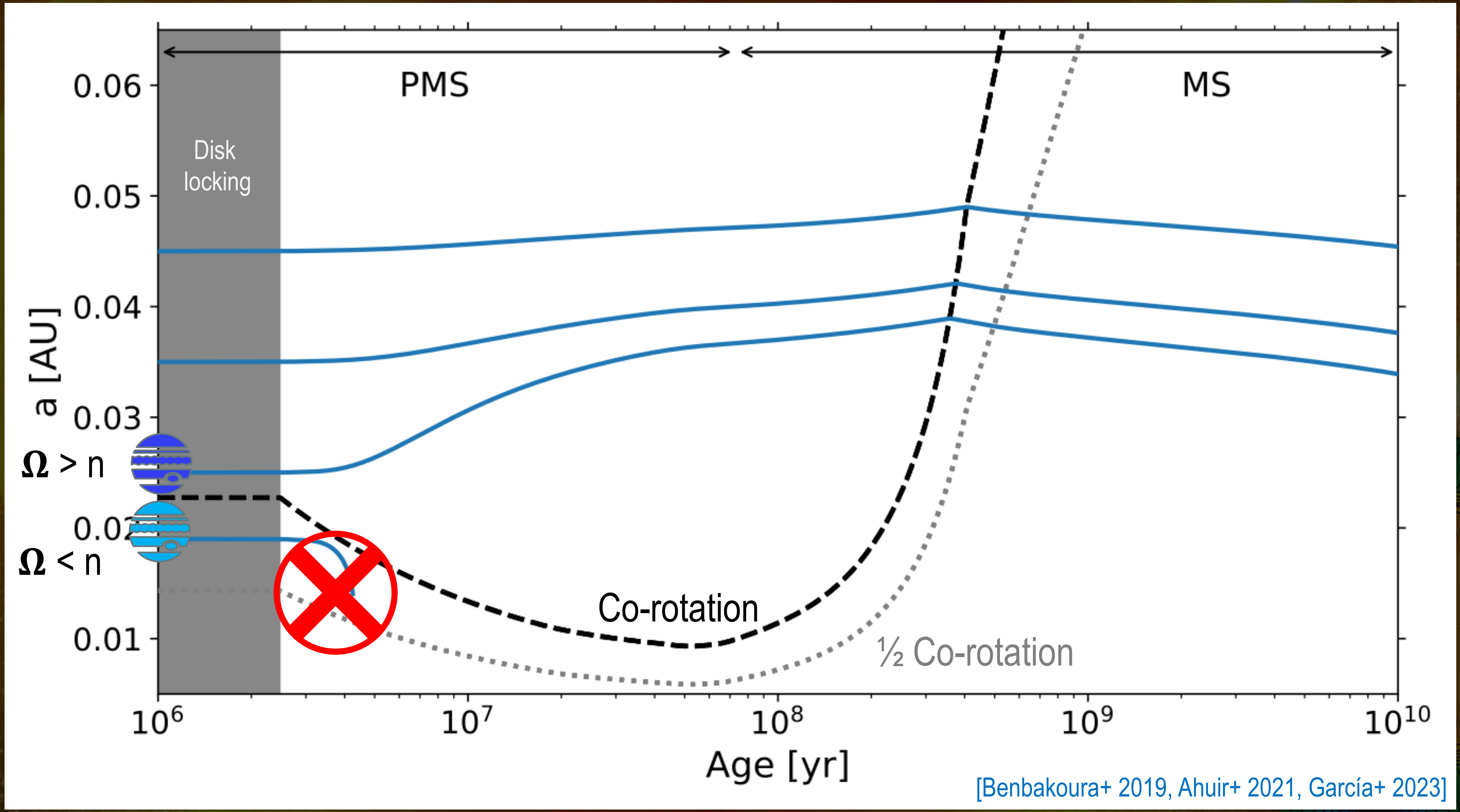


[Eggenberger+ 2022]

- Internal rotation profile
  - Not yet completely understood
    - What happens:
      - at the poles?
      - At the core?

STELLAR ROTATION: KEY INGREDIENT
TO UNDERSTAND EXOPLANET EVOLUTION

- Close-in planets migration due to tidal and magnetic torques: Depends on stellar rotation evolution



[Benbakoura+ 2019, Ahuir+ 2021, García+ 2023]

[see also: Lai 2012, Damiani & Mathis 2018, Ahuir+ 2021, Ferraz-Mello & Beaugé 2023]

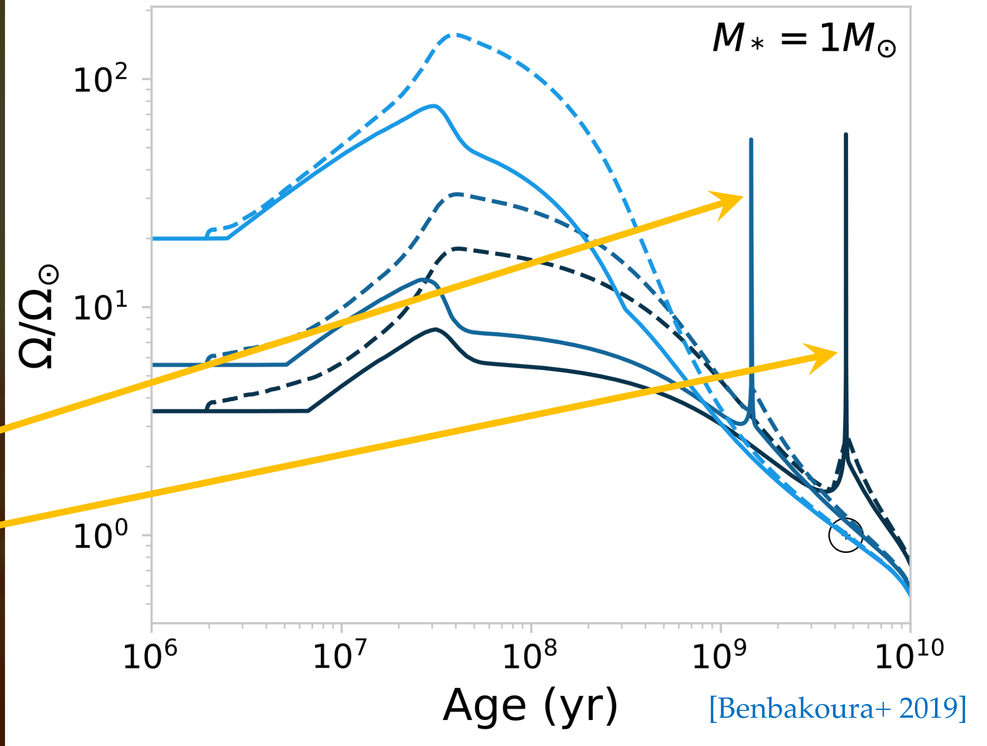
- The fast migration phase is dominated by the tides
- After co-rotation, the magnetic torque is responsible for the migration towards the star

STELLAR ROTATION: KEY INGREDIENT

TO UNDERSTAND EXOPLANET EVOLUTION

- Close-in planets migration due to tidal and magnetic torques: Depends on stellar rotation evolution

Perturbation by a planet of mass 1 M_{Jup}
Initial semi-major axis : 0.03 AU



[Benbakoura+ 2019]

Signature of the destruction of a planet

STELLAR ROTATION: KEY INGREDIENT
TO UNDERSTAND EXOPLANET EVOLUTION

- Close-in planets migration due to tidal and magnetic torques: Depends on stellar rotation evolution

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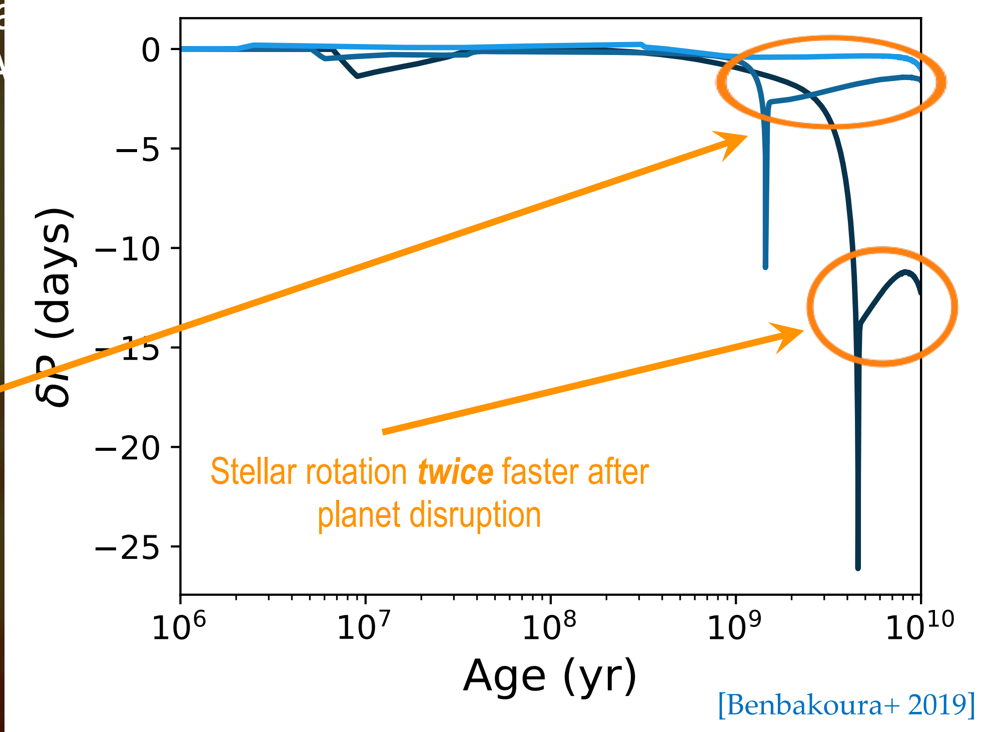


Stellar rotation ***twice*** faster after planet disruption

[Benbakoura+ 2019]

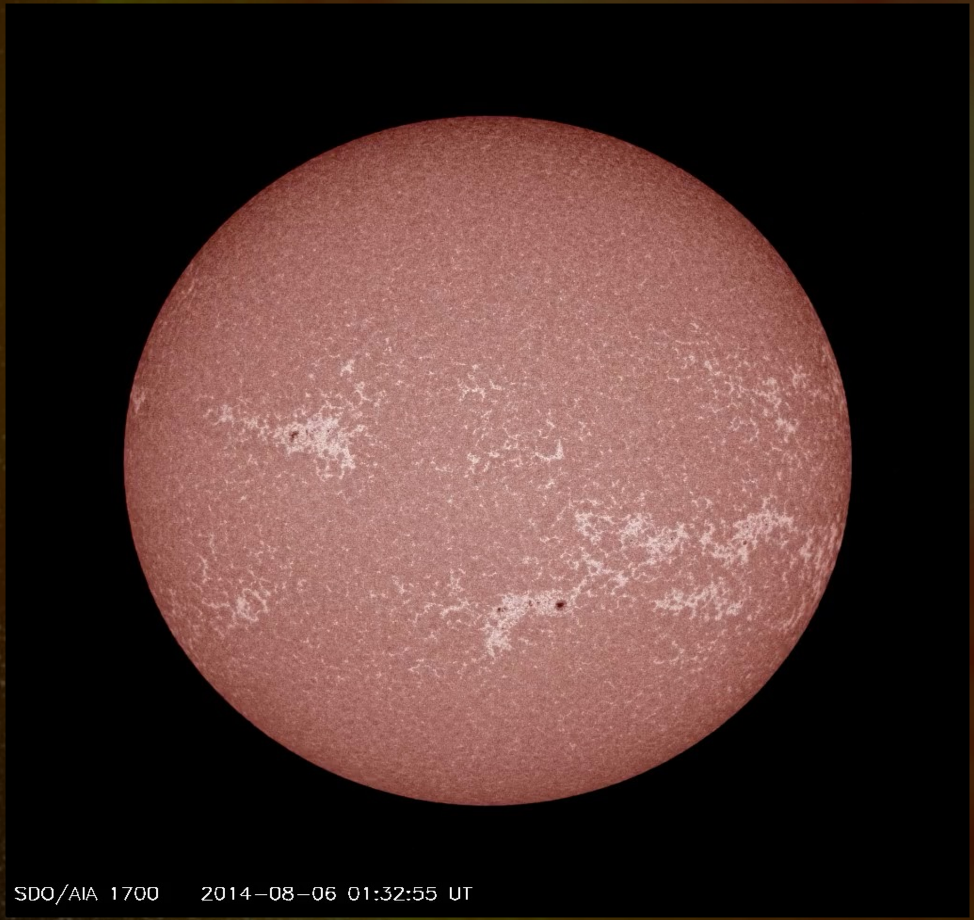
Difference < 5 days
(~15%)

II- Measuring

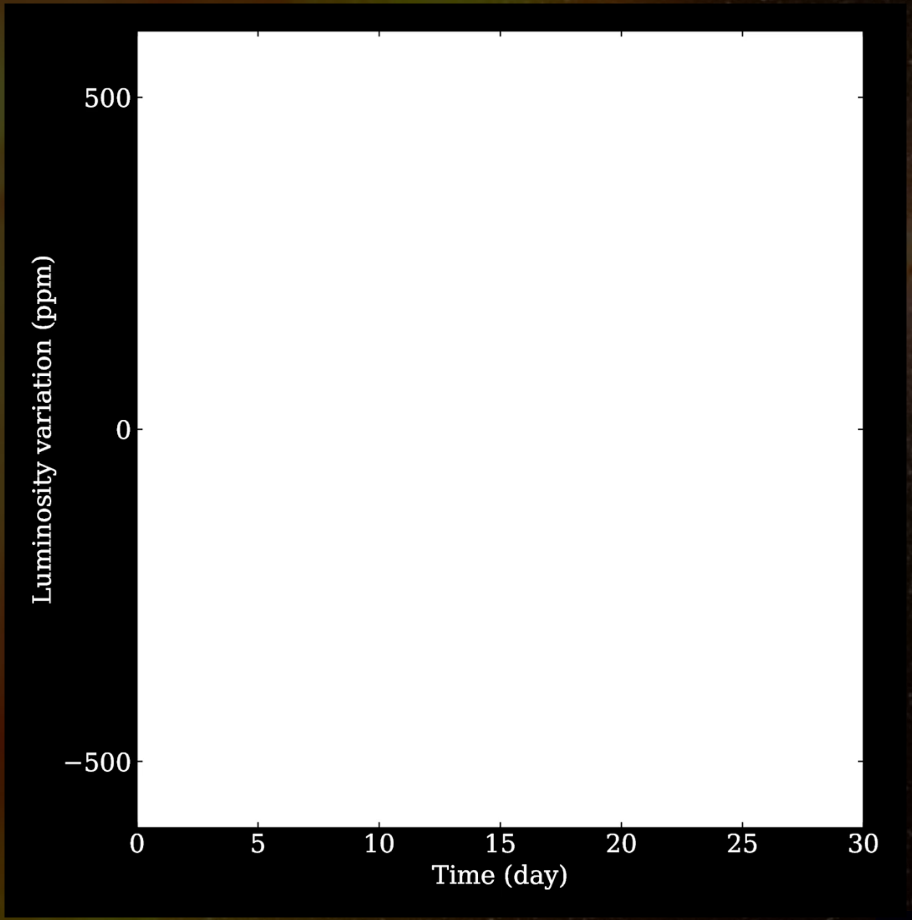
Surface and Internal rotation

II-MEASURING ROTATION: SURFACE

- Tracking features on the surface of the Sun
  - SDO/AIA 1700 Å

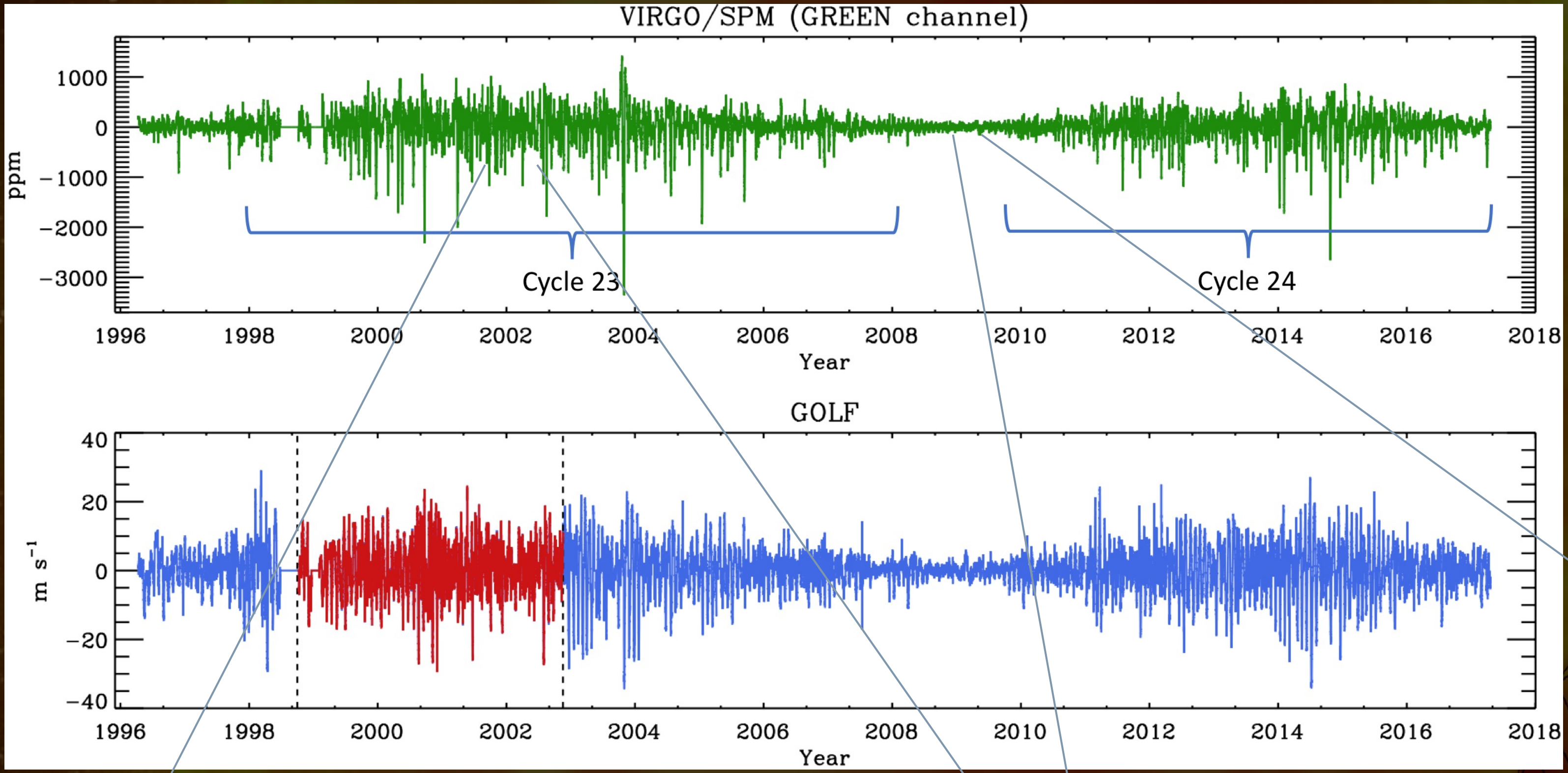


- Integrating the total flux
  - SoHO/VIRGO/SPM (visible light)

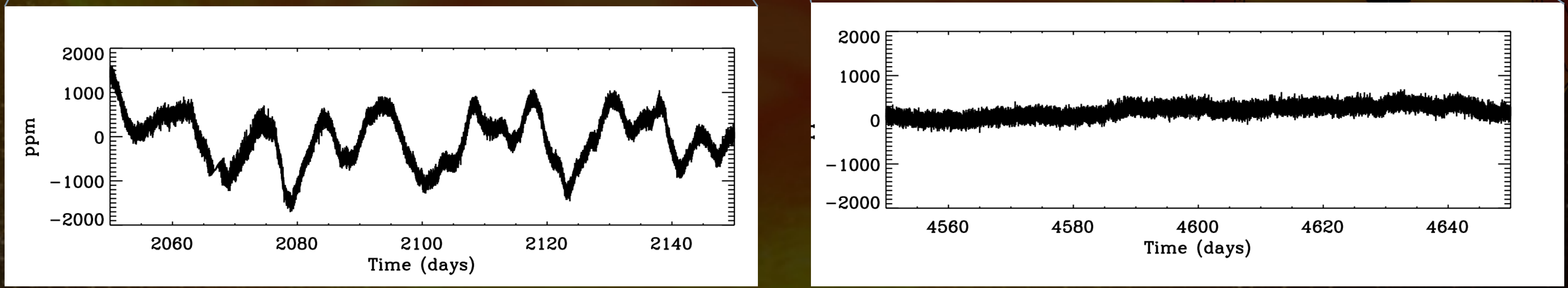


[Credits: S.N. Breton]

II-MEASURING ROTATION: SURFACE



[García & Ballot 2019]

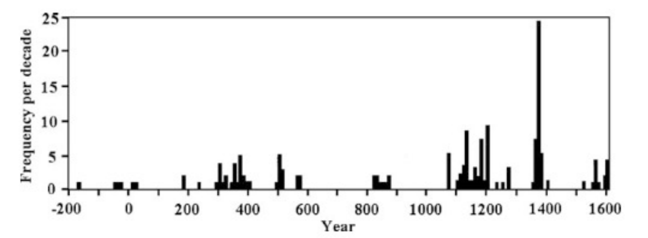


- SoHO Observations
  - VIRGO/SPM Green channel
  - GOLF: Doppler velocity

II-MEASURING ROTATION: SURFACE

A BIT OF HISTORY

- **Oriental Observations:**
  - Naked eye observation of sunspots started in China
    - Book of Changes, oldest surviving Chinese book compiled at around 800 BC
      - The text reads "A ***dou/mei*** is seen in the Sun". From the context, the words (i.e., Chinese characters) "***dou***" and "***mei***" mean “darkening” or “obscuration”.

