Solar seismic models

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A seismic Sun?

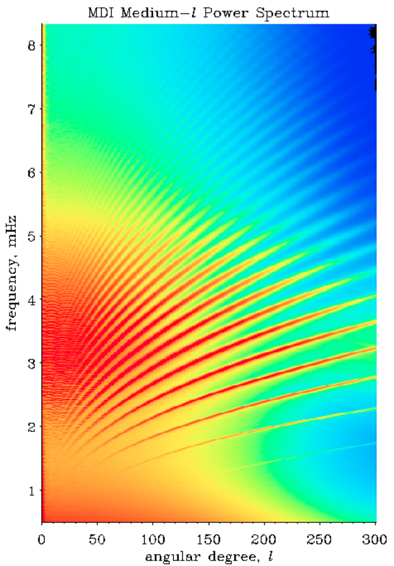
Helioseismic inversions allow to determine c^2, ρ, ... $\Rightarrow$ structure can be reintegrated!

- Construct a reference model.
- Correct it using helioseismic constraints.
- Improve the fit with data.

Outcome: a map of the Sun independent from the starting point.

"The Sun as seen by the waves propagating inside it."

Available constraints



Credit:https://sohowww.nascom.nasa.gov/gallery/Helioseismology/mdi005.html

- Thousands of modes (+-7000 (Reiter et al. 2020))
- Neutrinos (Orebi-Gann et al. 2021)
- Global parameters: R,L,M, T_{eff}, age
- Composition? (see talks on $Z + Y_{CZ}$ determination)

A lot of physical constraints to exploit to "map" the interior of Sun.

Mechanical model - Directly from data

Assuming hydrostatic equilibrium:

$$\frac{dm}{dr} = 4\pi\rho r^2, \quad \frac{dP}{dr} = \frac{-Gm\rho}{r^2}$$

Neglects turbulent pressure in the outermost layers, rotation, magnetic fields.

Thermal model

Assuming thermal equilibrium:

$$\frac{dL}{dr} = 4\pi r^2 \varepsilon, \quad \frac{dT}{dr} = \frac{-3\kappa\rho L}{16\pi acr^2T^3}\varepsilon$$

Only valid in radiative zone, assuming energy generation, EOS, composition (at least).

A tool only as good as its use:

Strengths:

- No dependency on history,
- No dependency on transport formalism,
- Can be used to test "crazy" hypotheses.

Simplifications:

- Underlying equations,
- Limited resolution,
- Dependency on data and methods.

While very powerful, inversions are not an absolute truth: formalism, cross-term, surface-effects, ...

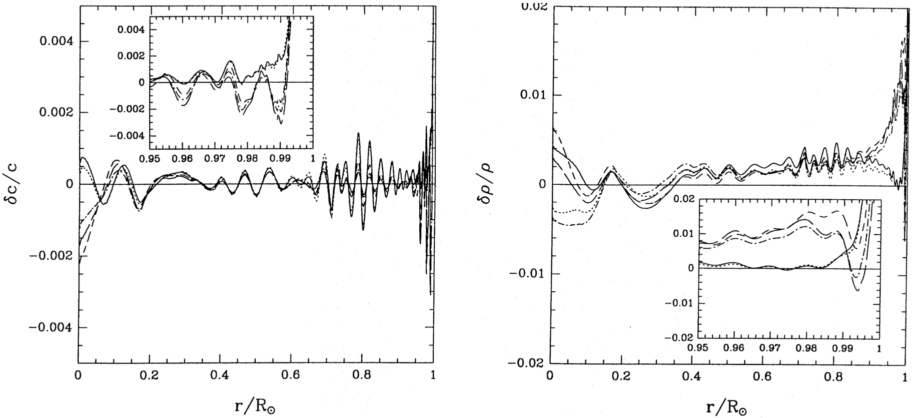
Various references in litterature:

Formalism: often based from seismic reconstruction using c^2 or ρ from variational equations:

