Parametric resonance after hilltop inflation caused by an inhomogeneous inflaton field

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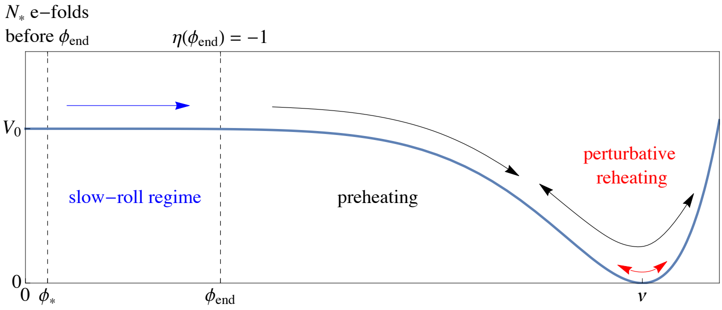
Based on
S. Antusch, F. Cefala, D. Nolde and S. Orani, arXiv:1510.04856 [hep-ph]
and references therein.

$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \qquad \text{with } v \ll m_{\rm Pl}$$

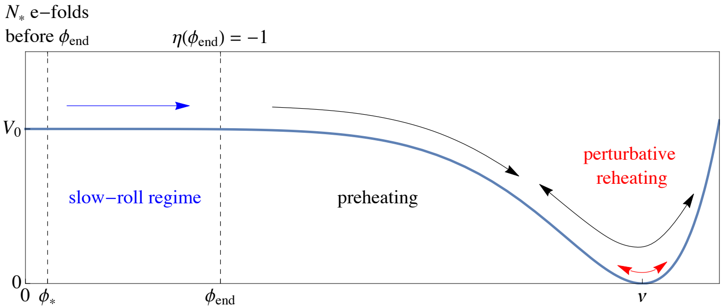


slow-roll inflation

- universe inflates as ϕ rolls slowly away from the hilltop towards v
- end of inflation: $\eta(\phi) = m_{\rm Pl}^2 V_{,\phi\phi}/V \simeq -1$
- model is compatible with recent Planck bounds on the primordial spectrum:

$$n_{\rm s} \simeq 0.96, \qquad r < 4 \times 10^{-6}$$

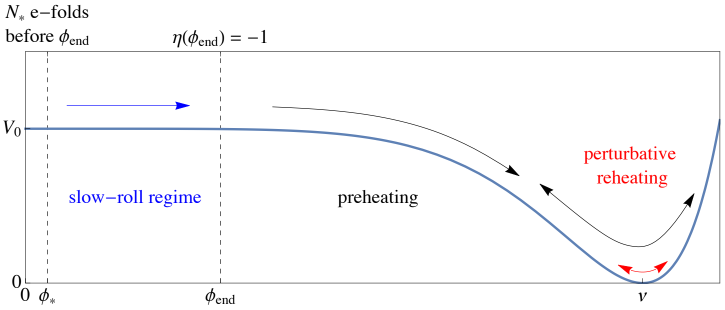
$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \qquad \text{with } v \ll m_{\rm Pl}$$



preheating

- starts when $|\eta| \gtrsim 1$
- non-perturbative
- may be non-linear (in terms of field fluctuations)
- dynamics may be very sensitive to model parameters
- sets initial conditions for perturbative reheating

$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \qquad \text{with } v \ll m_{\rm Pl}$$



perturbative reheating

- for sufficiently small amplitude → oscillations around $\phi = v$
  $\hat{=}$ collection of ϕ particles
- perturbative description of ϕ decays
- inflationary d.o.f vanish → beginning of radiation domination

$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 , \qquad \text{with } v \ll m_{\rm Pl}$$

preheating dynamics

$$\boxed{\begin{gathered} H(t)^2 = \rho_\phi/3 \\ \ddot{\phi}(t,\vec{x}) - \frac{\vec{\nabla}^2}{a^2}\phi(t,\vec{x}) + 3H(t)\,\dot{\phi}(t,\vec{x}) + \frac{\partial V}{\partial \phi} = 0 \end{gathered}}$$

