Born-Infeldizing gravity

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In collaboration with:
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outline

- Born-Infeld & Born-Infeld inspired gravity.
- Generalised Born-Infeld inspired gravity. Minimal extension.
- Perfect fluid and cosmological solutions.
- Dust inflation.

Born-Infeld electromagnetism

$$\mathcal{S}_{\rm Maxwell} = -\frac{1}{4}\int d^4x F_{\mu\nu}F^{\mu\nu}$$

Principle of finiteness

$$\frac{1}{2}m^2\int \mathrm{d}t\, v^2 \to mc^2\int \mathrm{d}t\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)$$

M. Born and L. Infeld. Proc.Roy.Soc.Lond. A144 .(1934)

$$\mathcal{S}_{\rm BIE} = -\lambda^4\int \mathrm{d}^4x\left[\sqrt{-\det\left(\eta_{\mu\nu}+\lambda^{-2}F_{\mu\nu}\right)}-1\right]$$

For small electromagnetic fields it recovers Maxwell's theory:

$$\mathcal{S}_{\rm BIE}(F_{\mu\nu}\ll\lambda^2)\simeq\mathcal{S}_{\rm Maxwell}$$

For large electromagnetic fields it differs so that it regularizes the self-energy of point-like charged particles.

Born-Infeld inspired gravity

$$\mathcal{S}_{\rm DG} = \int {\rm d}^4x \sqrt{-\det\left(a g_{\mu\nu} + b R_{\mu\nu} + c X_{\mu\nu}\right)}$$

S. Deser and G. Gibbons, CQG15 (1998)

- $X_{\mu\nu} =$ Higher order curvature terms to be tuned to avoid ghosts
- $X_{\mu\nu}$ contains terms of quadratic and higher orders in $R_{\mu\nu}$
- There is a large freedom in the choice of $X_{\mu\nu}$ and no clear immediate criterion.

Born-Infeld inspired gravity

$$\mathcal{S}_{\text{Ed}} = \lambda^4 \int \mathrm{d}^4x \sqrt{\det R_{(\mu\nu)}(\Gamma)}$$

Determinantal actions for gravity were considered by Eddington (1924) as a purely affine theory.

Couplings to matter: The metric enters as an auxiliary field that can then be integrated out.

M. Ferraris and J. Kijowski, Letters in Mathematical Physics, 5 127-135, (1981)

$$\mathcal{S}_{\text{BIP}} = \lambda^4 \int \mathrm{d}^4x \left[\sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}(\Gamma)\right)} - \sqrt{-\det(g_{\mu\nu})}\right]$$

D. N. Vollick, PRD 69 (2004) 064030.

In the Palatini formulation the ghost can be avoided without further corrections

Extended Born-Infeld gravity

We can rewrite the action as

$$\mathcal{S} = \lambda^4 \int \mathrm{d}^4 x \sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}\right)} = \lambda^4 \int \mathrm{d}^4 x \sqrt{-g}\,\det\sqrt{\delta^\mu{}_\nu + \lambda^{-2} g^{\mu\alpha} R_{\alpha\nu}}$$

$$\mathcal{S} = \lambda^4 \int \mathrm{d}^4 x \sqrt{-g}\,\det\sqrt{\hat{g}^{-1}\hat{q}}$$

$$q_{\alpha\nu} \equiv g_{\alpha\nu} + \lambda^{-2} R_{\alpha\nu}(\Gamma)$$

This reminds of the massive gravity potential:

$$\mathcal{S}_{MG} = \int \mathrm{d}^4 x \sqrt{-g} \sum_{n=0}^{4} \frac{\beta_n}{n!(4-n)!} e_n\left(\sqrt{g^{-1} f}\right)$$

C. de Rham, G. Gabadaze, A.J.Tolley, PRL106 (2011)

S.F. Hassan, R.A. Rosen JHEP1107 (2011)

elementary symmetric polynomials

Extended Born-Infeld gravity

...and so, a natural generalization of BI inspired gravity is

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JCAP 11 (2014) 004

$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

$$\begin{aligned}
e_0(\hat{M}) &= 1, \\
e_1(\hat{M}) &= [\hat{M}], \\
e_2(\hat{M}) &= \frac{1}{2!}\left([\hat{M}]^2 - [\hat{M}^2]\right), \\
e_3(\hat{M}) &= \frac{1}{3!}\left([\hat{M}]^3 - 3[\hat{M}][\hat{M}^2] + 2[\hat{M}^3]\right), \\
e_4(\hat{M}) &= \frac{1}{4!}\left([\hat{M}]^4 - 6[\hat{M}]^2[\hat{M}^2] + 8[\hat{M}][\hat{M}^3] + 3[\hat{M}^2]^2 - 6[\hat{M}^4]\right).
\end{aligned}$$

with matter minimally coupled.

Low curvature limit

$$\mathcal{S} \simeq \int \mathrm{d}^4x \sqrt{-g}\left[\tilde{\lambda}^4\left(\beta_0 + 4\beta_1 + 6\beta_2 + 4\beta_3 + \beta_4\right) + \frac{\tilde{\lambda}^4}{2\lambda^2}\left(\beta_1 + 3\beta_2 + 3\beta_3 + \beta_4\right) g^{\mu\nu} R_{\mu\nu}(\Gamma)\right]$$

Cosmological constant

Newton's constant

Minimal Born-Infeld extension

$$\mathcal{S}_{\text{min}} = \lambda^2 M_{\text{Pl}}^2 \int \mathrm{d}^4x \sqrt{-g}\,\mathrm{Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Metric field equations

$$(M^{-1})^{\alpha}{}_{(\mu}R_{\nu)\alpha} - \mathrm{Tr}(\hat{M} - \mathbb{1})\lambda^2 g_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}$$

$$\hat{R} = \lambda^2\hat{g}(\hat{M}^2 - \mathbb{1})$$

$$\frac{1}{2}\left[\hat{g}(\hat{M} - \hat{M}^{-1}) + (\hat{M} - \hat{M}^{-1})^T\hat{g}\right] - \mathrm{Tr}(\hat{M} - \mathbb{1})\hat{g} = \frac{1}{\lambda^2 M_{\text{Pl}}^2}\hat{T}$$

This equation allows to express $M^{\alpha}{}_{\beta}$ as a function of the matter content and the metric tensor.

Minimal Born-Infeld extension

$$\mathcal{S}_{\rm min} = \lambda^2 M_{\rm Pl}^2 \int {\rm d}^4 x \sqrt{-g}\,{\rm Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Connection field equations

$$\nabla_\lambda\left(\sqrt{-g}W^{\beta\nu}\right) - \delta^\nu_\lambda \nabla_\rho\left(\sqrt{-g}W^{\beta\rho}\right) + 2\sqrt{-g}\left(\mathcal{T}^\kappa_{\lambda\kappa}W^{\beta\nu} - \delta^\nu_\lambda \mathcal{T}^\kappa_{\rho\kappa}W^{\beta\rho} + \mathcal{T}^\nu_{\lambda\rho}W^{\beta\rho}\right) = 0$$

$$\hat{W} = \hat{M}^{-1}$$

We will consider solutions without torsion $\mathcal{T}^\alpha{}_{\mu\nu} = 0$

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho(\nu}W^{\beta)}{}_\rho\right) = 0 \Longrightarrow \Gamma = \Gamma(\tilde{g})$$

$$\tilde{g}^{\mu\nu} = \sqrt{\det\hat{M}}\,g^{\alpha\mu}(\hat{M}^{-1})^\nu{}_\alpha$$

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho[\nu}W^{\beta]}{}_\rho\right) = 0 \Longrightarrow$$

We set torsion to zero a posteriori. This is a consistency equation for this Ansatz.