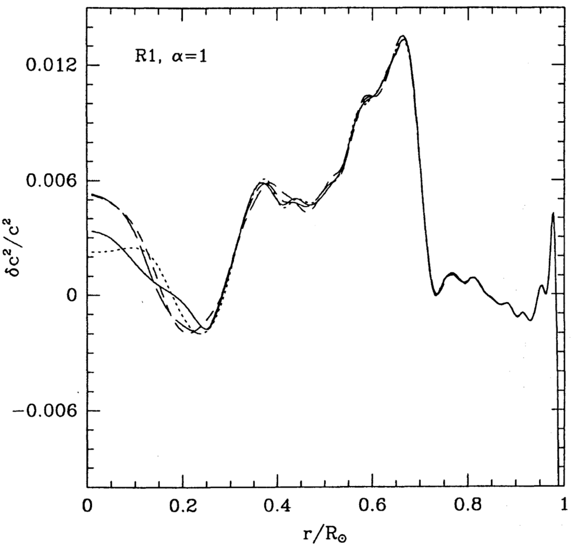
$$\frac{\delta \nu^{n,l}}{\nu^{n,l}} = \int_0^R K^{n,l}_{\rho,c^2} \frac{\delta \rho}{\rho} dr + \int_0^R K^{n,l}_{c^2,\rho} \frac{\delta c^2}{c^2} dr + \mathscr{F}(\nu) \tag{1}$$

Estimate of $\rho_\odot$ or $c^2_\odot$ $\Rightarrow$ injected in the hydrostratic equilibrium equations, using the corrections.

Numerous references of iterative methods (essentially seismic models): Antia (1996), Basu & Thompson (1996), Takata & Shibahashi 1998, Marchenkov et al. (2000), Gough (2004). **Envelope models** (e.g Vorontsov et al. 2013 and 2014) also fall within the category of "seismic models".



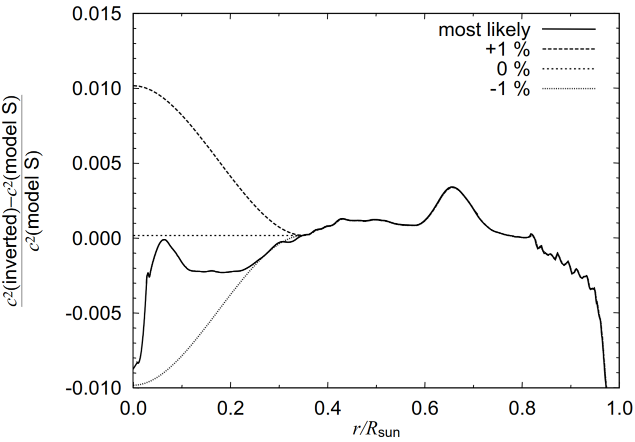
Iterated RLS on ρ using ρ and Γ_1, stop when χ^2 reincreases. Test of neutrinos following Antia & Chitre (1995). Mention the importance of systematics.



Fitting ν from successive RLS inversions on both ρ and c^2 from variational equations.

Conclusion: limited by surface effects. No energetic considerations.

Example 3 - Takata & Shibahashi (1998)



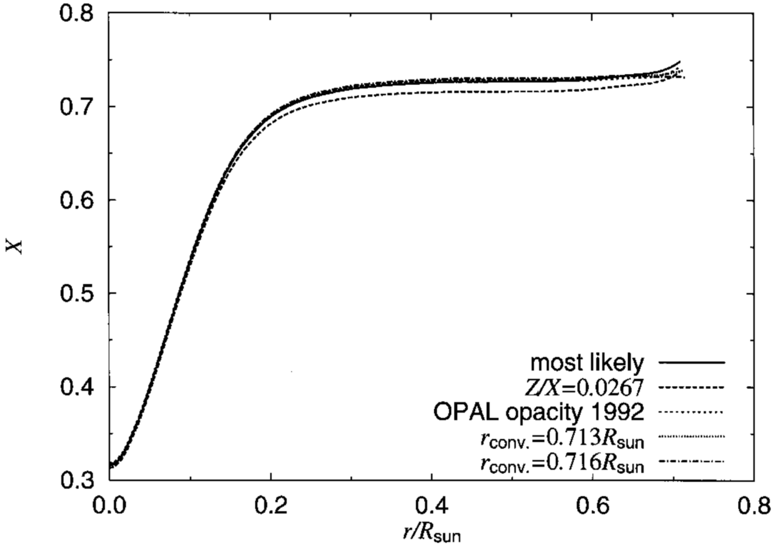
Linear inversion of sound speed and shooting technique to reintegrate hydrostatic structure.

