Multi-Dimensional Modelling of Stellar Interiors

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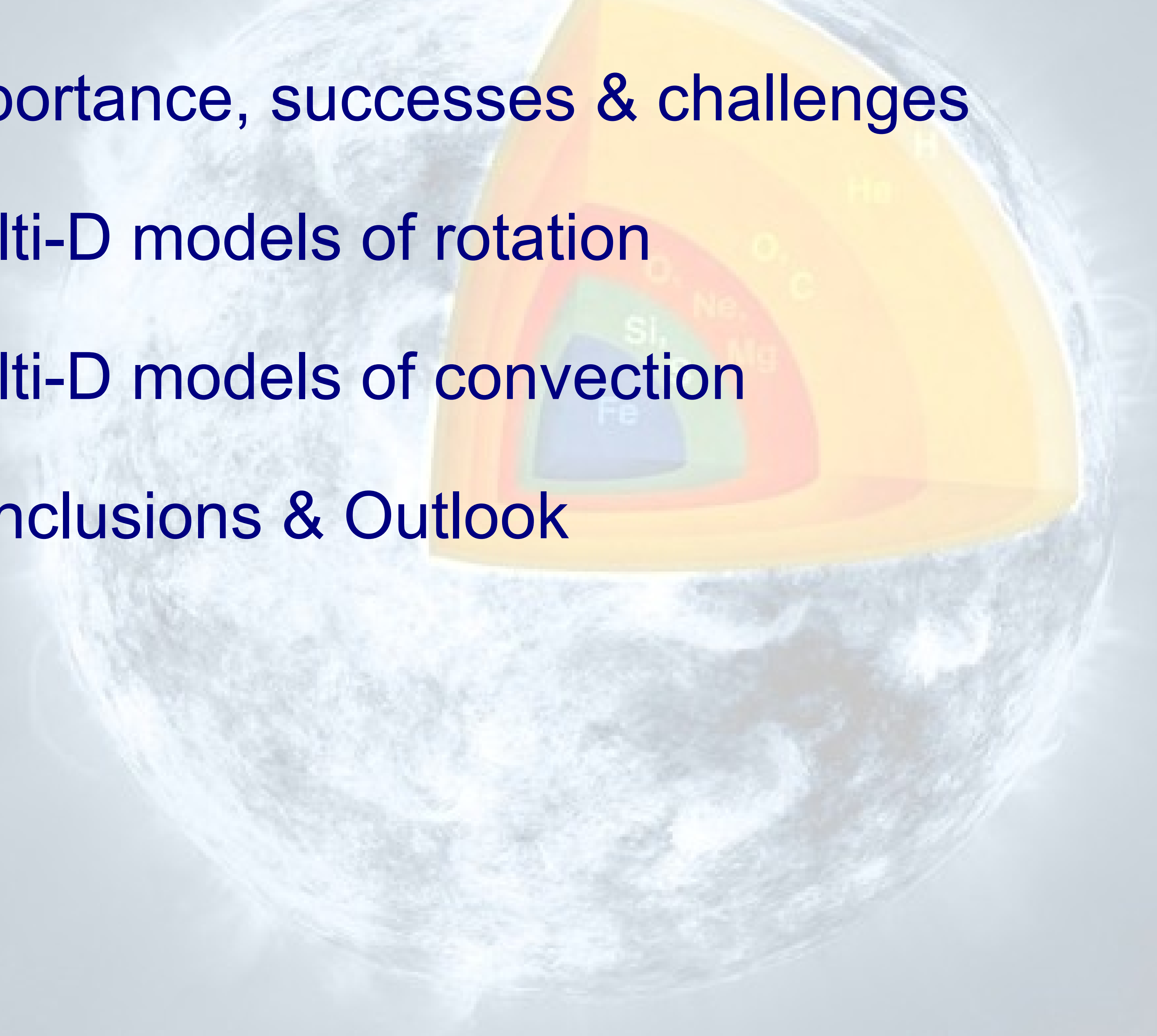
Plan

- Importance, successes & challenges

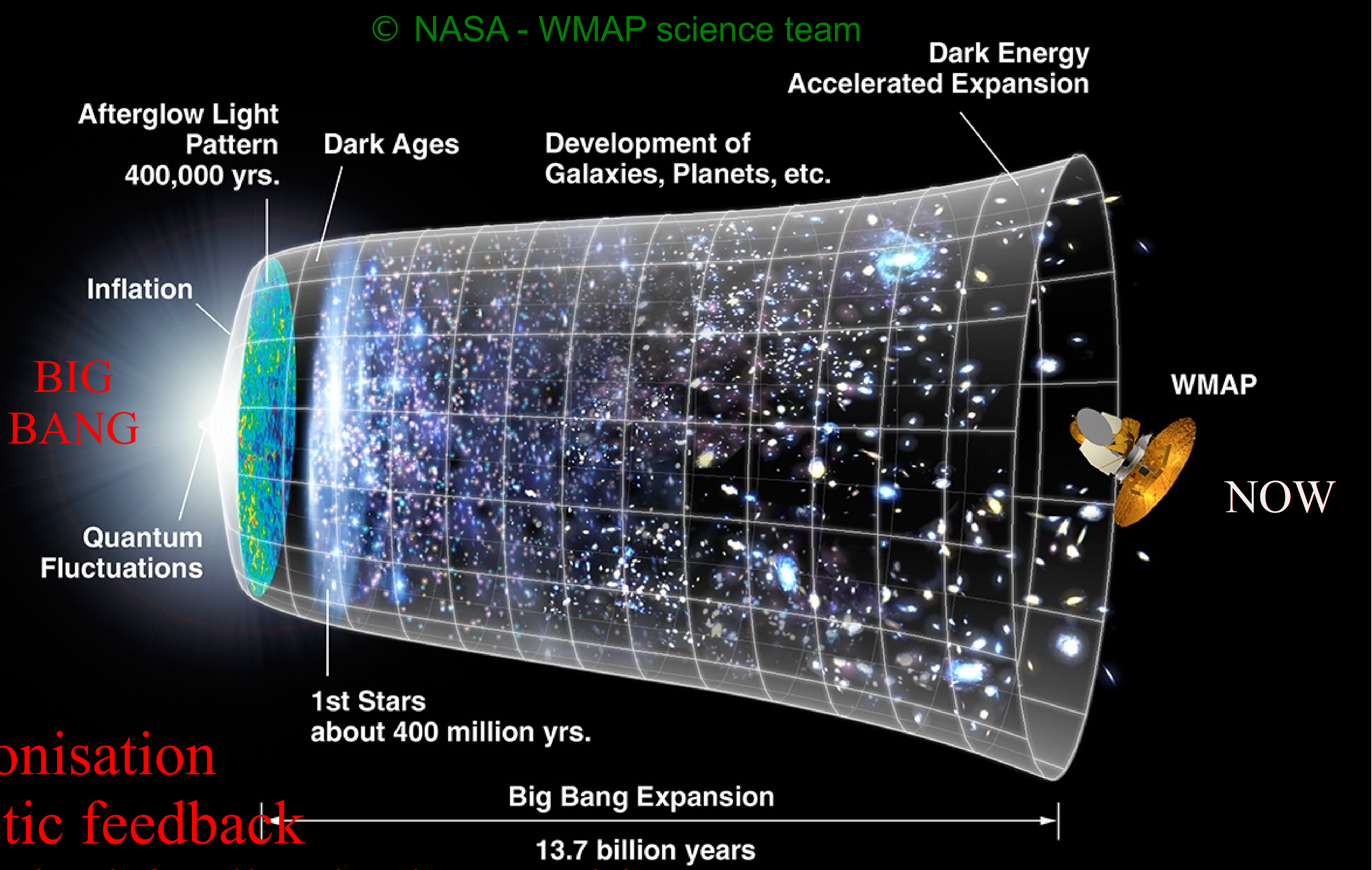
- Multi-D models of rotation

- Multi-D models of convection

- Conclusions & Outlook



Stars' Role in Universe



- Re-ionisation
- Kinetic feedback
- Chemical feedback observed in EMP stars

First Stellar Generations: Importance

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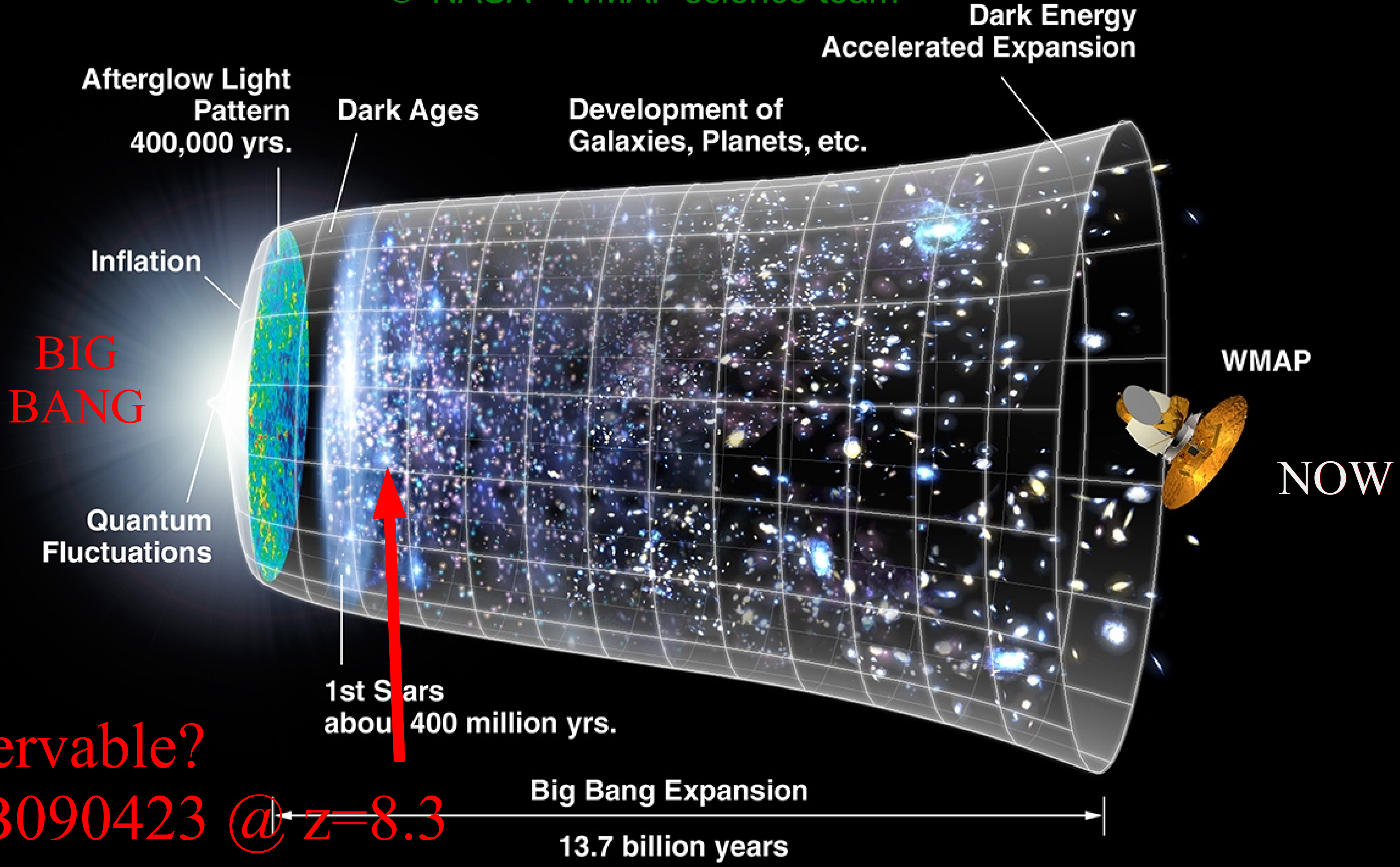
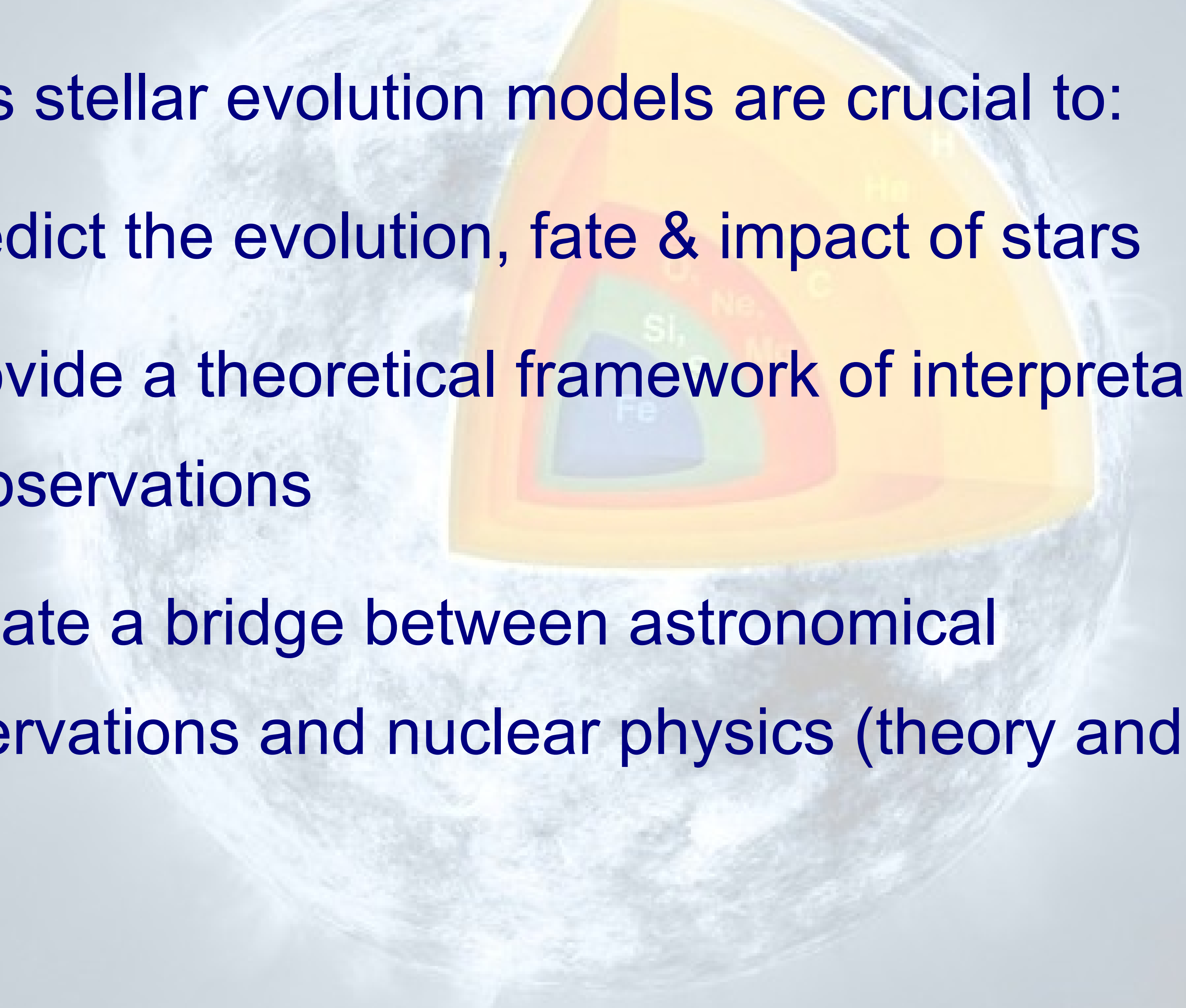