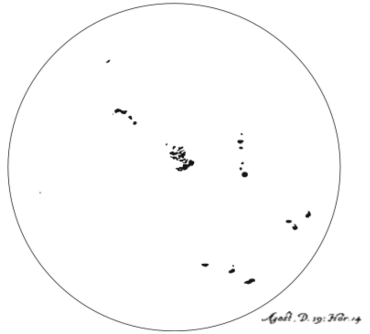


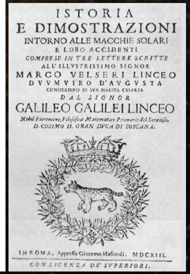
[Yau, 1988 , Vaquero & Vázquez, 2009]

- **Occidental Observations**
  - December 8, 1128 by John of Worcester
    - “…there appeared from the morning right up to the evening two black spheres against the sun. The first was in the upper part and large, the second in the lower and small, and each was directly opposite the other as this diagram shows…”
  - Observations by telescope
    - 1612 by Galileo Gallilei
  - Rotation of the Sun & the inclination of the rotation axis
    - 1630 Christoph Scheiner
      - Depicted a slight inclination of the Sun’s axis of rotation with respect to the ecliptic (7^o)
      - P_{rot} of 25 to 35 d from equator to near the poles

[https://www2.hao.ucar.edu/education/scientists/christoph-scheiner-1575-1650]

[Vaquero & Vázquez, 2009]

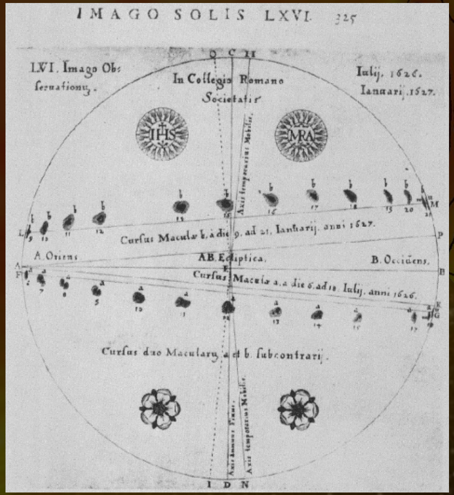


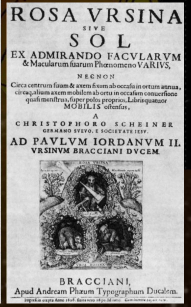


ISTORIA
E DIMOSTRAZIONI
INTORNO ALLE MACCHIE SOLARI
E LORO ACCIDENTI
COMPRESE IN TRE LETTERE SCRITTE
ALL'ILLVSTRISSIMO SIGNOR
MARCO VELSERI LINCEO
DVVMVIRO D'AVGVSTA
CONSIGLIERO DI SVA MAESTA CESAREA
DAL SIGNOR
GALILEO GALILEI LINCEO
Nobil Fiorentino, Filoſofo, e Matematico Primario del Sereniſs.
D. COSIMO II. GRAN DVCA DI TOSCANA.

IN ROMA, Appreſſo Giacomo Maſcardi. MDCXIII.
CON LICENZA DE' SVPERIORI.

[Galileo 1613]





ROSA VRSINA
SIVE
SOL
EX ADMIRANDO FACVLARVM
& Macularum ſuarum Phœnomeno VARIVS,
NECNON
Circa centrum ſuum & axem fixum ab occaſu in ortum annua, circaq. alium axem mobilem ab ortu in occaſum conuerſione quaſi menſtrua, ſuper polos proprios, Libris quatuor
MOBILIS oſtenſus,
A
CHRISTOPHORO SCHEINER
GERMANO SVEVO, E SOCIETATE IESV.
AD PAVLVM IORDANVM II.
VRSINVM BRACCIANI DVCEM.

BRACCIANI,
Apud Andream Phæum Typographum Ducalem.
Impreſſio cœpta Anno 1626. finita vero 1630. Id Iunij.

[Scheiner 1630]

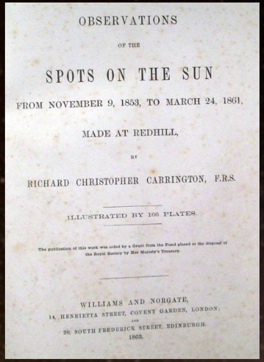
[see review in Appourchaux & Grundhal 2018]

II-MEASURING ROTATION: SURFACE

A BIT OF HISTORY

- Richard Christopher Carrington:
  - Devised a system of heliographic coordinates to track the motion of sunspots across the solar disk.
  - Defined a reference meridian and assigned a fixed longitude to it.
    - Introducing a synodic rotation period of 27.2753 days
      - corresponds to the average motion of sunspots at a latitude of 26 degrees
        - This period is still used today as a standard unit for solar rotation and is known as a **Carrington rotation**

[Appourchaux & Grundhal 2018]



OBSERVATIONS

OF THE

SPOTS ON THE SUN

FROM NOVEMBER 9, 1853, TO MARCH 24, 1861,

MADE AT REDHILL,

BY

RICHARD CHRISTOPHER CARRINGTON, F.R.S.

ILLUSTRATED BY 166 PLATES.

The publication of this work was aided by a Grant from the Fund placed at the disposal of the Royal Society by Her Majesty's Treasury.

WILLIAMS AND NORGATE,
14, HENRIETTA STREET, COVENT GARDEN, LONDON;
AND
20, SOUTH FREDERICK STREET, EDINBURGH.
1863.

[Carrington 1863]

  - Other discoveries:
    - Discovery of Sun's differential latitudinal rotation
    - Discovery of equatorward migration of spots in the course of the cycle
    - Determination of the Sun's rotation axis with an accuracy previously unprecedented
    - First and serendipitous observation of a white-light flare.

} ~ simultaneously & independently discovered by Gustav Spörer in Germany

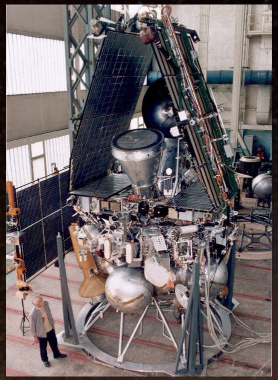
[Bhattacharya+ 2021]

[https://www2.hao.ucar.edu/education/scientists/richard-christopher-carrington-1826-1875]

II-MEASURING ROTATION: SEISMOLOGY

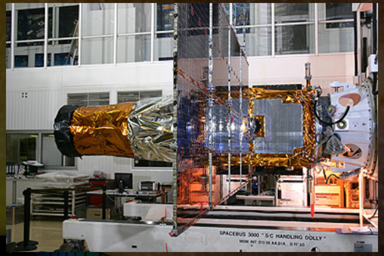
A BIT OF HISTORY: SPACE INSTRUMENTATION

- The development of space instrumentation for asteroseismology
  - Linked to the study of rotation
- CNES Instrument: Étude de la Variabilité, de la Rotation et des Intérieurs Stellaires (EVRIS)
  - A bord de la mission MARS96
  - A failure of the second fourth-stage burn
    - Burnt over the Pacific Ocean

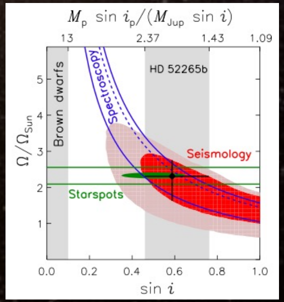


[Baglin+ 1993]

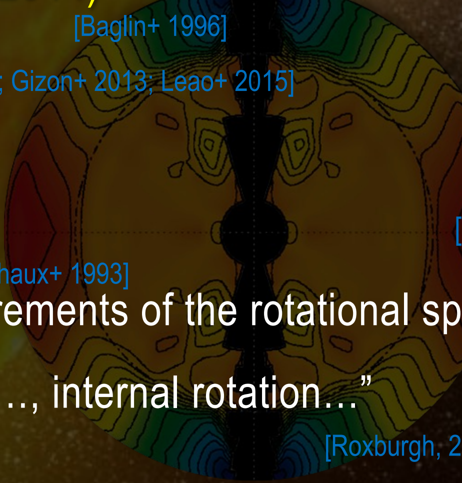
- CNES/ESA: Convection, Rotations et Transits planétaires mission (CoRoT, 2006-2014)
  - Success for asteroseismic studies and rotation [e.g. Mosser+ 2009; García+ 2010, Ballot+ 2011; Gizon+ 2013; Leao+ 2015]



[Baglin+ 1996]



[Gizon+ 2013]

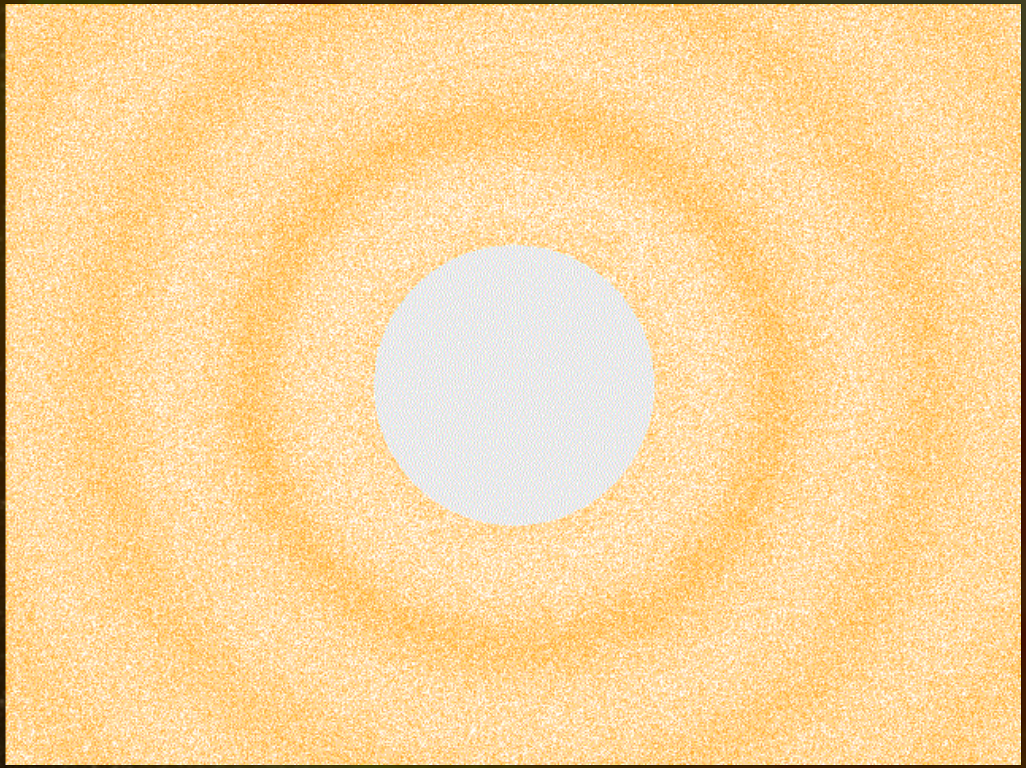


- Studies for ESA prior to PLATO:
  - PRISMA: Probing Rotation and Interior of Stars: Microvariability and Activity [Appourchaux+ 1993]
  - STARS: "…will determine an average ot the internal angular celocity from measurements of the rotational splittings…" [Fridlund+ 1995]
  - EDDINGTON: "…quantitatively determining, e.g., the size of convective regions, …, internal rotation…" [Roxburgh, 2003;Favata 2004]

[see also Appourchaux & Grundhal 2015]

II-MEASURING ROTATION: SEISMOLOGY

Mode family:	p modes	g modes	Inertial modes
Restoring force:	Compressibility	Buoyancy	Coriolis force



Credits: T.Dugnolle, Wikimedia Commons

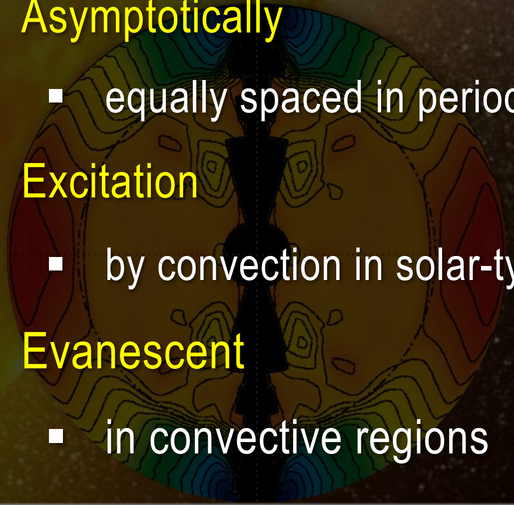
- Asymptotically
  - equally spaced in frequency
- Excitation
  - by convection in solar-type stars

Each perturbation can be expressed:

$$f'(r,\theta,\phi,t) = \sqrt{4\pi}Re\left[f'(r)Y_l^m(\theta,\phi)e^{-i\omega_{n,l,m}t}\right]$$

II-MEASURING ROTATION: SEISMOLOGY

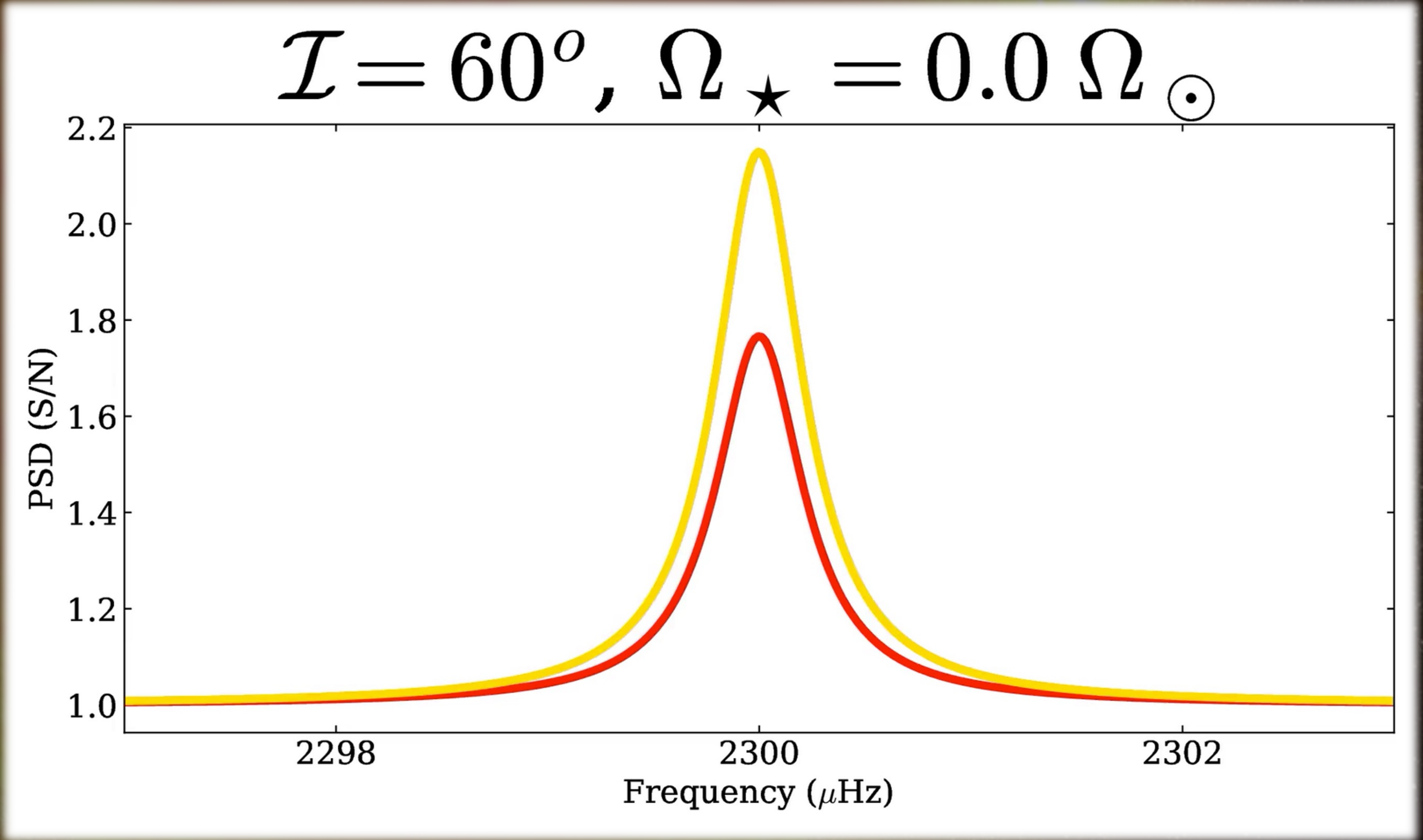
Mode family	p modes	g modes	~~Inertial modes~~
Restoring force	Compressibility	Buoyancy	~~Coriolis force~~



- Asymptotically
  - equally spaced in period
- Excitation
  - by convection in solar-type stars
- Evanescent
  - in convective regions

II-MEASURING ROTATION: SEISMOLOGY

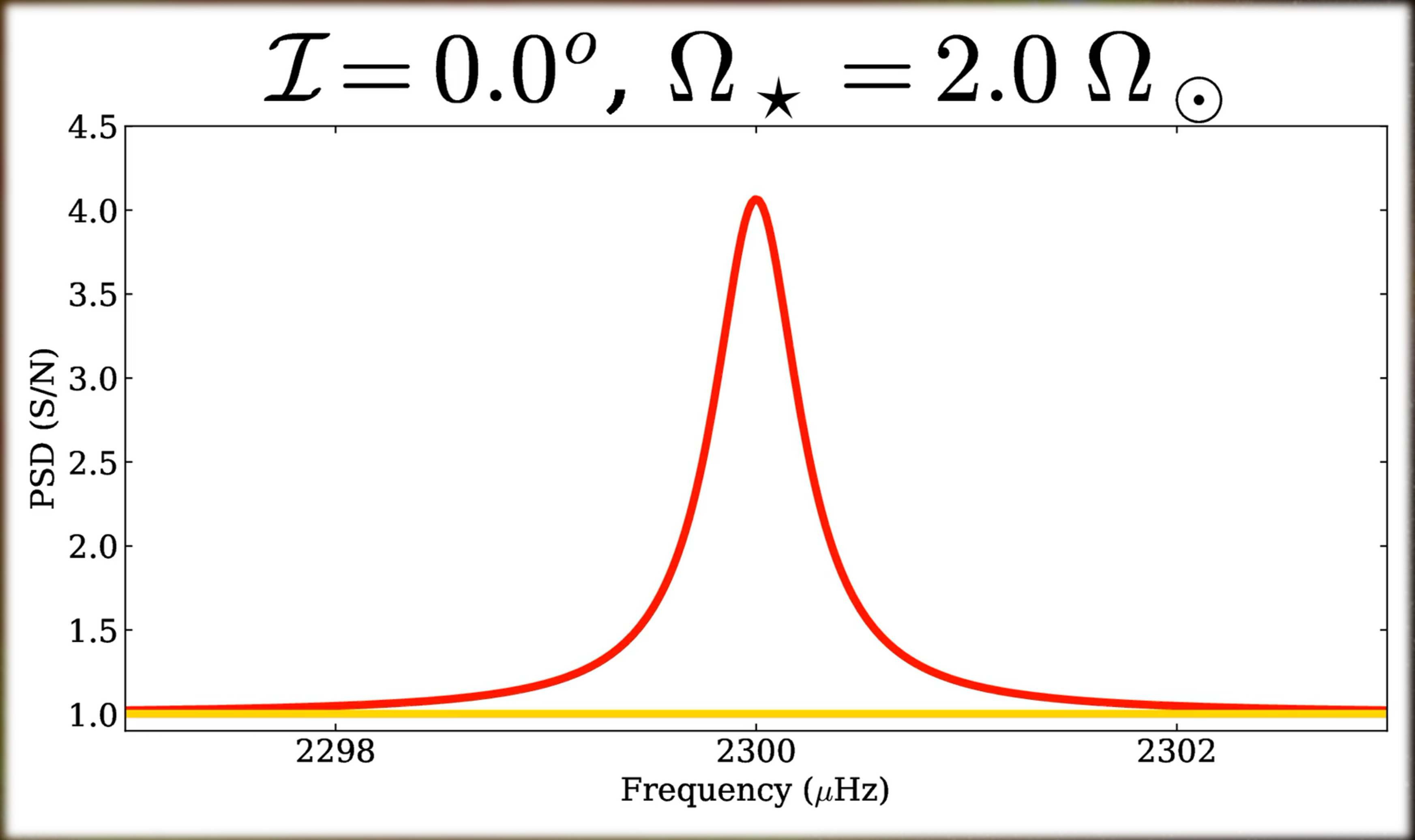
- Effect of the rotation on the m components of an l=1 mode