Energetics considered from constant Z and assumed opacity profile.

See also Shibahashi and Tamura (2006).

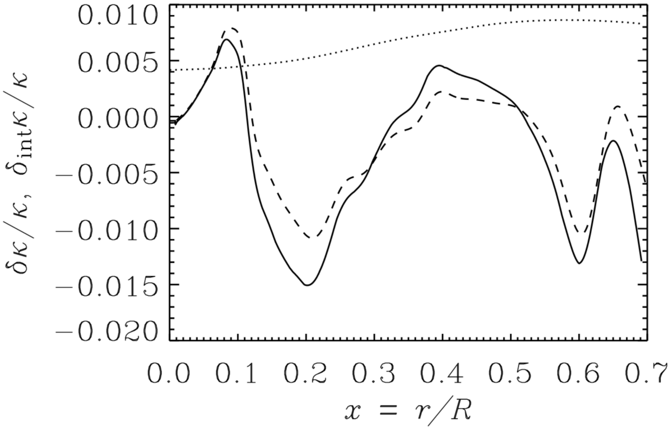
Focus on neutrinos and abundances.

Example 3 - Takata & Shibahashi (1998)



Study of the sensitivity to various: BCZ, Z/X, opacity, ...

Example 4 - Gough (2004)



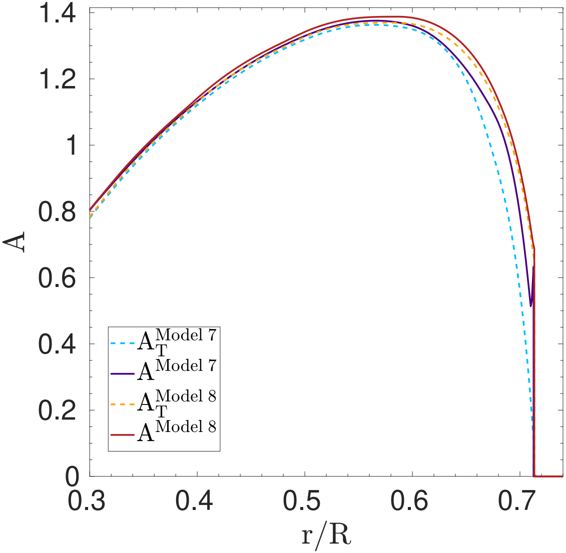
Full seismic structure from c^2 inversion.

Chemical composition from Model S.

Luminosity fit via variation of the total helium core mass.

Full tabulated structure available: unfortunately outdated physics and no uncertainties.

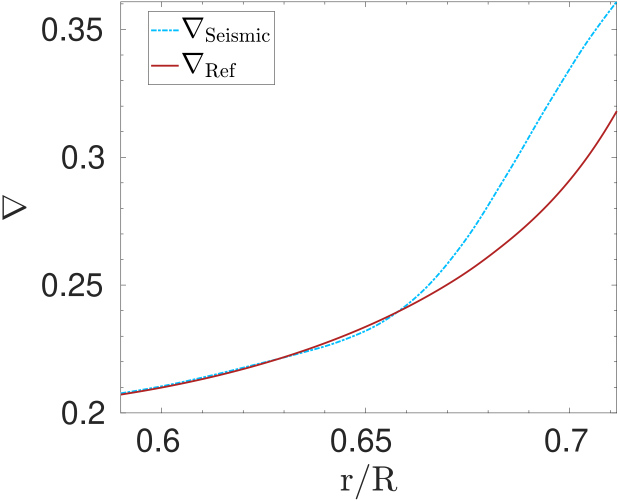
Determining seismic models from A inversions (Buldgen et al. 2020)



$A = A^T + A^\mu \propto \nabla T$:

1. Determine $A_{\mathrm{Sun}} - A_{\mathrm{Mod}}$;
2. Integrate the structure satisfying equilibrium;
3. Compute oscillations;
4. Back to 1.