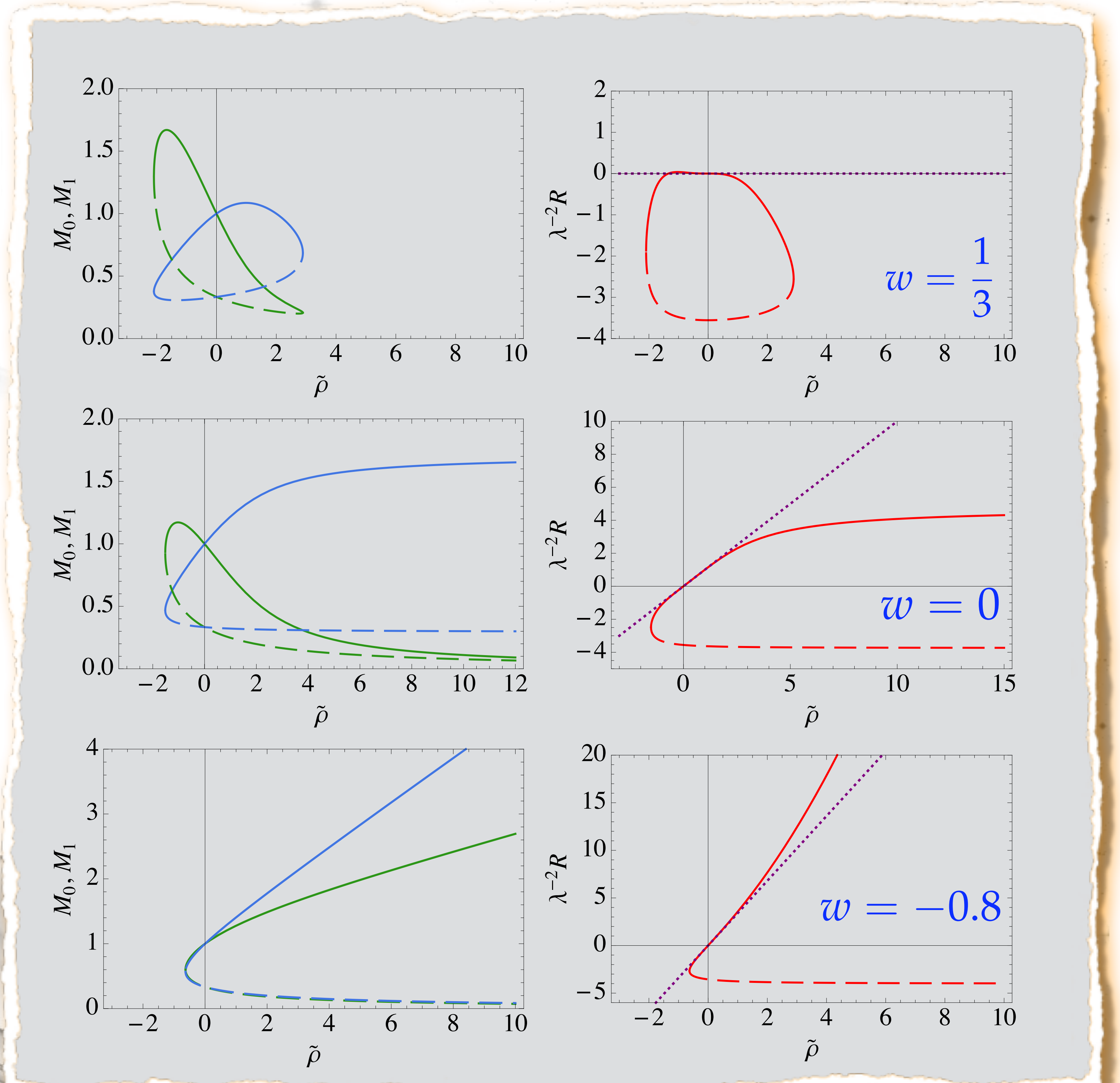
Perfect fluid solutions

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & \mathbf{0} \\ \mathbf{0} & p\mathbb{1}_{3\times 3} \end{pmatrix}$$

$$M^{\mu}{}_{\nu} = \begin{pmatrix} M_0 & \mathbf{0} \\ \mathbf{0} & M_1\mathbb{1}_{3\times 3} \end{pmatrix}$$

$$\begin{aligned} \frac{1}{M_0} + 3M_1 &= 4 + \tilde{\rho} \\ M_0 + 2M_1 + \frac{1}{M_1} &= 4 - \tilde{\rho} \end{aligned}$$

Metric field equations



In general we find 3 branches of solutions, but only two of them are physical. Out of those two, only one is continuously connected with GR.

Perfect fluid solutions

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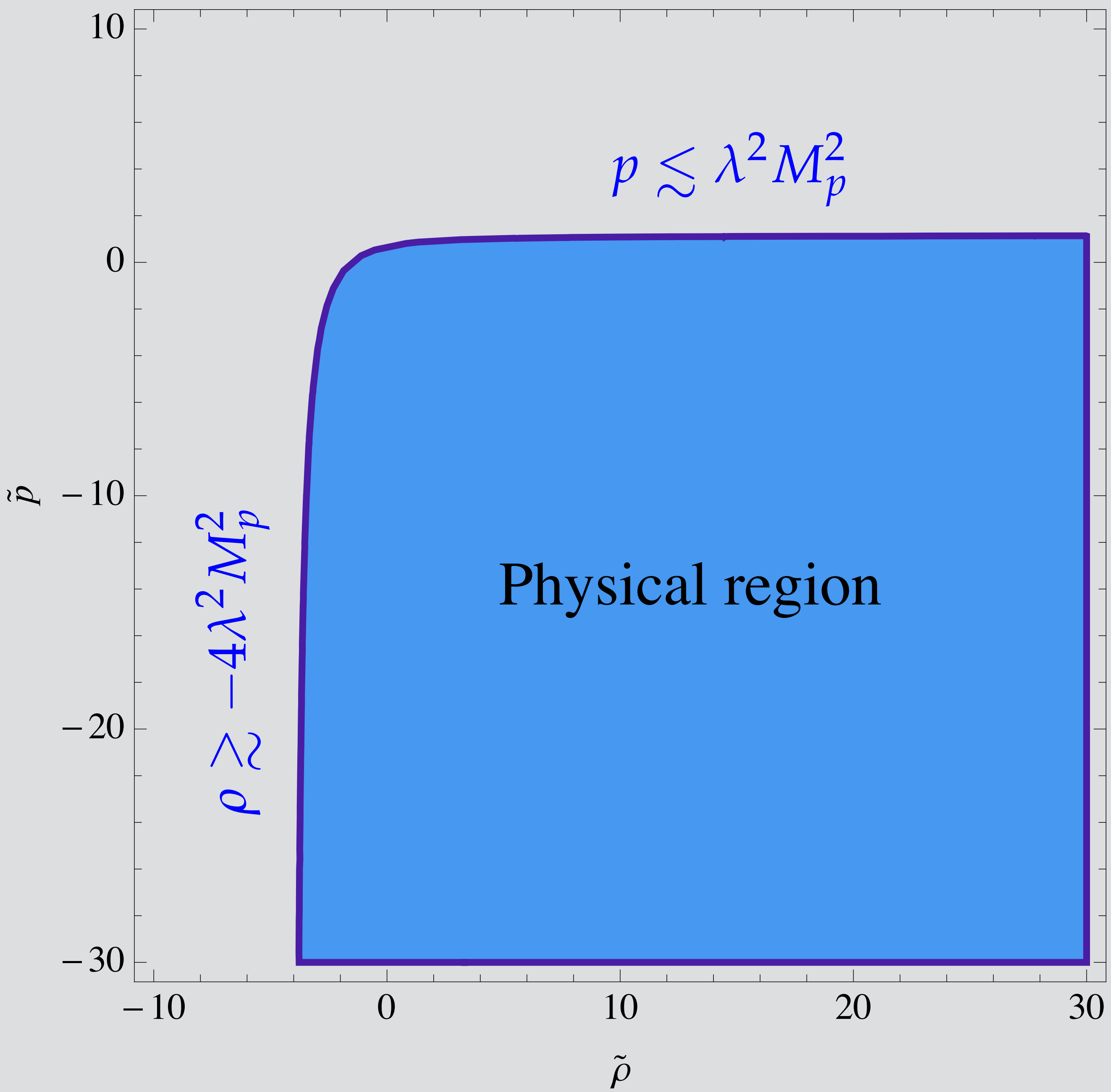
Metric field equations



In general we find 3 branches of solutions, but only two of them are physical. Out of those two, only one is continuously connected with GR.

