A unifying description of Dark Energy (& Modified Gravity)

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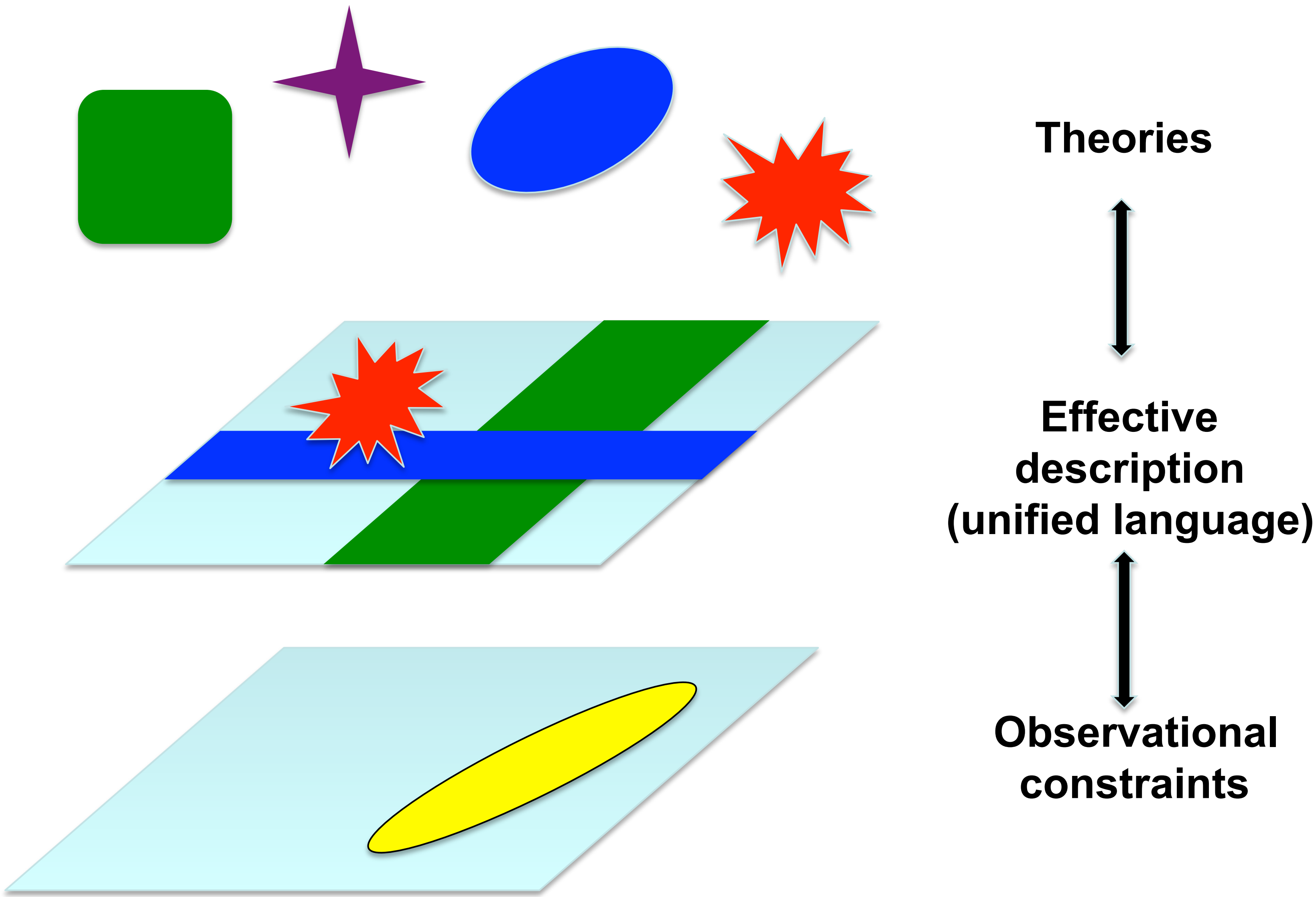
Astroparticules
et Cosmologie

Introduction & motivations

- **Plethora of models** of **dark energy** & **modified gravity**:
  - Cosmological constant
  - quintessence, K-essence
  - f(R) gravity
  - **Horndeski & beyond Horndeski**
  - Massive gravity
  - …

- Large amount of data from future cosmological surveys (DES, LSST, eBOSS, DESI, Euclid, …)

- **General framework** to confront models with data


Theories
Effective description (unified language)
Observational constraints

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- Large amount of data from future cosmological surveys (DES, LSST, eBOSS, DESI, Euclid, …)

- **General framework** to confront models with data:
  - Parametrized modified Einstein equations
  - **Effective action**