[Credits: S.N. Breton]

II-MEASURING ROTATION: SEISMOLOGY

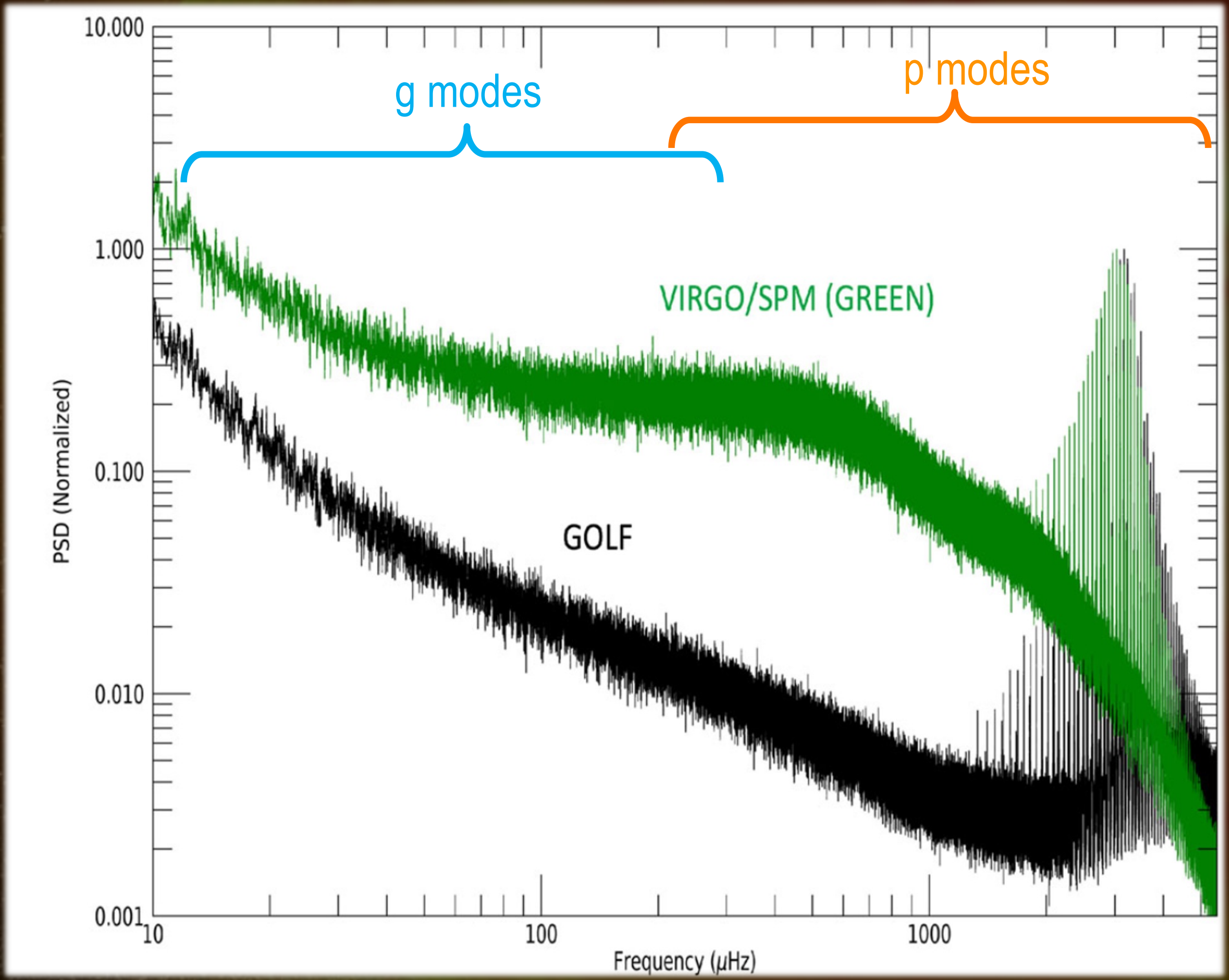
- Effect of the rotation inclination axis on the visibility of the m components of an l=1 mode



[Credits: S.N. Breton]

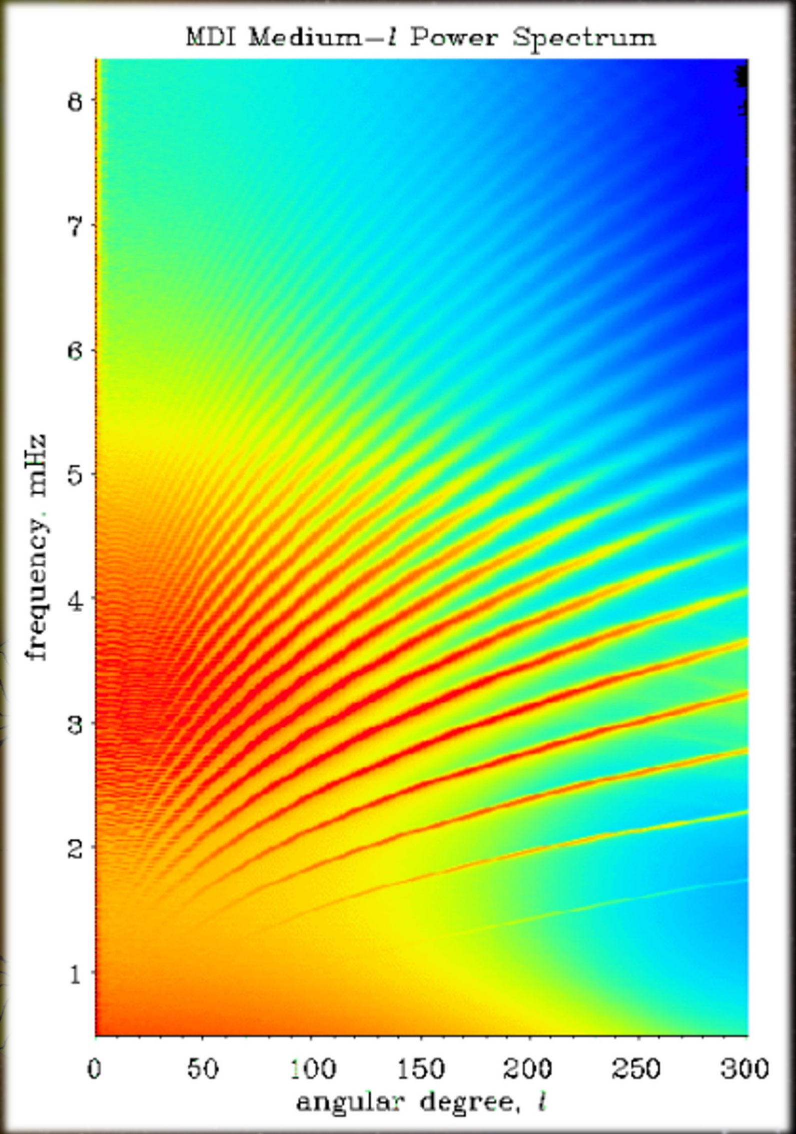
II-MEASURING ROTATION: SEISMOLOGY

- Integrated (Sun-as-a-star) instruments
  - Low-degree modes



[adapted from García+ & Ballot 2019]

- Imaged instruments
  - High-degree modes



http://www2.mps.mpg.de/projects/sun-climate/var_body.html

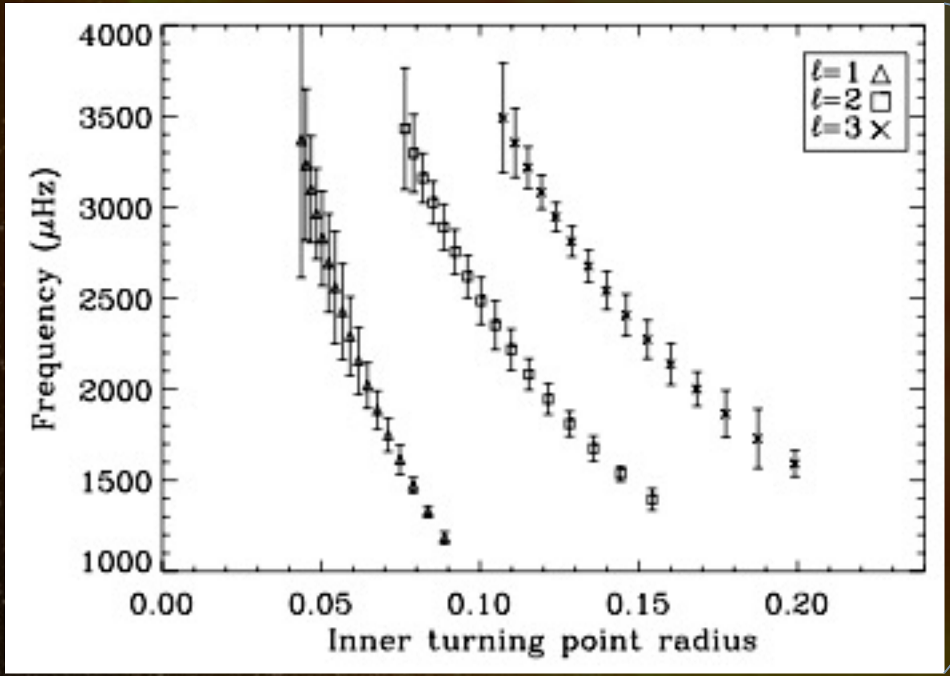
II-MEASURING ROTATION: SEISMOLOGY

- Lower turning point of p modes depend on:
  - Degree
  - Frequency

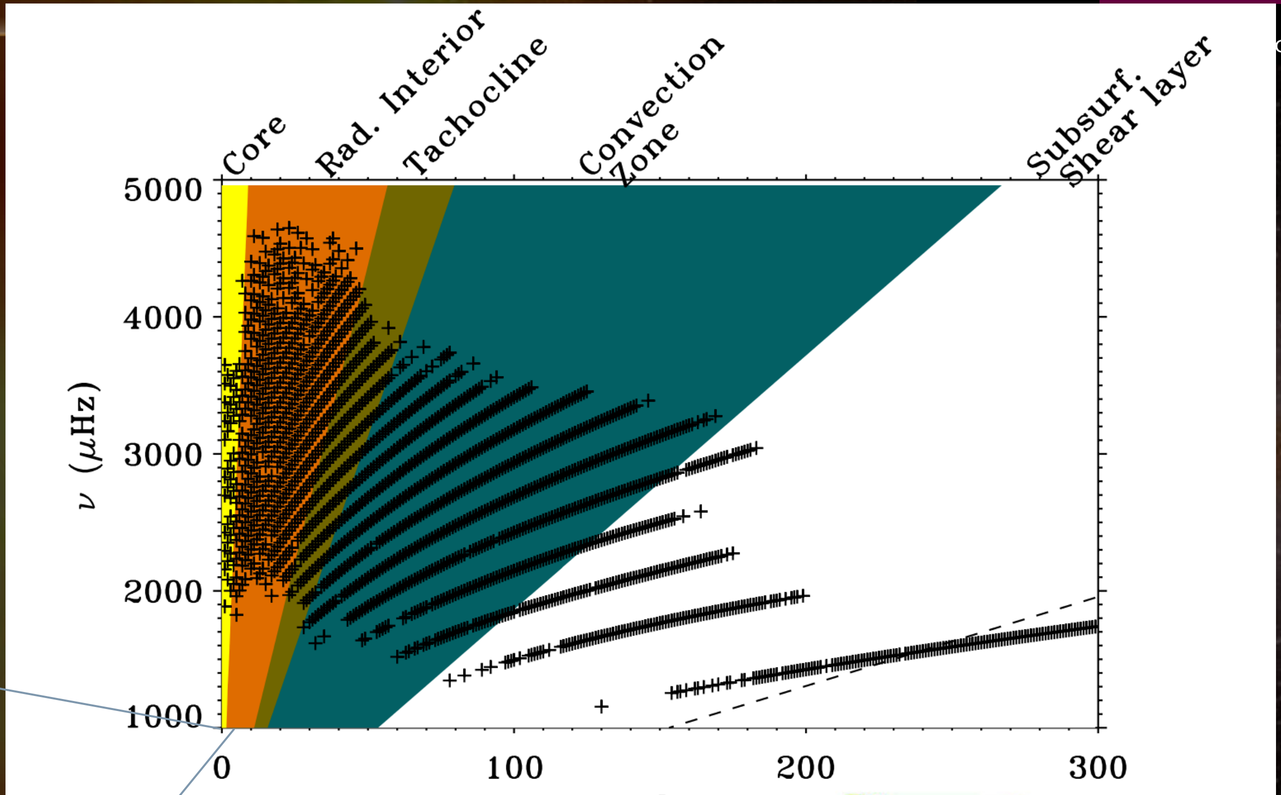
$$r_t = c_t L/(2\pi \nu_{n\ell})$$

$$L = \ell + 1/2$$

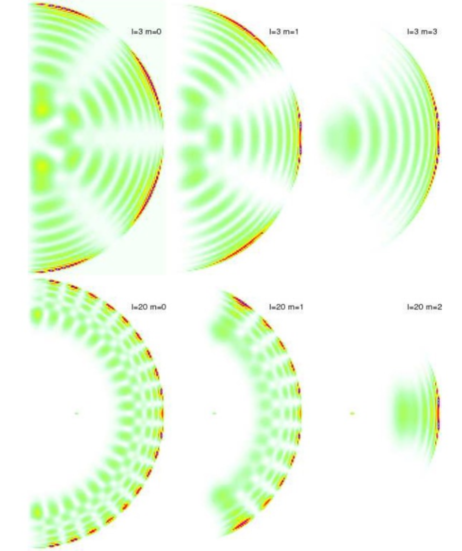
$$c_t = c(r_t)$$



[García+ 2008]

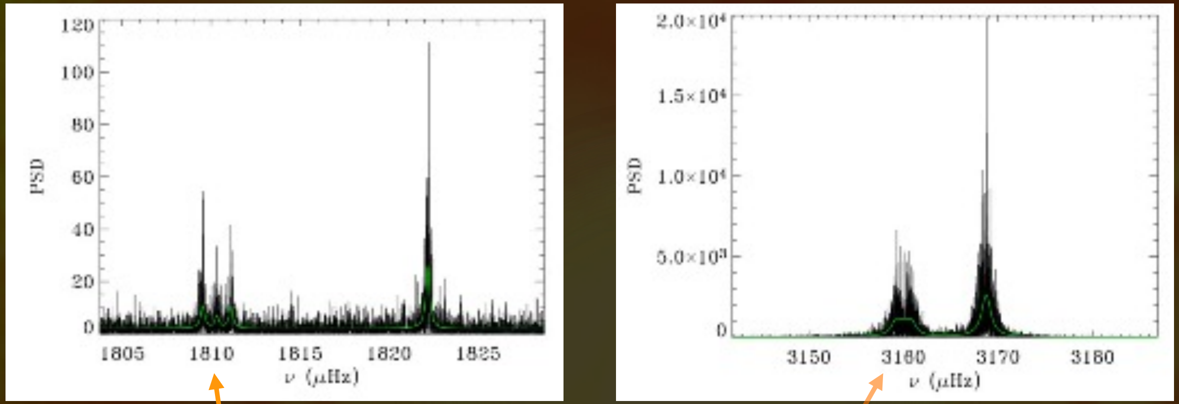


[Howe, 2009]



II-MEASURING ROTATION: SEISMOLOGY

- Which are the best modes to extract information on the inner radiative regions?
  - Balance between intrinsic sensibility and precision on the measurements

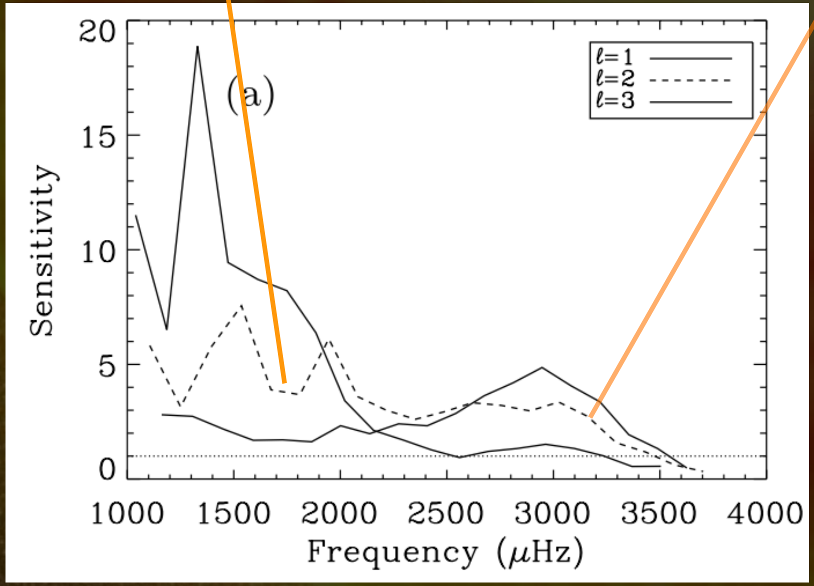


[García+ & Ballot 2019]

- For low-degree modes
  - Splittings are directly fitted to the data
    - Multi-Lorentzian profile

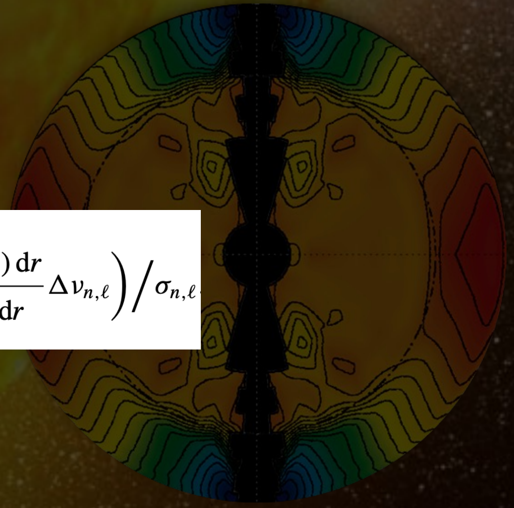
$$\nu_{n,\ell,m} = \nu_{n,\ell} - m\nu_s$$

Sensitivity of the splittings to the inner 0.2 R$_\odot$



[García+ 2008]

$$\text{Sensitivity} = \left(\frac{\int_0^{0.25R_\odot} K_{n,\ell}(r)\Omega(r)\,\mathrm{d}r}{\int_0^{R_\odot} K_{n,\ell}(r)\Omega(r)\,\mathrm{d}r} \Delta\nu_{n,\ell} \right) \Big/ \sigma_{n,\ell}$$



II-MEASURING ROTATION: SEISMOLOGY

- Measuring mid- and high-degree splittings
  - By fitting polynomials to the ridges

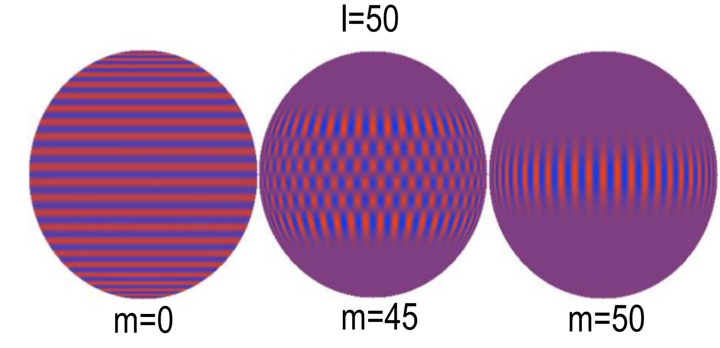
$$\nu_{nlm} = \nu_{nl} + \sum_{j=1}^{j_{\max}} a_j(n,l)\,\mathcal{P}_j^{(l)}(m)$$

$$\mathcal{P}_j^{(l)}(m) = \frac{l\sqrt{(2l-j)!(2l+j+1)!}}{(2l)!\sqrt{2l+1}} C_{j0lm}^{lm}$$

→ Clebsch–Gordan coefficients

[Ritzwoller and Lavely, 1991]

- Leakage problem
  - Spherical harmonic masks are used to separate the contributions from modes of different degrees and azimuthal orders



$$M_{l,m} \propto Y_{l,m}(\theta,\phi)A(\rho)$$

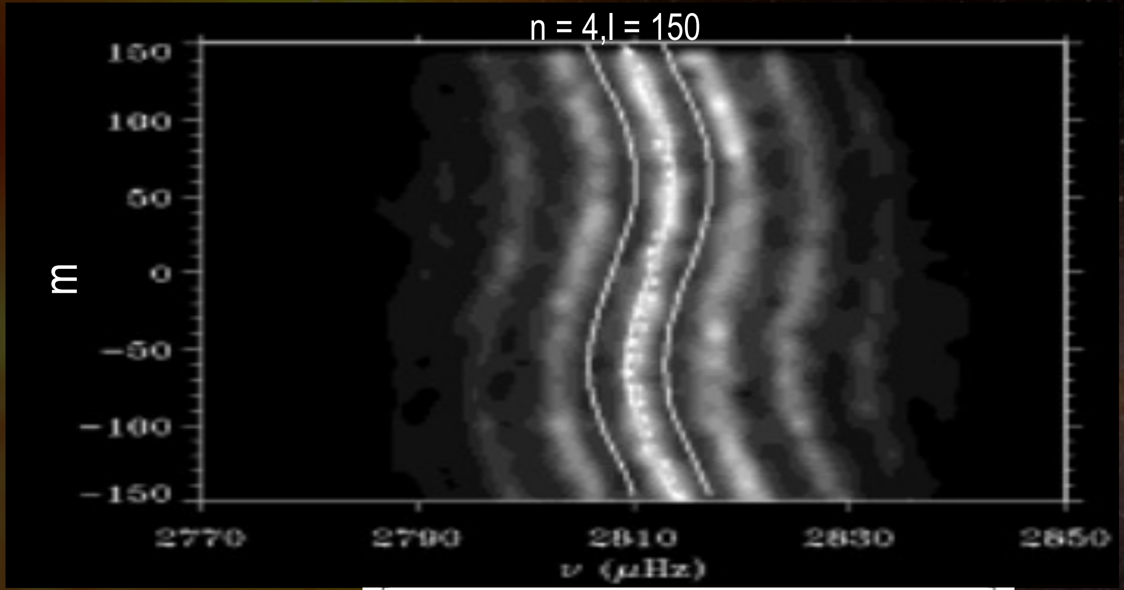
→ Fractional distance from disk center

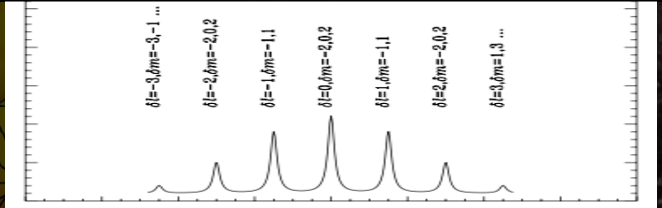
→ Apodization function

→ Longitude

→ Latitude

  - Because only part of the solar surface is visible at any time:
    - Masks are not completely orthogonal
      - Modes “leak” into neighboring spectra.