Cosmological solutions

$$\bar{H}^2 = \frac{1}{3}\frac{M_0^2 + \frac{3}{2}(\bar{P}+\bar{\rho})M_0 - 1}{\left\{1 - \frac{M_0}{4[(4+\bar{\rho})M_0-1]}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}N}\left[1+(4+\bar{\rho})\left(\frac{\partial \ln M_0}{\partial\bar{\rho}} + c_s^2\frac{\partial \ln M_0}{\partial \bar{P}}\right)\right]\right\}^2}$$

$$\mathrm{d}s^2 = -N(t)^2\mathrm{d}t^2 + a(t)^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

$$\mathrm{d}\tilde{s}^2 = -N^2(M_0M_1^{-3})^{1/2}\mathrm{d}t^2 + \frac{a(t)^2}{\sqrt{M_0M_1}}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

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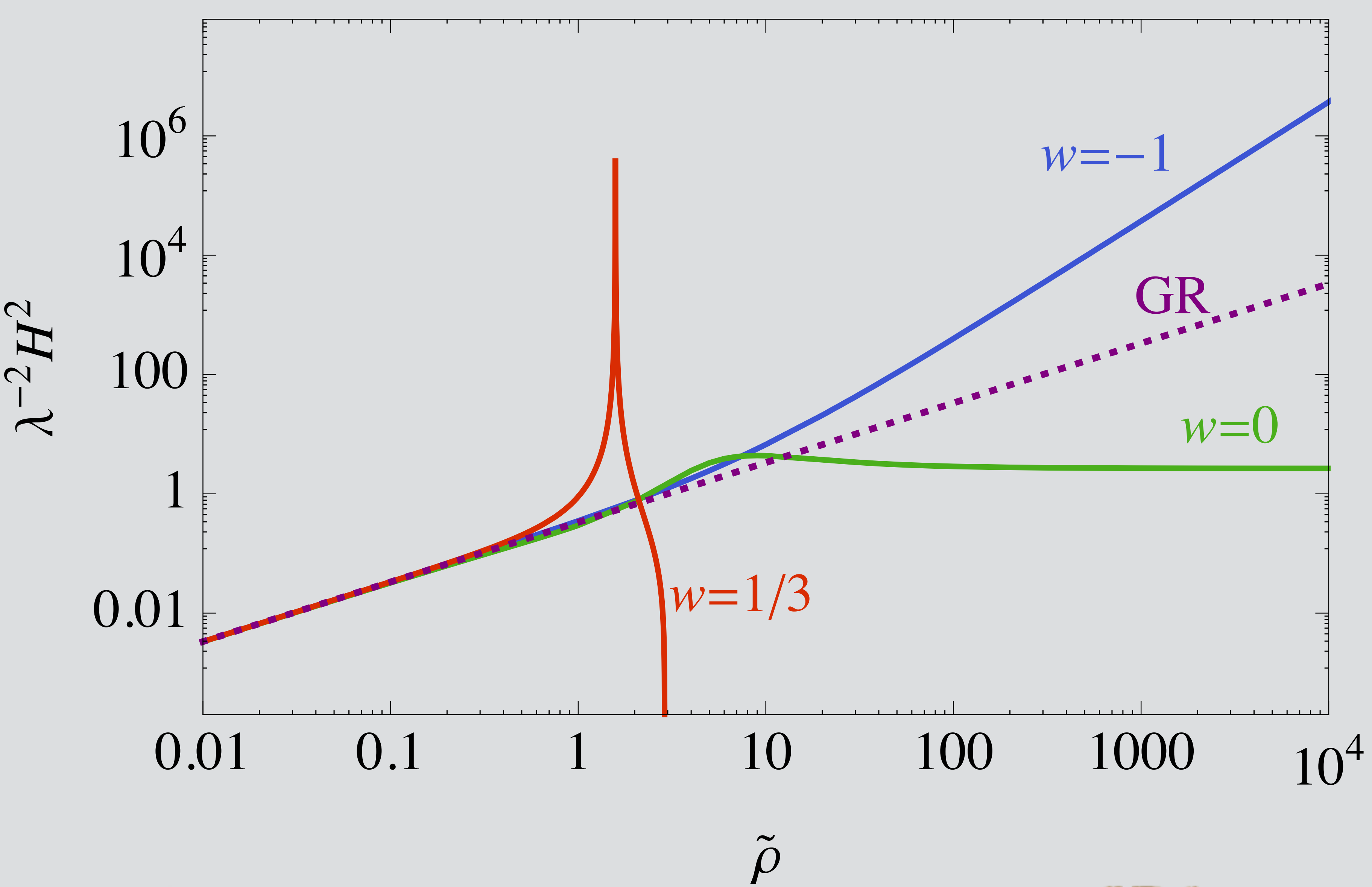
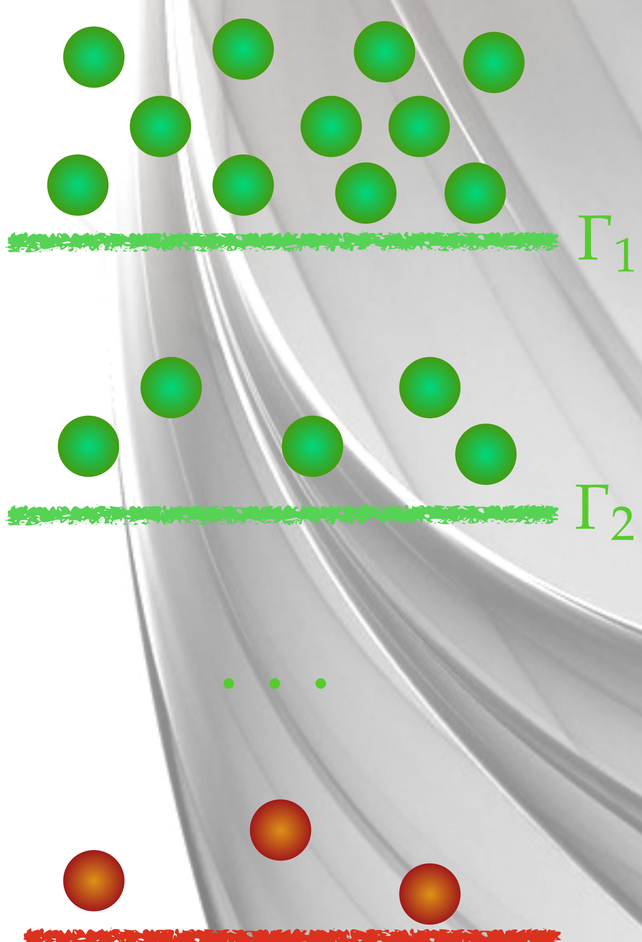
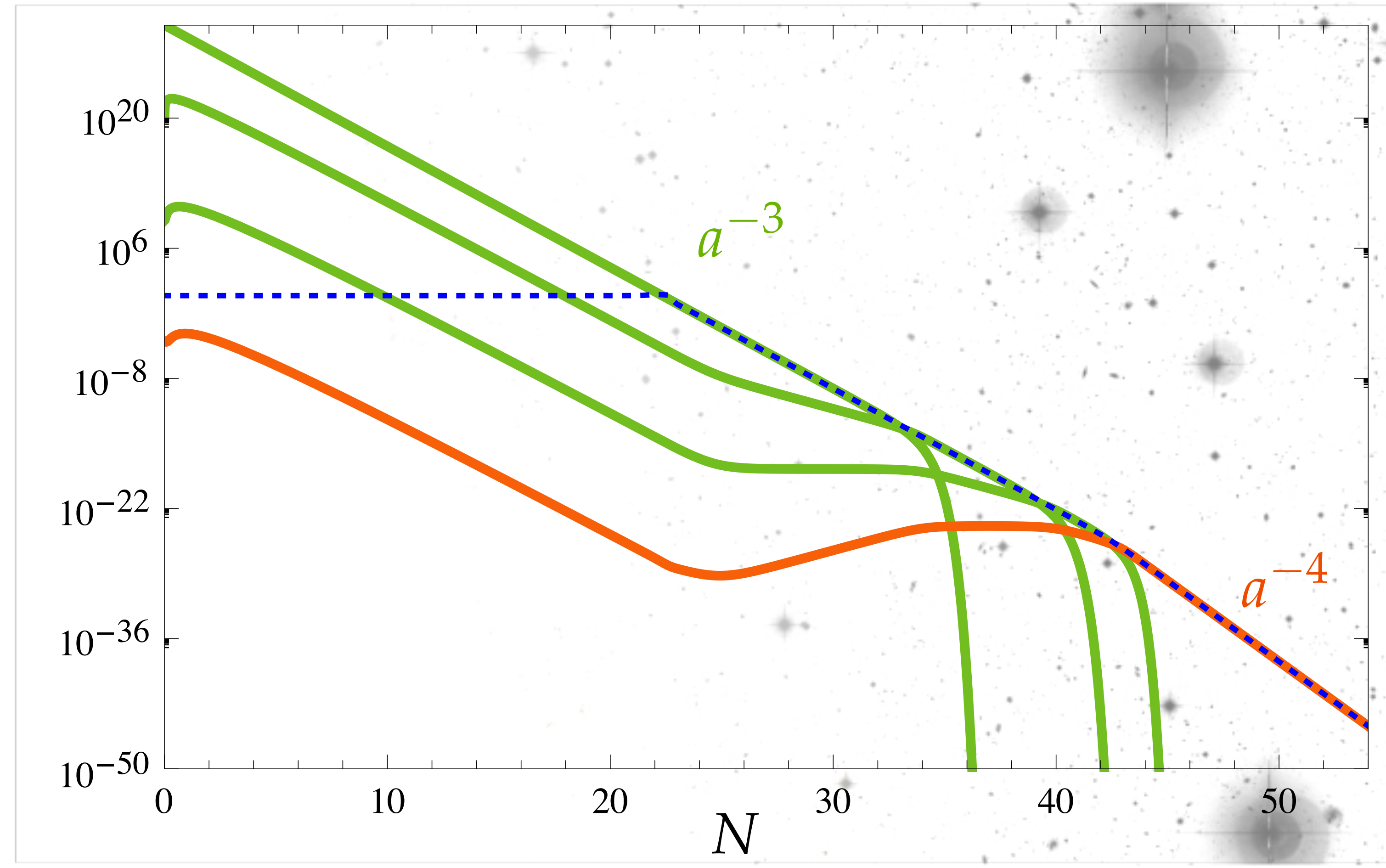