- Observable?
- GRB090423 @ z=8.3

Universe age ~ 600 Myr (Tanvir et al 09, Nature: arXiv:0906.1577)

Stellar Evolution Models



Thus stellar evolution models are crucial to:

- predict the evolution, fate & impact of stars

- provide a theoretical framework of interpretations of observations

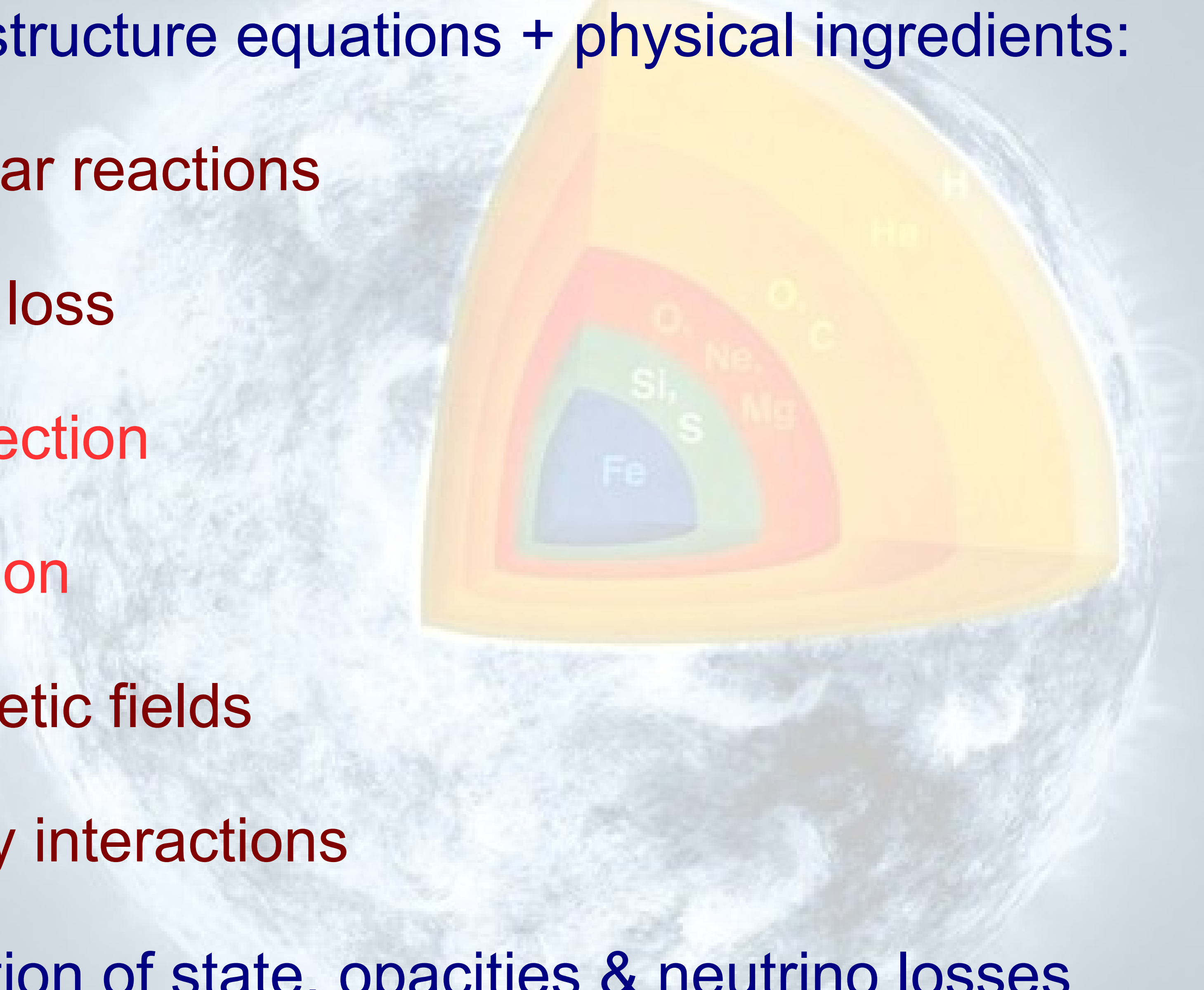
- create a bridge between astronomical observations and nuclear physics (theory and exp.)

1D Stellar Evolution Models

Stellar structure equations + physical ingredients:

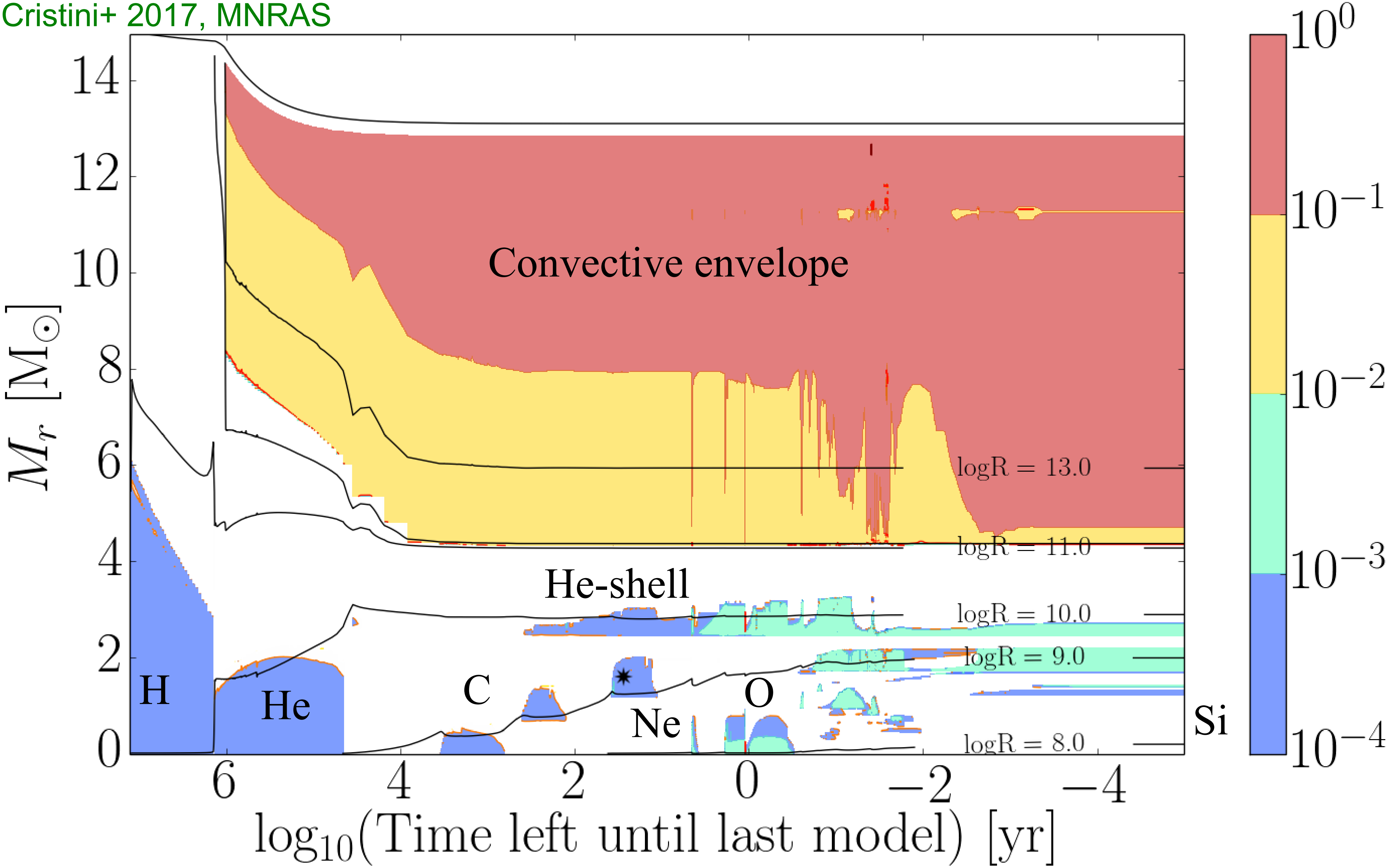
- Nuclear reactions
- Mass loss
- Convection
- Rotation
- Magnetic fields
- Binary interactions
- Equation of state, opacities & neutrino losses

including metallicity dependence



Structure Evolution of Massive Stars

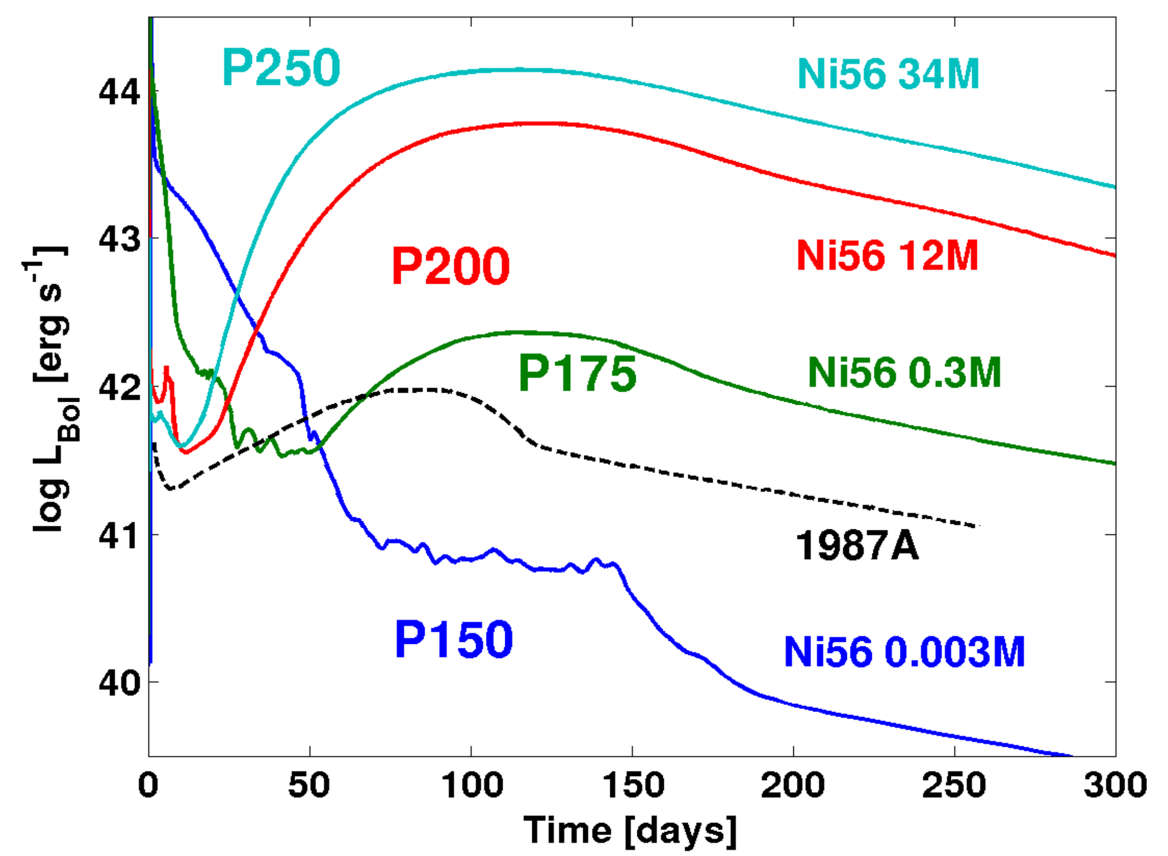
Cristini+ 2017, MNRAS



Convection takes place during most burning stages

PISNe Models: from ZAMS to LCs (at Z=0.001)

(Kozyreva+RH+ 2017MNRAS.464.2854K, Gilmer+RH+ 2017ApJ.846.100, ArXiV170607454G)

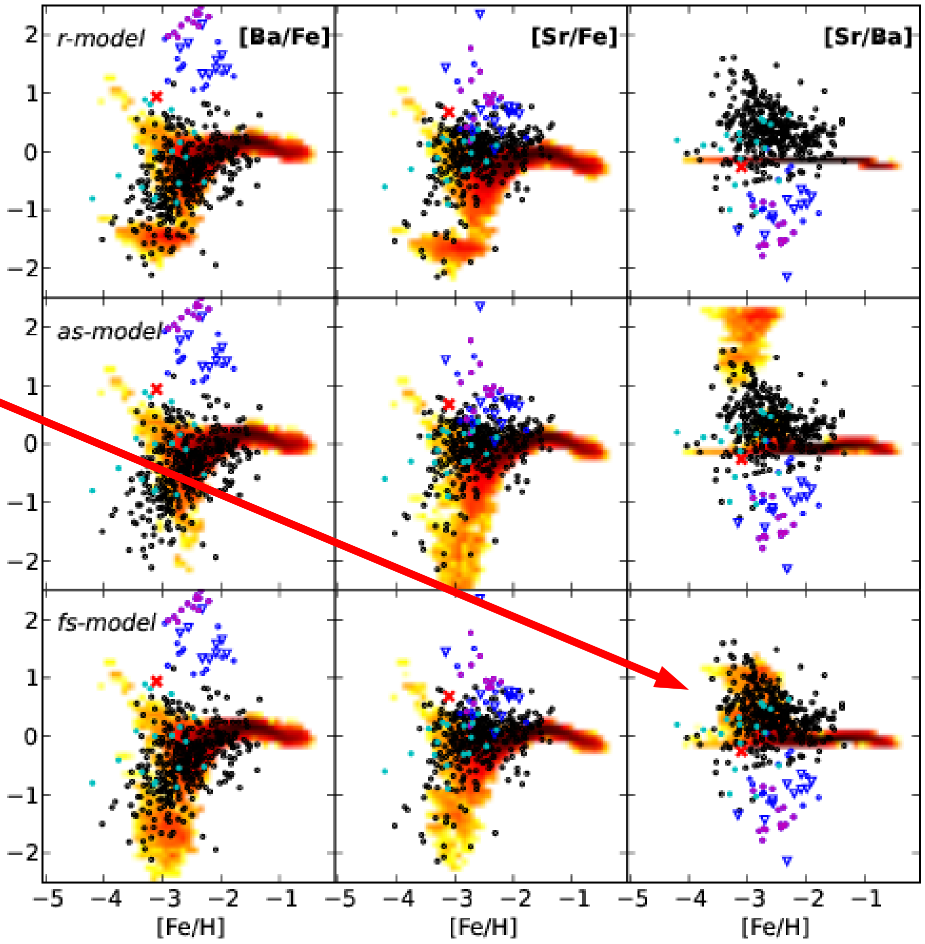


Some (many?) PISNe and most PPISNe are faint!!

Successes: e.g. [Sr/Ba] in EMP stars

Strong variations in [Sr/Ba] > 1 matches well observed range for EMP stars (black circles)!