with $\phi(t,\vec{x}) = \bar{\phi}(t) + \delta\phi(t,\vec{x})$

linearize and Fourier transform

$\downarrow$

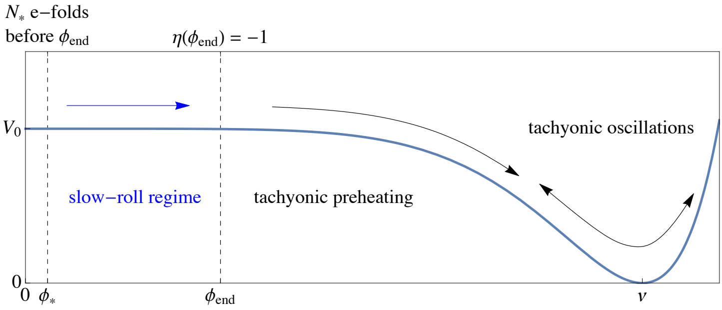
$$\boxed{\delta\ddot{\phi}_{\vec{k}}(t) + 3H(t)\,\delta\dot{\phi}_{\vec{k}}(t) + \left(\frac{\partial^2 V(\bar{\phi})}{\partial \phi^2} + \frac{\vec{k}^2}{a^2}\right)\delta\phi_{\vec{k}}(t) = 0}$$

BUT: only valid as long as

$$\langle \delta\phi(t,\vec{x})^2 \rangle = \int d\ln k \, \underbrace{\frac{k^3}{2\pi^3}|\delta\phi_{\vec{k}}(t)|^2}_{\equiv \mathcal{P}_\phi(k)} \ll \bar{\phi}^2(t)$$

If $\sqrt{\langle \delta\phi^2 \rangle} \sim \bar{\phi} \rightarrow$ need to solve the full EOM!

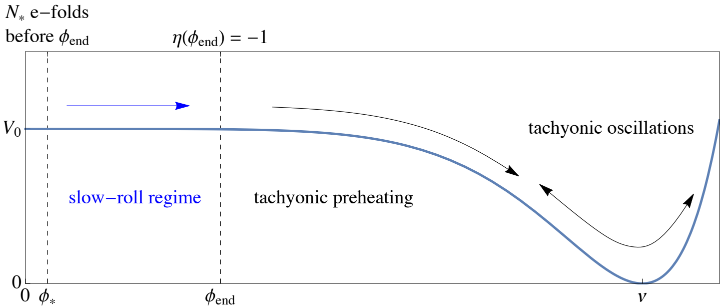
$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \qquad \text{with } v \ll m_{\rm Pl}$$



I Tachyonic preheating:

- exponential growth of all $\phi_{\vec{k}}$ for which $|\vec{k}|/a < \sqrt{-\partial^2 V/\partial\phi^2}$ due to tachyonic amplification
- growth is most efficient for very small v.
- For $v \lesssim 10^{-5} m_{\rm Pl} \to \langle \delta\phi^2 \rangle \gtrsim v^2 \to$ dynamics become non-linear.

$$V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \qquad \text{with } v \ll m_{\rm Pl}$$



II **Tachyonic oscillations**:

- periodic entering into the tachyonic region ($\partial^2 V/\partial\phi^2 < 0$)
  $\rightarrow$ interplay between **growth** of the $\phi_{\vec{k}}$ around $|\vec{k}_{\rm peak}|$ and **damping** due to Hubble friction
- For $v \gtrsim 10^{-1} m_{\rm Pl}$ $\rightarrow$ strong damping
- For $10^{-5}\, m_{\rm Pl} < v < 10^{-1}\, m_{\rm Pl}$ $\rightarrow$ fluctuations eventually grow non-linear $\rightarrow$ system eventually develops localized bubbles which oscillate between the two minima $\phi = \pm v$, typically separated by a distance $\lambda_{\rm peak} \sim 2\pi/k_{\rm peak}$

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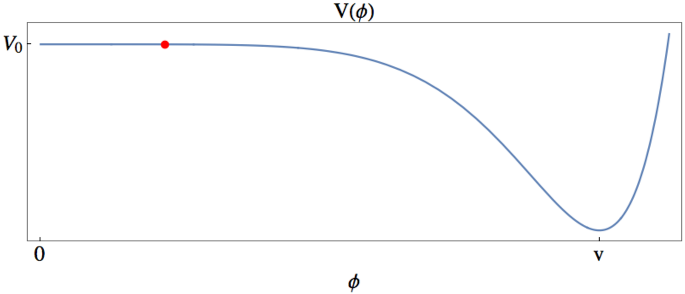