[Howe & Thompson 1998,
see also Schou & Brown 1994]

[see a review in Howe 2009]

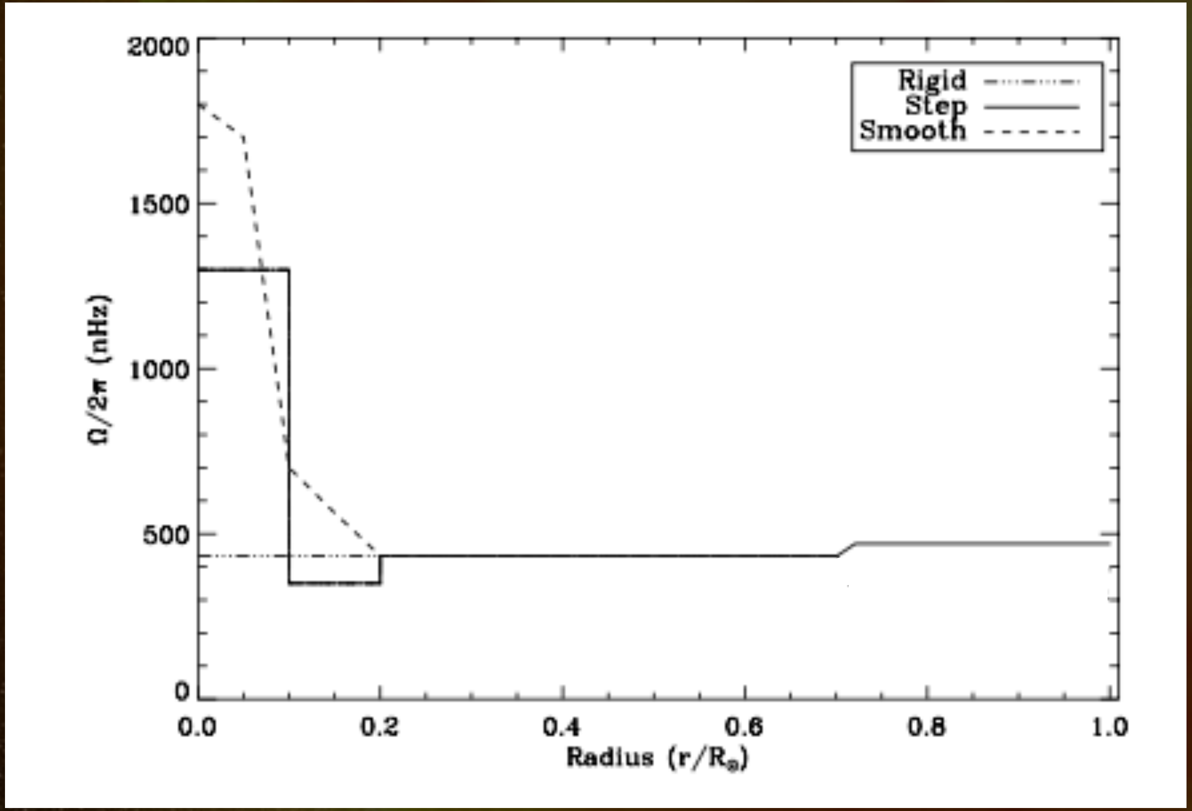
II-MEASURING ROTATION: SEISMOLOGY

- From splittings to internal rotation profile:

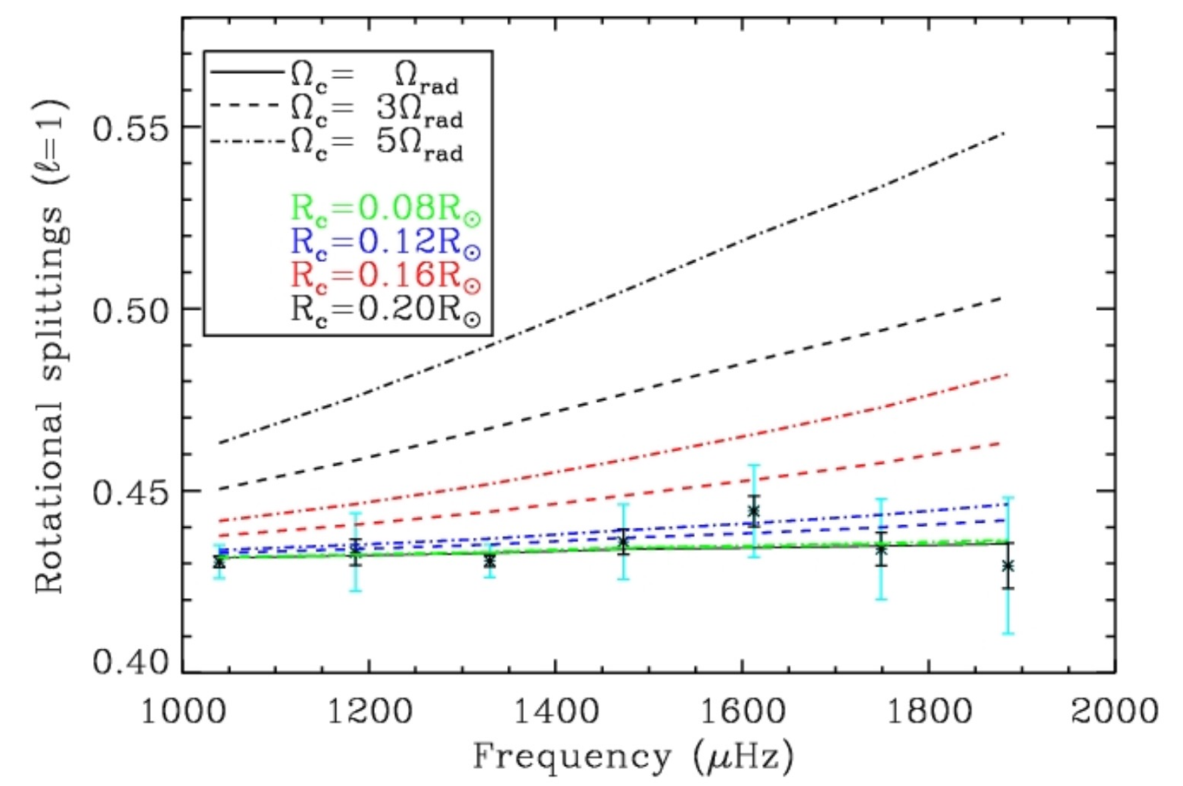
$$\text{Model} \xrightarrow{\text{Forward modeling}} \text{Observables}$$

Simulated rotation profiles

Theoretical an observed splittings



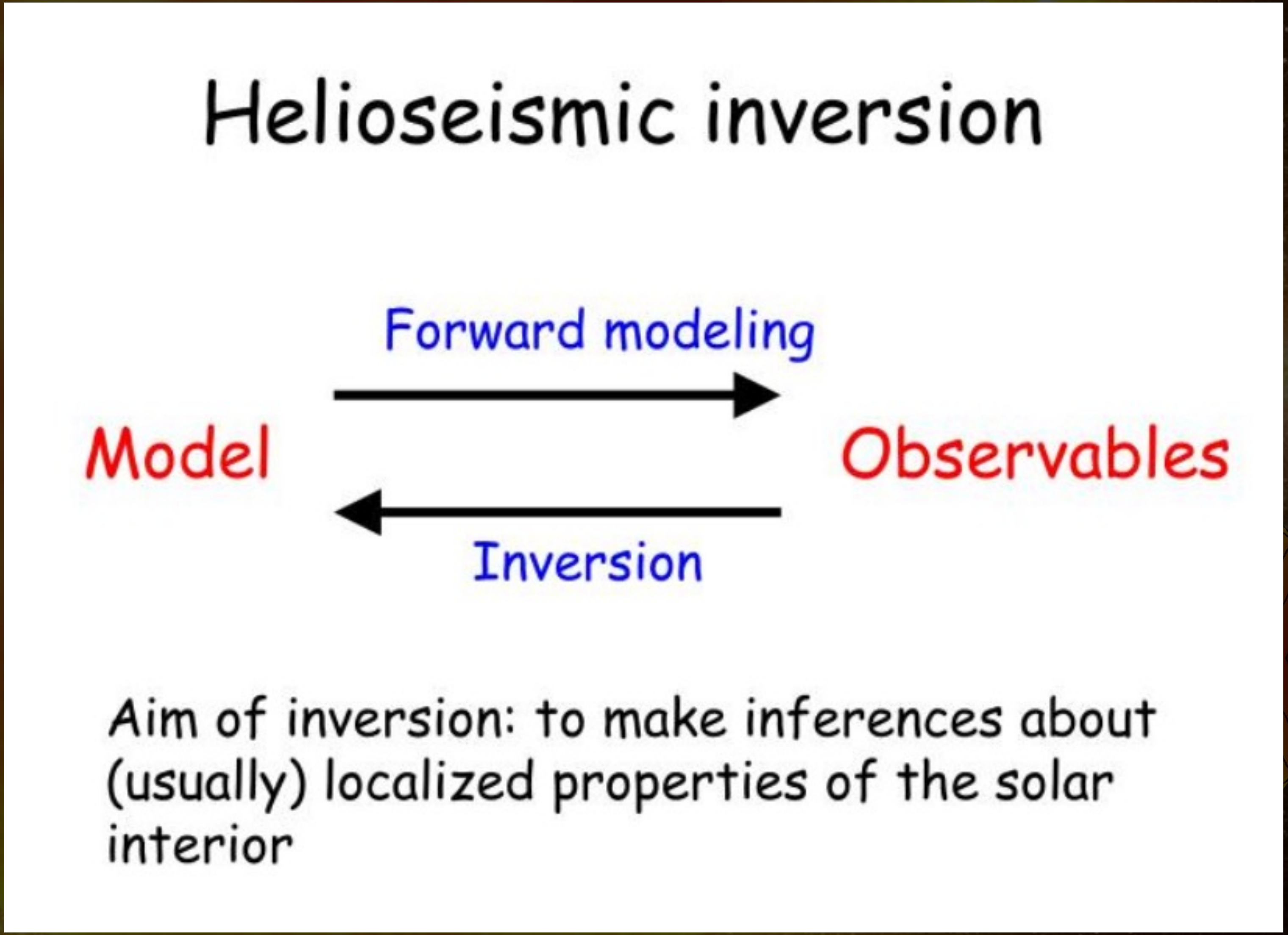
[Mathur 2007]



[García+ 2007]

II-MEASURING ROTATION: SEISMOLOGY

- From splittings to internal rotation profile: [Thompson, 2014]



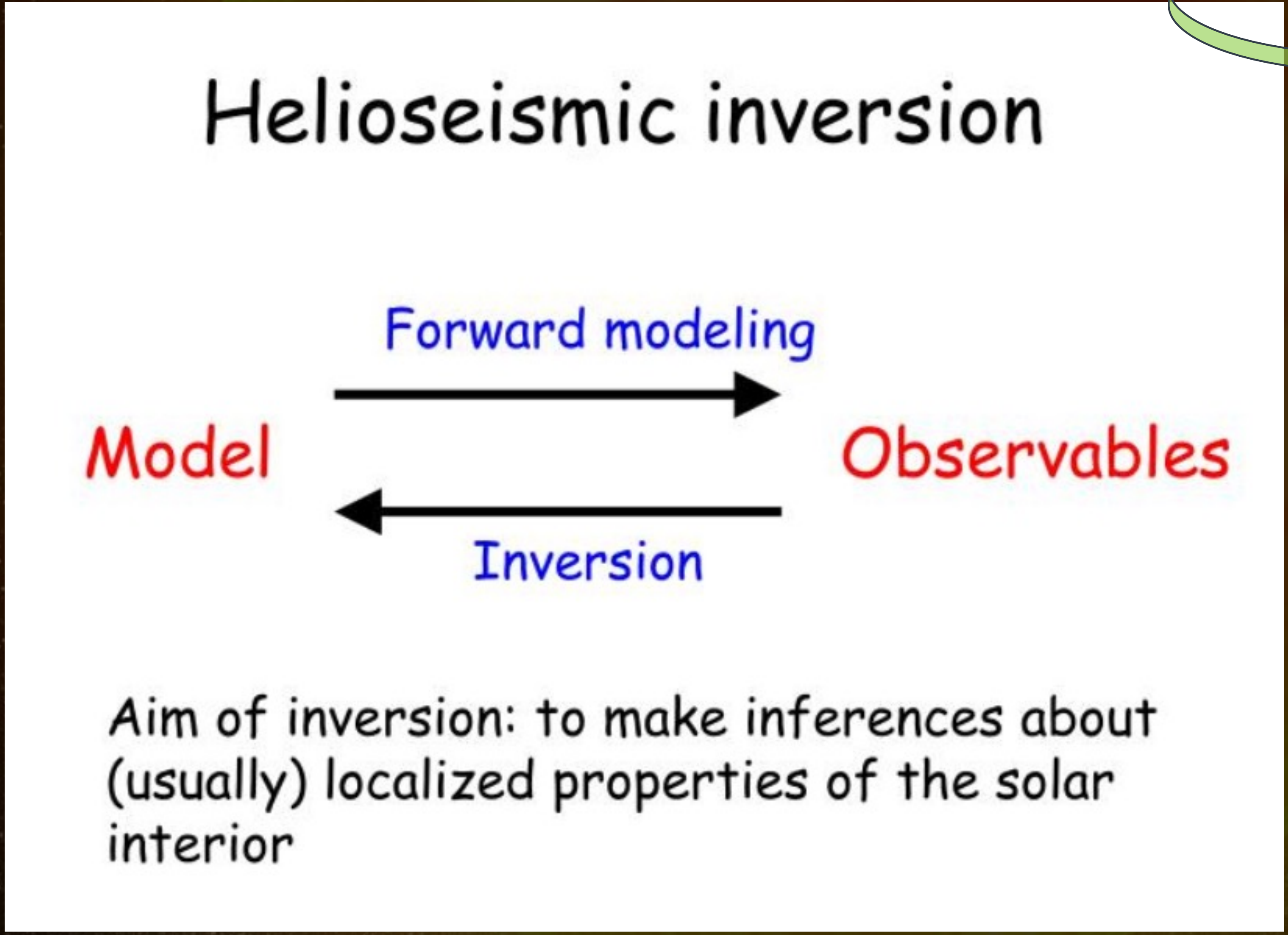
Helioseismic inversion

Aim of inversion: to make inferences about (usually) localized properties of the solar interior

II-MEASURING ROTATION: SEISMOLOGY

- From splittings to internal rotation profile:

[Thompson, 2014]



Helioseismic inversion

Forward modeling

Model

Observables

Inversion

Aim of inversion: to make inferences about (usually) localized properties of the solar interior

Splittings for 1-D rotation

Let $d_{nl} = (\omega_{nlm} - \omega_{nl0})/m$ be our data. Then

$$d_{nl} = \int K_{nl}(r)\Omega(r)\,dr + \epsilon_{nl}$$

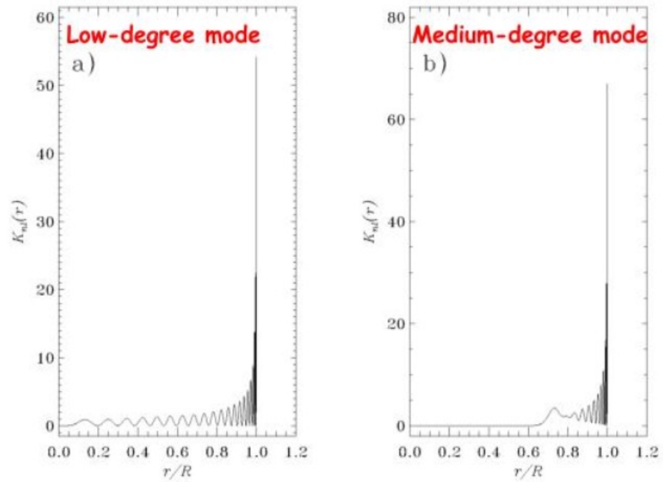
where ε_{nl} are noise in the data, each with with standard deviation (s.d.) σ_{nl}.

Kernels

[Hansen+ 1977, Cuypers 1980, Schou+1994

Model-dependent spatial weighting functions

Kernels $K_{nl}(r)$ for 1-D rotation

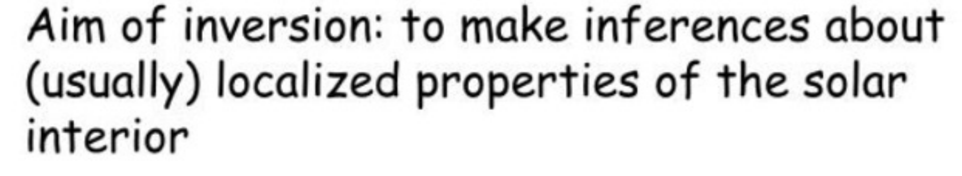


2D

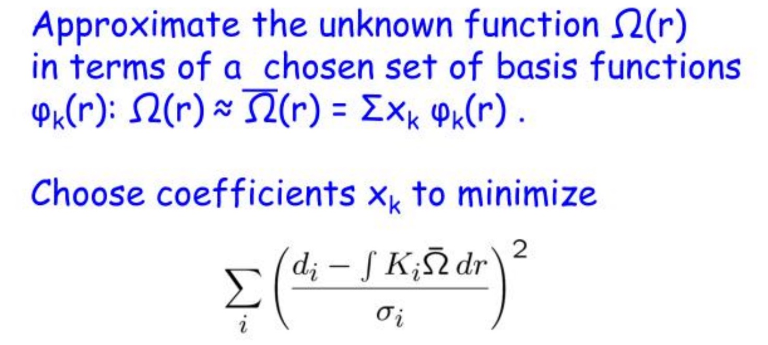
$$\Delta\nu_{n\ell m} = \frac{1}{2\pi}\int_0^R \int_0^\pi K_{n\ell m}(r,\theta)\Omega(r,\theta)dr d\theta + \epsilon_{n\ell m}$$

II-MEASURING ROTATION: SEISMOLOGY

- From splittings to internal rotation profile:
  - Inversions (see talk on Wednesday by J. Bétrisey)
  - Several methods:
    - Least Squares (LS) fitting



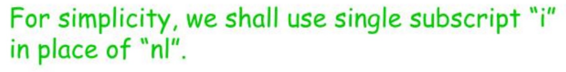
Aim of inversion: to make inferences about (usually) localized properties of the solar interior



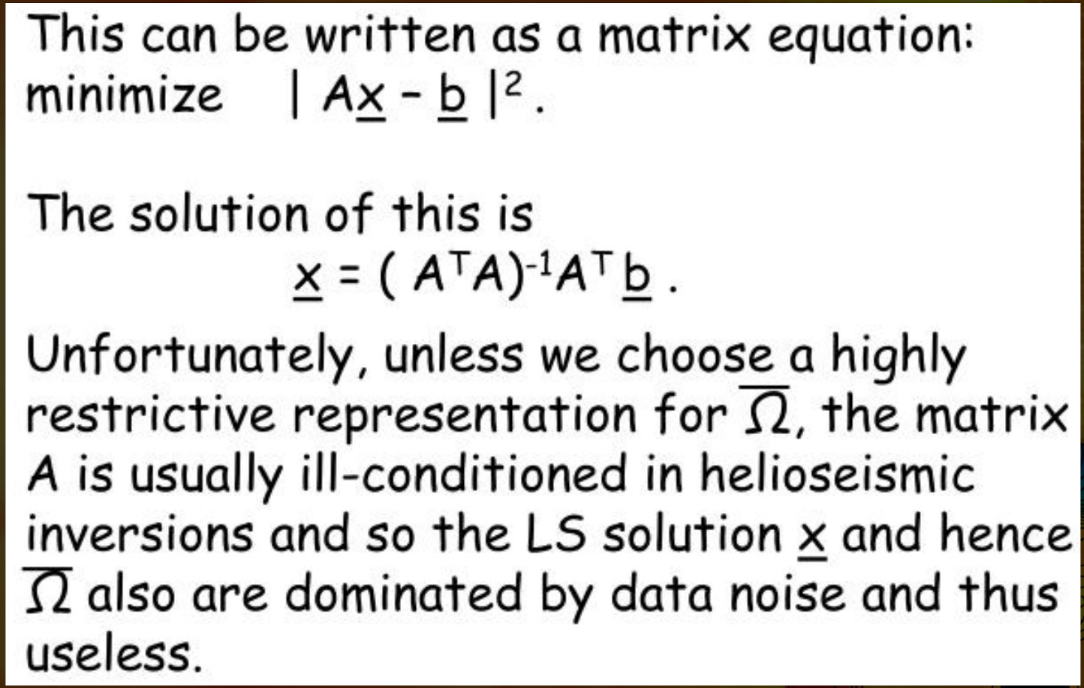
Approximate the unknown function $\Omega(r)$ in terms of a chosen set of basis functions $\varphi_k(r)$: $\Omega(r) \approx \overline{\Omega}(r) = \Sigma x_k \varphi_k(r)$.