(no main s-process included so cannot explain CEMP-s stars in blue)



Cescutti *et al.* (2013), see also 2015

Model name	panels in Fig. 5	s-process	r-process
r-	Upper	No s-process from massive stars	standard + extended r-process site (8 - 30 $M_\odot$)
as-	middle	average rotators (v_{ini}/v_{critic} = 0.4)	standard r-process site (8 - 10 $M_\odot$)
fs-	lower	fast rotators (v_{ini}/v_{critic} = 0.5) and 1/10 for $^{17}O(\alpha,\gamma)$ reaction rate	standard r-process site (8 - 10 $M_\odot$)

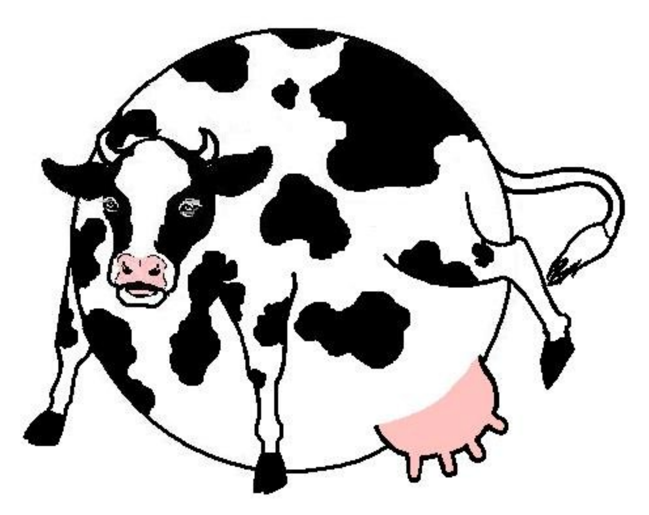
Advantages:

- model entire evolution ($\Delta t \sim 10^3$ yrs)
- compare to observations
- progenitor models
- large grids (M, Z)

Disadvantages:

- parametrized physics (e.g. convection)
- missing multi-D processes
- incapable of modelling turbulence

1D stellar models



What's missing?

- self-consistent physical descriptions of mass loss, **convection**, **rotation**, magnetic fields, opacity, binarity and their interplay

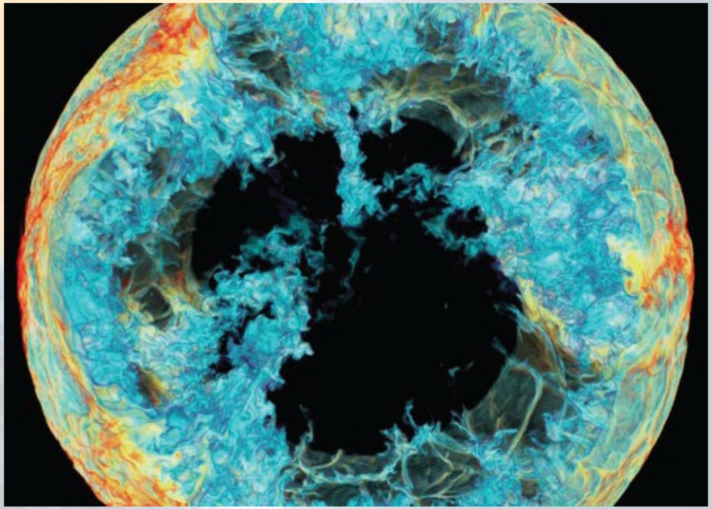
3D stellar models

Advantages:

- model fluid instabilities (e.g. Rayleigh-Taylor)
- modeling 3D processes
- model diffusive and advective processes

Disadvantages:

- computational cost
- limited to dynamical timescales (t_{conv} ~ 1s – days)
- resolution dependent?
- initial condition dependent?



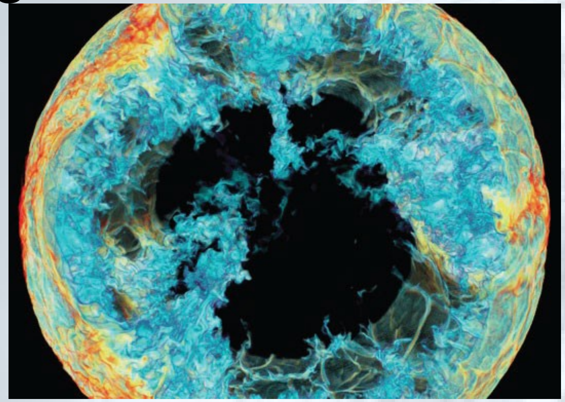
Herwig, Woodward et al 2013

What's missing?

- full star or lifetime simulations
- Large scale (LES) and small scale (DNS) cannot be followed simultaneously

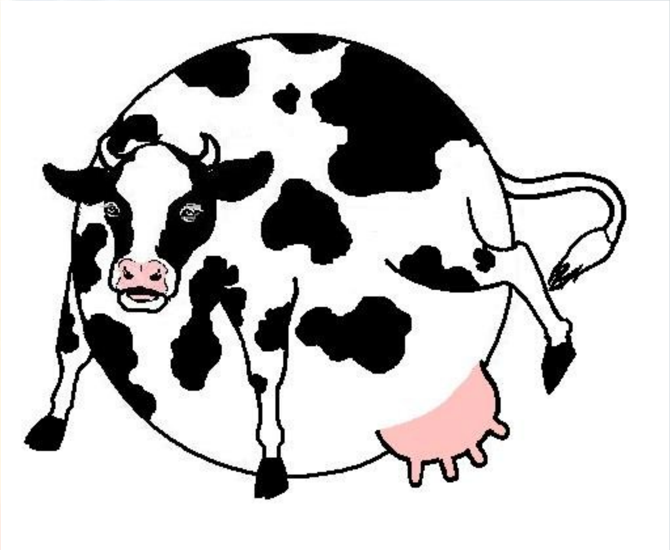
Way Forward: 1 to 3 to 1D link

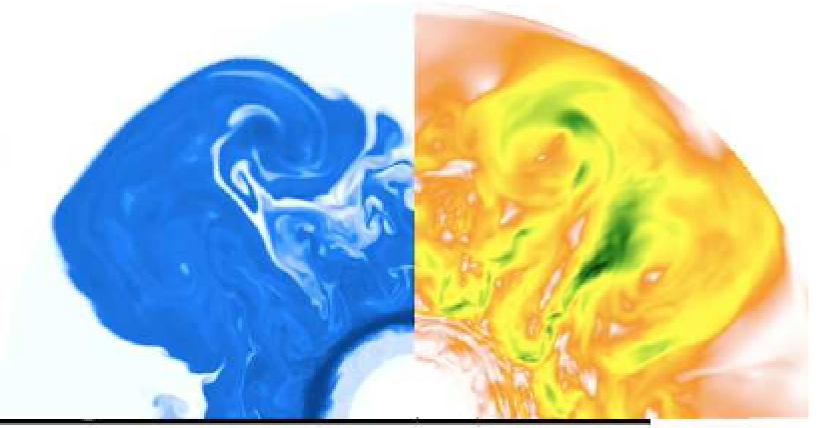
Targetted 3D simulations



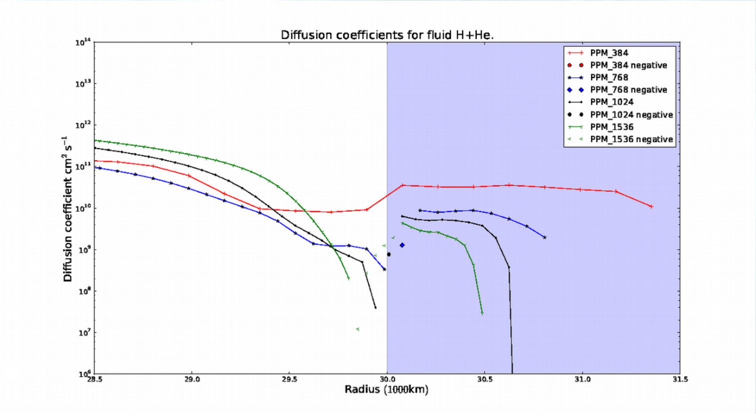
Herwig et al 06, Herwig, Woodward et al 2013

Uncertainties in 1D





e.g. Arnett & Meakin 2011, ...
Mocak et al 2011,
Viallet et al 2013, ...



Meakin et al 09 ; Bennett et al (thesis), Jones et al 16

→ Determine effective coefficient / improve theoretical prescriptions

2.3. Dynamical shear

Hirschi et al 2004

The criterion for stability against dynamical shear instability is the Richardson criterion:

$$\boxed{Ri = \frac{N^2}{(\partial U/\partial z)^2} > \frac{1}{4} = Ri_c,} \tag{1}$$

where U is the horizontal velocity, z the vertical coordinate and N^2 the Brunt-Väisälä frequency:

$$N^2 = \frac{g\delta}{H_P}\left[\nabla_{ad} - \nabla + \frac{\varphi}{\delta}\nabla_\mu\right] \tag{2}$$

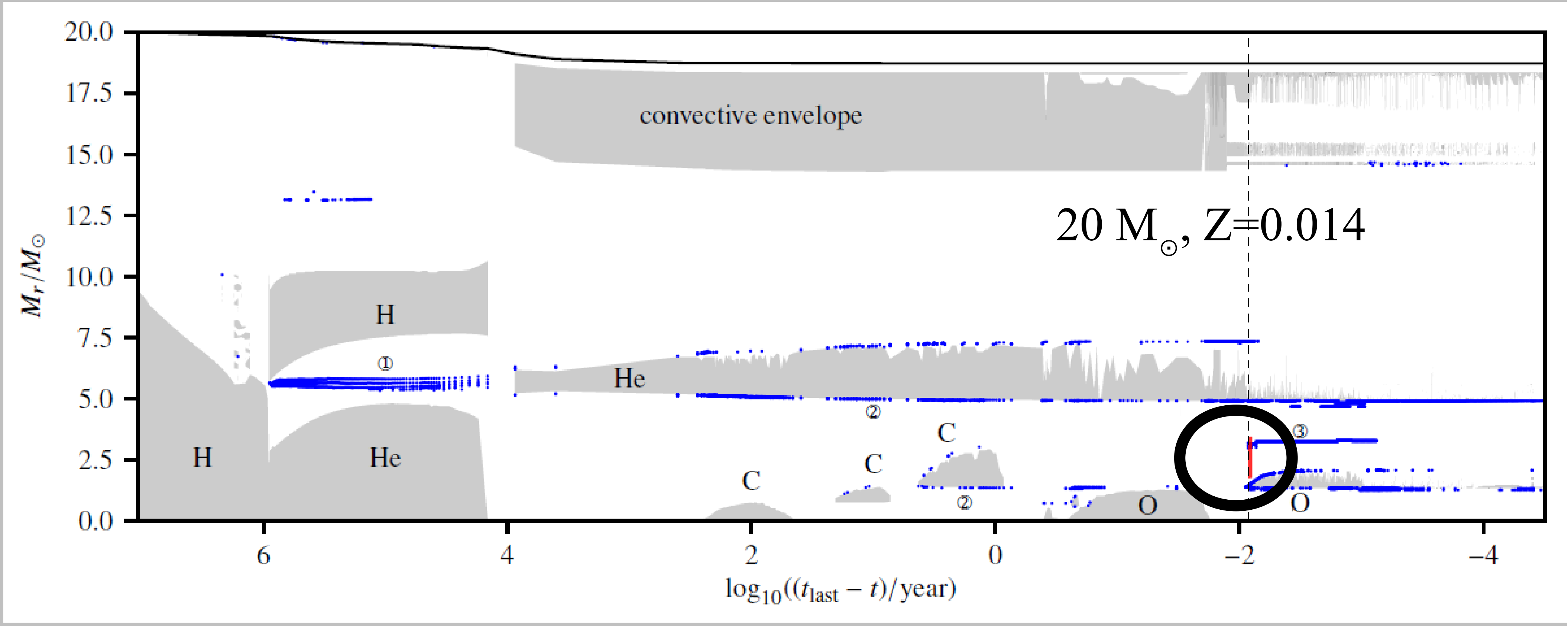
2.3.1. The recipe

The following dynamical shear coefficient is used, as suggested by J.–P. Zahn (priv. comm.):

$$\boxed{D = \frac{1}{3}vl = \frac{1}{3}\ \frac{v}{l}\ l^2 = \frac{1}{3}\ r\frac{\mathrm{d}\,\Omega}{\mathrm{d}\,r}\ \Delta r^2 = \frac{1}{3}\ r\Delta\Omega\ \Delta r} \tag{5}$$

where r is the mean radius of the zone where the instability occurs, $\Delta\Omega$ is the variation of Ω over this zone and Δr is the extent of the zone. The zone is the reunion of