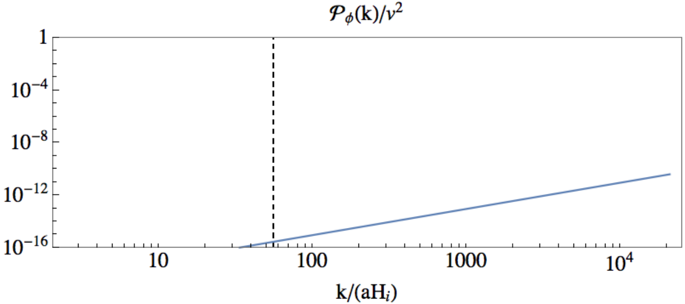

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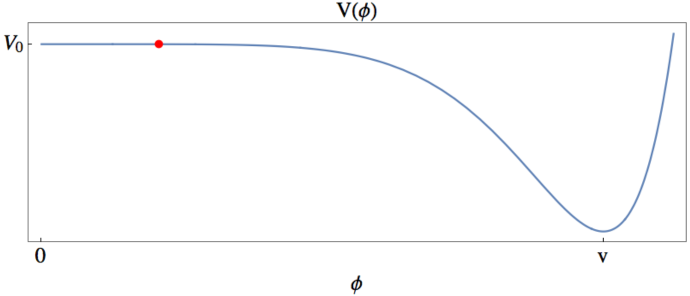
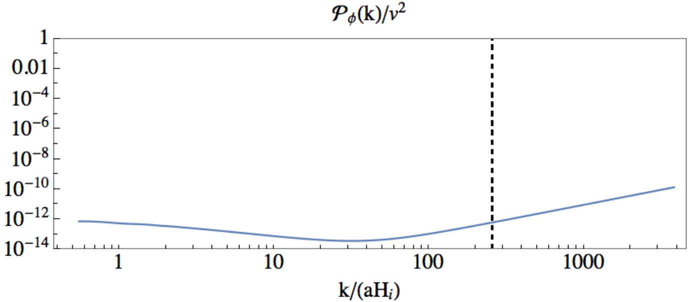
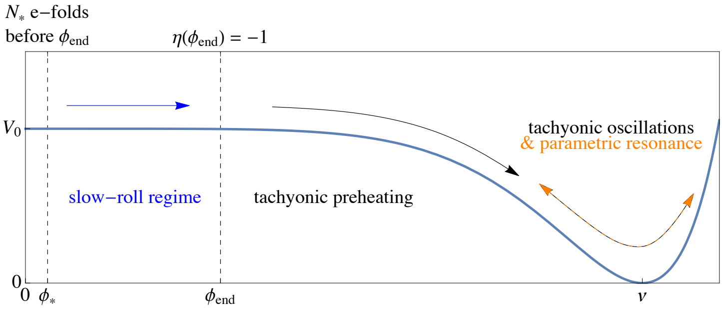




$$V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2), \qquad \text{with } v = 10^{-2}m_{\rm Pl}$$



III Parametric resonance of χ caused by inhomogeneous ϕ:

- possible amplification of χ fluctuations which is characterised by:
  - **exponential growth**
  - **high sensitivity to lambda λ, or equivalently to the mass-ratio**

$$\boxed{m_\chi/m_\phi \;=\; \lambda\, v^3/\sqrt{72\,V_0} \;\propto\; \lambda}$$

- the resonance takes place **after** ϕ has become inhomogeneous (in contrast to a standard parametric resonance)

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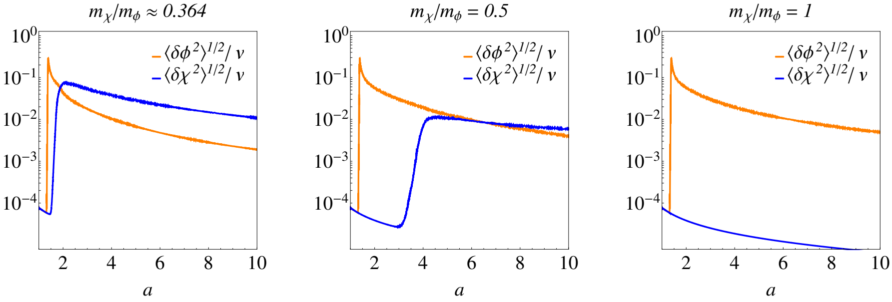

