

# Gaussian processes regression networks for the analysis of RV data

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March 19, 2019



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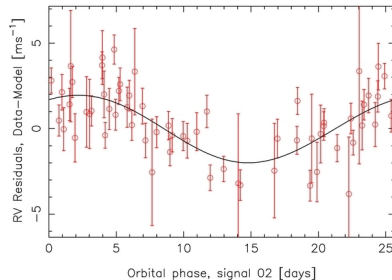
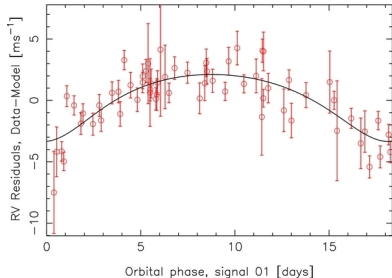
# Stellar noise problem

## HD 41248

- Jenkins et al. (2013); ApJ, 771, 41
- Santos et al. (2014); A&A 566, A35

Stellar activity can induce RV signals:

- Periodicity from stellar rotation
- Aperiodicity from active region evolution



Jenkins et al. (2013)

# Gaussian Processes<sup>1</sup>

A Gaussian process (GP) is a generalization of the multivariate Gaussian distribution.

$$f(\mathbf{x}) \sim \mathcal{GP}(m(\mathbf{x}), k(\mathbf{x}, \mathbf{x}'))$$

Has been used for exoplanet detection:

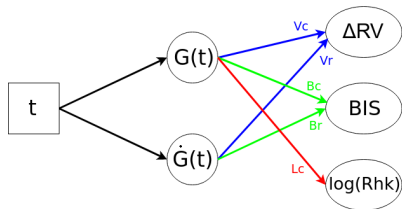
- Haywood et al. (2014) → CoRoT-7
- Grunblatt et al. (2015) → Kepler-78
- Rajpaul et al. (2015) → Gliese 15 and Alpha Centauri B
- Faria et al. (2016) → CoRoT-7
- Cloutier et al. (2018) → K2-18
- etc...

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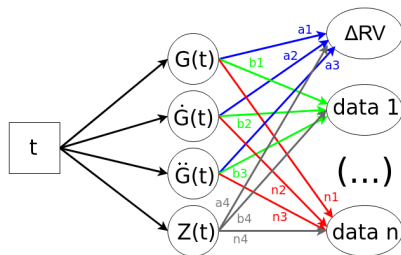
<sup>1</sup>Rasmussen and Williams (2006)

# Some existing multi-output Gaussian processes frameworks

Before explaining my work there are two important GP frameworks to mention.



Rajpaul et al. (2015)



Jones et al. (2017)

$$\begin{cases} \Delta RV = V_c G(t) + V_r \dot{G}(t) \\ BIS = B_c G(t) + B_r \dot{G}(t) \\ \log(R'_{HK}) = L_c G(t) \end{cases}$$

$$G(t) \equiv F^2(t),$$

$F(t)$  is the fraction of the visible hemisphere covered in spots.  
(Aigrain et al. 2012).

- It is a physically-motivated GP framework capable of modelling RV-induced by stellar activity jointly with activity indicators, by assuming the same GP is responsible for all stellar activity signals.

$$\begin{cases} \Delta RV = a_1 G(t) + a_2 \dot{G}(t) + a_3 \ddot{G}(t) + a_4 Z_0(t) \\ data_1 = b_1 G(t) + b_2 \dot{G}(t) + b_3 \ddot{G}(t) + b_4 Z_1(t) \\ \dots \\ data_n = n_1 G(t) + n_2 \dot{G}(t) + n_3 \ddot{G}(t) + n_4 Z_l(t) \end{cases}$$

- Extends the Rajpaul et al. (2015) framework to a larger class of models.
- Other stellar activity indicators can carry similar information to the BIS and  $\log(R'_{HK})$ , and thus help further characterize the RV-induced by stellar activity.

# Gaussian processes regression network (GPRN)<sup>2</sup>

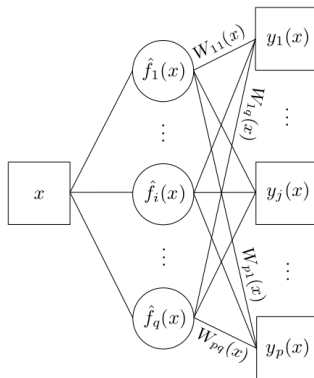
A general GP framework capable of taking into account multiples inputs and outputs

- Combines a Bayesian neural network with the flexibility of GPs

$$\mathbf{y}(\mathbf{x}) = \mathbf{W}(\mathbf{x}) [\mathbf{f}(\mathbf{x}) + \sigma_f \boldsymbol{\epsilon}] + \sigma_y \mathbf{z}$$

$$f_i(x) \sim \mathcal{GP}(0, k_f)$$

$$W_{ij} \sim \mathcal{GP}(0, k_w)$$



Wilson (2014)

A GPRN is in effect an adaptive mixture of GPs, that accommodates input dependent signal and noise correlations between multiple output variables.