Dust inflation

$$\dot{\rho}_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot{\rho}_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
$$\cdots$$
$$\dot{\rho}_r + 4H\rho_r = \Gamma_n\rho_n$$





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Dust inflation

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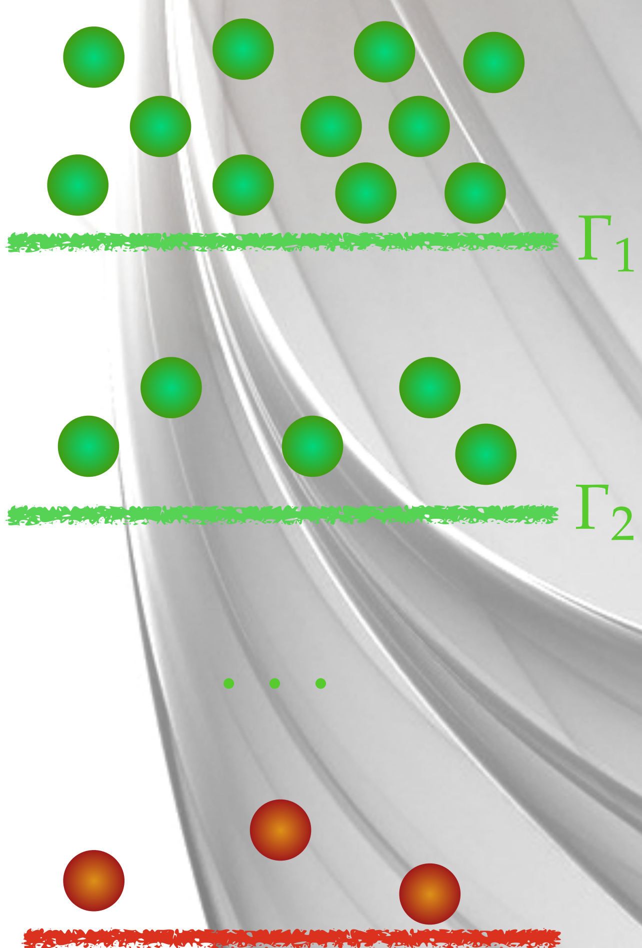