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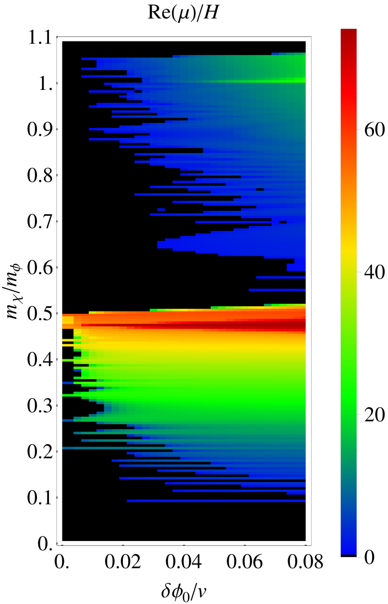
- parametric resonance caused by inhomogeneous ϕ

  → formally equivalent to multi-field case for parametric resonance

  → $\delta\chi_{\vec{k}}(t) \propto e^{\mu\, t}$
- approximation best for $\delta\phi_0 \lesssim 0.01v$ (left part of the plot)



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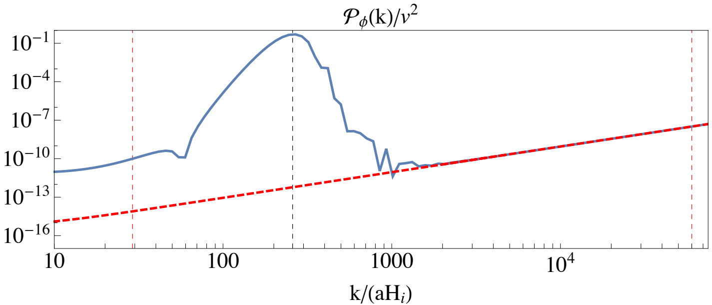
Using the program LATTICEEASY:

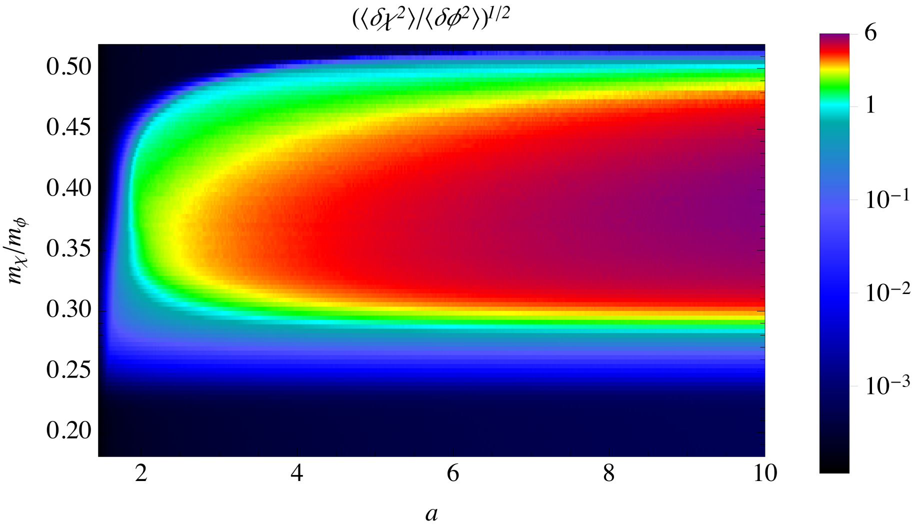
$$\ddot{\phi}(t,\vec{x}) - \frac{\vec{\nabla}^2}{a^2}\phi(t,\vec{x}) + 3H(t)\,\dot{\phi}(t,\vec{x}) + \frac{\partial V}{\partial \phi} = 0$$

$$\ddot{\chi}(t,\vec{x}) - \frac{\vec{\nabla}^2}{a^2}\chi(t,\vec{x}) + 3H(t)\,\dot{\chi}(t,\vec{x}) + \frac{\partial V}{\partial \chi} = 0$$

$$H(t)^2 = \frac{1}{3m_{\rm Pl}^2}\left\langle V + \frac{1}{2}\left(\dot{\phi}^2 + \dot{\chi}^2\right) + \frac{1}{2a^2}\left(\left|\vec{\nabla}\phi\right|^2 + \left|\vec{\nabla}\chi\right|^2\right)\right\rangle$$

Initialized at the end of inflation with $\langle\chi\rangle = \langle\dot{\chi}\rangle = 0$.