Choose coefficients x_k to minimize

$$\sum_i \left(\frac{d_i - \int K_i \bar{\Omega}\, dr}{\sigma_i}\right)^2$$



For simplicity, we shall use single subscript “i” in place of “nl”.

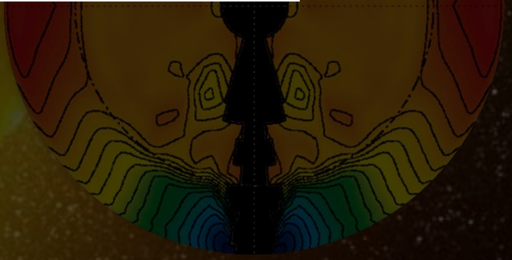


This can be written as a matrix equation: minimize $| A\underline{x} - \underline{b} |^2$.

The solution of this is

$$\underline{x} = (A^T A)^{-1} A^T \underline{b} .$$

Unfortunately, unless we choose a highly restrictive representation for $\overline{\Omega}$, the matrix A is usually ill-conditioned in helioseismic inversions and so the LS solution $\underline{x}$ and hence $\overline{\Omega}$ also are dominated by data noise and thus useless.



[Thompson, 2014]

II-MEASURING ROTATION: SEISMOLOGY

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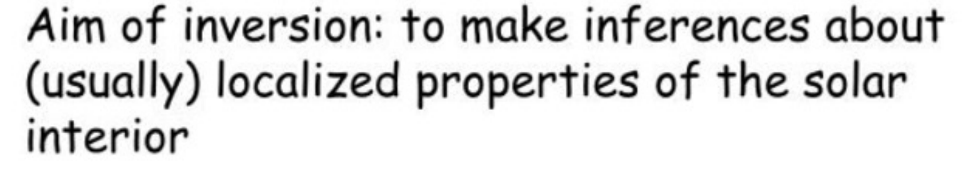
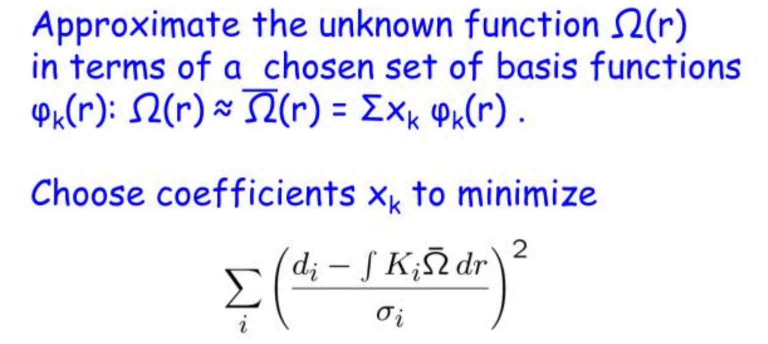


Aim of inversion: to make inferences about (usually) localized properties of the solar interior

[Thompson, 2014]



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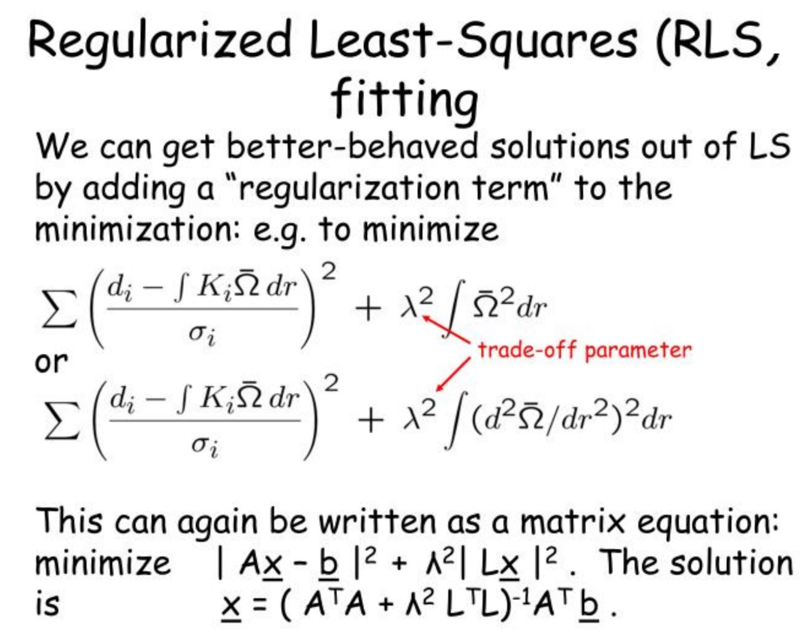
For simplicity, we shall use single subscript "i" in place of "nl".

Regularized Least Squares (RLS) fitting

[Craig & Brown 1986]



Regularized Least-Squares (RLS, fitting

We can get better-behaved solutions out of LS by adding a "regularization term" to the minimization: e.g. to minimize

$$\sum \left(\frac{d_i - \int K_i \bar{\Omega}\, dr}{\sigma_i}\right)^2 + \lambda^2 \int \bar{\Omega}^2 dr$$

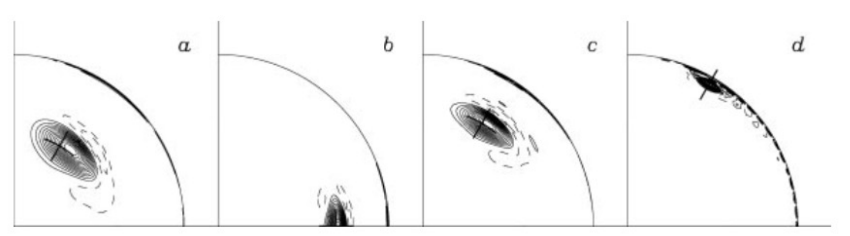
trade-off parameter

or

$$\sum \left(\frac{d_i - \int K_i \bar{\Omega}\, dr}{\sigma_i}\right)^2 + \lambda^2 \int (d^2\bar{\Omega}/dr^2)^2 dr$$

This can again be written as a matrix equation: minimize $| A\underline{x} - \underline{b} |^2 + \lambda^2 | L\underline{x} |^2$. The solution is $\underline{x} = (A^T A + \lambda^2 L^T L)^{-1} A^T \underline{b}$.

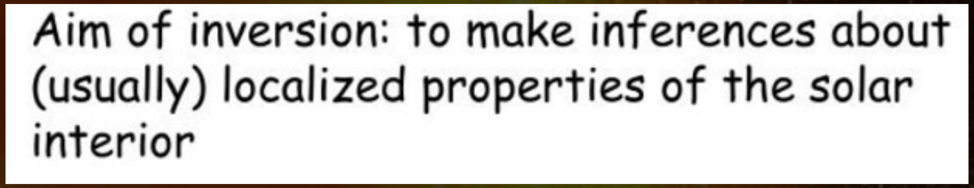
Example of averaging kernels for a RLS inversion



[Schou+ 1998, Thompson+2003]

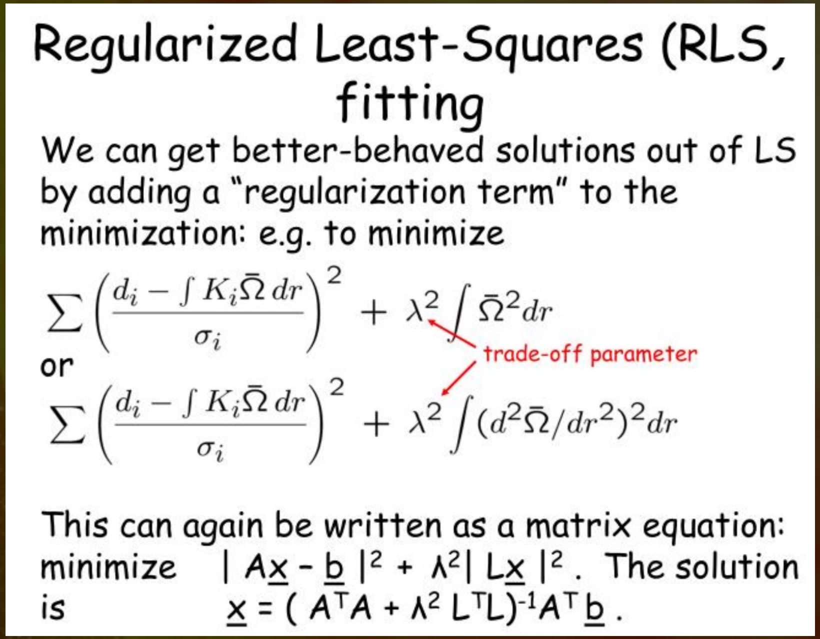
II-MEASURING ROTATION: SEISMOLOGY

- From splittings to internal rotation profile:
  - Inversions (see talk on Wednesday by J. Bétrisey)
  - Several methods:



Aim of inversion: to make inferences about (usually) localized properties of the solar interior

- Regularized Least Squares (RLS) fitting [Craig & Brown 1986]



Regularized Least-Squares (RLS, fitting

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trade-off parameter

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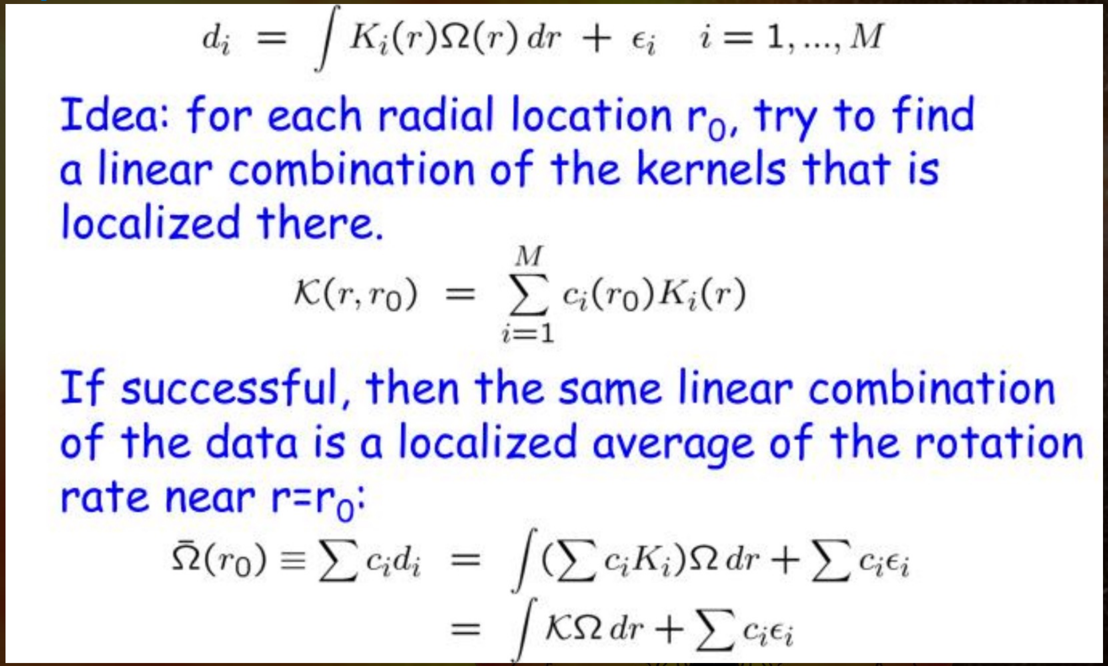
$$\sum\left(\frac{d_i-\int K_i\bar{\Omega}\,dr}{\sigma_i}\right)^2+\lambda^2\int(d^2\bar{\Omega}/dr^2)^2dr$$

This can again be written as a matrix equation: minimize $|A\underline{x}-\underline{b}|^2+\lambda^2|L\underline{x}|^2$. The solution is $\underline{x}=(A^TA+\lambda^2L^TL)^{-1}A^T\underline{b}$.

Optimally Localized Averages (OLA) [Backus and Gilbert, 1968, 1970]



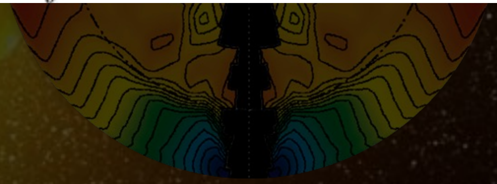
$$d_i=\int K_i(r)\Omega(r)\,dr+\epsilon_i\quad i=1,\ldots,M$$

Idea: for each radial location r_0, try to find a linear combination of the kernels that is localized there.

$$\mathcal{K}(r,r_0)=\sum_{i=1}^{M}c_i(r_0)K_i(r)$$

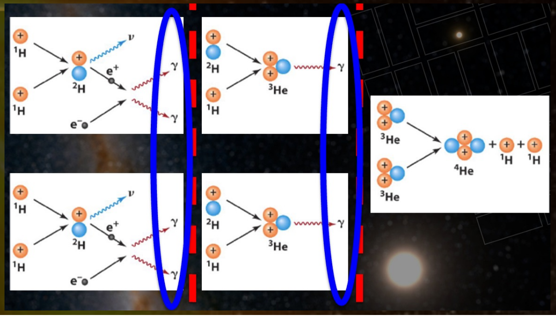
If successful, then the same linear combination of the data is a localized average of the rotation rate near $r=r_0$:

$$\bar{\Omega}(r_0)\equiv\sum c_id_i=\int\left(\sum c_iK_i\right)\Omega\,dr+\sum c_i\epsilon_i$$
$$=\int\mathcal{K}\Omega\,dr+\sum c_i\epsilon_i$$

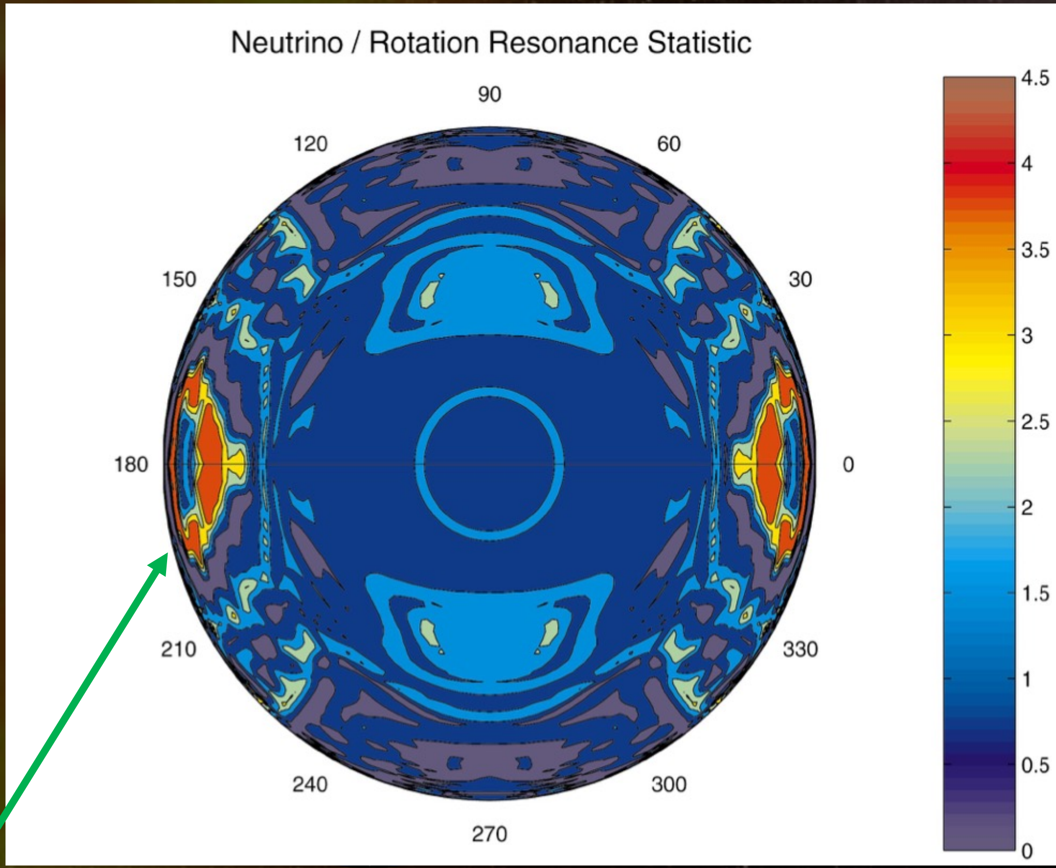


[Thompson, 2014]

II-MEASURING ROTATION: NEUTRINOS



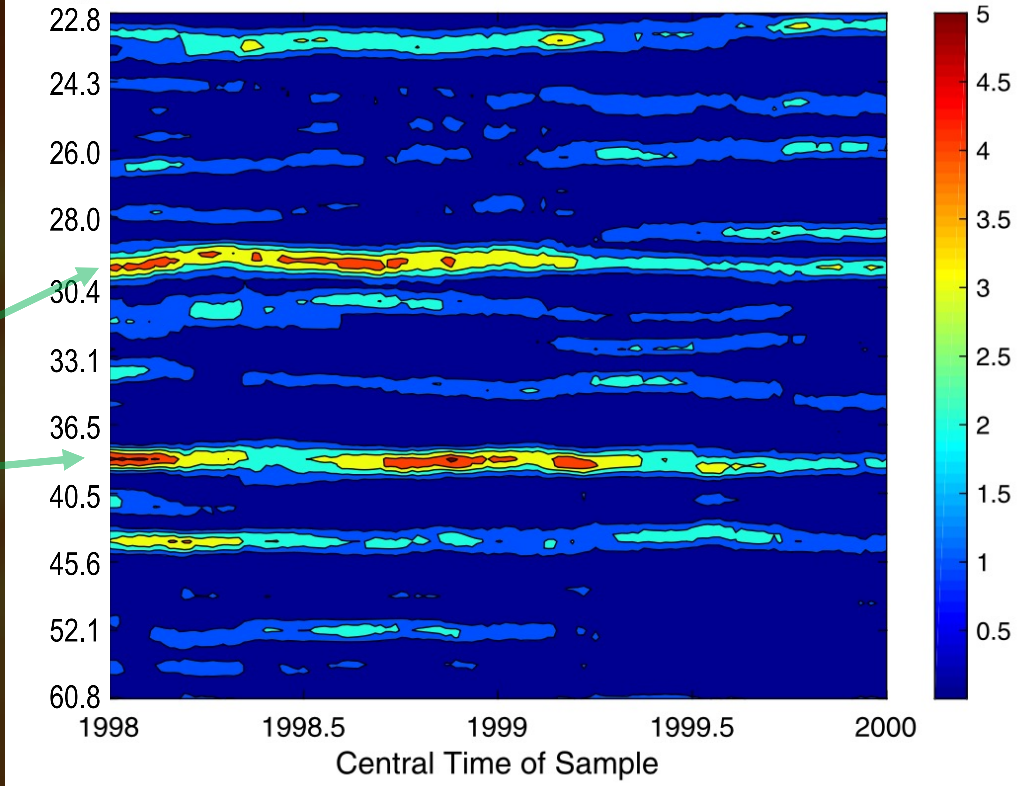
- Neutrinos:
  - Another way to extract information of the solar interior
    - Depends on the thermodynamical conditions of the core
    - And the nuclear reactions
- Variability of solar neutrino flux
  - Time scales of weeks (GALLEX-GNO and SAGE)
  - If neutrinos have nonzero magnetic moment:
    - Their flux could be modulated by inhomogeneous magnetic fields
      - Time scales of solar rotation
  - Comparison with SoHO/MDI data:
    - Common peaks in the L-S periodogram in the CZ
    - Resonance statistics:
      - Degree of resonance of oscillations in the neutrino flux and the solar rotation as a function of radius and latitude
      - The modulation is occurring in the convection zone, near the equator