Dynamical Shear

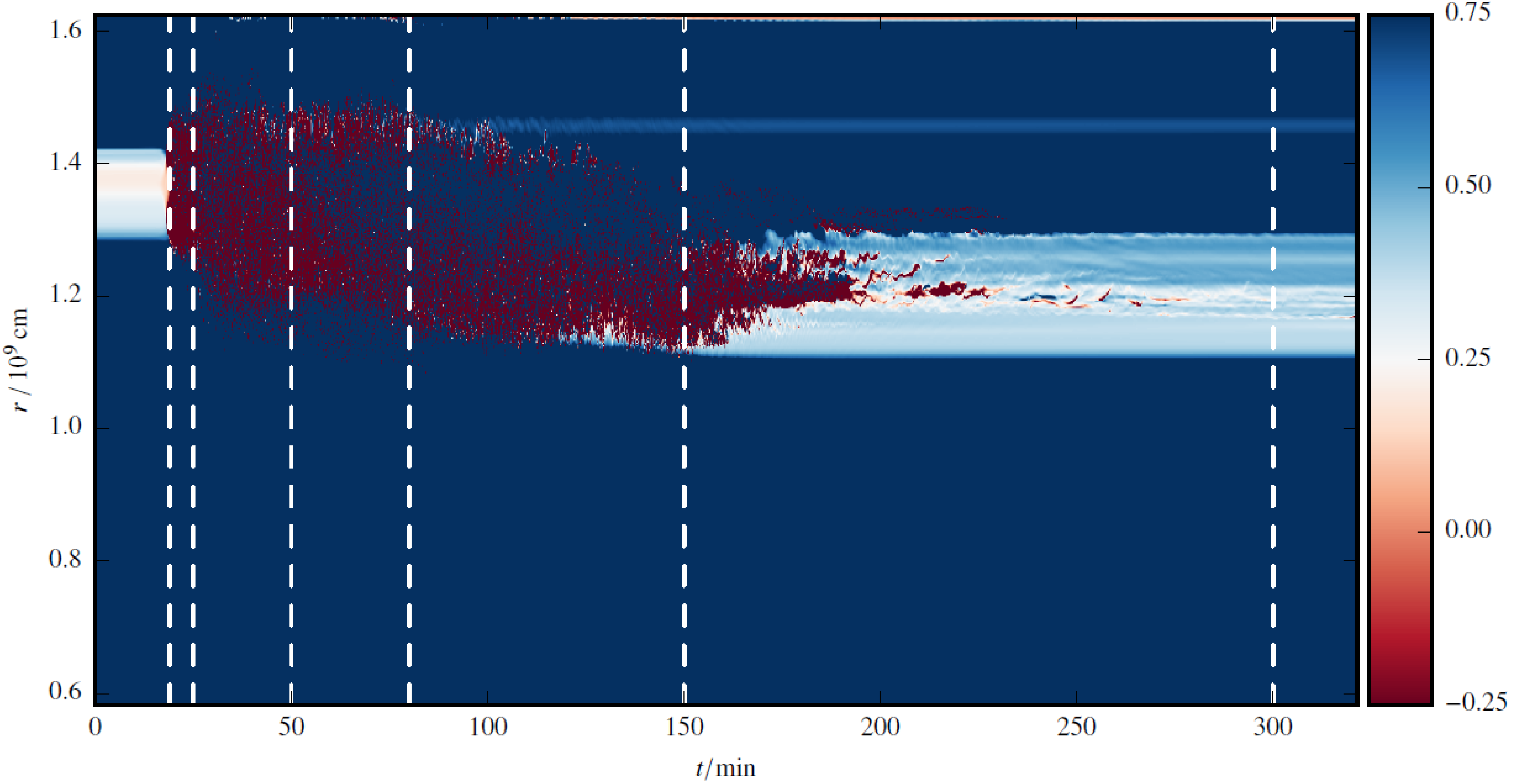


Edelmann et al 2016, A&A

2D simulations with SLH

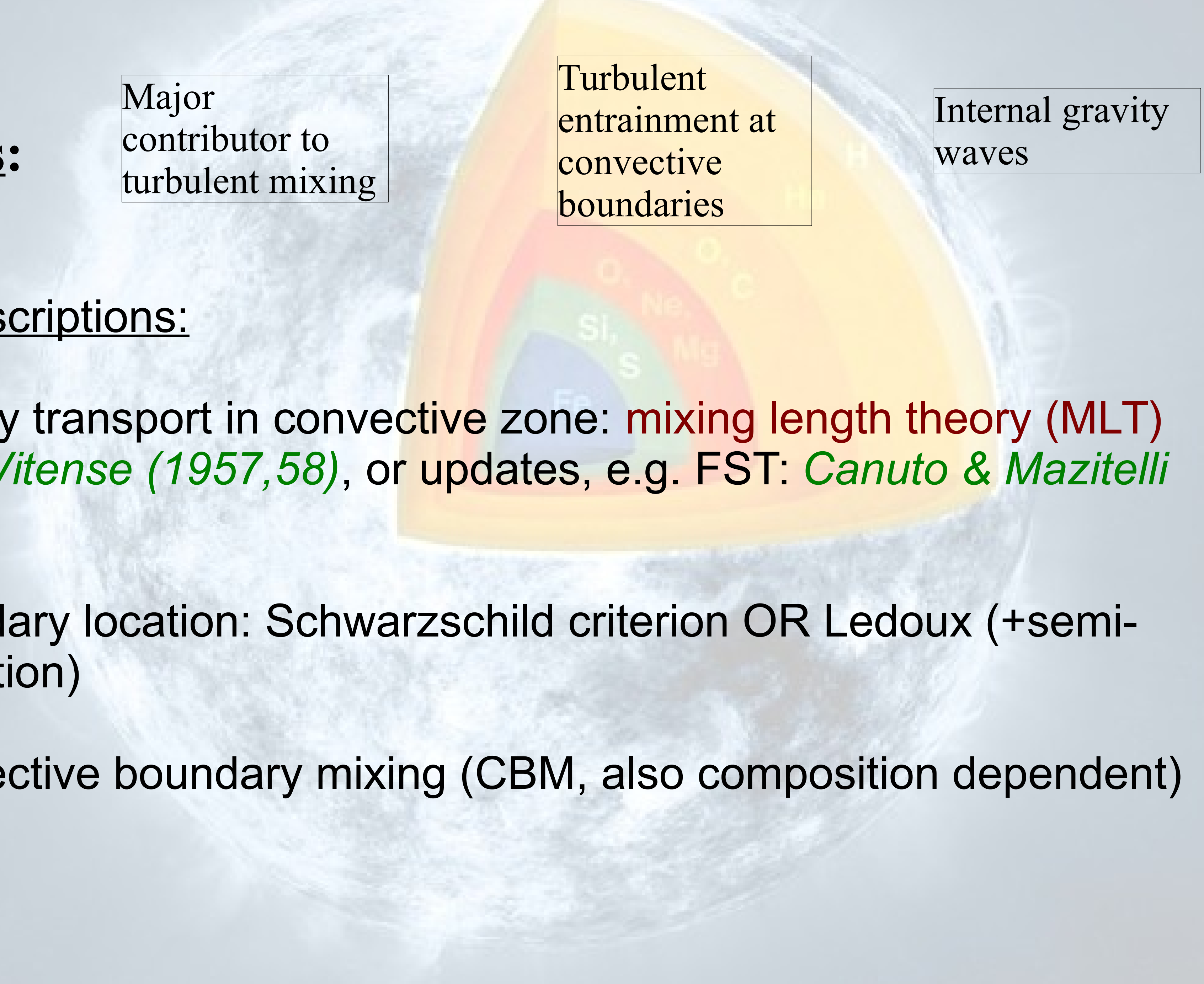
Edelmann et al 2016, A&A

Dynamical Shear: Ri Time Evolution in 2D



- Threshold (Ri_c) confirmed.
- Mixing/transport rate is tricky to implement correctly in 1D codes

Convection: Current Implementation in 1D Codes



Multi-D processes:

Major contributor to turbulent mixing

Turbulent entrainment at convective boundaries

Internal gravity waves

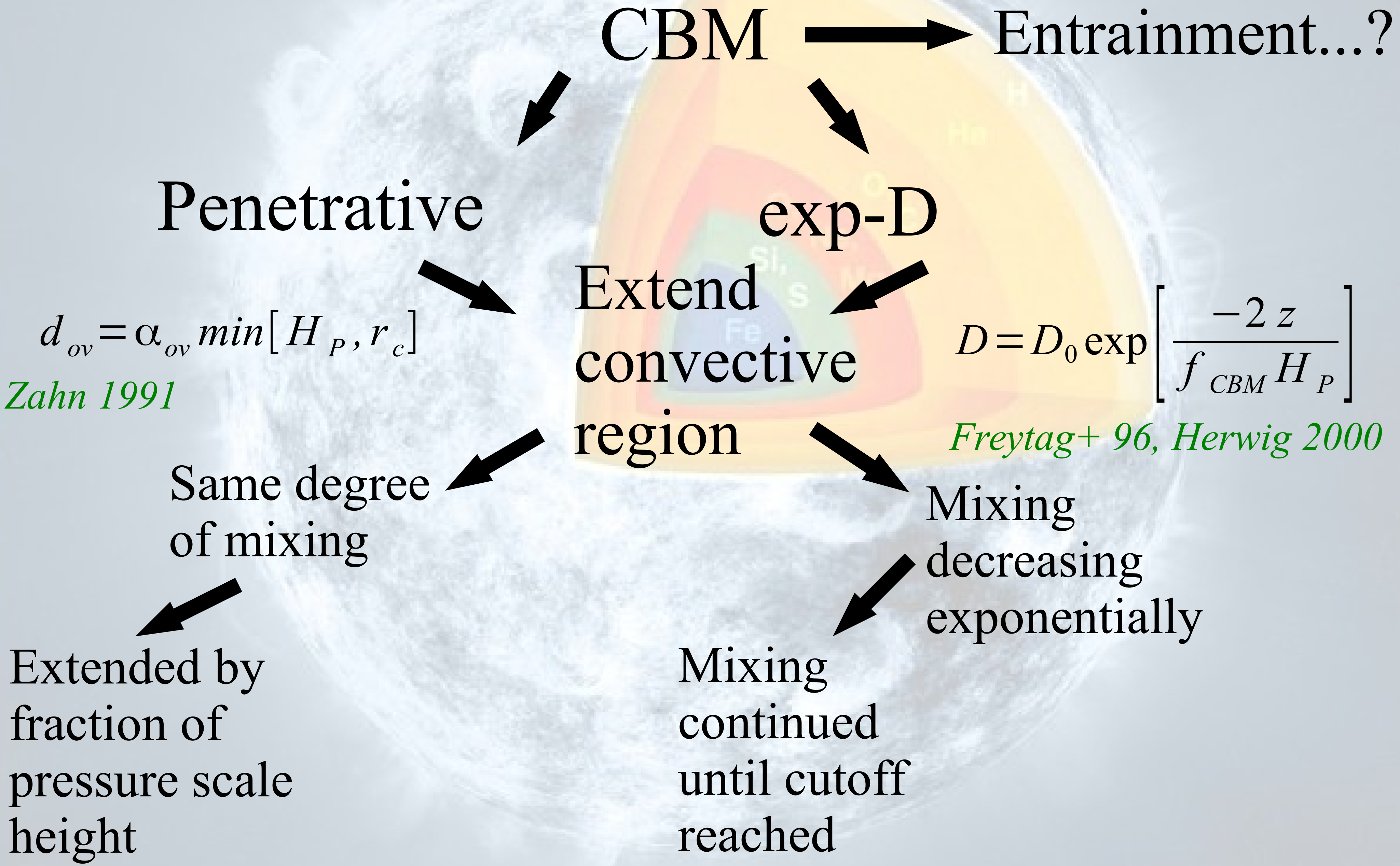
1D prescriptions:

- Energy transport in convective zone: mixing length theory (MLT) *Bohm-Vitense (1957,58)*, or updates, e.g. FST: *Canuto & Mazitelli (1991)*

- Boundary location: Schwarzschild criterion OR Ledoux (+semi-convection)

- Convective boundary mixing (CBM, also composition dependent)

Convective Boundary Mixing (CBM)



CBM → Entrainment...?

Penetrative

exp-D

Extend convective region

$$d_{ov} = \alpha_{ov} \, min[H_P, r_c]$$

Zahn 1991

$$D = D_0 \exp\left[\frac{-2z}{f_{CBM} H_P}\right]$$

Freytag+ 96, Herwig 2000

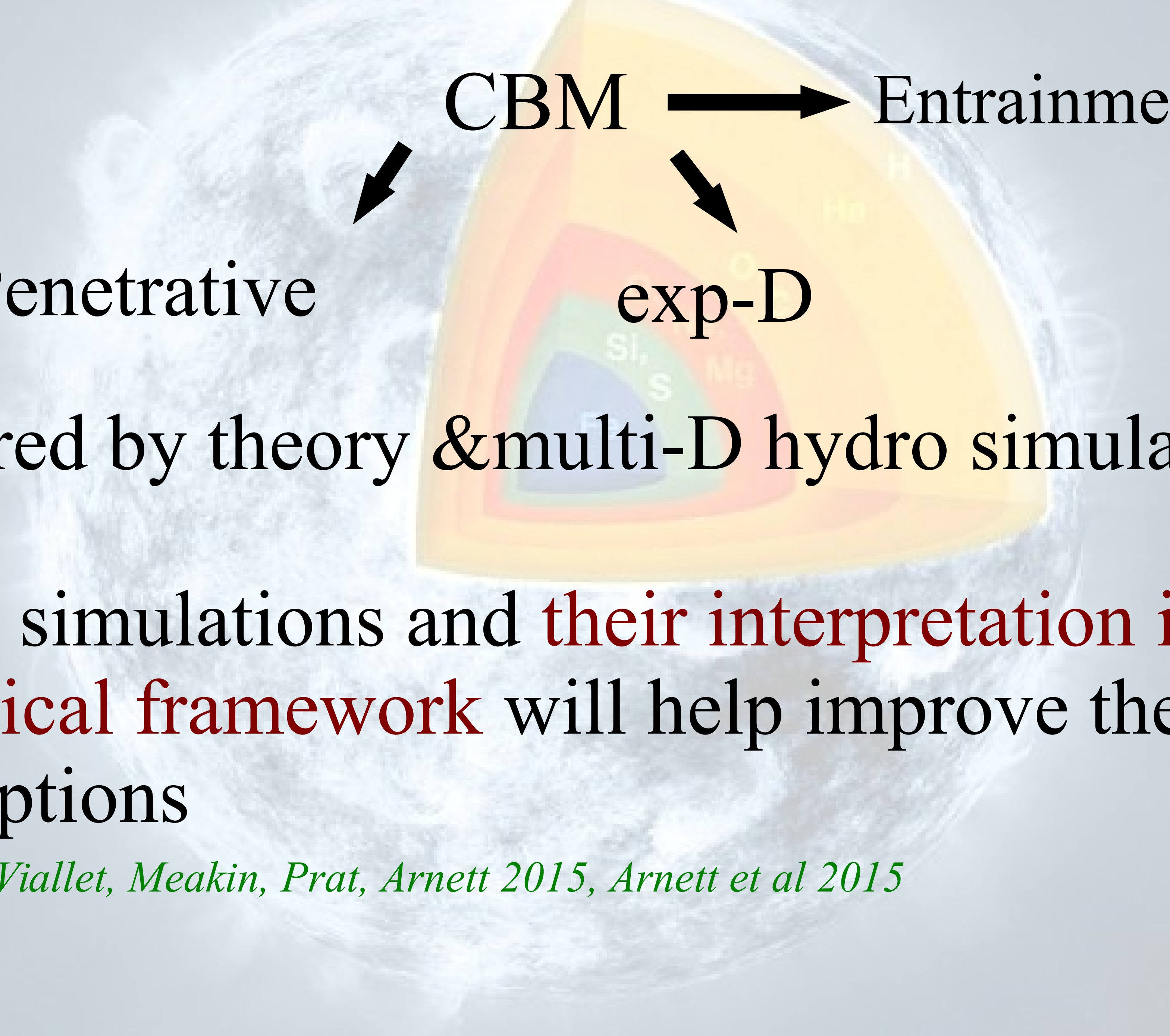
Same degree of mixing

Extended by fraction of pressure scale height

Mixing decreasing exponentially

Mixing continued until cutoff reached

Convective Boundary Mixing (CBM)



CBM → Entrainment...?

Penetrative

exp-D

- Inspired by theory &multi-D hydro simulations

- More simulations and their interpretation in a theoretical framework will help improve these prescriptions

Viallet, Meakin, Prat, Arnett 2015, Arnett et al 2015

3D Hydro Efforts/Priority List

* Convective boundary mixing during core hydrogen burning:
- +: many constraints (HRD, astero, ...)
- -: difficult to model due to important thermal/radiative effects
- -: long time-scale

- * Silicon burning:
- +: important to determine impact on SNe of multi-D structure in progenitor (Couch et al 2015a,b, Mueller & Janka aph1409.4783, Mueller et al ArXiV1605.01393)
- +: possible shell mergers occurring after core Si-burning (e.g. Tur et al 2009ApJ702.1068; Sukhbold & Woosley 2014ApJ783.105) strongly affect core compactness
- +: radiative effects small/negl.
- -: ~ 10^9 CPU hours needed for full silicon burning phase will be ok soon;
- -: might be affected by convective shell history

- * AGB thermal pulses/H-ingestion:
- +: already doable (e.g. Herwig et al 2014ApJ729.3, 2011ApJ727.89, Mocak et al 2010A&A520.114, Woodward et al 2015)
- +: thermal/radiative effects not dominant
- ?: applicable to other phases?