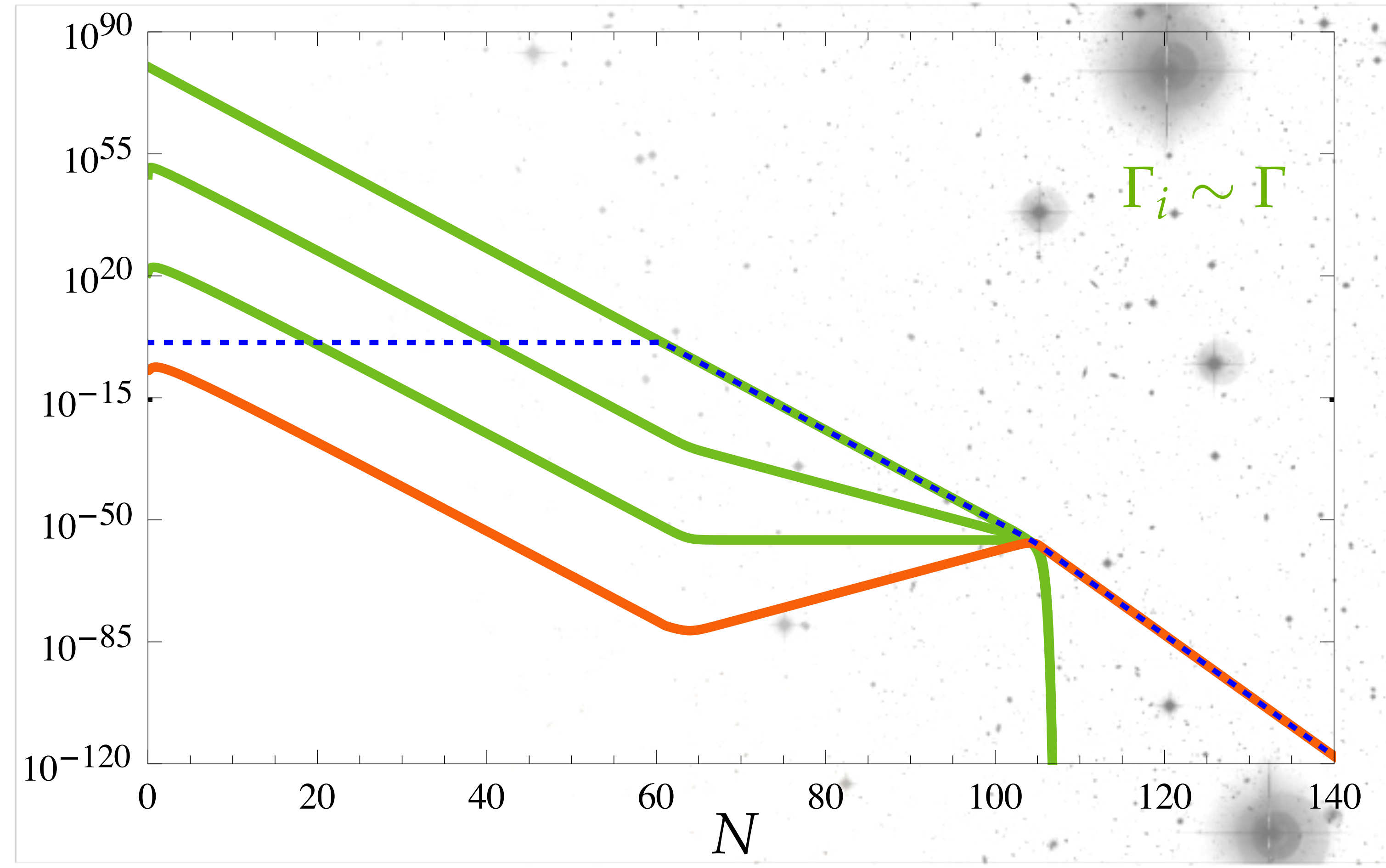

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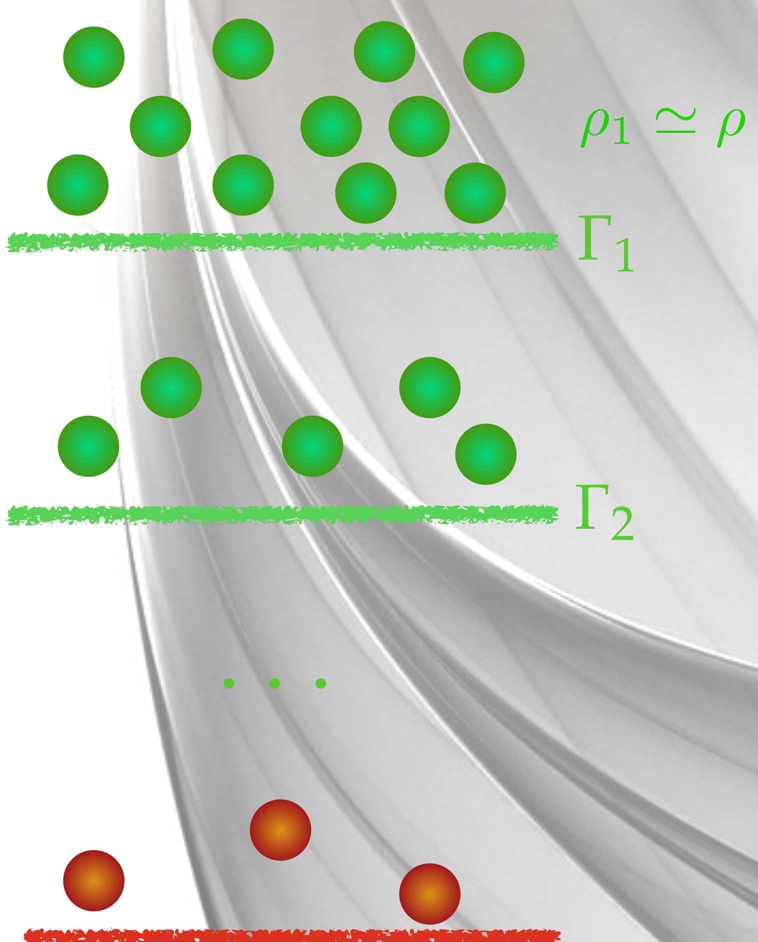

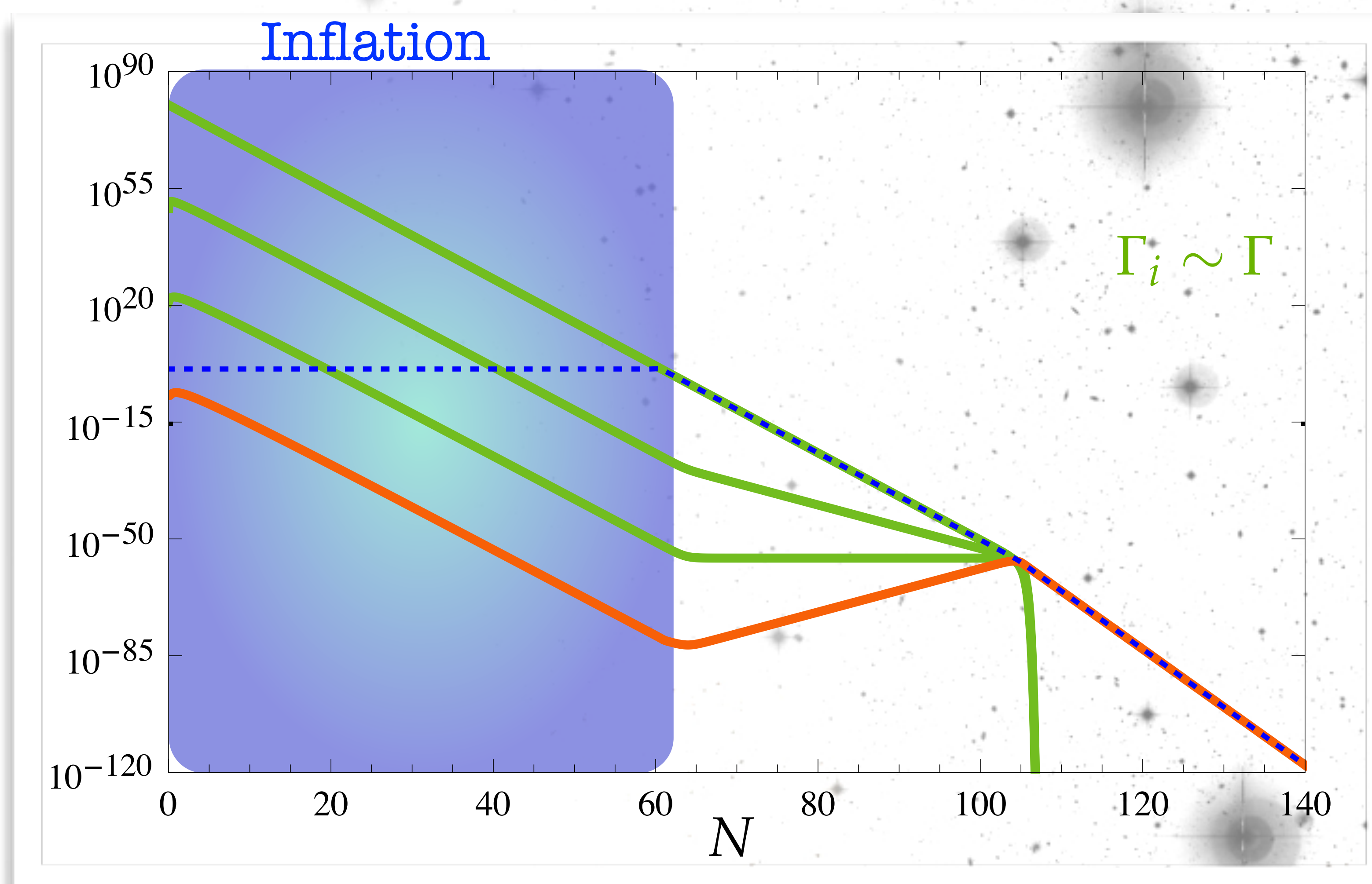

quasi de Sitter phase

$$\bar{H} = \sqrt{\frac{8}{3}} + \frac{8\sqrt{3} - 3\sqrt{6}}{\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}\right) \qquad \bar{H} = \frac{H}{\lambda} \qquad \bar{\rho} = \frac{\rho}{\lambda^2 M_{\rm Pl}^2}$$

$$\epsilon_1 \equiv -\frac{d\log H}{dN} = -\frac{9(4\sqrt{2}-3)}{2\bar{\rho}} + \mathcal{O}\left(\frac{1}{\bar{\rho}^2}\right)$$