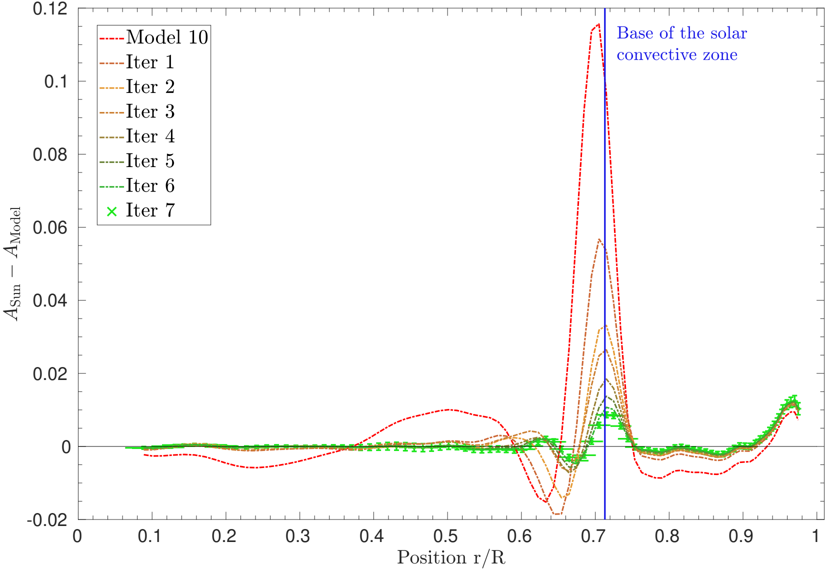
Impact on temperature gradient in a solar model



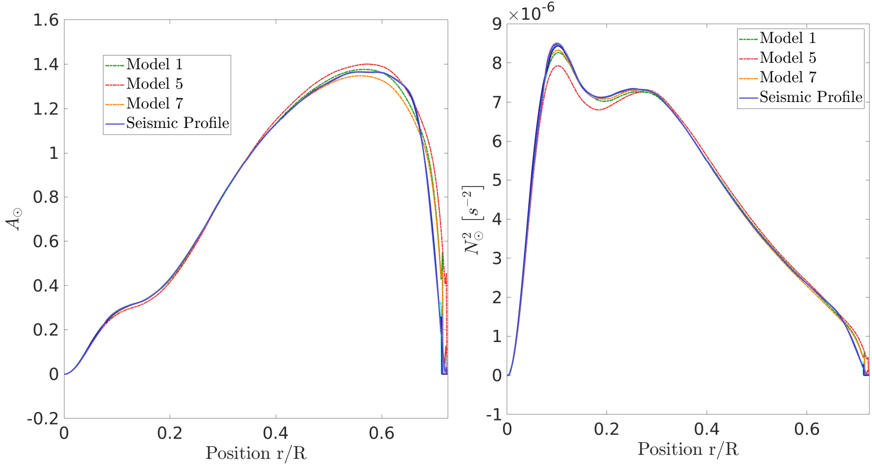
Assuming $\delta A \propto \delta \nabla T$

- Steeper gradients,
- Extension at medium temperatures,
- Compatible with broad "peak" feature.