<sup>2</sup>Wilson et al. (2012)



# Our GPRN implementation

- **gprn**<sup>3</sup> → eventually I'll give it a prettier name.
  - ▷ Python implementation that uses either **emcee** (affine-invariant ensemble sampler) or **dynesty** (dynamic nested sampling).
  - ▷ Since I'm currently developing this package, it is possible that I'll mess up something as I work on it and break everything!
  - ▷ To be used in a future GPRN implementation in **kima**<sup>4</sup>, a package for the analysis of radial velocity (RV) data (Faria et al. 2018).

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<sup>3</sup><https://github.com/jdavidrcamacho/gprn>

<sup>4</sup><https://github.com/j-faria/kima>

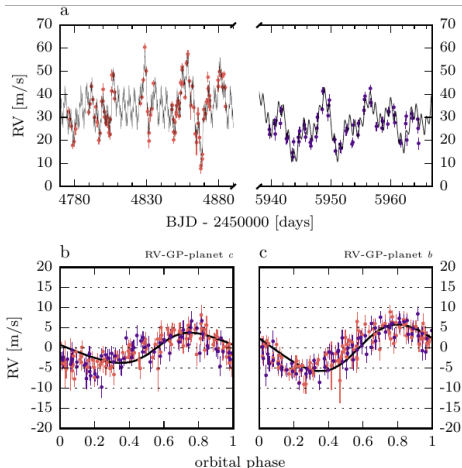
# CoRoT-7 - previous works

A G9V type star, with an average temperature  $\sim 5250$  K, and age  $\sim 1.2 - 2.3$  Gyr (Léger et al. 2009).

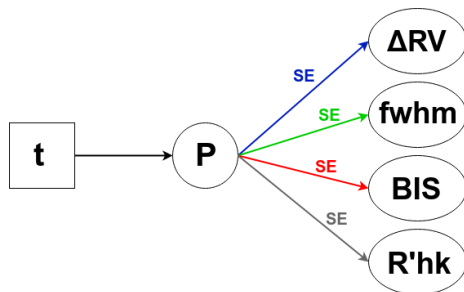
- Queloz et al. (2009)
- Haywood et al. (2014)
- Faria et al. (2016)

Very active star with two confirmed planets.

- CoRoT-7b  
P  $\sim 0.85$  days and K  $\sim 3.97$  m/s
- CoRoT-7c  
P  $\sim 3.70$  days and K  $\sim 5.55$  m/s



Faria et al. (2016)



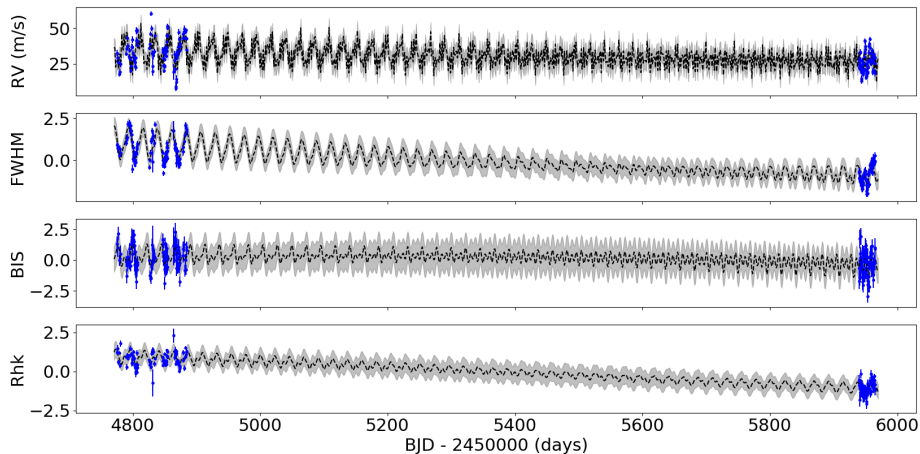
P → periodic kernel

$$P(x, x') = \theta_P^2 \left[ -\frac{2}{l_P^2} \sin^2 \left( \frac{\pi}{P} |x - x'| \right) \right]$$