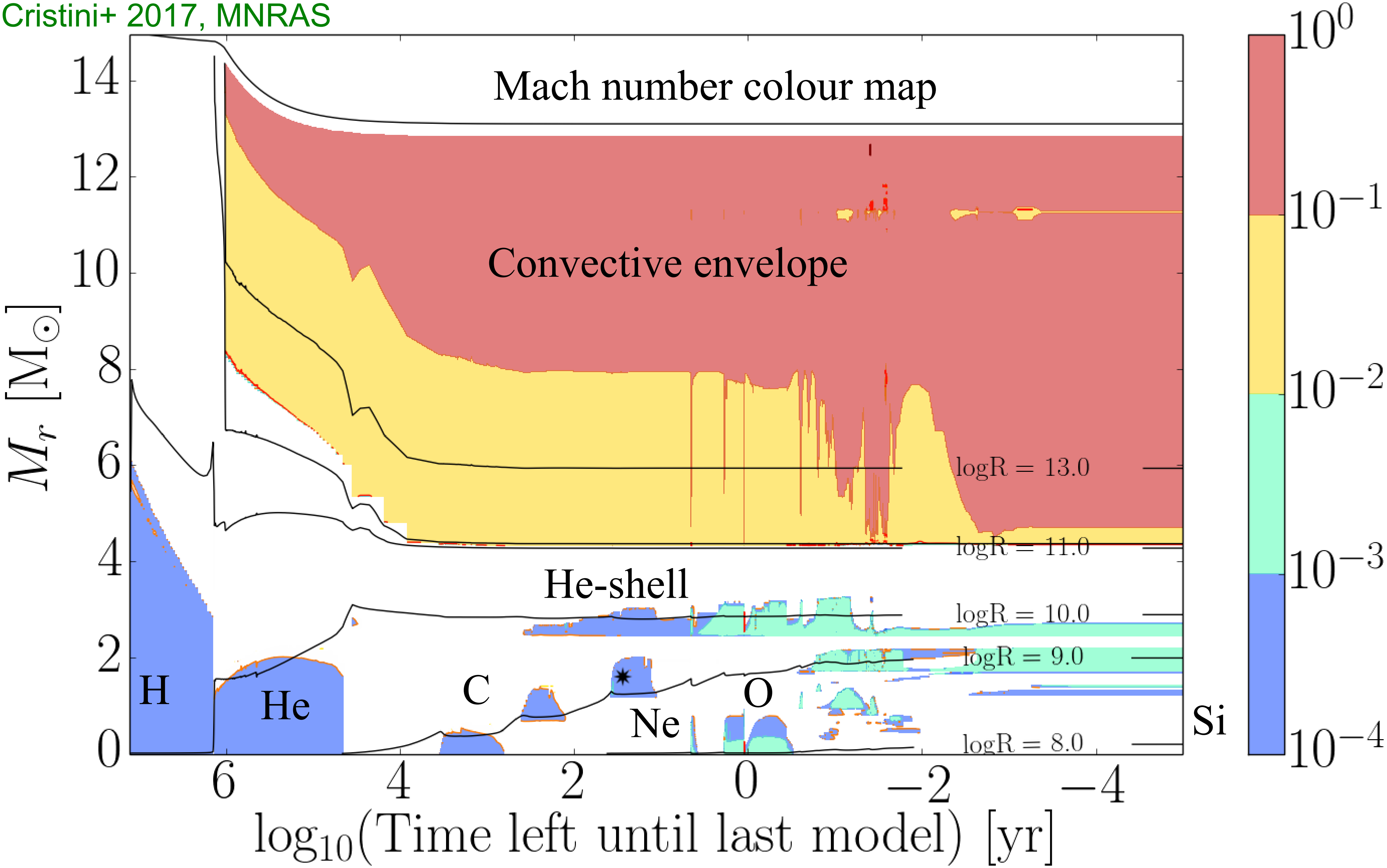
- * Oxygen shell: (Meakin & Arnett 2007ApJ667.448/665.448, Viallet et al 2013ApJ769.1, Jones et al ArXiV1605.03766)
- +: similar to silicon burning but smaller reaction network needed
- -: might be affected by convective shell history

- * Carbon shell: (PhD A. Cristini)
- +: not affected by prior shell history
- +: first stage for which thermal effects become negligible

- * Envelope of RSG (e.g. Viallet et al. 2013, Chiavassa et al 2009-2013),
- * Solar-type stars (e.g. Magic et al. 2013A&A557.26, ...)

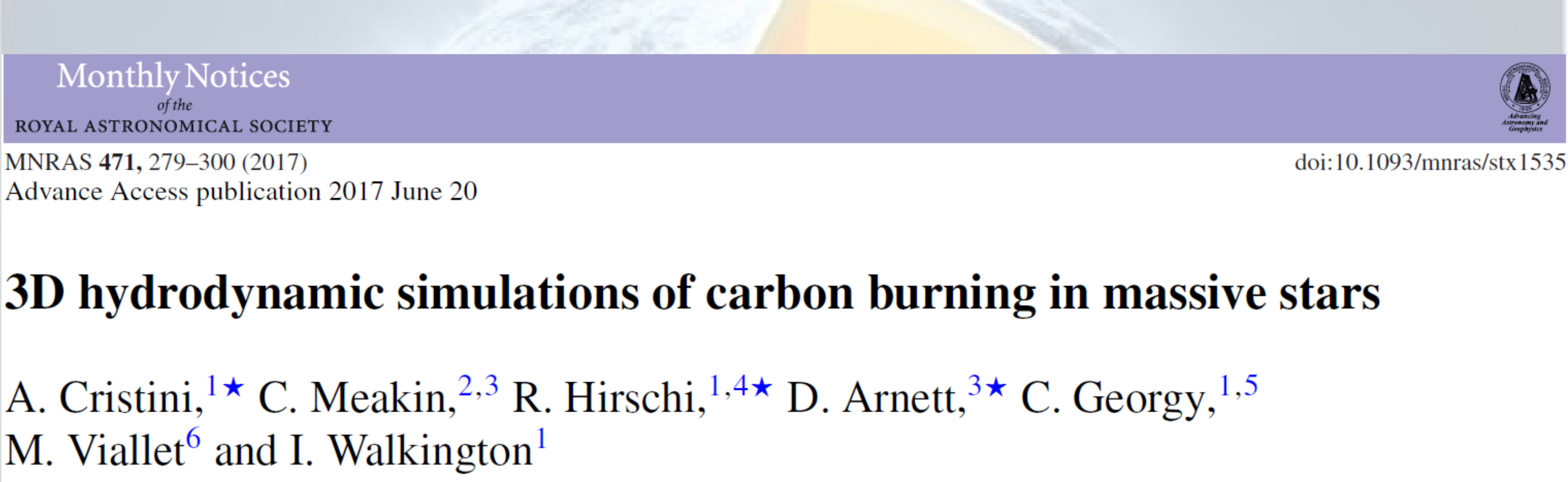
Where to Start?

Cristini+ 2017, MNRAS



Convection takes place during most burning stages

Plan



Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

MNRAS **471,** 279–300 (2017) doi:10.1093/mnras/stx1535
Advance Access publication 2017 June 20

3D hydrodynamic simulations of carbon burning in massive stars

A. Cristini,[1]★ C. Meakin,[2,3] R. Hirschi,[1,4]★ D. Arnett,[3]★ C. Georgy,[1,5]
M. Viallet[6] and I. Walkington[1]

Monthly Notices
of the
ROYAL ASTRONOMICAL SOCIETY

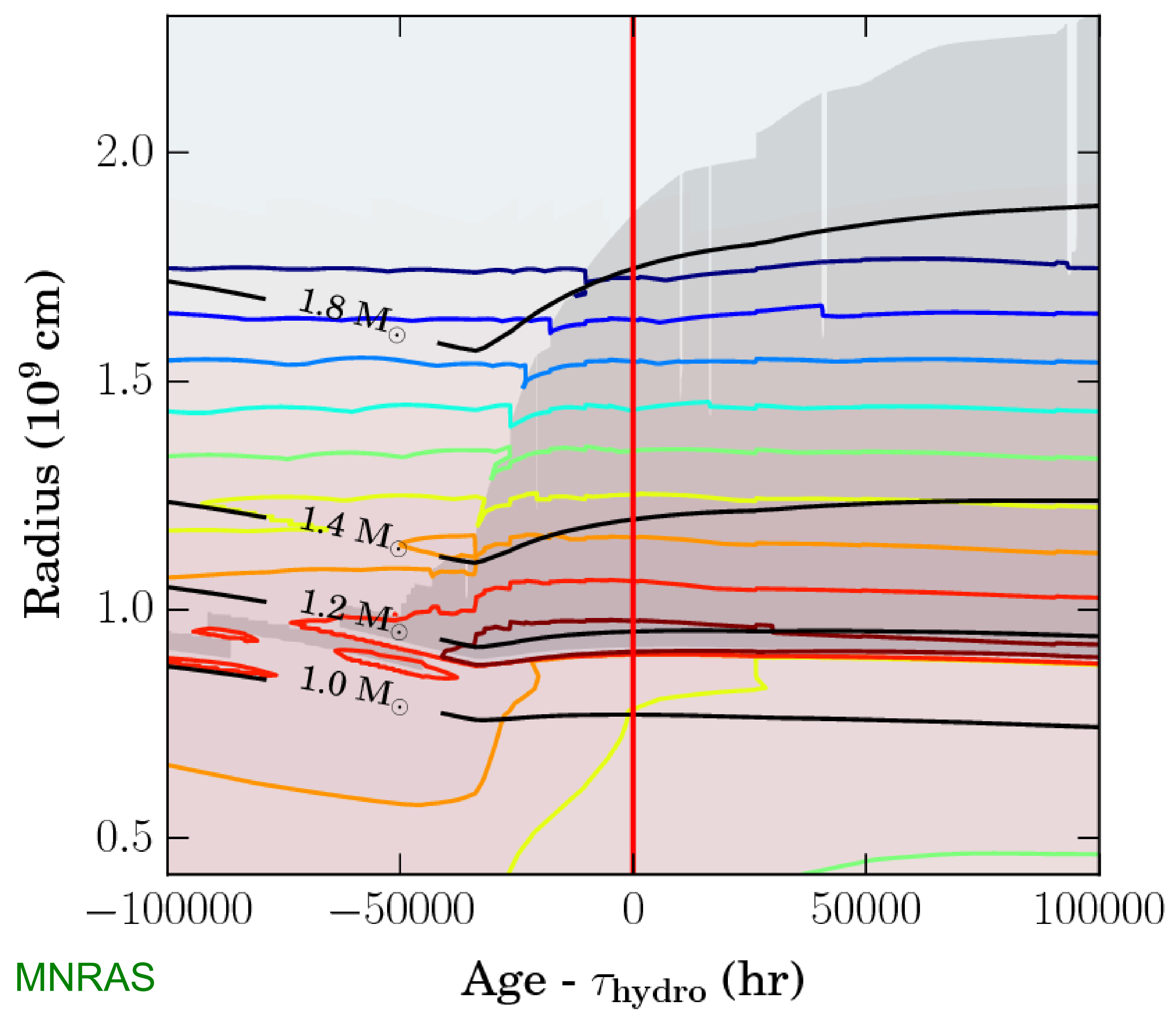
MNRAS **484,** 4645–4664 (2019) doi:10.1093/mnras/stz312
Advance Access publication 2019 February 1

Dependence of convective boundary mixing on boundary properties and turbulence strength

A. Cristini ,[1]★ R. Hirschi,[1,2]★ C. Meakin,[3,4] D. Arnett,[3] C. Georgy[1,5] and
I. Walkington[1]

From 1D to 3D

C-shell in 15 $M_\odot$, Z=0.014 1D stellar evolution model



Cristini+ 2017, MNRAS

From 1D to 3D

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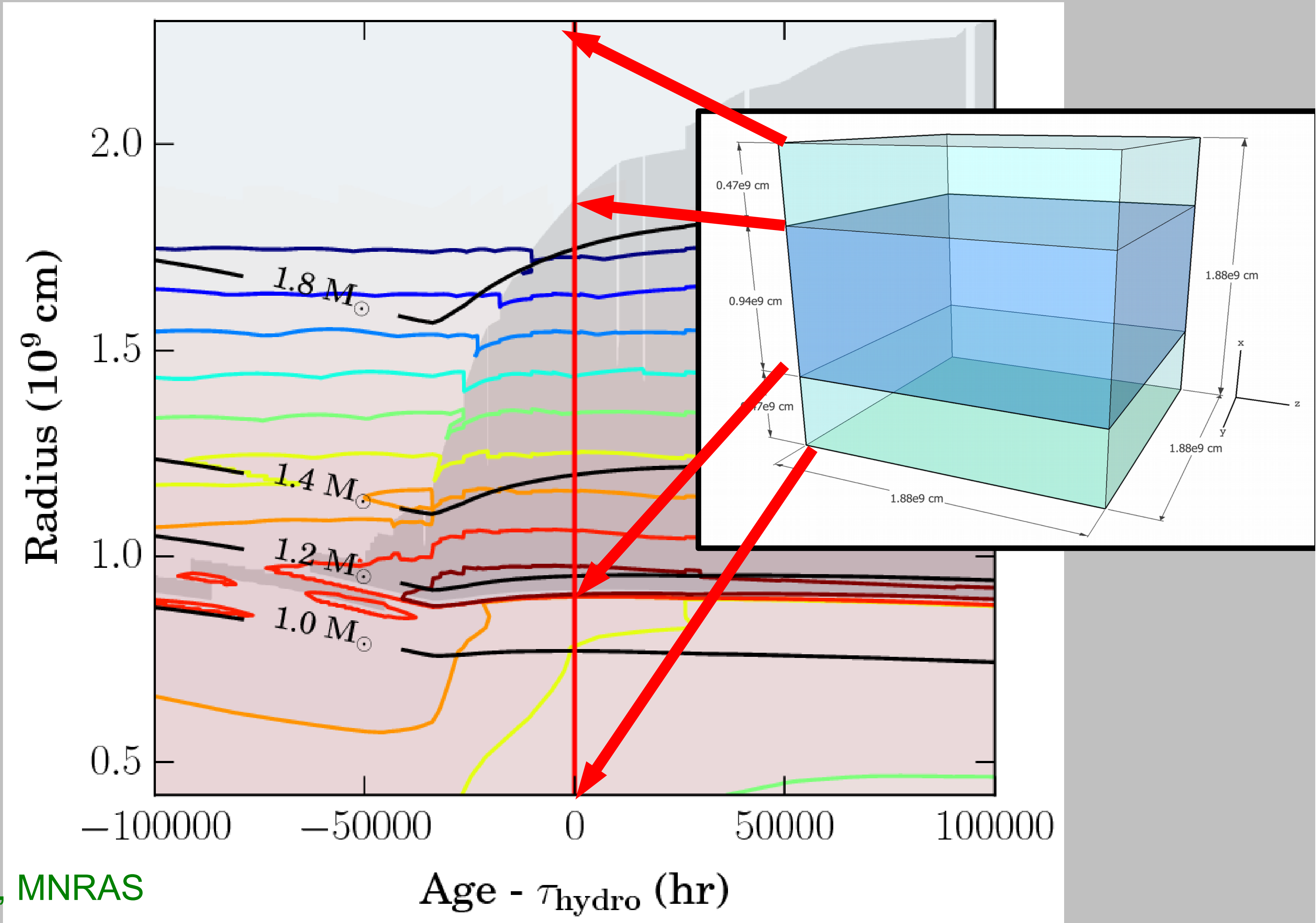
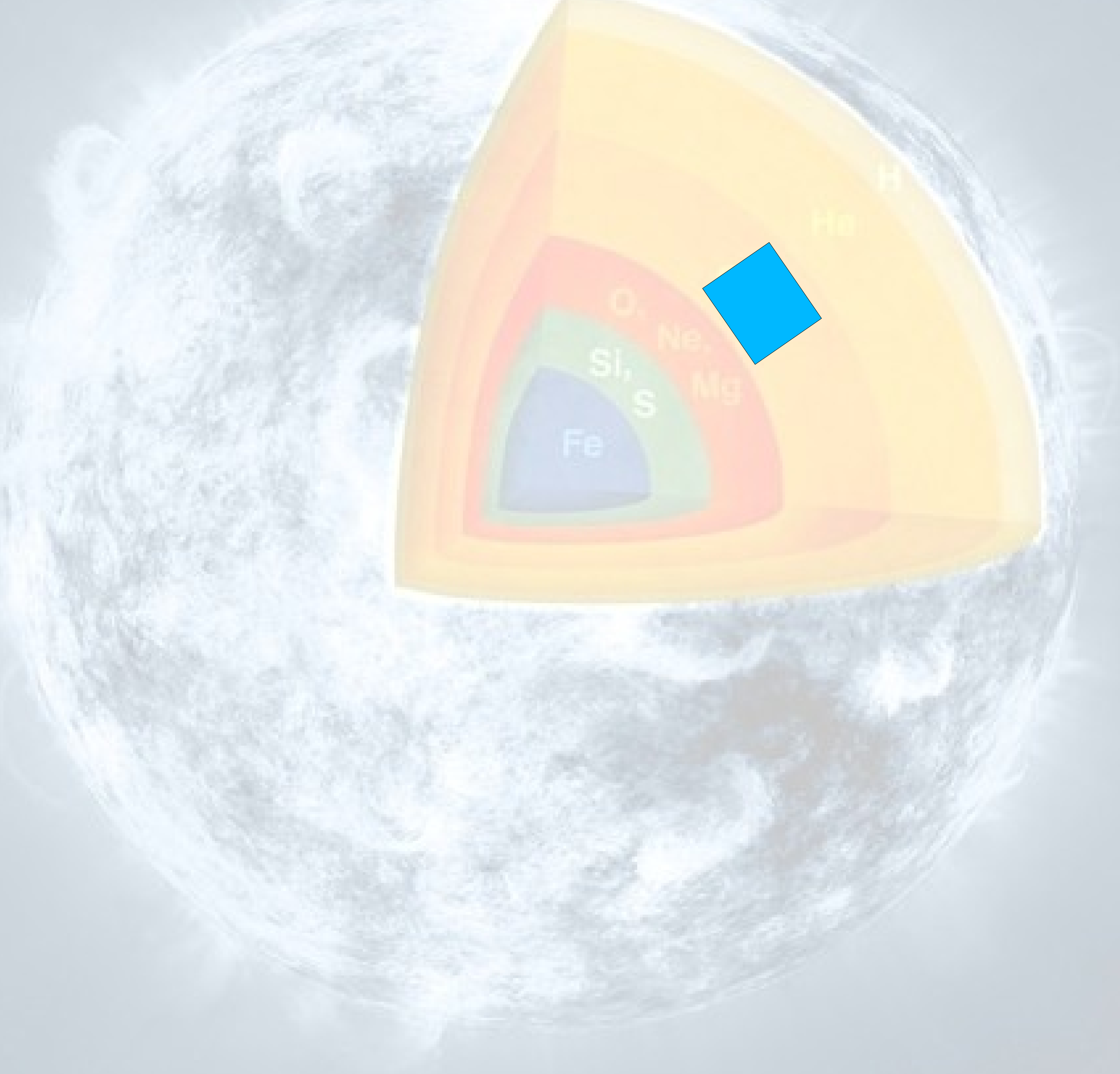