[Sturrock & Weber 2002]

II-MEASURING ROTATION: NEUTRINOS

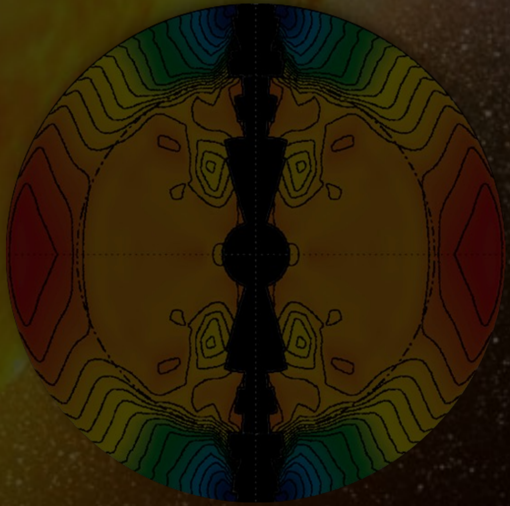
- Neutrinos:
  - Another way to extract information of the solar interior
    - Depends on the thermodynamic conditions of the core
    - And the nuclear reactions
- Variability of solar neutrino flux
  - Time scales of weeks (Super-Kamiokande)
  - Two main stable peaks at 38.71 and 28.97 days



- Variability found in nuclear-decay experiments
  - Found in different laboratories in the world
  - Modulations at around solar rotation rate found
- Rotational modulations due to an influence of the solar internal magnetic field by the RSFP (Resonant Spin-Flavour Precession) process.
  - Detection of doublets and triplet separated by 1 year
    - Could be due to misalignments internal rotation axes with respect to the normal to the ecliptic

[Sturrock 2022]

III- Results



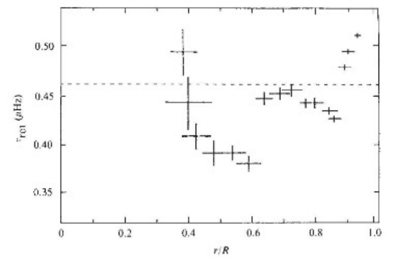
III-RESULTS

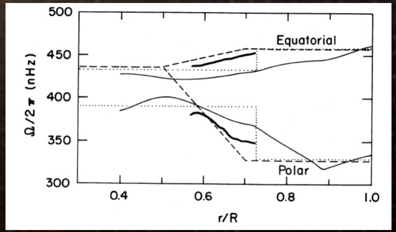
- Some key periods of time:
  - 1960s: Before seismology
    - Measurement of the oblatness of the Sun that could be caused by a fast internal rotation but
      - Controversy raised [Dicke 1964, Dicke and Goldenberg 1967]
        - E.g. could be explained by differential rotation [Roxburgh 1967]
    - SCLERA found a much smaller oblatness and some temporal variation [Hill and Stebbins 1975, Hill 1974]
      - Temporal variations confirmed with SoHO/MDI observations (higher at maximum activity) [Kuhn+ 1998, Emilio+ 2007]
        - But the temporal variations could be removed by a magnetic term [Fivian+ 2008]

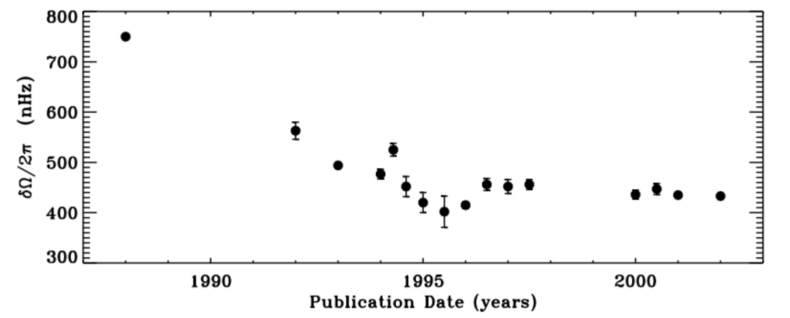
[See reviews e.g. by Howe, 2009, Thompson+ 2003 and references therein]

III-RESULTS: SEISMOLOGY

- Some key periods of time:
  - 1960s: Before seismology
    - Measure of the oblatness of the Sun that could be caused by a fast internal rotation but
    - SCLERA found a much smaller oblatness and some temporal variation
  - 1980s: With Seismology
    - First measurements of global splittings [Claverie+ 1981, Isaak 1982]
      - But the values given were too high and not confirmed, ruled out later by the large width of the modes [e.g. Fossat 1981, Woodard & Hudson 1983]
    - And from SCLERA [Hill+ 1982, Hill 1985]
    - First results on rotation nearly solar equator [Duvall Jr+ 1984] →
      - Small variation with r but slower in the RZ [Brown 1985, Duvall Jr+ 1986]
    - Dependence of splittings with m
      - First constraints on latitudinal rotation [e.g., Brown & Morrow 1987, Brown+ 1989, Christensen-Dalsgaard & Schou 1988, Kosovichev 1988, Libbrecht 1988, Rhodes+ 1990]
  - >1990s
    - BiSON + IRIS + IPHIR + SoHO/(GOLF, MDI, VIRGO) + LOW-l + SDO/HMI







[Howe, 2009]

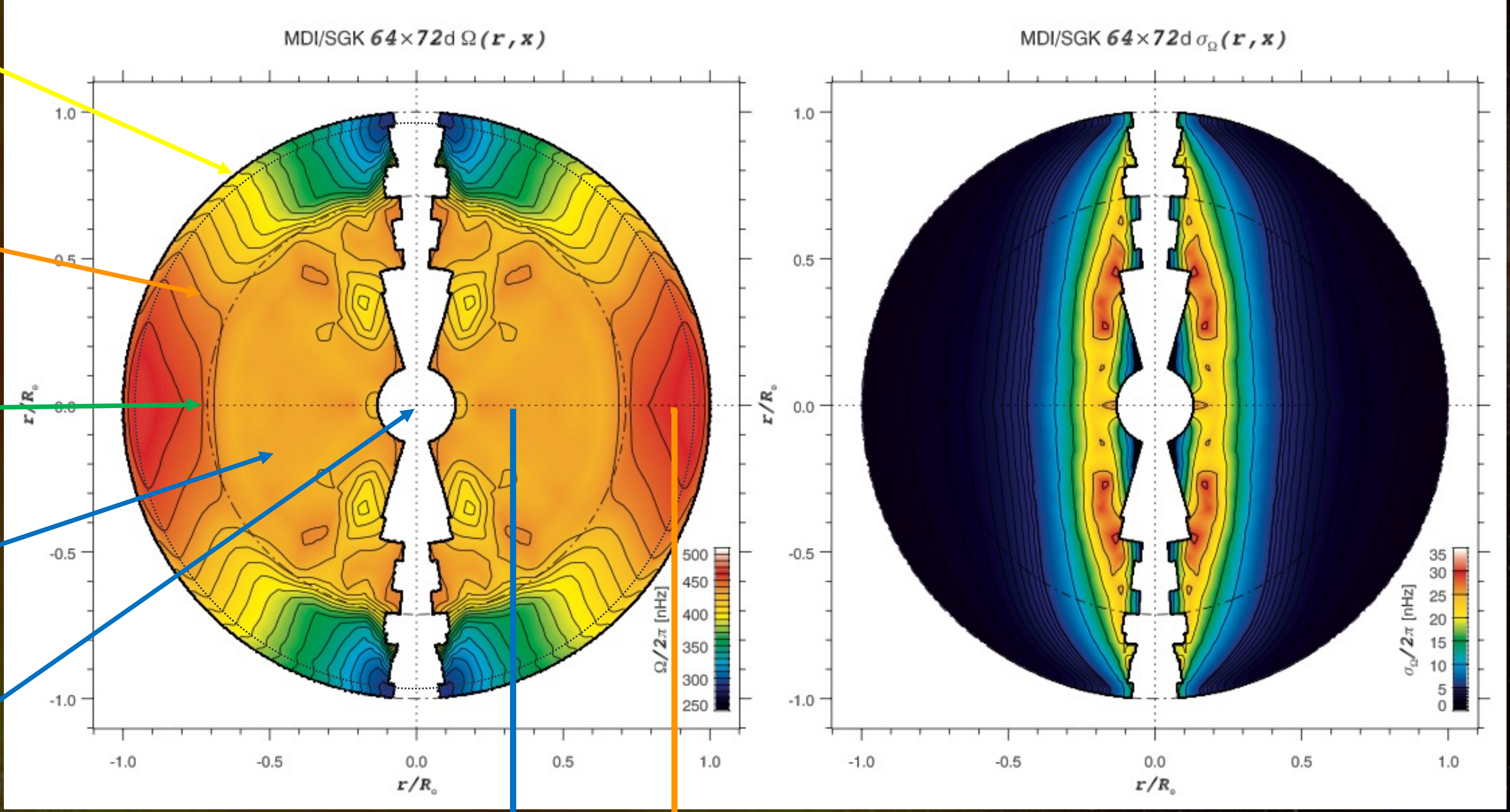
[See reviews e.g. by Howe, 2009, Thompson+ 2003 and references therein]

III-RESULTS: SEISMOLOGY

- Medium degree modes (up to l ~200)

Cycle 23: 12.6 years

[Korzennick & Eff-Darwich 2012]



Near-surface shear

CZ

Tachocline

RZ

Core

CZ: Differential rotation

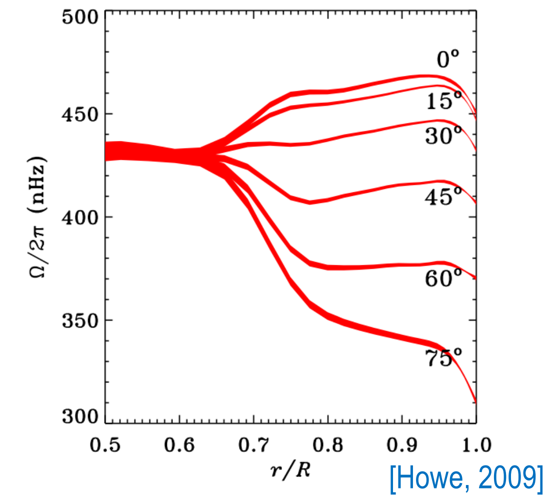
RZ: Rigid rotation

III-RESULTS: SEISMOLOGY

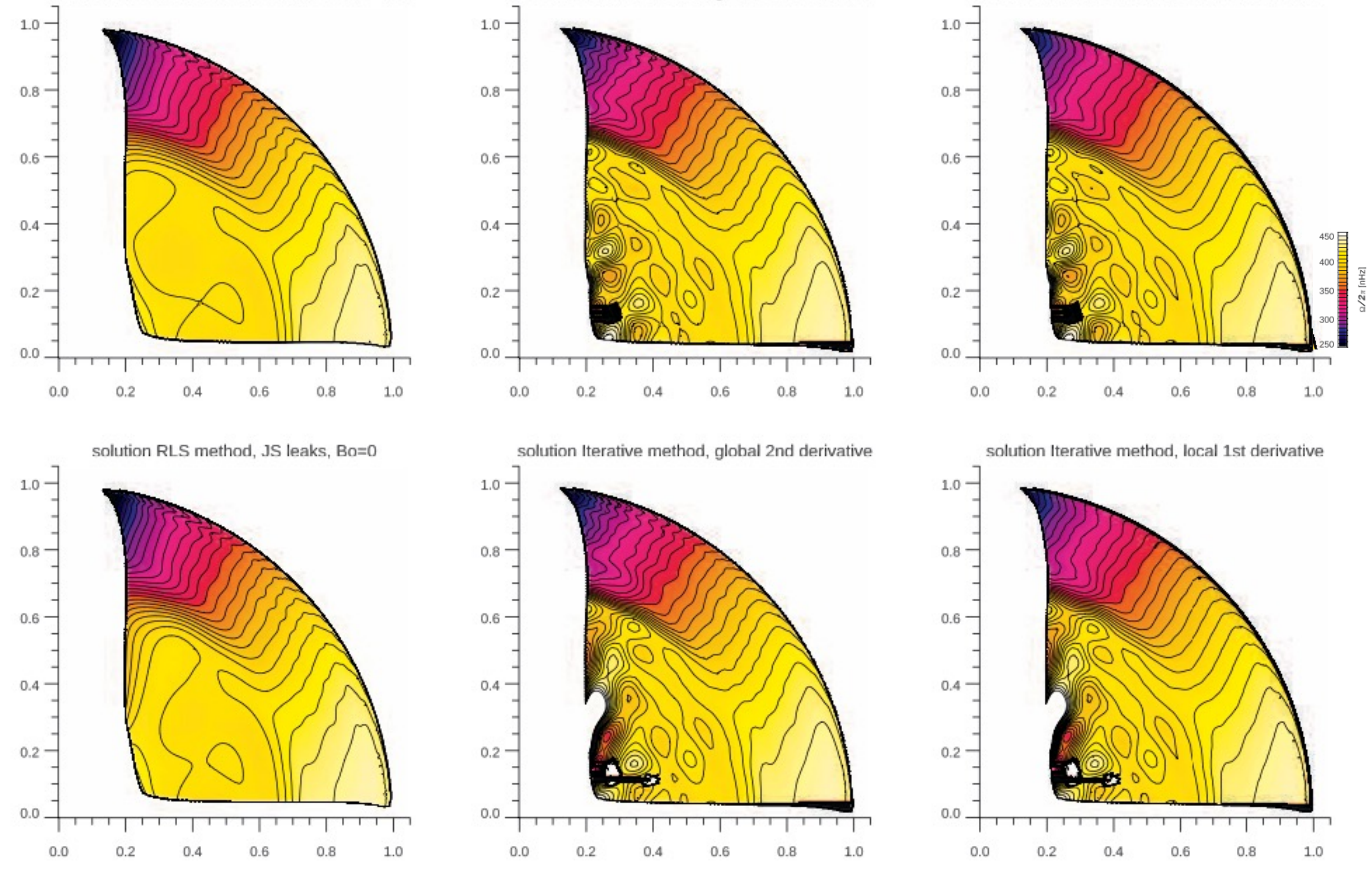
- Medium degree modes (up to l ~200)
  - Need to use the longest time series (SNR follows sqrt(T))
    - But posible biases due to magnetic activity
      - Depending on current uncertainties

[García+ 2009]

  - GONG 25 years
  - Compared to other sets of data
    - MDI & HMI
  - In general, differences are
    - ~< 10%



[Howe, 2009]



solution RLS method, SGK leaks, Bo=a2 + PSF

solution Iterative method, global 2nd derivative

solution Iterative method, local 1st derivative

solution RLS method, JS leaks, Bo=0

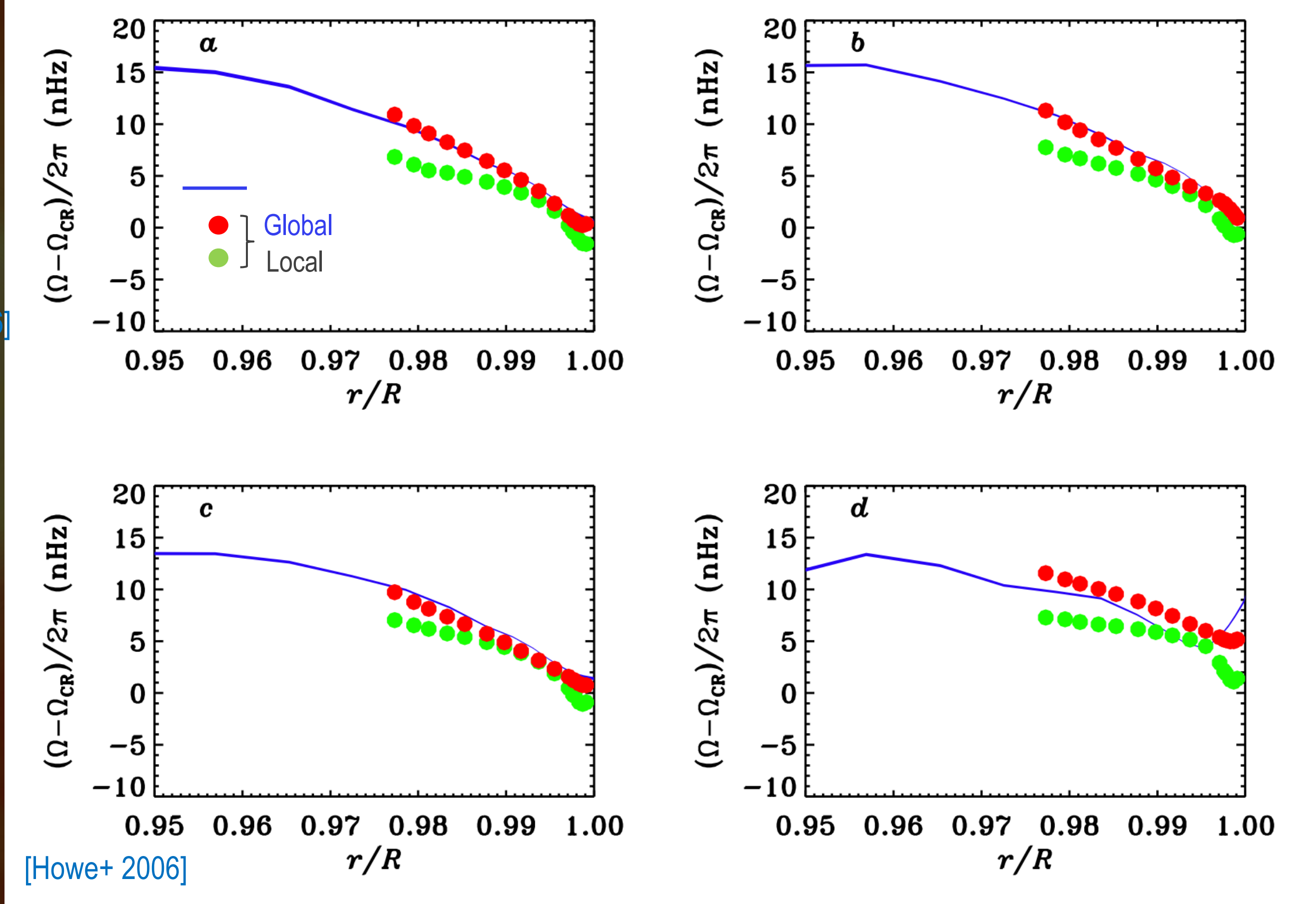
solution Iterative method, global 2nd derivative

solution Iterative method, local 1st derivative

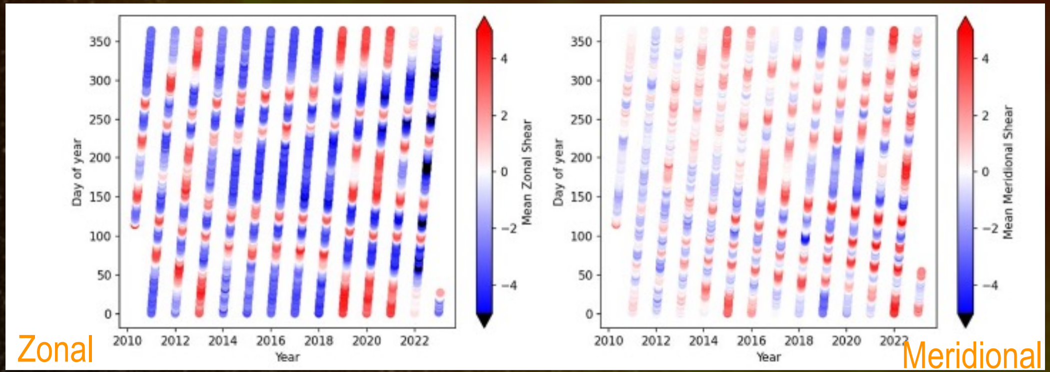
[Korzennick & Eff-Darwich 2023]

III-RESULTS: SEISMOLOGY
NEAR-SURFACE SHEAR

- Visible in modern inversions
  - Not visible at higher latitudes towards the poles
  - Similar results independently of
    - Instruments
    - And techniques:
      - Global vs Local seismology [Basu +1999, Howe+ 2006]
  - Quasiperiodic Global-Scale Oscillations
    - Large scale shear variations
      - Both zonal and meridional flows
      - Reverses with a period of a solar rotation
    - Small scales anomalies around active regions
      - Support solar origin [Bogart+ 2023]



[Howe+ 2006]