- variances → meassure for energy density of the respective field
- χ produced from initial vacuum fluctuations well after the system has become non-linear

We have studied preheating for the hilltop inflation model

$$V(\phi,\chi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2), \qquad \text{with } v = 10^{-2} m_{\rm Pl}$$

where we found that:

- χ can be amplified from initial vacuum fluctuations up to amplitudes of the order of ϕ and even larger!
- the amplification is characterised by an exponential growth and happens only for certain values of λ within a band
- the amplification happens after the inflaton has become completely inhomogeneous

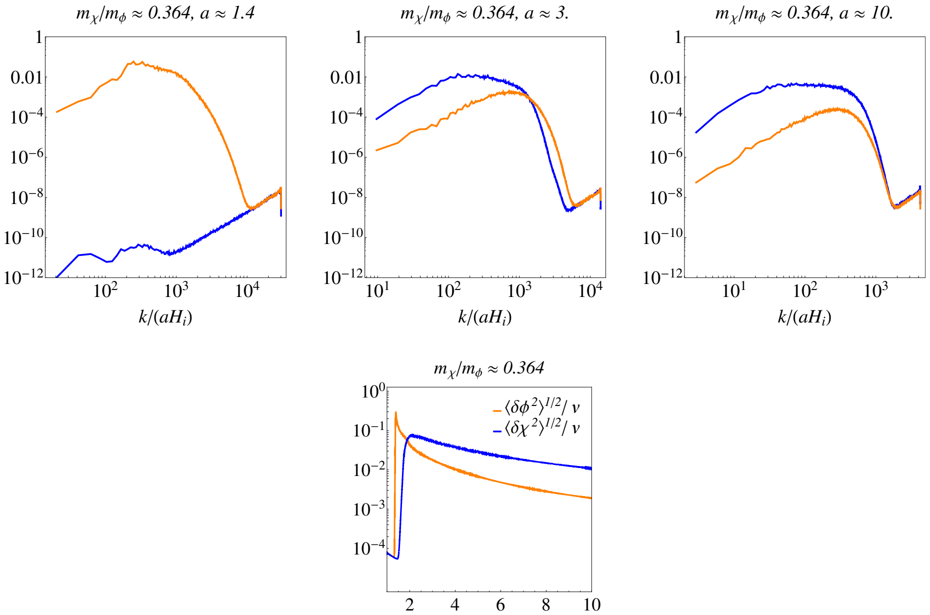
→ parametric resonance of χ caused by inhomogeneous ϕ

It is important to study preheating until well after non-linear dynamics become dominant:

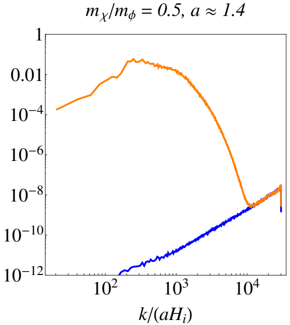
- resonant amplification of χ can occur even 1 e-folds after ϕ became fully inhomogeneous → may have an impact on the perturbative reheating
- dynamics of χ may have a significant influence on ϕ → oscillons,...

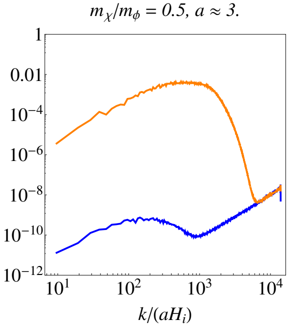
Backup

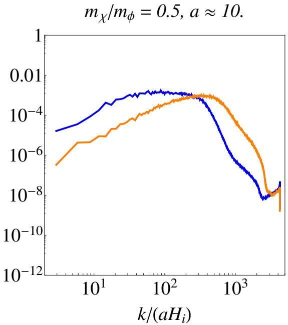
Spectra for $V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2)$, $v = 10^{-2}m_{\rm Pl}$, $\lambda = 1\times10^{-3}m_{\rm Pl}^{-1}$

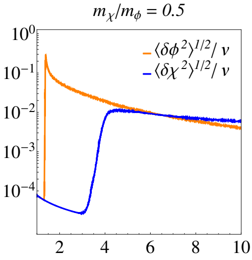


Spectra for $V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2)$, $v = 10^{-2} m_{\rm Pl}$, $\lambda \approx 1.375\times 10^{-3} m_{\rm Pl}^{-1}$