Level of agreement for seismic models I

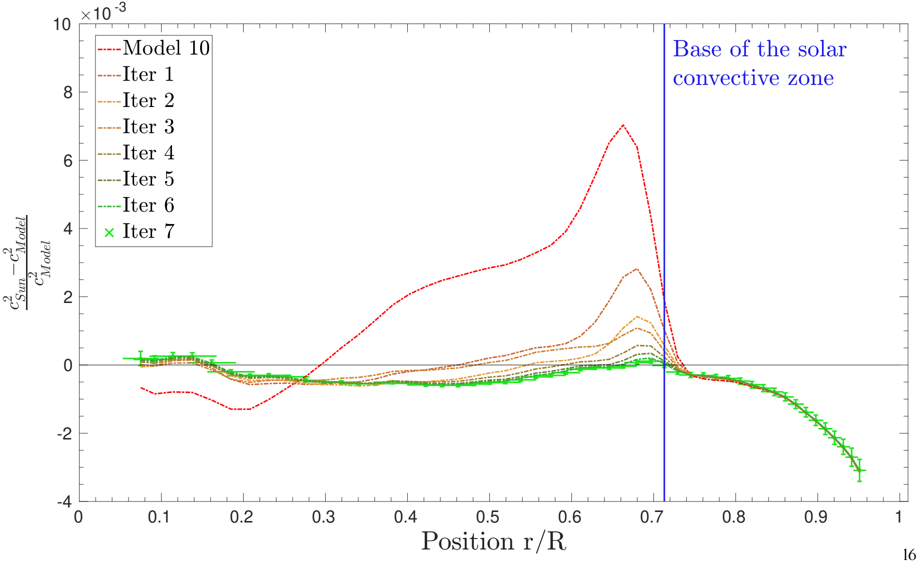


Level of agreement for seismic models II

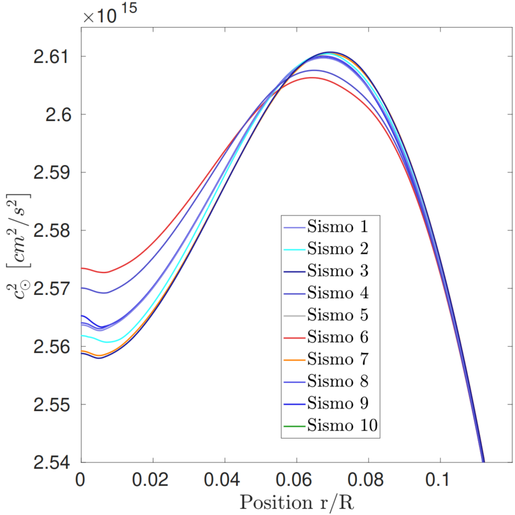


Same A and B-V profile ⇒ c^2, ρ, S also agree within 0.1%. ⇒ excellent acoustic structure

Level of agreement for seismic models III



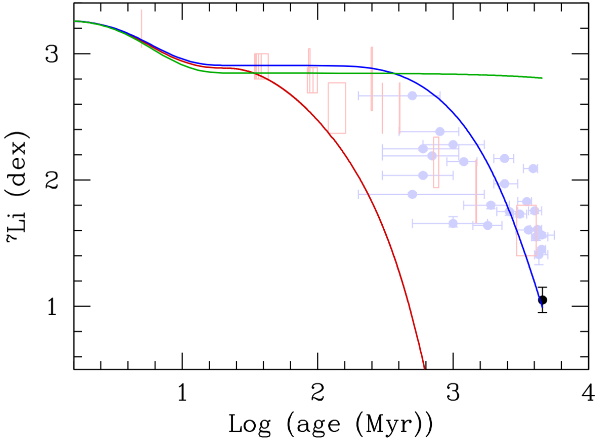
Pushing for the core regions - constraints on period spacing



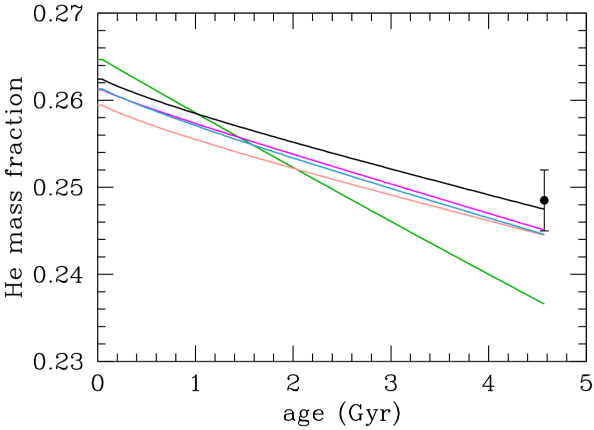
- Constrain core from full structure inversions (as low as $0.05R_\odot$
- M and R are fixed.
- Amount of variation limited?

Variations too small... need gravity modes to push down.

Maybe neutrinos can help?