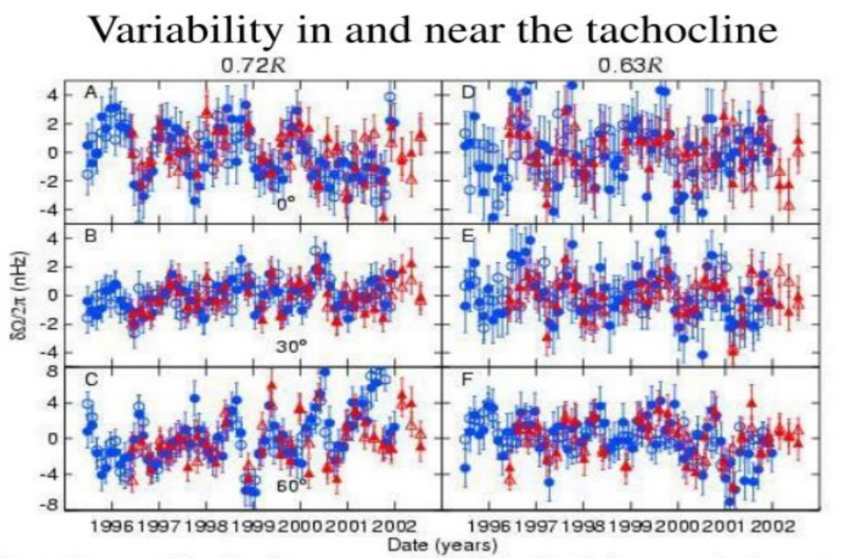


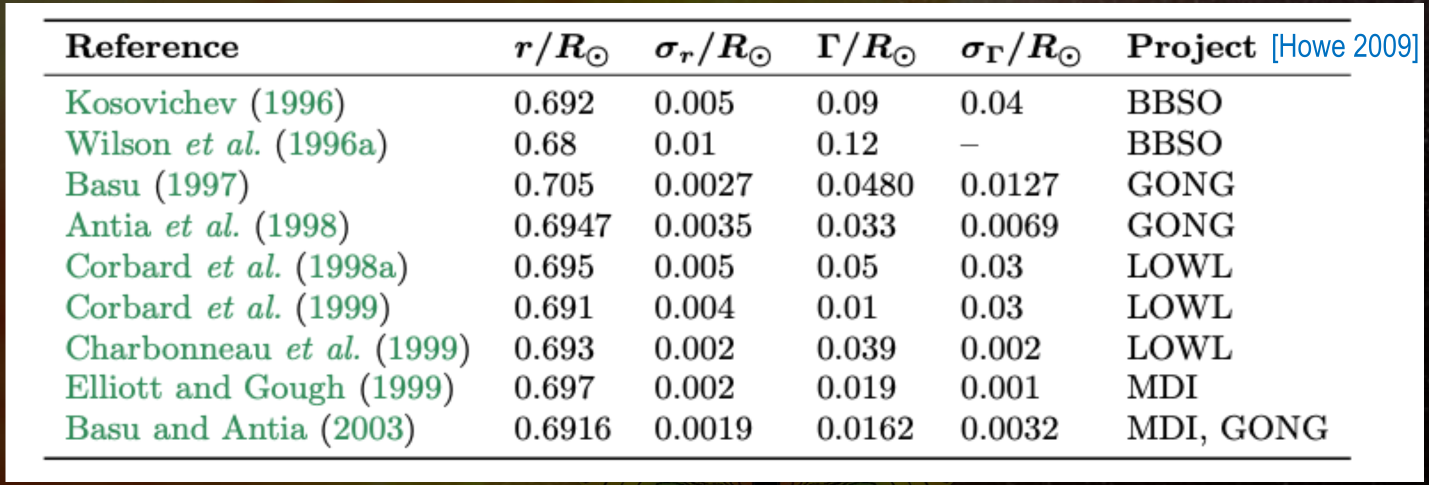
Zonal

Meridional

III-RESULTS: SEISMOLOGY TACHOCLINE

- Late 80s: Presence of shear layer [Brown+1989, Dziembowski+ 1989]
  - Modes of l ~20
    - lower turning points at this depth but with poor resolution in the inversion (1-10% in the radial direction)
  - General consensus stablishes the centroid of the shear layer right below the base of the CZ
  - The thickness is ~0.05$R_\odot$
    - With some possible change with latitude [Charbonneau+ 1999, Basu & Antia 2003]
  - Possible variations with time
    - But still controversial





Reference	$r/R_\odot$	$\sigma_r/R_\odot$	$\Gamma/R_\odot$	$\sigma_\Gamma/R_\odot$	Project
Kosovichev (1996)	0.692	0.005	0.09	0.04	BBSO
Wilson *et al.* (1996a)	0.68	0.01	0.12	–	BBSO
Basu (1997)	0.705	0.0027	0.0480	0.0127	GONG
Antia *et al.* (1998)	0.6947	0.0035	0.033	0.0069	GONG
Corbard *et al.* (1998a)	0.695	0.005	0.05	0.03	LOWL
Corbard *et al.* (1999)	0.691	0.004	0.01	0.03	LOWL
Charbonneau *et al.* (1999)	0.693	0.002	0.039	0.002	LOWL
Elliott and Gough (1999)	0.697	0.002	0.019	0.001	MDI
Basu and Antia (2003)	0.6916	0.0019	0.0162	0.0032	MDI, GONG

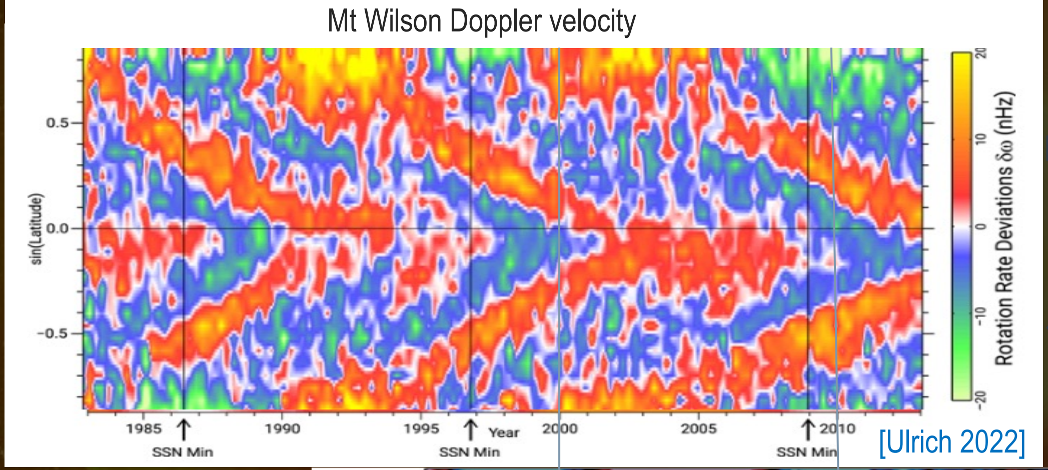
[Howe 2009]

III-RESULTS:

VARIATIONS: TORSIONAL OSCILLATIONS

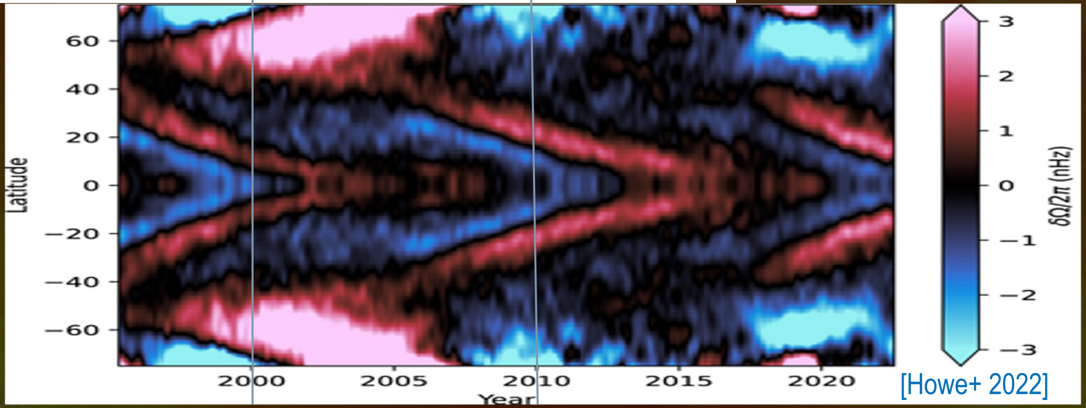
- Is a pattern of migrating bands faster/slower than average zonal (i.e., parallel to the equator) flow associated with the equatorward drift of the activity belts during the solar cycle.

[Howard and LaBonte 1980, Kosovichev & Schou 1997, Basu & Antia 2003, Vorontsov+ 2002..]



[Ulrich 2022]

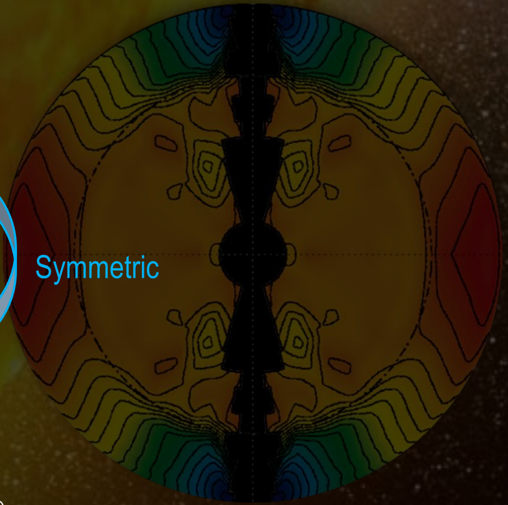
Differences between both hemispheres



[Howe+ 2022]

2dRLS inversion residuals of GONG, MDI, and HMI rotational splitting data, at a target depth of 0.99 $R_\odot$

Symmetric



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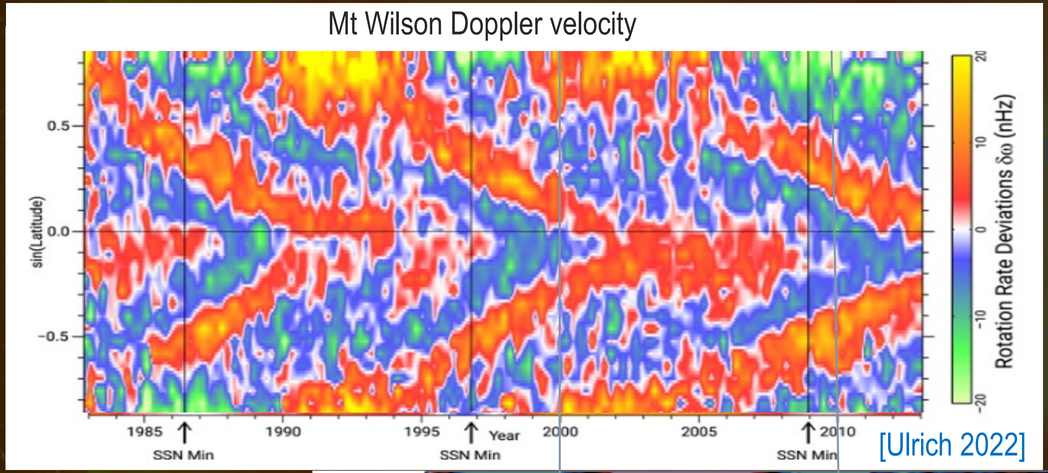
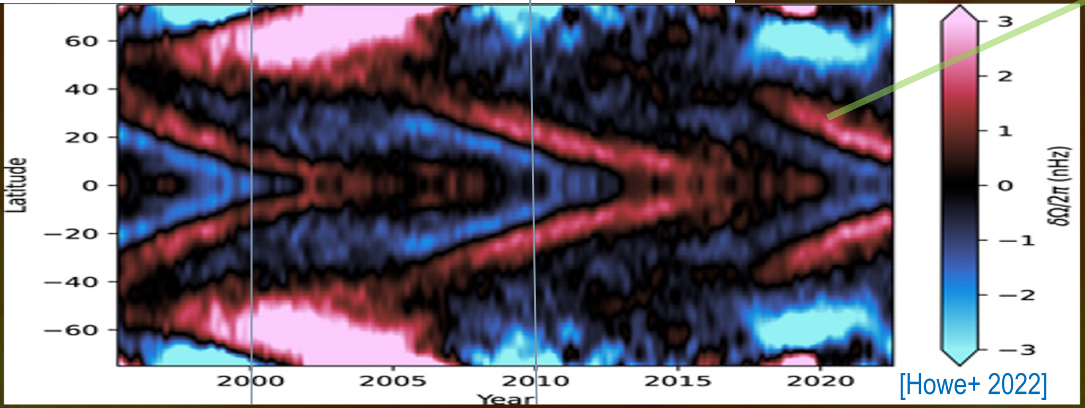


[Ulrich 2022]

Differences between both hemispheres

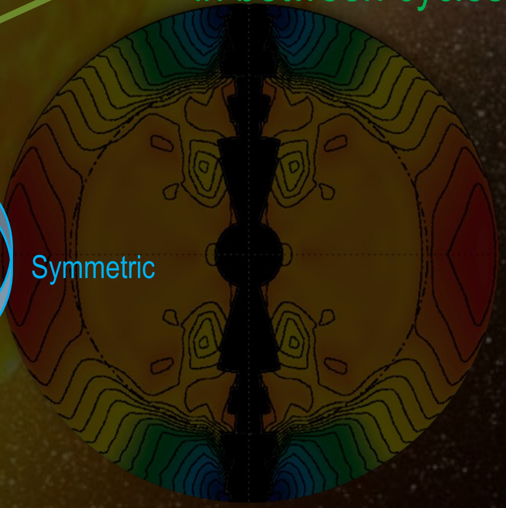
Cycle 25 speed seems to behave in between cycles 21-23 and 24



[Howe+ 2022]

2dRLS inversion residuals of GONG, MDI, and HMI rotational splitting data, at a target depth of 0.99 $R_\odot$

Symmetric

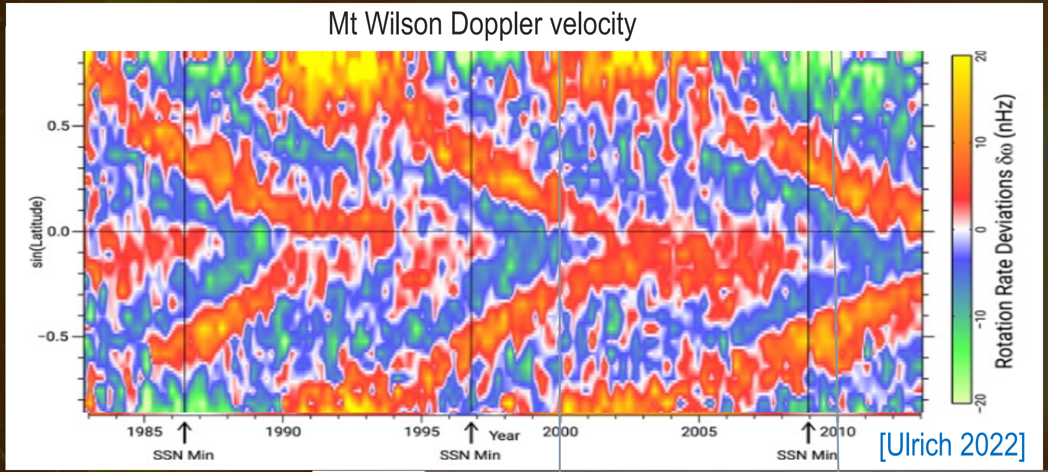


III-RESULTS:

VARIATIONS: TORSIONAL OSCILLATIONS

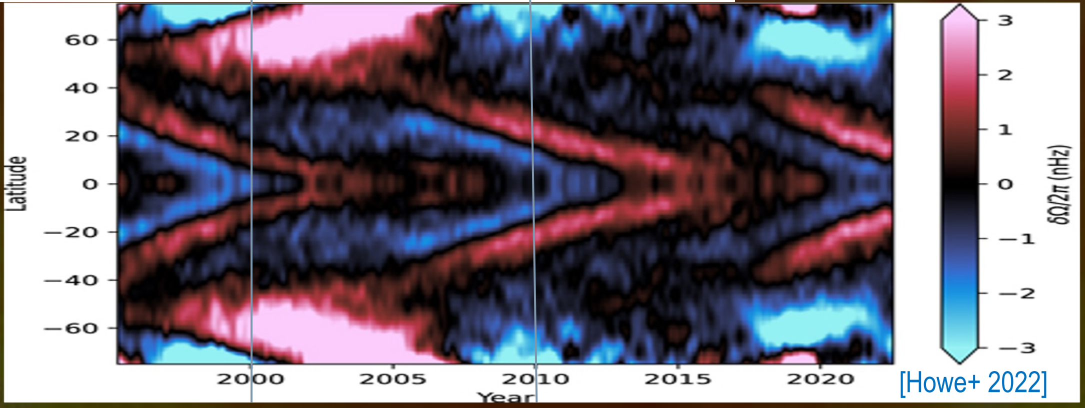
- Is a pattern of migrating bands faster/slower than average zonal (i.e., parallel to the equator) flow associated with the equatorward drift of the activity belts during the solar cycle.

[Howard and LaBonte 1980, Kosovichev & Schou 1997, Basu & Antia 2003, Vorontsov+ 2002..]


Mt Wilson Doppler velocity

[Ulrich 2022]

Differences between both hemispheres



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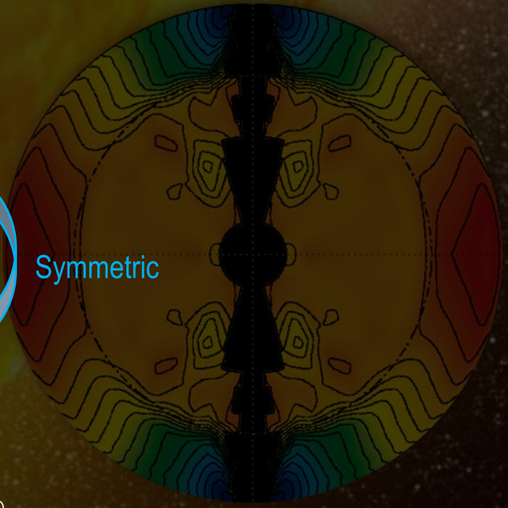

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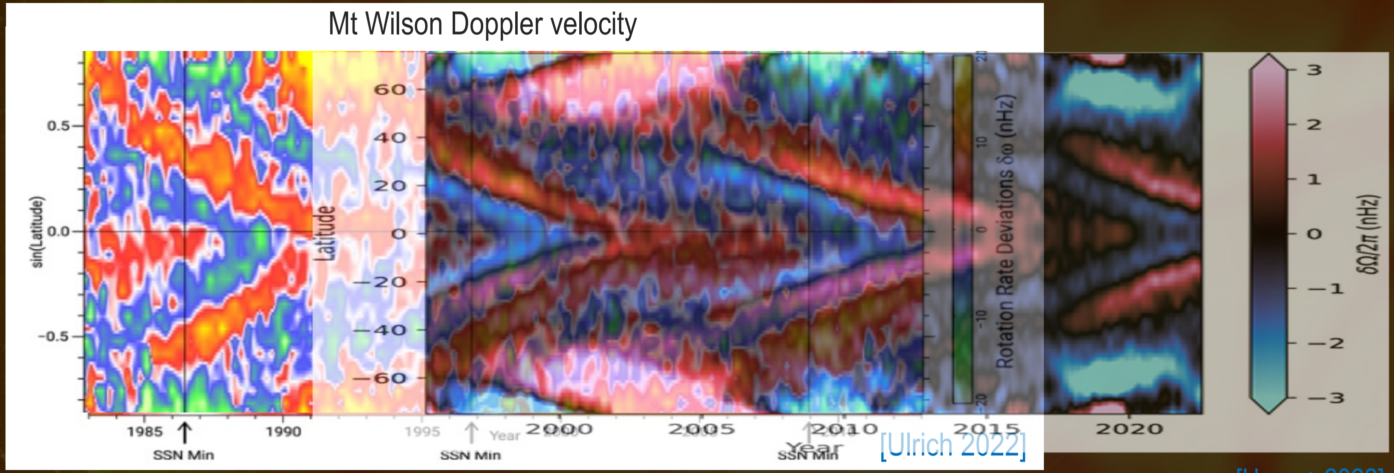
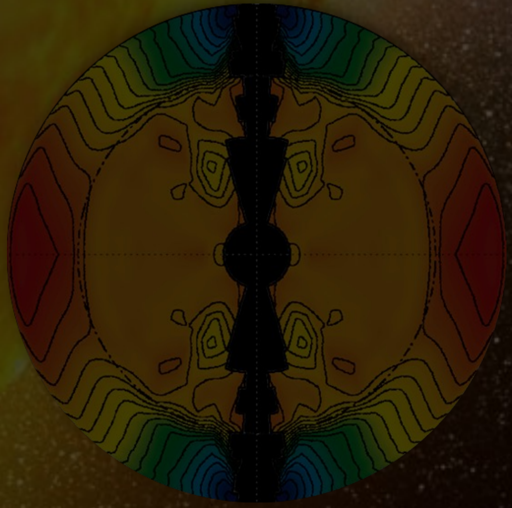

Mt Wilson Doppler velocity

[Ulrich 2022]

[Howe+ 2022]



The agreement is very good in the north hemisphere but not in the south

Why?