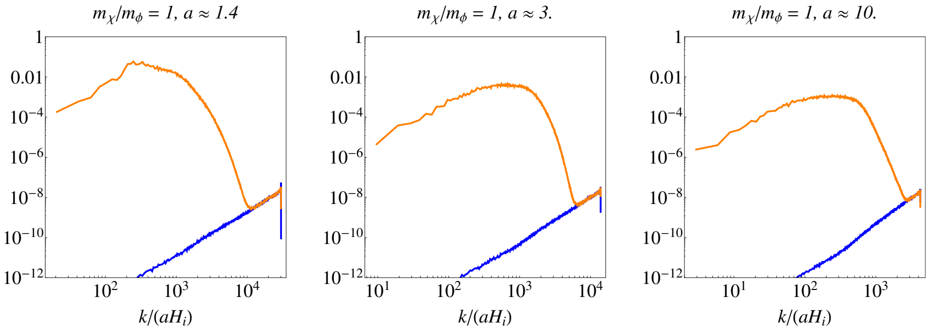


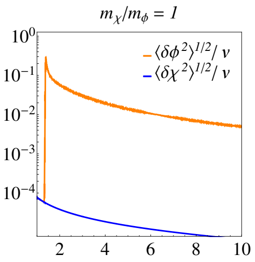






Spectra for $V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2)$, $v = 10^{-2} m_{\rm Pl}$, $\lambda \approx 2.750 \times 10^{-3} m_{\rm Pl}^{-1}$





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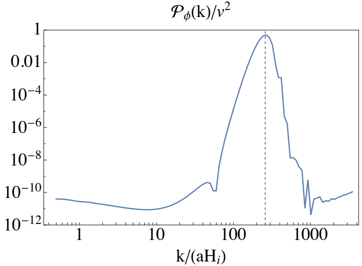
$$\boxed{\ddot{\chi}(t,\vec{x}) - \frac{\vec{\nabla}^2}{a^2}\chi(t,\vec{x}) + 3H(t)\,\dot{\chi}(t,\vec{x}) + \frac{\partial V}{\partial \chi} = 0}$$

$$\bar{\phi} \simeq v \text{ and } \bar{\chi} \simeq 0$$

Further assumptions:

1. $\delta\phi \ll v$
2. $\delta\chi \ll \delta\phi$
3. $\delta\phi(t,\vec{x}) = \delta\phi_0\,\cos\left(\vec{k}_{\rm p}\vec{x}\right)\cos(\omega_\phi t)$,
   with $\omega_\phi = \sqrt{\vec{k}_p^2 + m_\phi^2}$
4. $H(t) \simeq 0$



+ expand EOM for χ linear in $\delta\phi$ and $\delta\chi$ & Fourier transform $\rightarrow$

$$\boxed{\delta\ddot{\chi}_{\vec{k}} + \left(k^2+\lambda^2 v^4\right)\delta\chi_{\vec{k}} + 2\lambda^2 v^3 \delta\phi_0 \cos(\omega_\phi t)\left(\delta\chi_{\vec{k}+\vec{k}_{\rm p}} + \delta\chi_{\vec{k}-\vec{k}_{\rm p}}\right) = 0}$$

$$V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2), \qquad \text{with } v = 10^{-2}m_{\rm Pl}$$

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$\widehat{=}$ set of equations which couples each mode with momentum $\vec{k}$ to all other modes with momentum $\vec{k'} = \vec{k}\pm\vec{k}_p, \vec{k}\pm 2\vec{k}_p, ..., \vec{k}\pm N\vec{k}_p$

defining $\vec{k}_n := \vec{k}_0 + n\cdot\vec{k}_{\rm p}$, and $f(t) := 2\lambda^2v^3\delta\phi_0\cos(\omega_\phi t)$:

$$\begin{pmatrix} \delta\ddot{\chi}_{\vec{k}_N} \\ \delta\ddot{\chi}_{\vec{k}_{N-1}} \\ \ldots \\ \delta\ddot{\chi}_{\vec{k}_{-(N-1)}} \\ \delta\ddot{\chi}_{\vec{k}_{-N}} \end{pmatrix} = \underbrace{\begin{pmatrix} k_N^2 & f(t) & 0 & 0 & \ldots & 0 \\ f(t) & k_{N-1}^2 & f(t) & 0 & \ldots & 0 \\ 0 & f(t) & k_{N-2}^2 & f(t) & \ldots & 0 \\ 0 & 0 & f(t) & k_{N-3}^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & f(t) \\ 0 & 0 & 0 & 0 & f(t) & k_{-N}^2 \end{pmatrix}}_{\mathcal{F}(t)} \begin{pmatrix} \delta\chi_{\vec{k}_N} \\ \delta\chi_{\vec{k}_{N-1}} \\ \ldots \\ \delta\chi_{\vec{k}_{-(N-1)}} \\ \delta\chi_{\vec{k}_{-N}} \end{pmatrix}$$