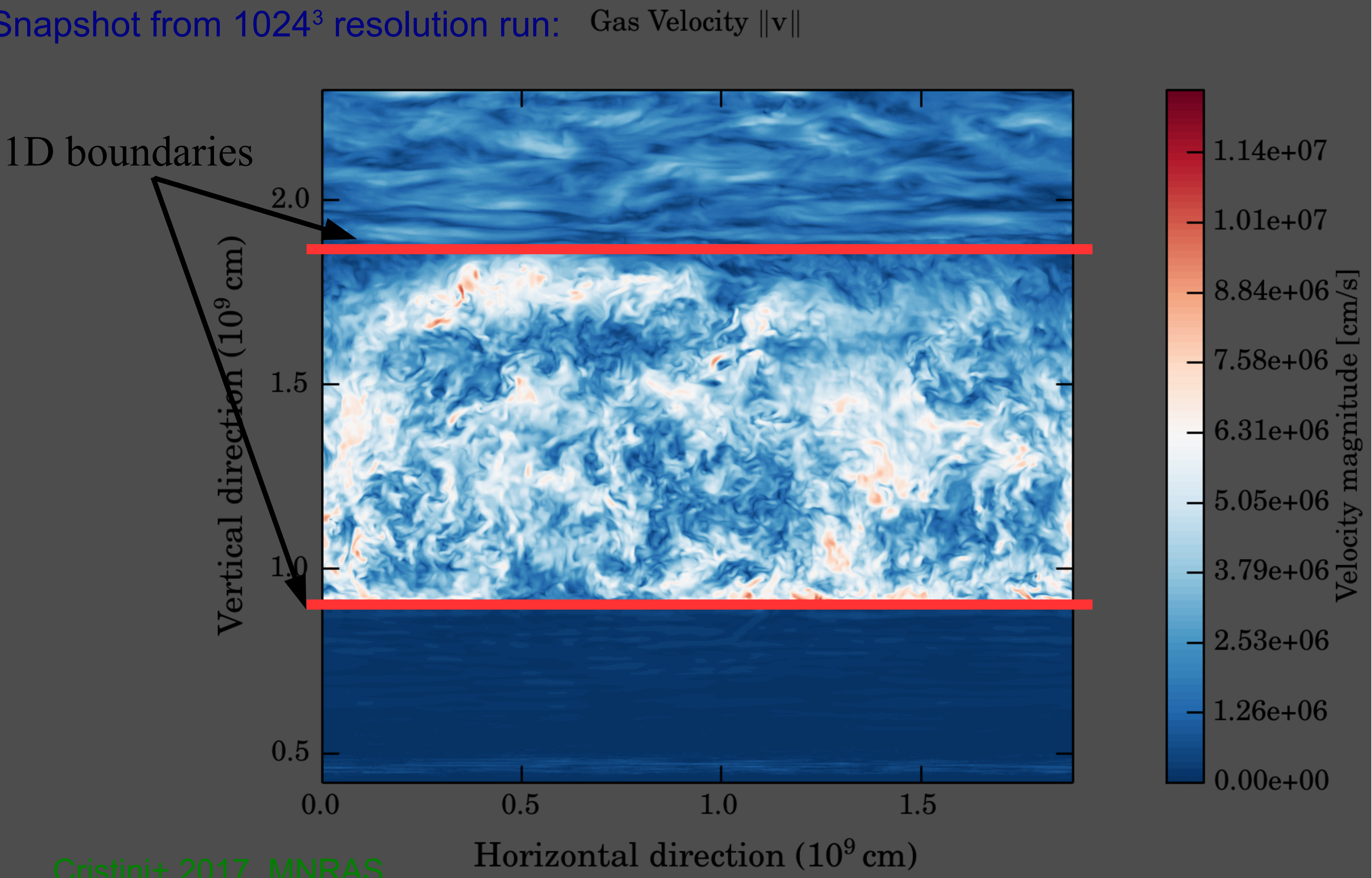
Cristini+ 2017, MNRAS

3D simulations of Carbon burning



3D C-shell Simulations

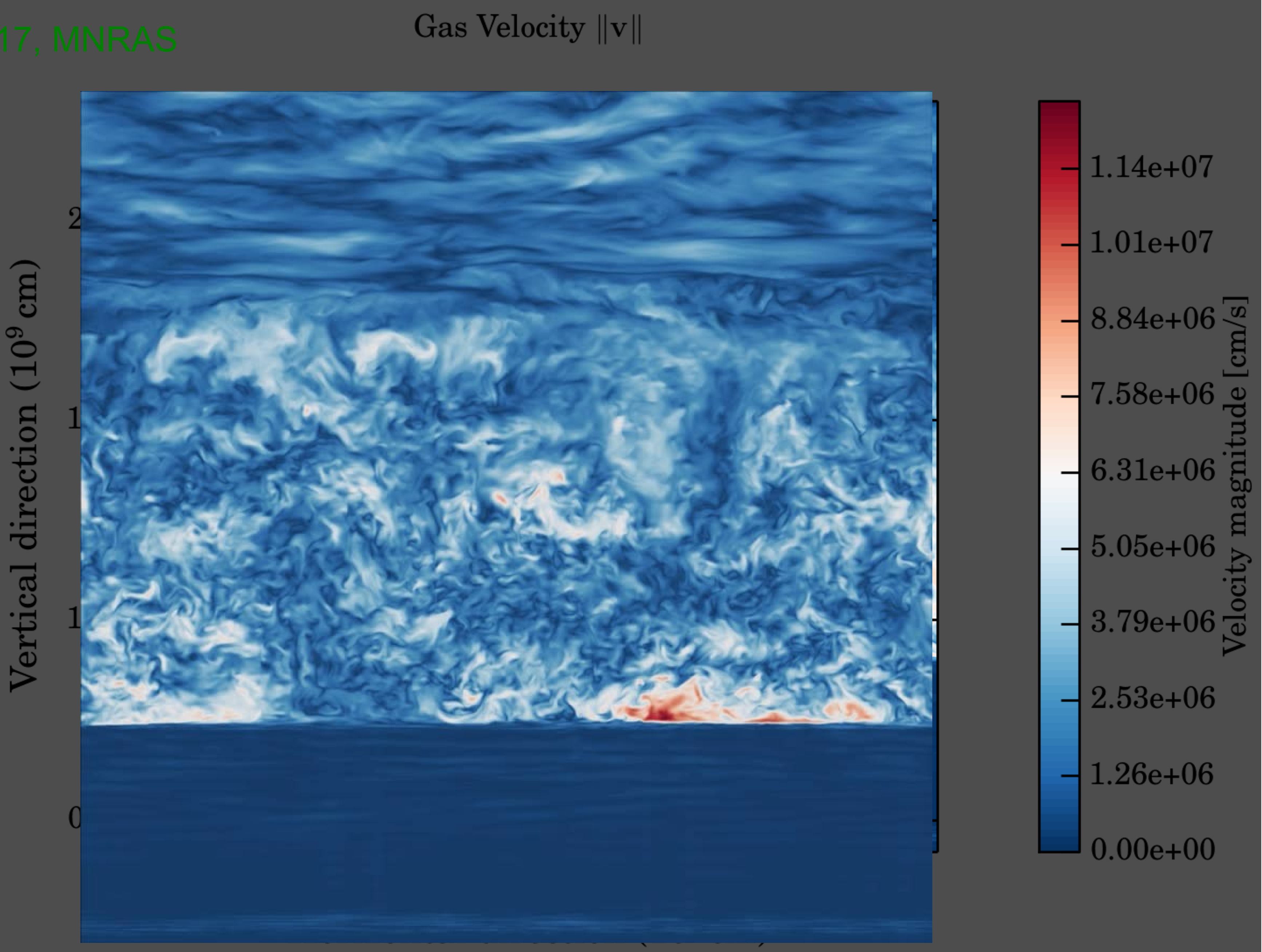
Snapshot from 1024^3 resolution run:



Cristini+ 2017, MNRAS

3D C-shell Simulations: |v| movie

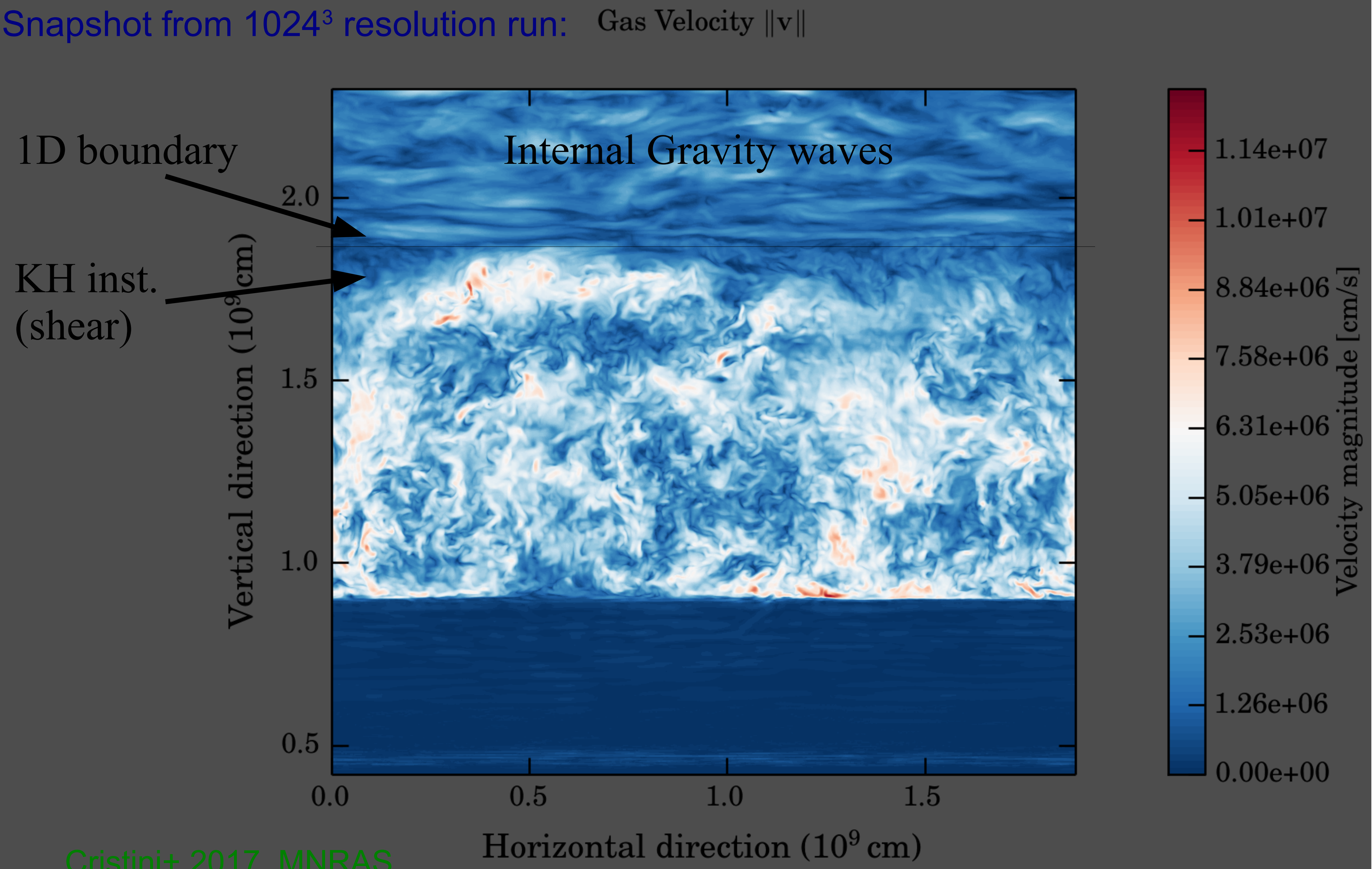
Cristini+ 2017, MNRAS



http://www.astro.keele.ac.uk/shyne/321D/convection-and-convective-boundary-mixing/visualisations