Blue spectrum

Dust inflation

$$\dot\rho_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot\rho_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
$$\dots$$
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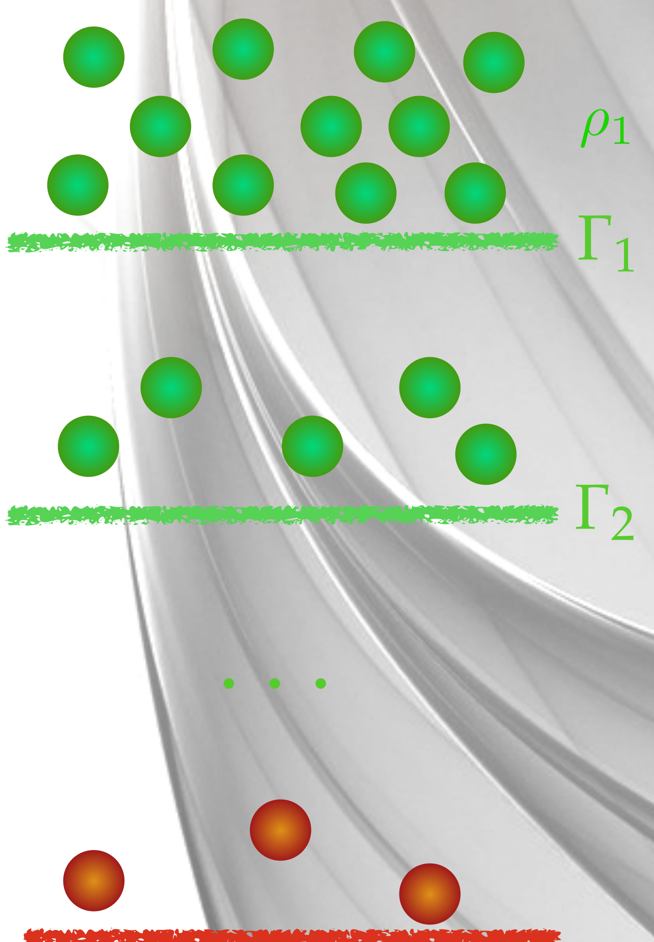

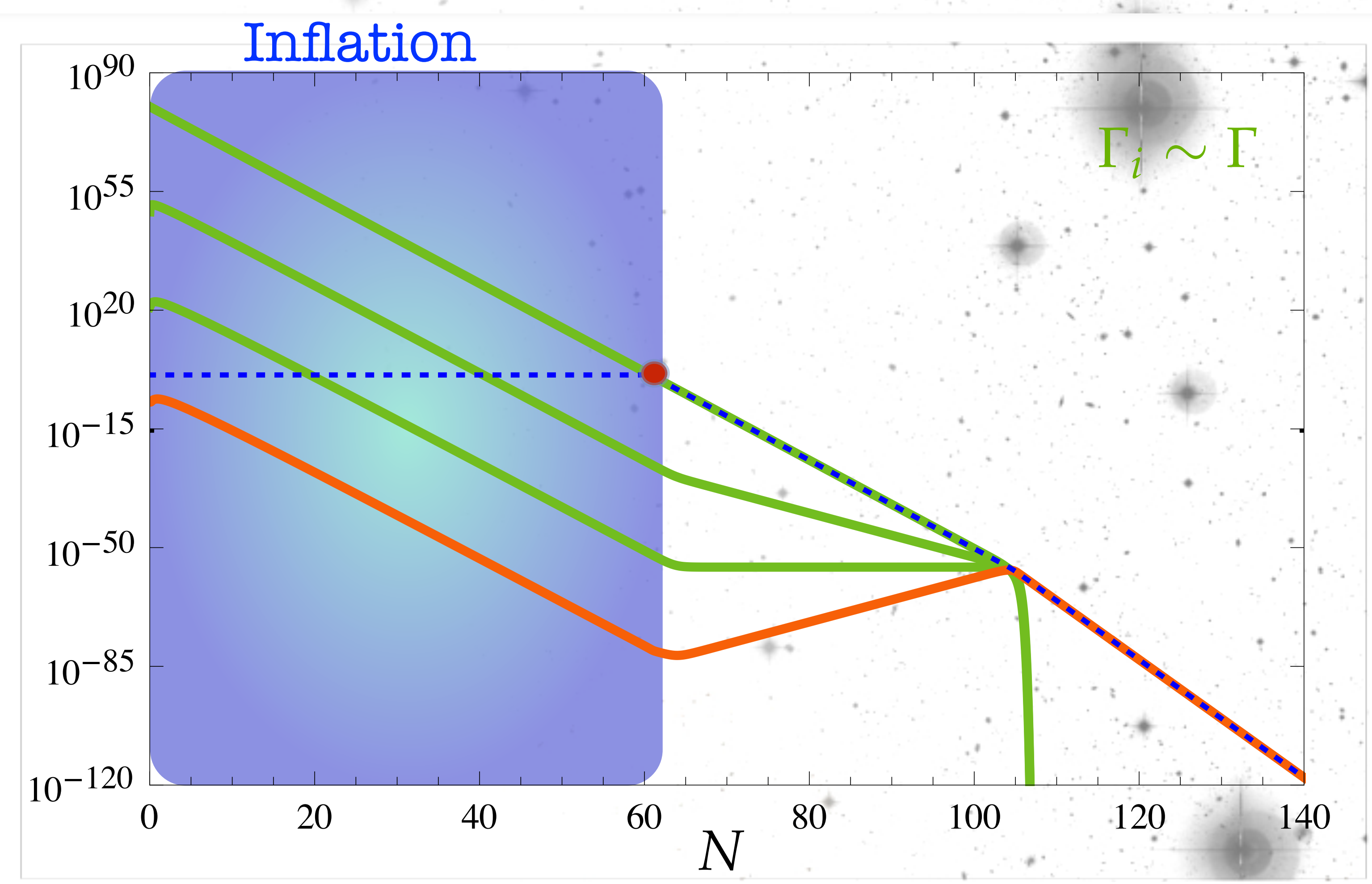

Duration of inflation

- Inflation ends when we exit the Born-Infeld regime
- The duration of inflation depends on all Γ_i and λ
- During inflation $\Gamma_i \ll H$
- There is an upper bound for the duration of inflation due $p_r \lesssim \lambda^2 M_{\rm Pl}^2$

Dust inflation

$$\dot{\rho}_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot{\rho}_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
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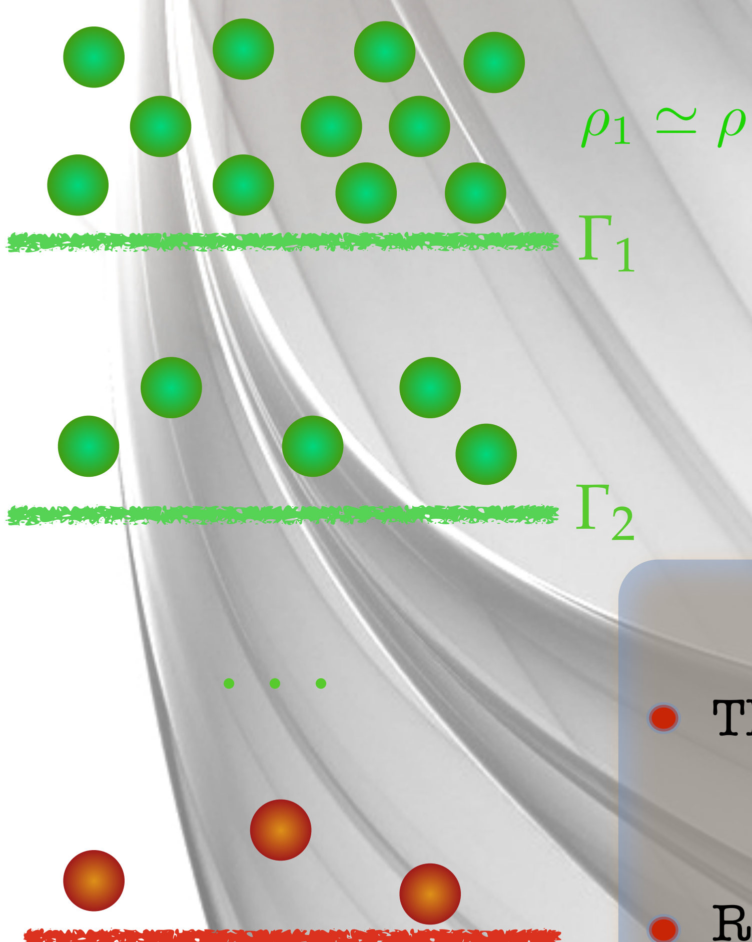