Introduce $\pi_{\vec{k}_n} := \dot{\chi}_{\vec{k}_n}$ and $y(t) := (\delta\chi_{\vec{k}_N}, \delta\chi_{\vec{k}_{N-1}}, ..., \delta\chi_{\vec{k}_{-N}}, \pi_{\vec{k}_N}, \pi_{\vec{k}_{N-1}}, ..., \pi_{\vec{k}_{-N}})^T$,

$$\dot{y}(t) = \begin{pmatrix} \mathbb{0} & \mathbb{1} \\ \mathcal{F}(t) & \mathbb{0} \end{pmatrix} y(t) \equiv U(t)y(t)$$

$$V(\phi,\chi) = V_0\left(1-\frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2), \qquad \text{with } v = 10^{-2}m_{\rm Pl}$$

$$\boxed{\delta\ddot{\chi}_{\vec{k}} + \left(k^2+\lambda^2 v^4\right)\delta\chi_{\vec{k}} + 2\lambda^2 v^3 \delta\phi_0 \cos(\omega_\phi t)\left(\delta\chi_{\vec{k}+\vec{k}_{\rm p}} + \delta\chi_{\vec{k}-\vec{k}_{\rm p}}\right) = 0}$$

$\widehat{=}$ set of equations which couples each mode with momentum $\vec{k}$ to all other modes with momentum $\vec{k'} = \vec{k}\pm\vec{k}_p, \vec{k}\pm 2\vec{k}_p, ..., \vec{k}\pm N\vec{k}_p$

$$\dot{y}(t) = \begin{pmatrix} \mathbb{0} & \mathbb{1} \\ \mathcal{F}(t) & \mathbb{0} \end{pmatrix} y(t) \equiv U(t)y(t) \qquad (1)$$

Comments:

- Eq. (1) is formally equivalent to the multi-field case: $\delta\chi_{\vec{k}\pm n\vec{k}_{\rm p}} \leftrightarrow \chi^{(n)}$
- **Floquet theorem:** growing solutions of eq. (1) can be found using

$$\dot{\mathcal{O}}(t) = U(t)\mathcal{O}(t). \qquad (2)$$

The Floquet exponents μ can be determined using the following algorithm:

- Solve eq. (2) with the initial value $\mathcal{O}(0) = \mathbb{1}$ up to time $T = 2\pi/\omega_\phi$.
- Find the eigenvalues σ of $\mathcal{O}(T)$.
- The Floquet exponents are $\mu = \frac{1}{T}\log\sigma$.
- Growth is effective if $\mathrm{Re}(\mu) \gg H$.

Considering the superpotential

$$W = \sqrt{V_0}\, S \left(1 - \frac{8\,\Phi^6}{v^6}\right) + \lambda \Phi^2 X^2 \,,$$

with Φ, X and S being chiral superfields, the scalar potential is given by:

$$\begin{aligned} V(\phi,\chi) &= \left|\frac{\partial W}{\partial S}\right|^2_{\theta=0} + \left|\frac{\partial W}{\partial \Phi}\right|^2_{\theta=0} + \left|\frac{\partial W}{\partial X}\right|^2_{\theta=0} + V_{\mathrm{SUGRA}} \\ &= V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2 + \frac{\lambda^2}{2}\phi^2\chi^2(\phi^2+\chi^2) + V_{\mathrm{SUGRA}} + \dots \,, \end{aligned}$$

where $\phi = \sqrt{2}\,\mathrm{Re}[\Phi]$, $\chi = \sqrt{2}\,\mathrm{Re}[X]$.

Comments:

- We assume $\mathrm{Im}[\Phi] = 0$ (indeed justified for most of the parameter space)
- Additional Kähler corrections (V_{SUGRA}) are assumed to be negligible
- The form of the superpotential can results if the fields are charged under a $U(1)_{\mathrm{R}} \times \mathbb{Z}_6$:

	$U(1)_{\mathrm{R}}$	$\mathbb{Z}_6$
S	2	0
X	1	2
Φ	0	1