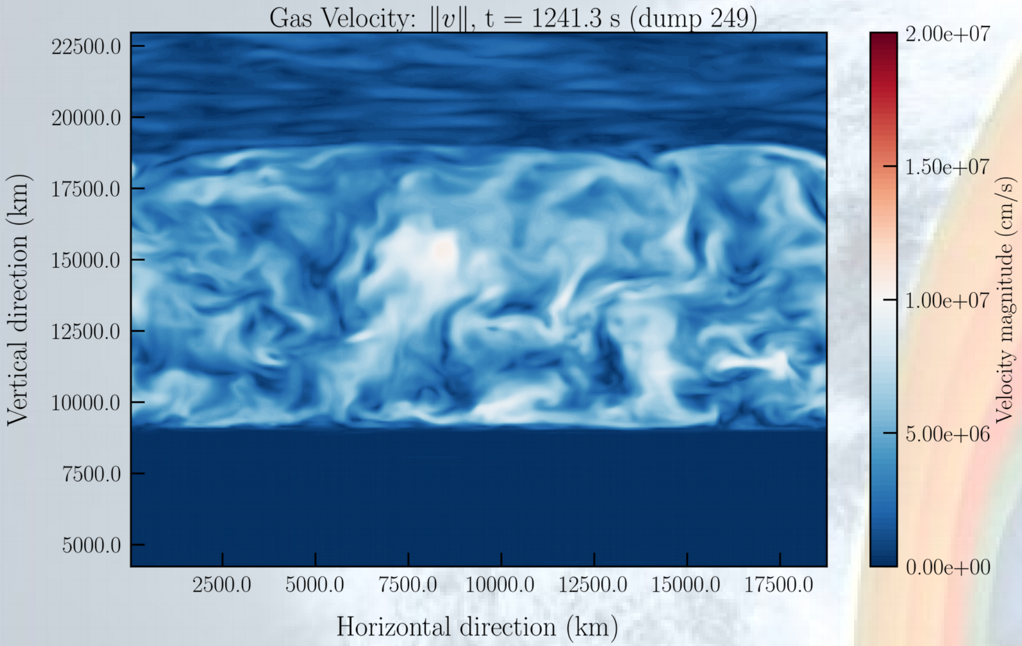
3D C-shell Simulations

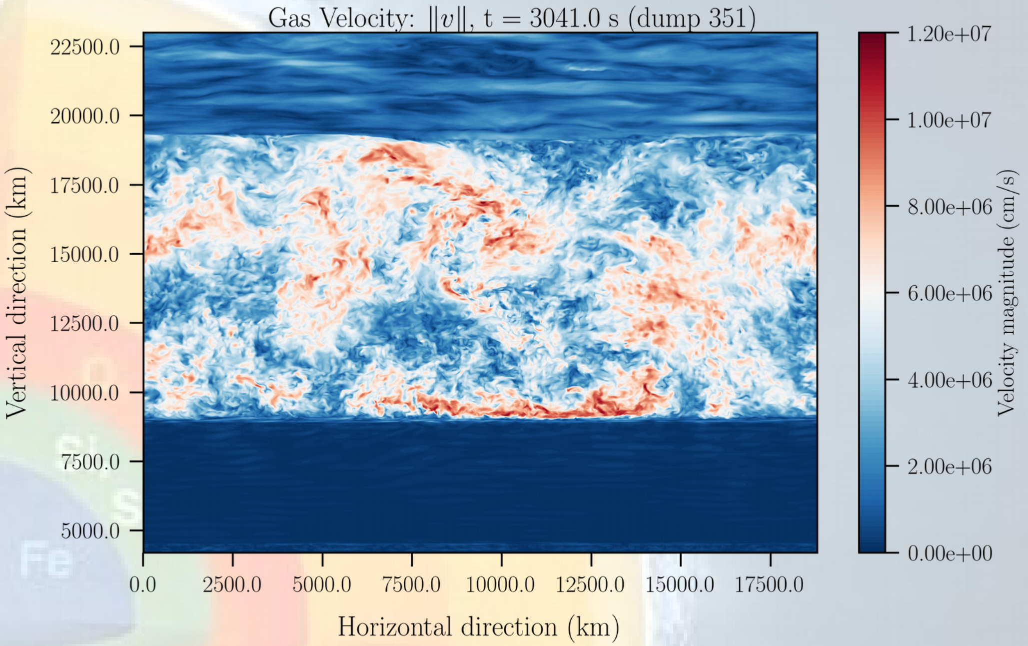
Snapshot from 1024^3 resolution run:



Cristini+ 2017, MNRAS

Resolution & Luminosity Study





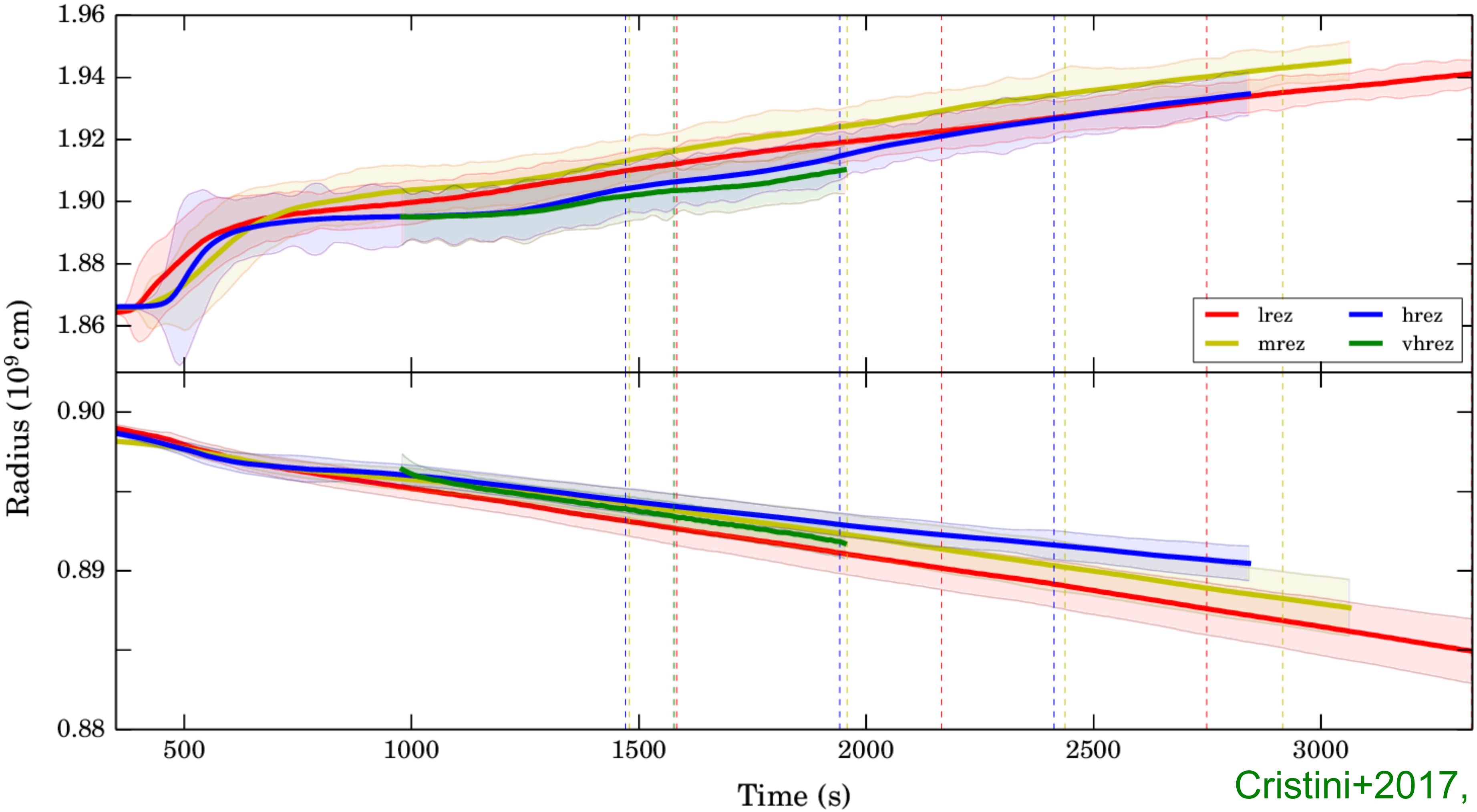
Luminosity **Resolution**	**128**	**256**	**512**	**1024**
1			eps1	
33			eps33	
100			eps100	
333			eps333	
1000	lrez	mrez	hrez/eps1k	vhrez
3333			eps3k	
10000			eps10k	
33333			eps33k	
			Luminosity study	

Cristini+2017, MNRAS

Resolution study

Cristini+2019, MNRAS

Boundary Entrainment



Cristini+2017, MNRAS

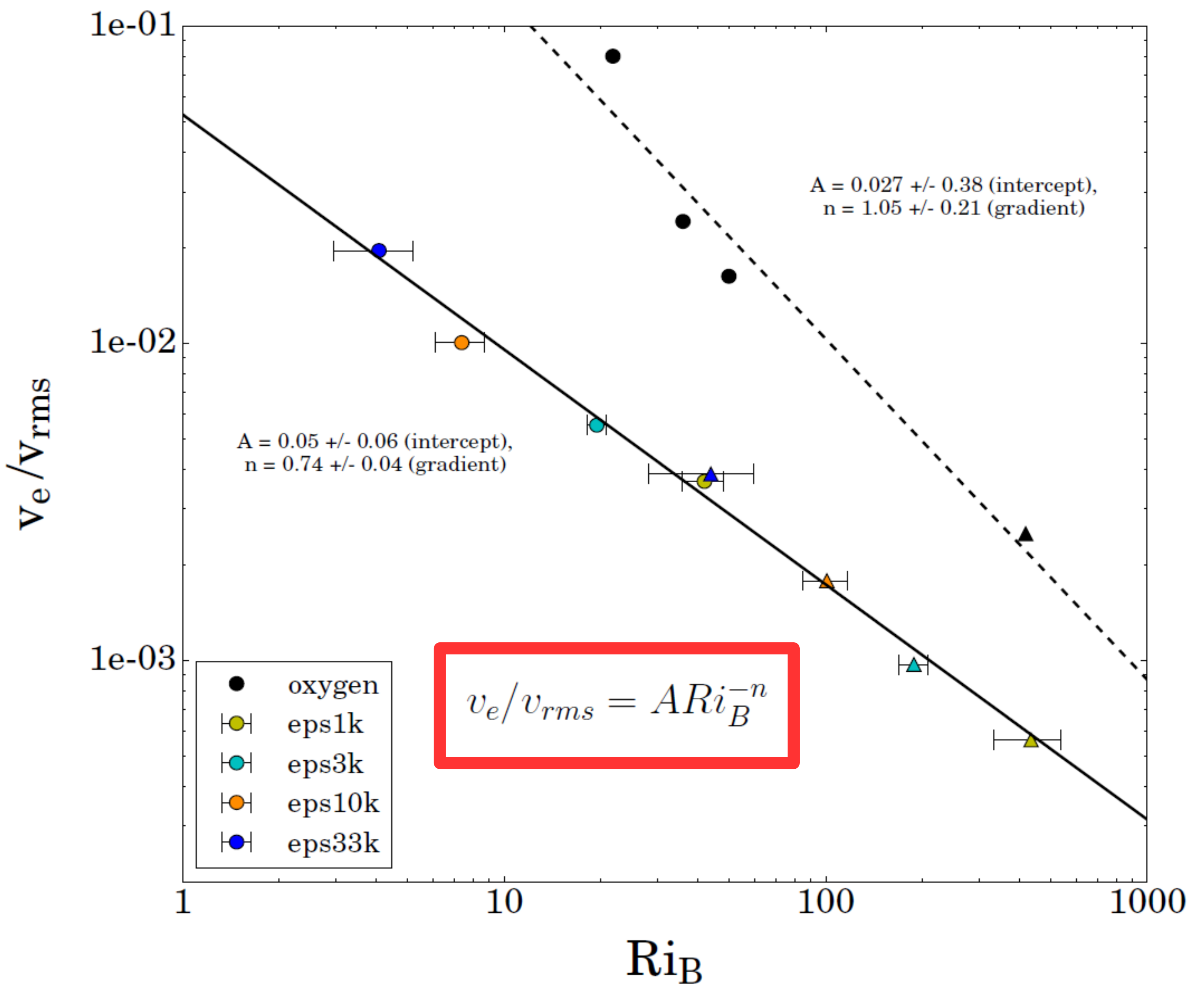
Top: u_e ~ 20,000 cm/s; Bottom: u_e ~ 3,000 cm/s. *Rescaled for eps_{burn} boosting (1/1000)*

→ In 1 year, top: ΔR ~ 6x10^8 cm, bottom: ΔR ~ 10^8 cm: large but reasonable

Consistent with oxygen-shell results and entrainment law.

Entrainment Law

Cristini et al 2019 (see also Garcia & Mellado, 2014, Deardor 1980, Chemel, Staquet and Chollet 2010, Fernando, 1991, Stevens and Lenschow, 2001, Jonker+ 2013)



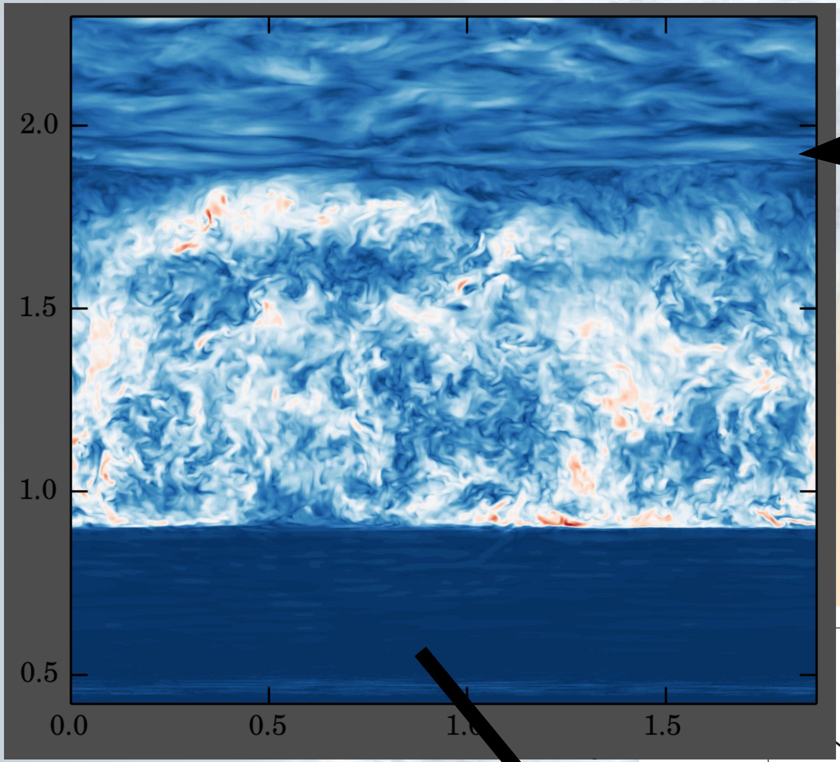
$$v_e / v_{rms} = A Ri_B^{-n}$$

$$Ri_B = \frac{\Delta B \times l}{v_{rms}^2}$$

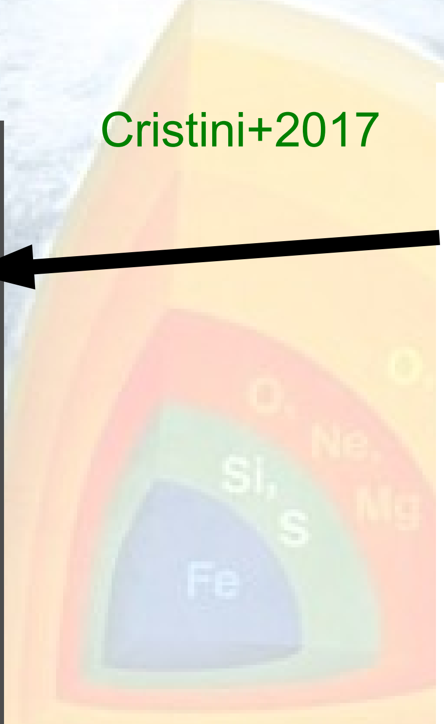
$$Ri_B = \frac{\text{stabilising potential}}{\text{turbulent kinetic energy}}$$