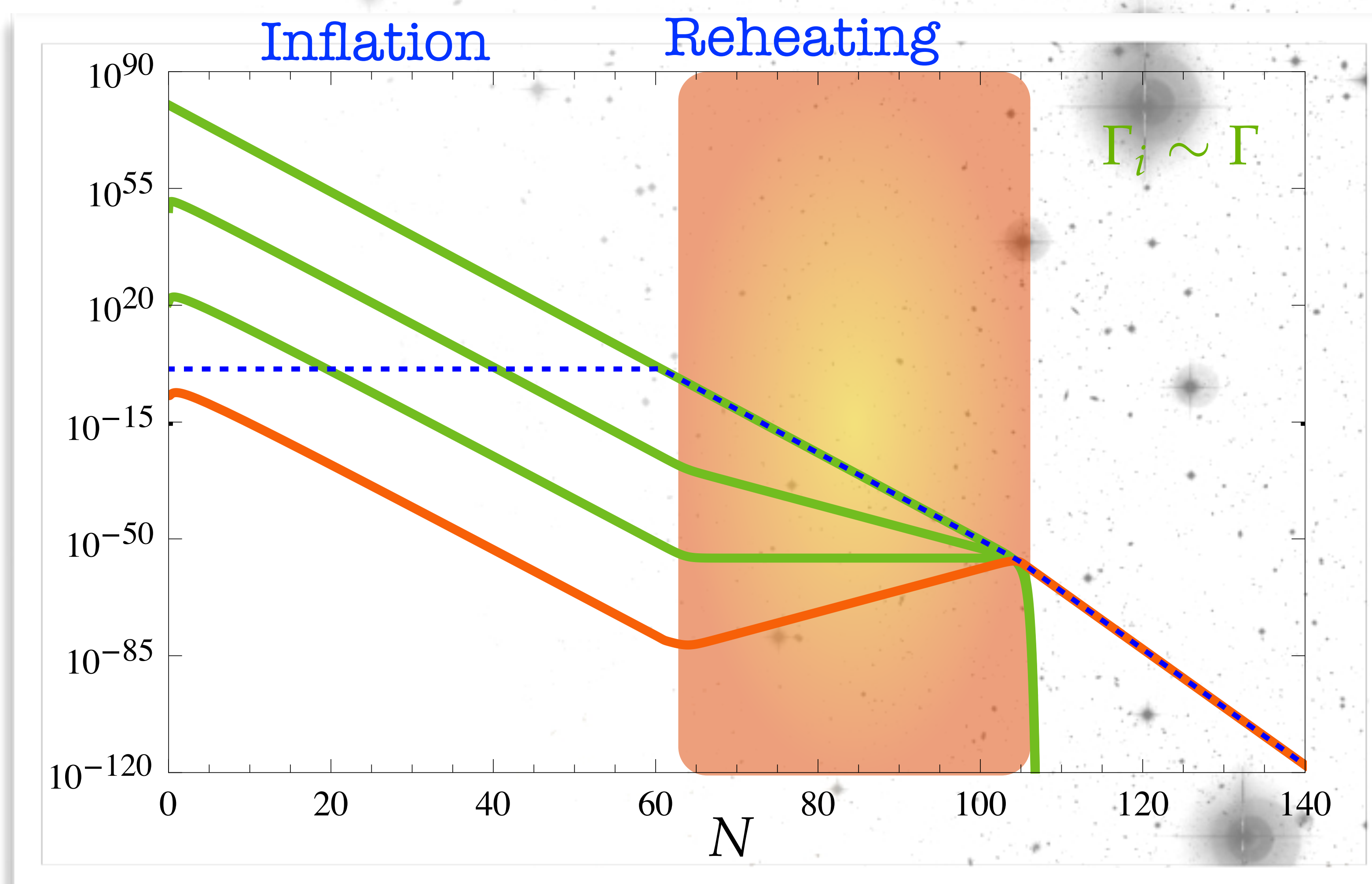

Duration of reheating

- The end of reheating is set by the smallest decay rate

  $\min(\Gamma_i) \simeq 3H$

- Reheating must end before BBN

Dust inflation

$$\dot{\rho}_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot{\rho}_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
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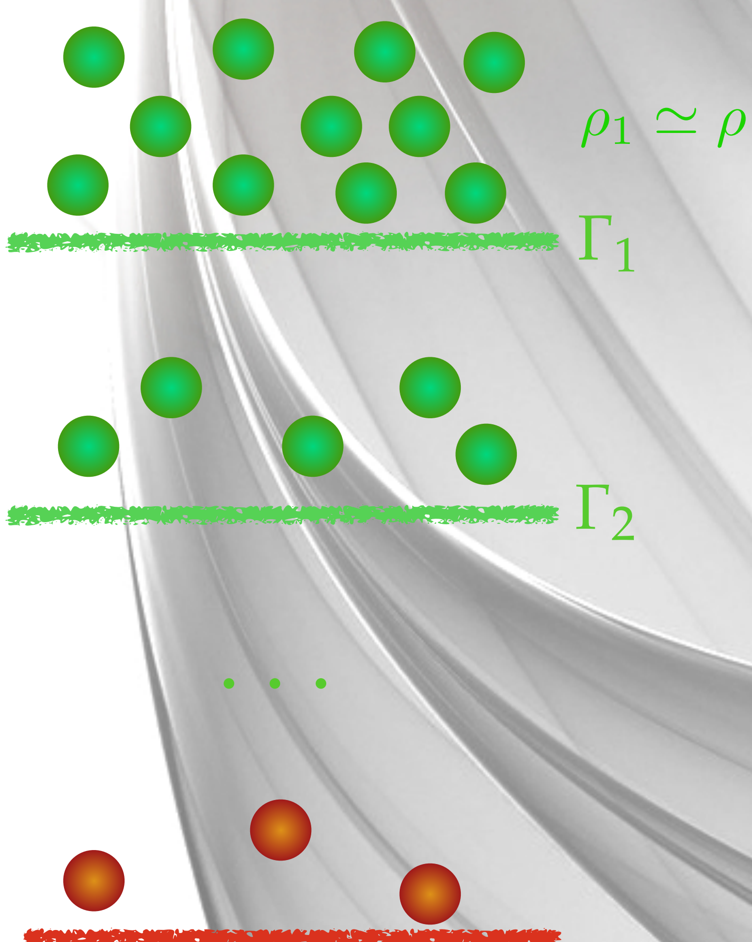