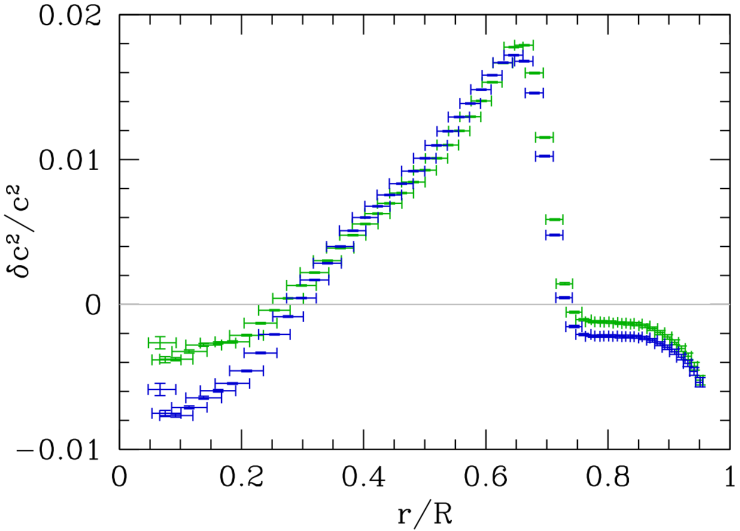


Lithium depletion is an issue since 1990s (Proffitt & Michaud 1991, Richard et al. 1996).



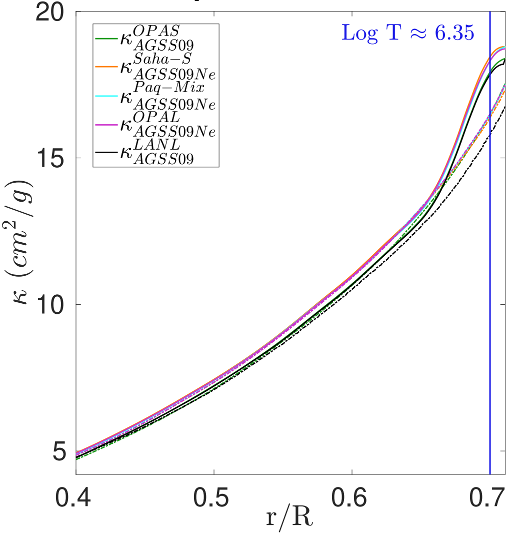
The helium-lithium correlation exists for multiple shapes of the transport coefficients. (Careful with the latest values however).

Sound speed at the BCZ and rotation



Sound speed at the BCZ not strongly affected by mixing.

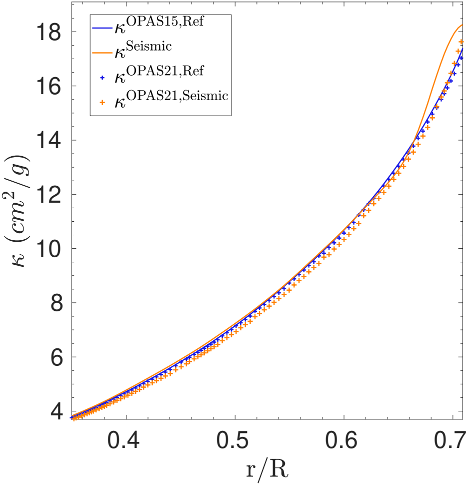
From the analysis of static models and non-standard models:

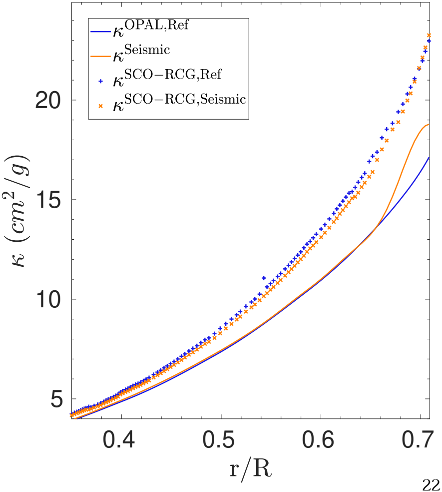


- Use chemical composition from non-Std models.
- Integrate and iterate to reproduce $L_\odot$.
- Determination of amount of "missing" opacity.