Way Forward: 1 to 3 to 1D link

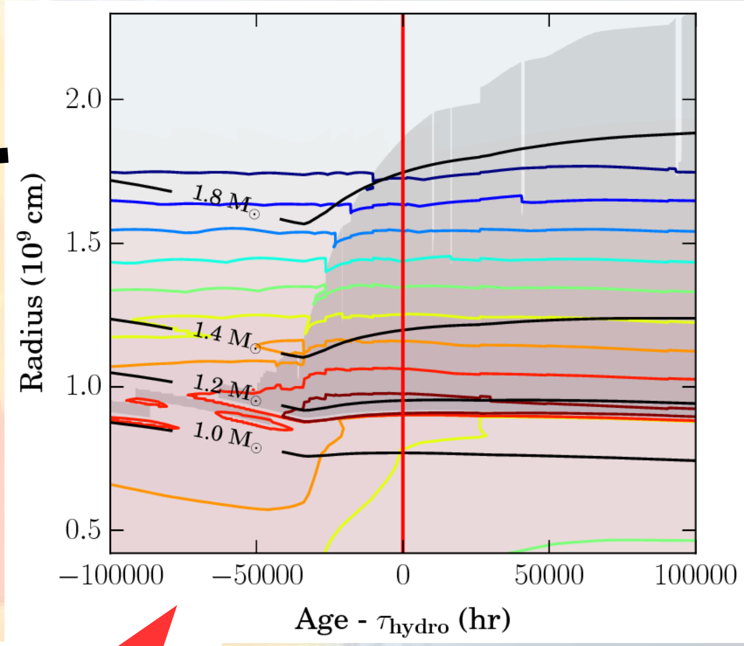
Targeted 3D simulations

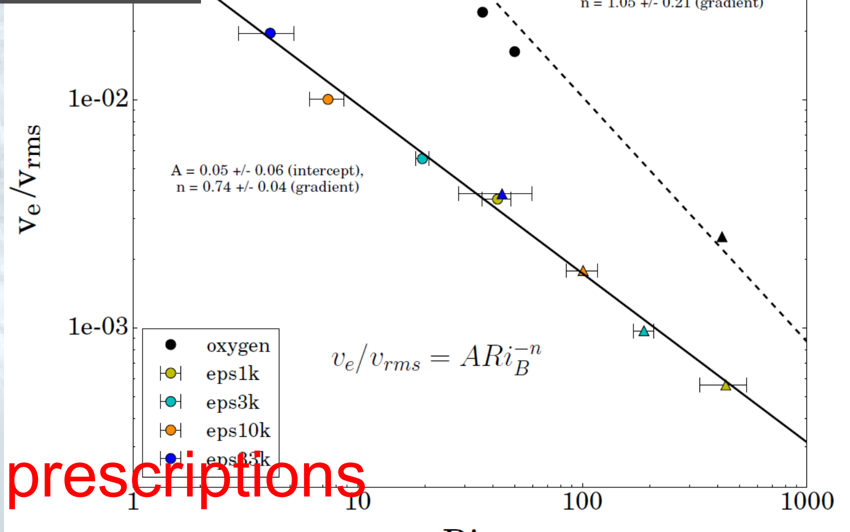


Cristini+2017



Uncertainties in 1D

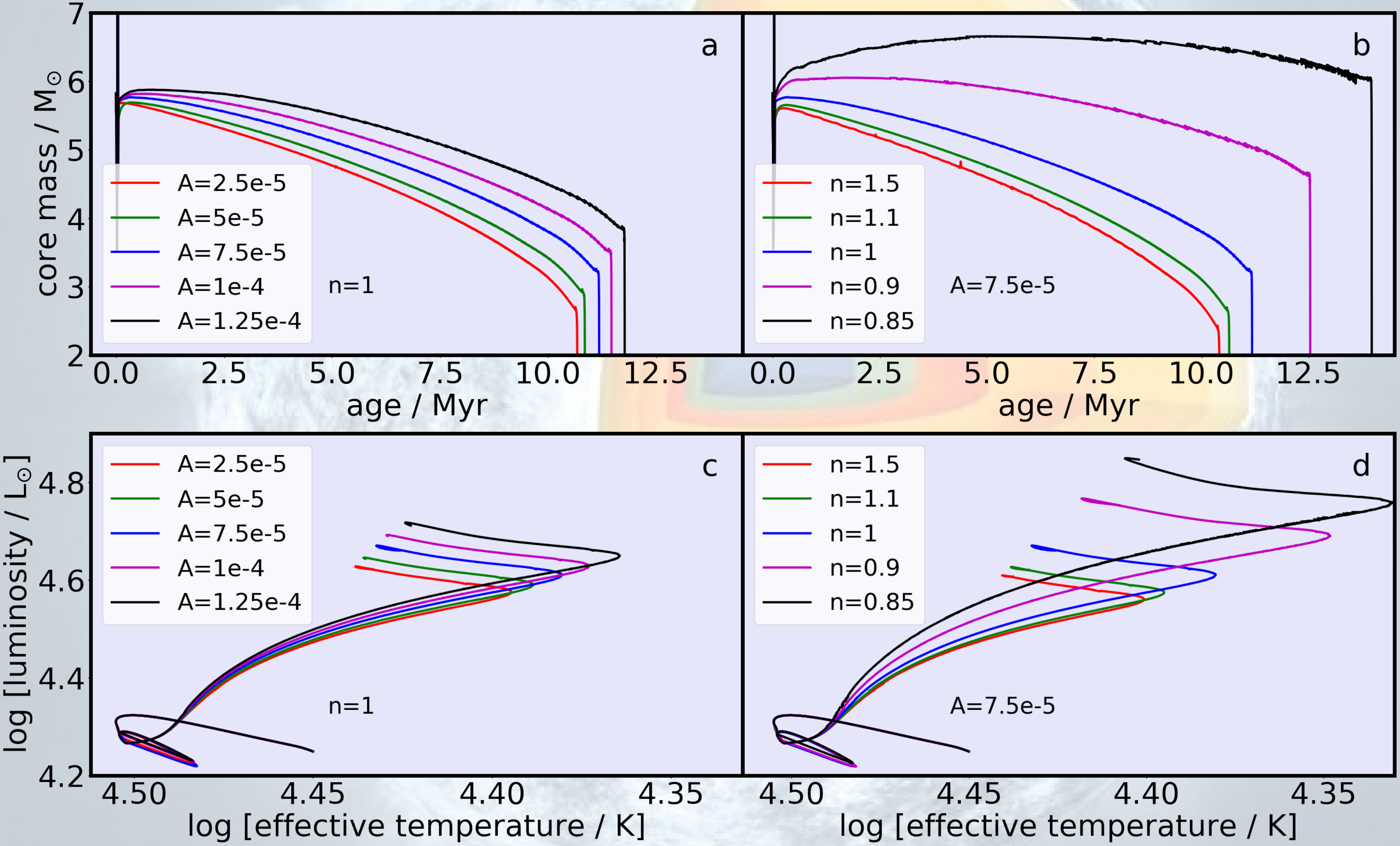




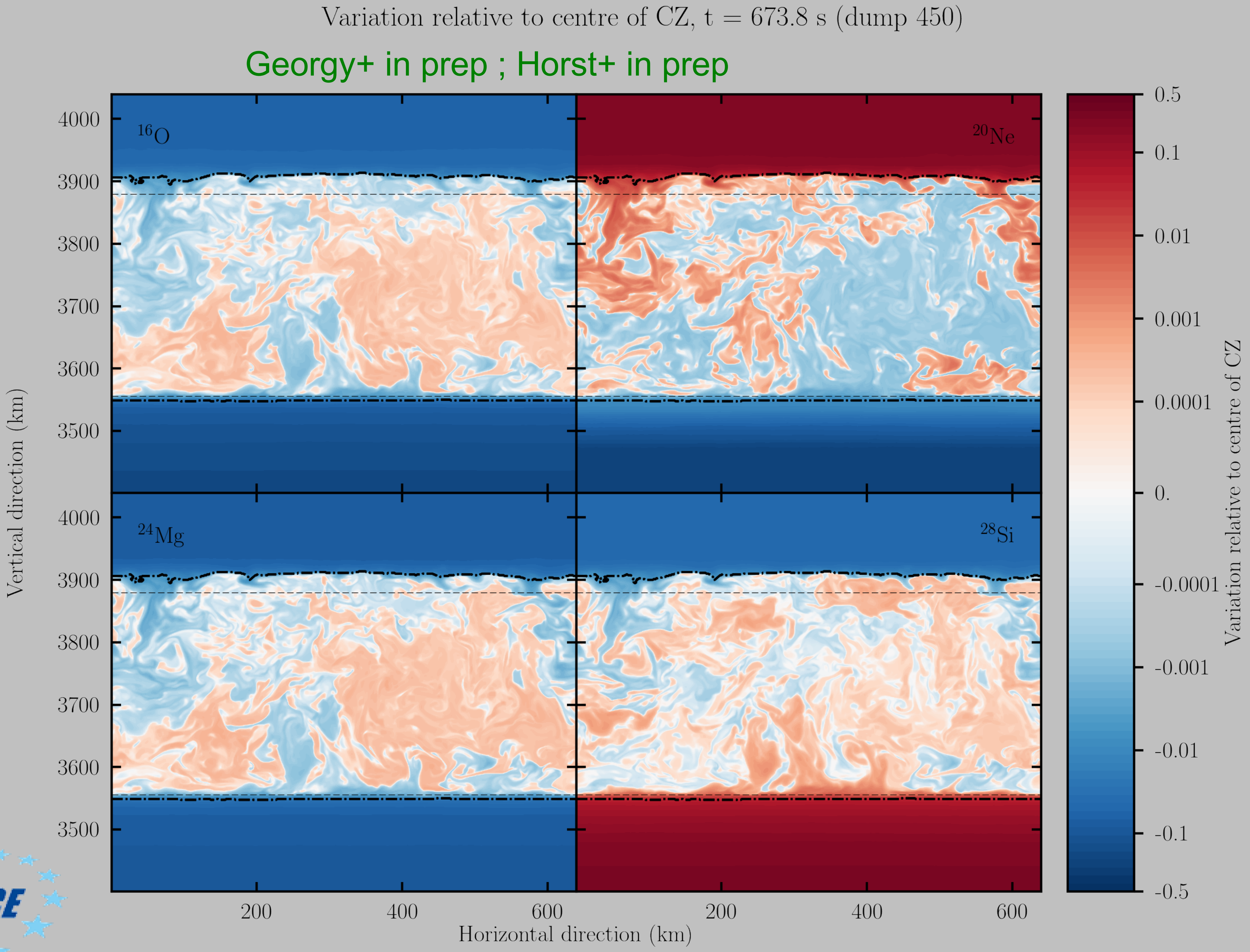
→ Improve theoretical prescriptions

Entrainment in 1D model

Scott+ in prep

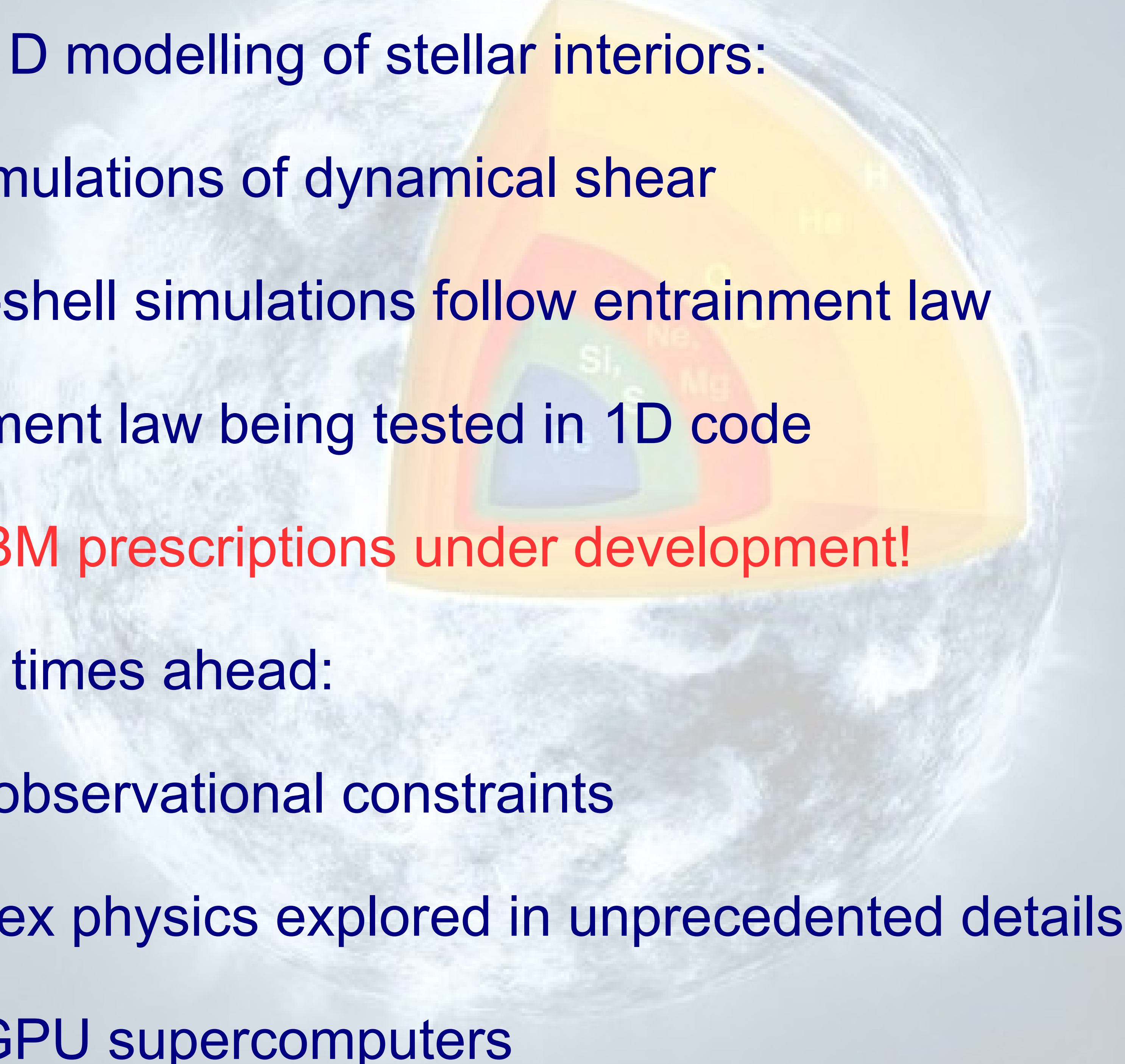


Ne-burning & He-burning Underway



Georgy+ in prep ; Horst+ in prep

Conclusions & Outlook



- 1 - 3 - 1 D modelling of stellar interiors:
  - 2D simulations of dynamical shear
  - 3D C-shell simulations follow entrainment law
- Entrainment law being tested in 1D code

& new CBM prescriptions under development!

- Exciting times ahead:
- - Great observational constraints
- - Complex physics explored in unprecedented details
- - CPU-GPU supercomputers