SE → squared exponential kernel

$$SE(x, x') = \theta_{SE}^2 \exp \left( -\frac{(x - x')^2}{2l_{SE}^2} \right)$$

# CoRoT-7 - results



# CoRoT-7 - results

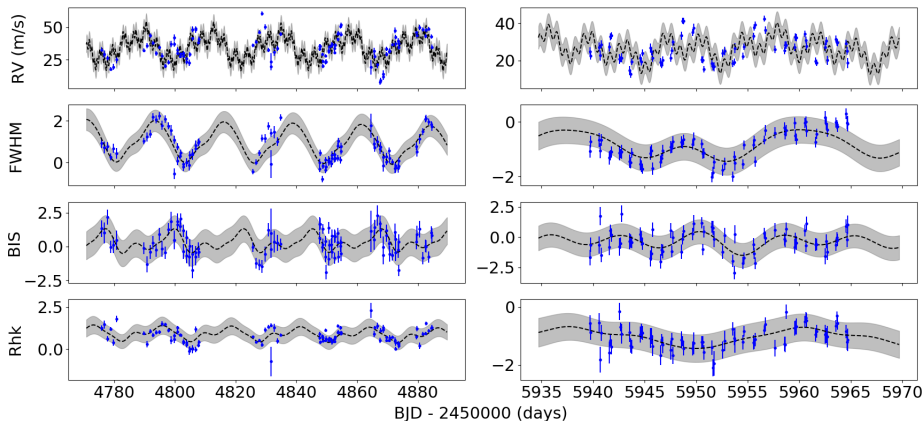
$$\text{Period (days)} = 22.704_{-0.168}^{+0.228}$$

$$\text{Periodic scale} = 1.197_{-0.151}^{+0.171}$$

$$\text{Decay timescale (days)} = 1100.187_{-670.123}^{+744.159}$$

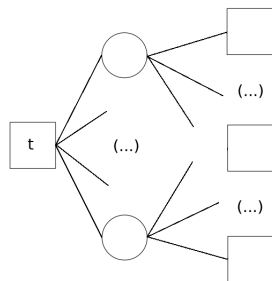
	<b>Our work</b>	<b>Faria et al.(2016)</b>	<b>Haywood et al.(2014)</b>
$P_b$ (days)	$0.85358_{-0.00009}^{+0.00030}$	$0.85424_{-0.00126}^{+0.0071}$	$0.085359165 \pm 5 \times 10^{-8}$
$K_b$ (m/s)	$3.58_{-0.21}^{+0.17}$	$3.97_{-0.55}^{+0.62}$	$3.42 \pm 0.66$
$e_b$	$0.046_{-0.005}^{+0.005}$	$0.045_{-0.027}^{+0.053}$	$0.12 \pm 0.07$
$P_c$ (days)	$3.6957_{-0.0233}^{+0.0024}$	$3.69686_{-0.00026}^{+0.00036}$	$3.70 \pm 0.02$
$K_c$ (m/s)	$5.58_{-1.02}^{+0.78}$	$5.55_{-0.31}^{+0.34}$	$6.01 \pm 0.47$
$e_c$	$0.020_{-0.005}^{+0.007}$	$0.026_{-0.017}^{+0.033}$	$0.12 \pm 0.06$

# CoRoT-7 - results



# Final thoughts

- A "true" GPRN will not be as simplistic as the example shown today. Different nodes should (in theory) be capable of dealing with other processes that might be affecting our data.



- How to interpret the GPRN parameters?
  - ▷ SE timescale  $\sim 1100$  days?  
It's not the average lifespan of the active regions.
  - ▷ If one of the datasets has no relation with the others should we see its weight  $\theta \rightarrow 0$  for a given node?

# Thank you!

This work is financed by FEDER - Fundo Europeu de Desenvolvimento Regional funds through the COMPETE 2020 - Programa Operacional Competitividade e Internacionalização (POCI), and by Portuguese funds through FCT - Fundação para a Ciência e a Tecnologia in the framework of the project POCI-01-0145-FEDER-028953 and POCI-01-0145-FEDER-032113.



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## Periodic kernel

Period  $\rightarrow \mathcal{U}(10, 40)$

Periodic scale  $\rightarrow \mathcal{U}(0, 2)$

## Squared Exponential kernel

Timescale  $\rightarrow \mathcal{U}(\sim 7, \sim 2378)$

RV " $\theta$ "  $\rightarrow \mathcal{U}(0, \sim 105.4)$

FWHM " $\theta$ "  $\rightarrow \mathcal{U}(0, \sim 8.5)$

BIS " $\theta$ "  $\rightarrow \mathcal{U}(0, \sim 10.4)$

$\log(R'_{hk})$  " $\theta$ "  $\rightarrow \mathcal{U}(0, \sim 8.8)$

**Jitters**  $\rightarrow \mathcal{U}(0, 2\sigma)$

## Keplerians

$P_b \rightarrow \mathcal{N}(0.85359165, 0.001)$

$K_b \rightarrow \mathcal{U}(3, 4)$

$e_b \rightarrow \mathcal{N}(0.04, 0.01)^5$

$\omega_b \rightarrow \mathcal{U}(0, 2\pi)$

$\phi_b \rightarrow \mathcal{U}(0, 2\pi)$

$P_c \rightarrow \mathcal{N}(3.70, 0.1)$

$K_c \rightarrow \mathcal{U}(5, 6)$

$e_c \rightarrow \mathcal{N}(0.03, 0.01)^5$

$\omega_c \rightarrow \mathcal{U}(0, 2\pi)$

$\phi_c \rightarrow \mathcal{U}(0, 2\pi)$

**Offsets**  $\rightarrow \mathcal{U}(min, max)$

<sup>5</sup>Truncated between 0 and 1