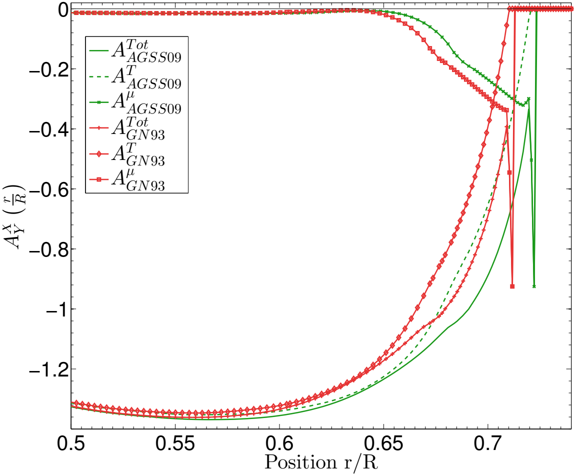
Consistent with experiments (Bailey et al. 2015)

Codes give conflicting results for similar conditions.





Further improvements and applications?



- Improve resolution at BCZ: non-linear RLS?
- Combine with envelope models for fully consistent composition?.
- Combine neutrinos and inversions using parametrized core?

All rely on updated physics: EOS, nuclear rates, transport of chemicals, opacities...

Testing underlying hypotheses

Seismic models are "evolution independent", but still have hidden dependencies:

- Dependencies on the inversion technique,
- Dependencies on the dataset,
- Dependencies on surface effect, activity, ...
- Integration scheme for the reconstruction, starting variable, ...

Full robustness assessment must be done to allow a good estimate of precision and thus of the relevance of the observed discrepancies. Similarly to the 10000 SSMs of Bahcall et al. (2005).

In conclusion

Still a problem: **Yes**. Will new opacity computations do it? Possibly.

What can we do? **Improve seismic models and constrain physics.**

Improvements expected?

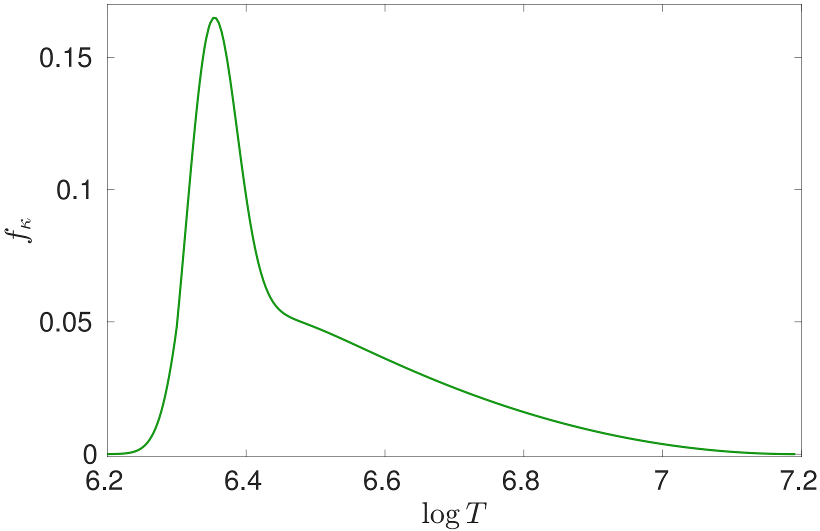
New MDI+HMI data (around 6400 modes) $\Rightarrow$ More constraints on fine structure.

Adapt inversion techniques $\Rightarrow$ sharp transitions: non-linear RLS, separate domains.

Global helioseismology is neither closed nor stuck.

Thank you for your attention!

Considered opacity modification



Other classical diagnostics

	$r_{Conv}/R_{\odot}$	Y_{Conv}
Helioseismic measurements	0.713 ± 0.001	0.2485 ± 0.0035
SSM (AGSS09, Free, OPAL)	0.720	0.236
SSM (AGSS09, Free, OPLIB)	0.718	0.230
SSM (AGSS09, Free, OPAS)	0.717	0.232
SSM (GN93, Free, OPAL)	0.711	0.245
SSM (GN93, Free, OPLIB)	0.708	0.240