III-RESULTS:

VARIATIONS: TORSIONAL OSCILLATIONS

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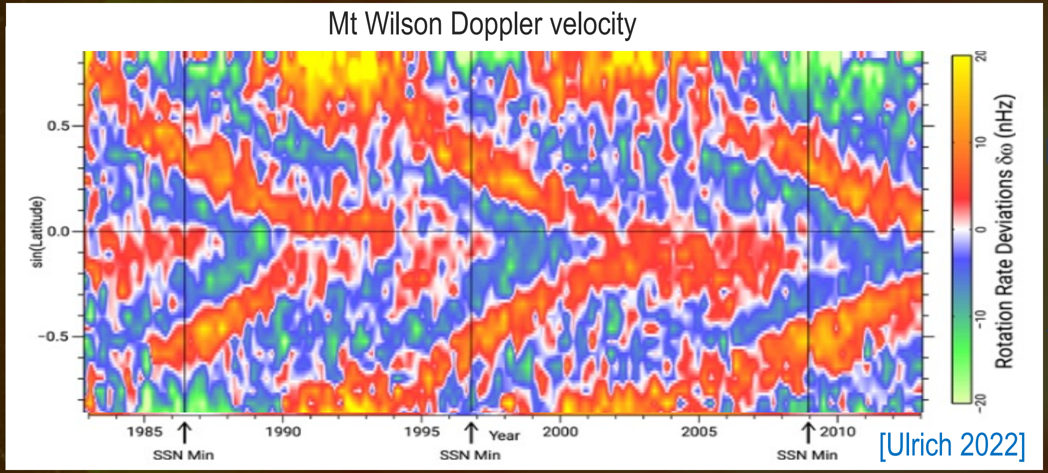
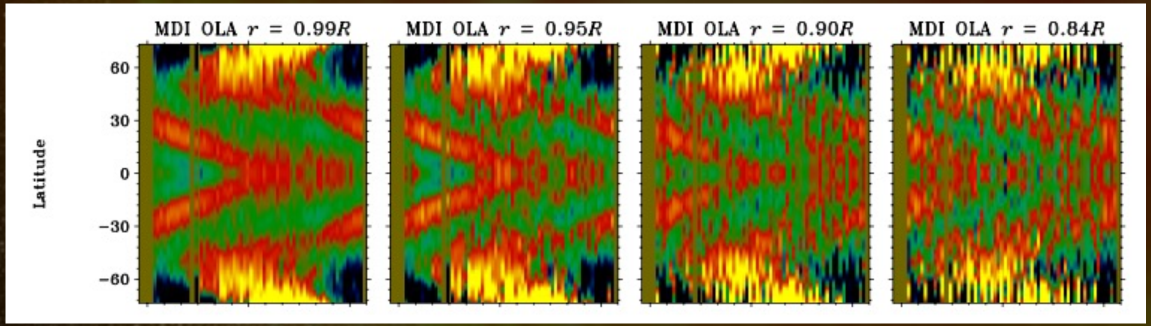


[Ulrich 2022]

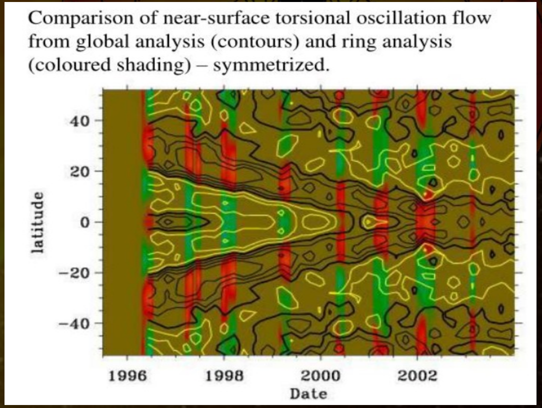
Differences between both hemispheres

- Strong dependence with depth



[Howe+ 2009]

Also observed with local seismology (ring diagrams)



Comparison of near-surface torsional oscillation flow from global analysis (contours) and ring analysis (coloured shading) – symmetrized.

Differences between both hemispheres

III-RESULTS: SEISMOLOGY

- Low- + medium-degree modes combined (up to 1< l< 200)
  - Improves the result in the radiative zone
  - P modes with current uncertainties
    - No information below 0.2 $R_\odot$
  - Only a few g modes are enough

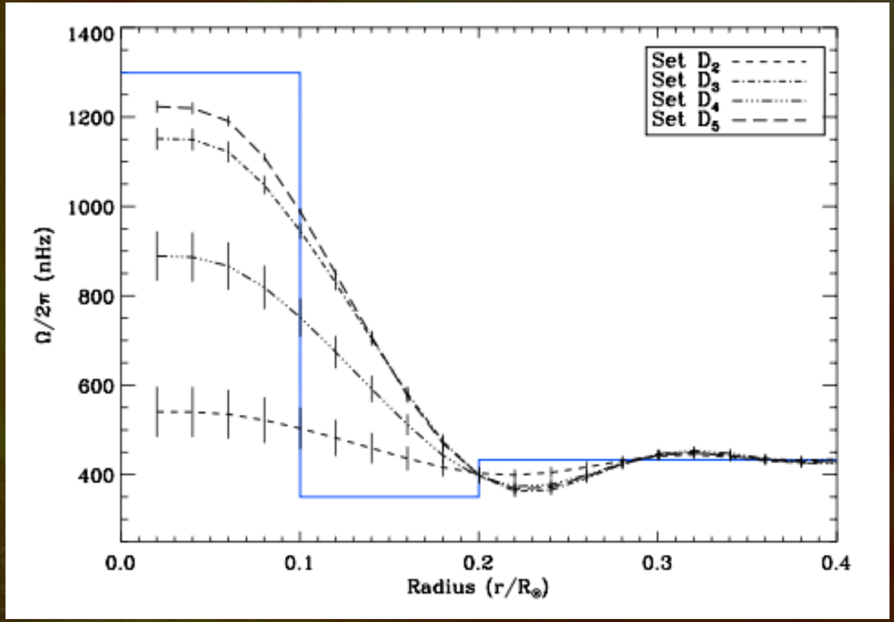
[Eff-Darwich & Korzennik 1998, Eff-Darwich+ 2002]

[García+ 2008]

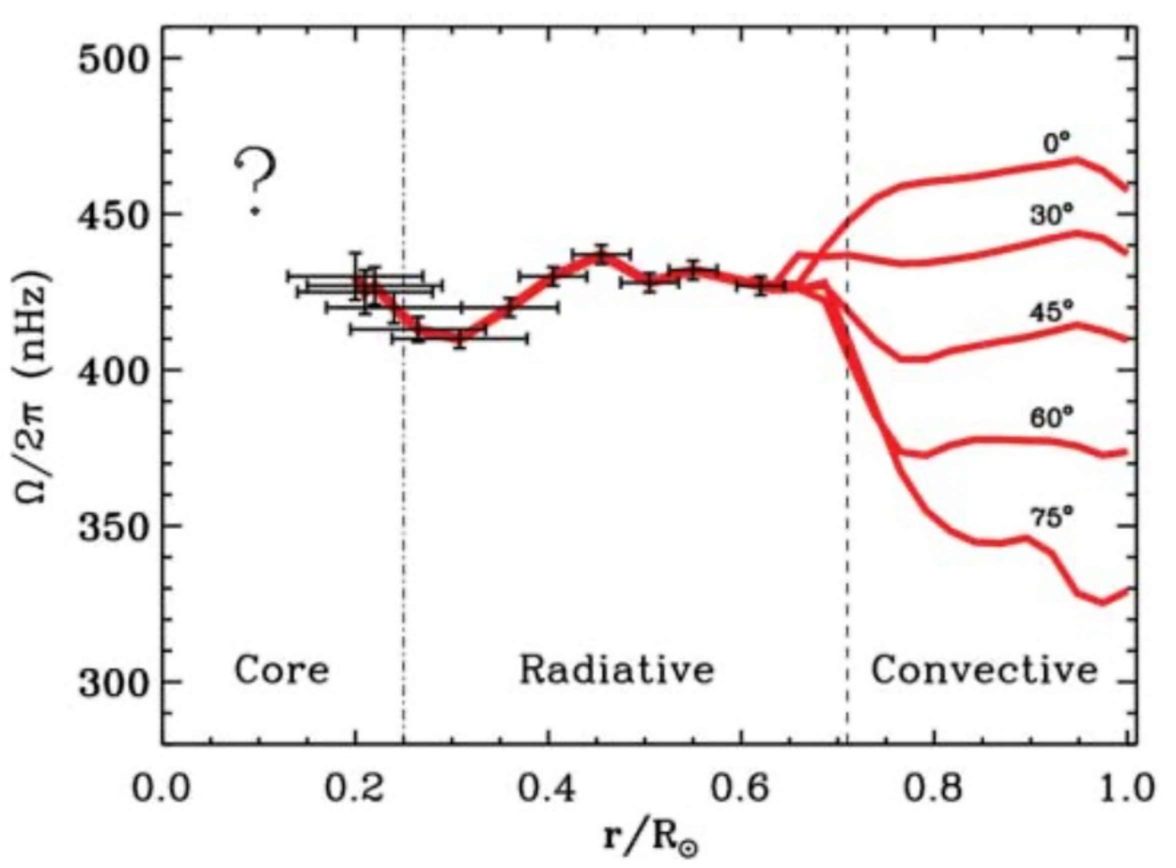
Errors 7.5 and 75 nHz

8 g modes

1 g modes
L=2, n=03



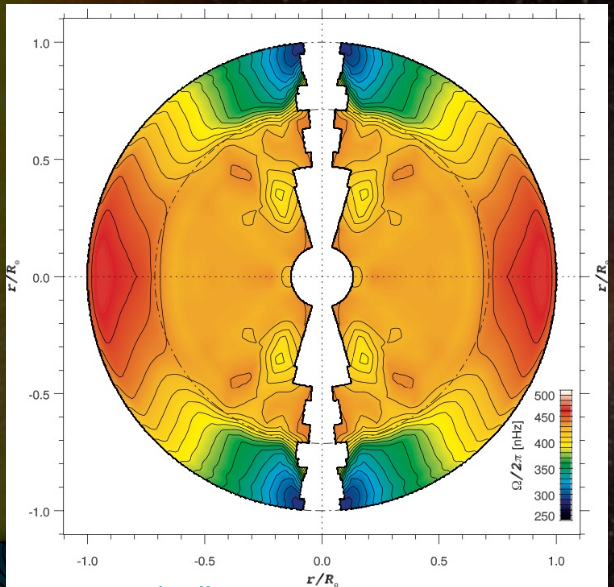
[Mathur+ 2008]



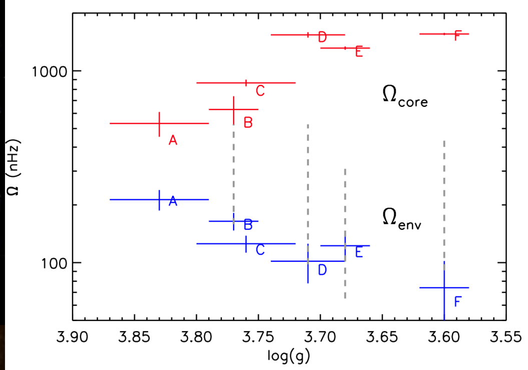
[García+2007]

III-RESULTS: SEISMOLOGY
CURRENT LIMITATIONS

- The shalower and deeper regions:
  - are limited by the deepest and shallowest turning-point radii of the available modes
- Equator
  - Higher sensitivity for sectoral modes l = |m|
- Rotation axis and poles:
  - No sensitivity to rotation on the axis
  - Close to the poles, need to improve overall uncertainties to have better localized kernels
- Radiative interior:
  - Latitudinal sensitivity of the radiative interior:
    - Only l ≤ 20 penetrate into the radiative interior (ν ≤ 3 mHz)
      - The latitudinal resolution is quite poor and becomes progressively worse with depth
- Core (below 0.2 R⊙)
  - Nearly no sensitivity with p modes
    - With current uncertainties
  - Need of g modes
    - Inversions of subgiants and giants with mixed modes
      - Allow to obtain the rotation of the core



[Korzennick & Eff-Darwich 2012]



[Deheuvels+ 2012, 2014]

IV- Stellar Rotation

Is the Sun rotation profile representative of other MS stars?

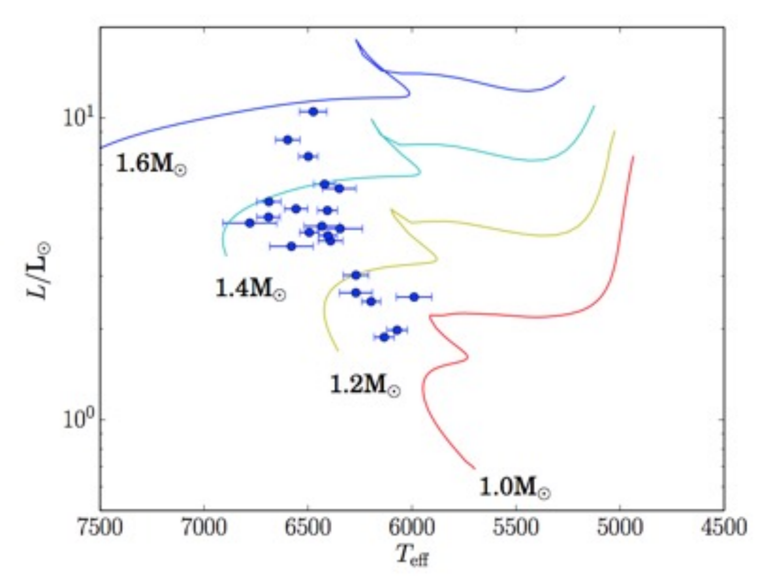
IV- STELLAR ROTATION

- Analysis of 22 main-sequence stars with masses between 1.0 and 1.6 $M_\odot$
  - Combining seismic analysis of *Kepler & CoRoT*

$$\delta\nu_{n,l} \simeq I_{\rm rad}\, f_{\rm rad} + I_{\rm conv}\, f_{\rm conv}$$

$$I_{\rm conv} + I_{\rm rad} = 1,$$

"I" are the integrals of the Kernels

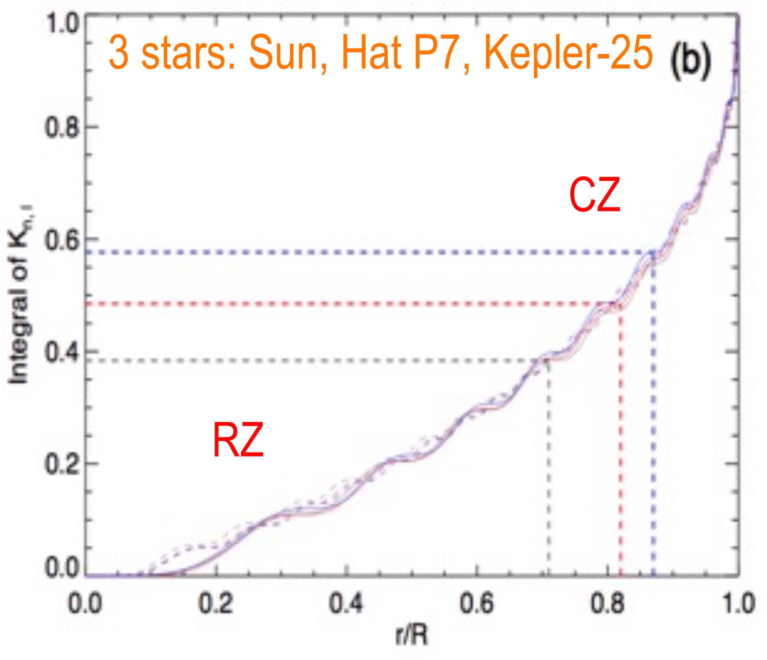


- Assuming

Two zones (CZ & RZ) rotate uniformly with different rates (about the same axis)

- $f_{\rm conv}$ is equal to the surface rotation (as in the solar case) => $f_{\rm conv} = f_{\rm surf}$
- Rotational splittings remain nearly constant over the observed ranges of n and

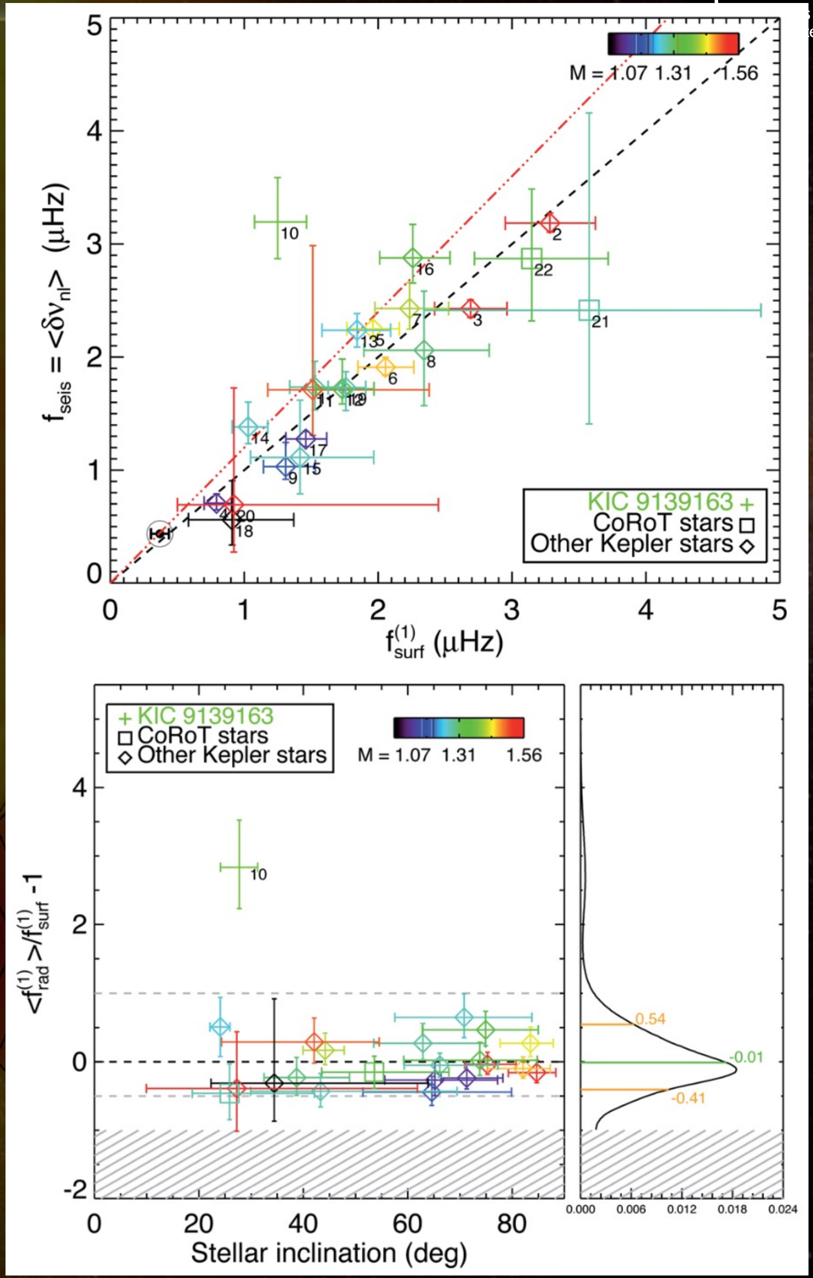
$$\langle f_{\rm rad}\rangle = f_{\rm surf} + \langle I_{\rm rad}\rangle^{-1}(f_{\rm seis} - f_{\rm surf}).$$


3 stars: Sun, Hat P7, Kepler-25

[Benomar et al. 2015]

IV- STELLAR ROTATION

- The ratio of the rotation rate between
  - radiative interior and the outer convective zone/surface
    - Is close to 1 in most of the stars independently of their ages.
    - This suggests that
      - Similar behaviour than the Sun
      - An efficient process of angular momentum transport operates during and/or before the MS stage of stars.



[Benomar et al. 2015]

V- Conclusions

And

Perspectives

VI-CONCLUSIONS & PERSPECTIVES

- The internal solar rotation profile
  - Is robust from the equator to near the poles and down to 0.2 $R_\odot$
- Tachocline
  - Need to improve the resolution in this region
    - Local seismology & different vantage points (several satellites (!))
- Need to improve uncertainties
  - data
    - Longer data sets (in progress)
      - Need to be carefull with systematic effects (magnetic activity cycle)
  - inversions
    - Improving the inversion techniques (in progress)
- The rate of the rotation in the RZ vs CZ (surface)
  - seems similar to those found in other stars with *Kepler* & CoRoT
- Look for gravity modes
  - In current datasets or with new instrumentation
- Comparison & improvements of 2D & 3D models

SOLAR-ROTATION GROUP

- Objective
  - To produce the best reference solar rotation inversion
- Originally organized by:
  - García, Jain, Mathur, & Thompson (07/2017)

    + Bertello, Hill, & Tripathy
- Coordinated by:
  - Theoretical/modeling:
    - Thompson/JCD
  - Data analysis
    - García
- Around 20-30 scientists involved
    - Results are slowly coming
    - 2024: papers on data analysis and observations
- Tackling some known issues:
  - Precision of the kernels
  - Uncertainties on the inversions
- Discussions about, e.g.:
  - Tachocline
  - Solar core
- On the data analysis side
  - Solve differences between “fitters” and “codes”
  - Compatibility of instruments and techniques
    - E.g. merging Sun-as-a-star and imaged instruments

If you want to get involved

… and you have time to work

Contact us !!

THANKS!