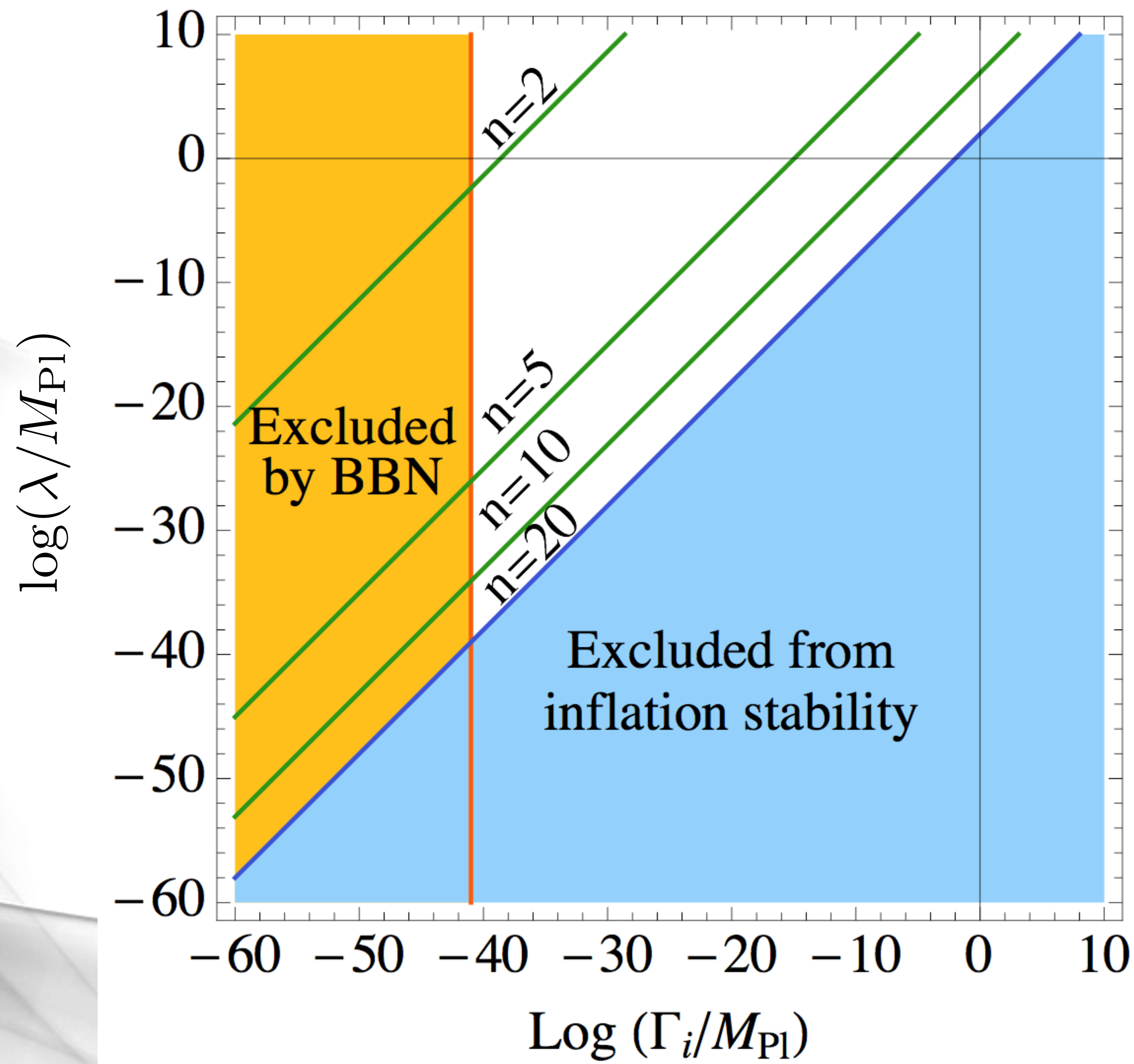

Tensor perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & a^2 h_{ij} \end{pmatrix} \qquad \delta T^{\mu}{}_{\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \Pi^i{}_j \end{pmatrix}$$

All matrices commute at first order in tensor perturbations.

Metric field equations

$$\frac{1}{2}\left[(\hat{M} - \hat{M}^{-1}) + \hat{g}^{-1}(\hat{M} - \hat{M}^{-1})^T \hat{g}\right] - \mathrm{Tr}(\hat{M} - \mathbb{1})\mathbb{1} = \frac{1}{\lambda^2 M_p^2}\hat{T} \Longrightarrow \delta M^i{}_j = \frac{1}{\lambda^2 M_p^2}\frac{1}{1 + M_1^{-2}}\Pi^i{}_j$$

Auxiliary metric

$$\tilde{g}^{\mu\nu} = \sqrt{\det \hat{M}}\, g^{\alpha\mu}(\hat{M}^{-1})^{\nu}{}_{\alpha} \Longrightarrow h^i{}_j = \tilde{h}^i{}_j - \frac{1}{\lambda^2 M_p^2}\frac{1}{M_1 + M_1^{-1}}\Pi^i{}_j$$

$\delta M^i{}_j$ vanishes and both metric perturbations coincide in the absence of anisotropic stresses .

An analogous result was found for the original Born-Infeld gravity theory in C. Escamilla-Rivera,M. Banados, P. G. Ferreira, PRD85 (2012)

It is actually true for any theory of the form

$$\mathcal{S} \sim \int \mathrm{d}^4 x \sqrt{-g} F(\hat{g}^{-1}, \hat{R})$$

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Tensor perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & a^2 h_{ij} \end{pmatrix} \qquad \delta T^{\mu}{}_{\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \Pi^i{}_j \end{pmatrix}$$

All matrices commute at first order in tensor perturbations.

$$(M^{-1})^{\alpha}{}_{(\mu} R_{\nu)\alpha} - \mathrm{Tr}(\hat{M} - \mathbb{1})\lambda^2 g_{\mu\nu} = \frac{1}{M_{\mathrm{Pl}}^2} T_{\mu\nu} \Rightarrow \delta R^i{}_j(\tilde{g}) = \frac{\sqrt{M_0 M_1^3}}{M_p^2} \Pi^i{}_j$$

Same equation as in GR with a modified Newton's constant.

$$\ddot{\tilde{h}}_{ij} + \left(3\tilde{H}(t) - \frac{\dot{\tilde{n}}(t)}{\tilde{n}(t)}\right)\dot{\tilde{h}}_{ij} - \frac{\tilde{n}(t)^2}{\tilde{a}(t)^2}\nabla^2 \tilde{h}_{ij} = 2\frac{\sqrt{M_0 M_1^3}}{M_P^2}\Pi_{ij} \Rightarrow \boxed{\tilde{\mathrm{h}}''_{ij} - \left(\nabla^2 + \frac{\tilde{a}''}{\tilde{a}}\right)\tilde{\mathrm{h}}_{ij} = 0}$$

$\tilde{n} = 1$

$\Pi^i{}_j = 0$

$\tilde{\mathrm{h}}_{ij} = \tilde{a}\tilde{h}_{ij}$

The tensor perturbations see the auxiliary metric. In the quasi de Sitter regime, we have

$$\tilde{H}^2 = \frac{1}{16}H^2 \simeq \frac{1}{16}H_I^2 n^2(t) \simeq \frac{1}{16}H_I^2 \tilde{n}^2(t)\sqrt{\frac{\rho_{\mathrm{m,ini}}}{(20 - 14\sqrt{2})\lambda^2 M_p^2}}\left(\frac{\tilde{a}}{\tilde{a}_{\mathrm{ini}}}\right)^{-6} \rightarrow \boxed{w_{\mathrm{eff}} = 1}$$

No generation of tensor perturbations!

$$a = a_{\mathrm{ini}}\left(\frac{\tilde{a}}{\tilde{a}_{\mathrm{ini}}}\right)^4 \qquad \frac{\tilde{n}^2}{n^2} \simeq \sqrt{\frac{(20 - 14\sqrt{2})\lambda^2 M_p^2}{\rho}} \qquad \frac{\tilde{a}^2}{a^2} \simeq \sqrt{\frac{(2 - \sqrt{2})\rho}{\lambda^2 M_p^2}}$$

prospects

- Bouncing cosmologies.
- Gravitational collapse. Black hole singularity.
- Scalar perturbations. Presence of instabilities.
- Role of torsion. Further explore the general action and possible extensions.
- ...