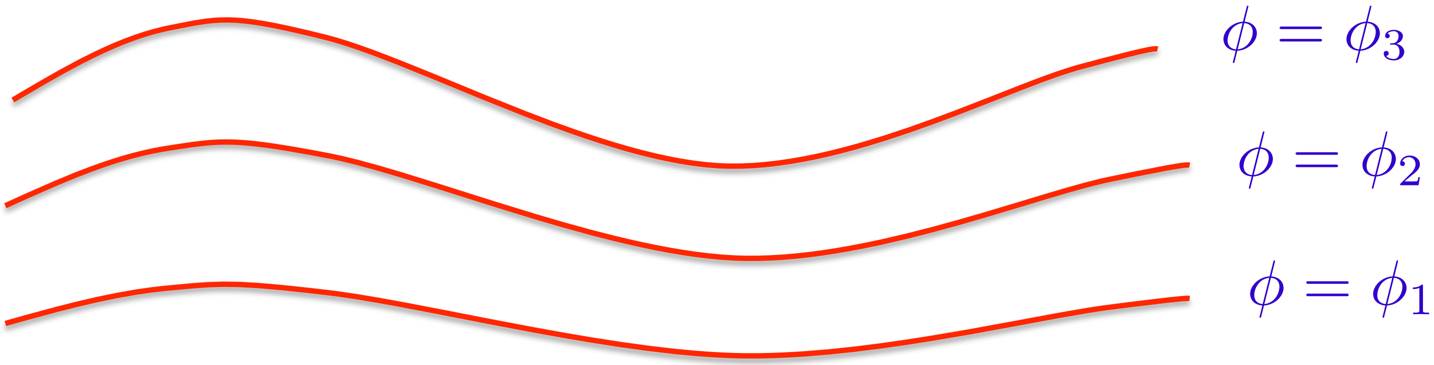
Uniform scalar field slicing

[Inflation: Creminelli et al. ’06; Cheung et al. ’07]

- Restriction: **single scalar field** models

- The scalar field defines a **preferred slicing**

  Constant time hypersurfaces = uniform field hypersurfaces



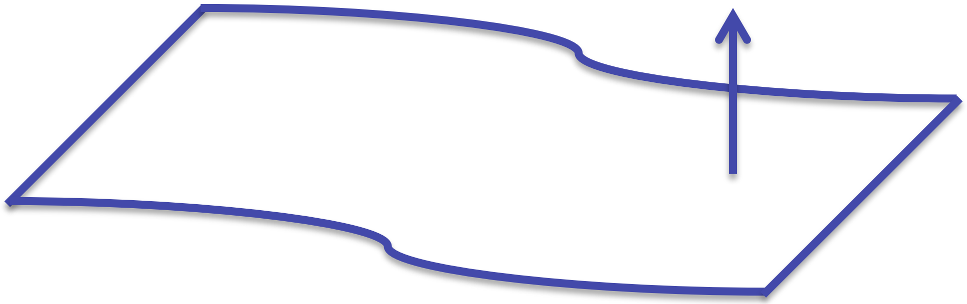
- All perturbations embodied by the metric only

Uniform scalar field slicing

- **3+1 decomposition** based on this preferred slicing

- Basic ingredients
  - **Unit vector normal** to the hypersurfaces

$$n^\mu = -\frac{\nabla^\mu \phi}{\sqrt{-(\nabla\phi)^2}}$$

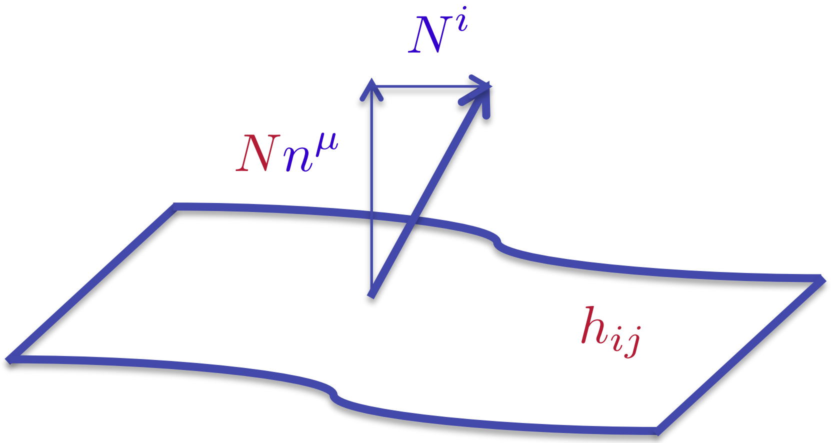


  - **Projection** on the hypersurfaces: $h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$

ADM formulation

- **ADM decomposition of spacetime**

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$



Extrinsic curvature:

$$K_{ij} = \frac{1}{2N}\left(\dot{h}_{ij} - D_i N_j - D_j N_i\right)$$

Intrinsic curvature: R_{ij}

$$X \equiv g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = -\frac{\dot{\phi}^2(t)}{N^2}$$

- **Generic Lagrangians of the form**

$$S_g = \int d^4x\, N\sqrt{h}\, L(N, K_{ij}, R_{ij}; t)$$

Example: GR + quintessence

- Consider a quintessence model

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2}\,{}^{(4)}R - \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

- In the uniform ϕ slicing, this leads to the Lagrangian

$$L = \frac{M_{\rm Pl}^2}{2}\left[K_{ij}K^{ij} - K^2 + R\right] + \frac{\dot\phi^2(t)}{2N^2} - V(\phi(t))$$

Homogeneous evolution

- FLRW metric: $ds^2 = -\bar{N}^2(t)\,dt^2 + a^2(t)\,\delta_{ij}dx^i dx^j$
- Extrinsic curvature: $K^i_j = \frac{\dot{a}}{\bar{N}a}\delta^i_j \equiv H\,\delta^i_j$
- Homogeneous Lagrangian

$$\bar{L}(a,\dot{a},\bar{N}) \equiv L\left[K^i_j = \frac{\dot{a}}{\bar{N}a}\,\delta^i_j, R^i_j = 0, N = \bar{N}(t)\right]$$

- One can include **matter** by adding the Lagrangian for matter (assumed to be minimally coupled to the metric).

Friedmann equations

- Variation of the action $\bar{S}_g = \int dt\, d^3x\, \bar{N} a^3\, \bar{L}(a, \dot{a}, \bar{N})$

  Using $\left(\frac{\partial L}{\partial K^j_i}\right)_{\text{bgd}} \equiv \mathcal{F}\,\delta^i_j$, one finds

$$a^{-3}\frac{\delta \bar{S}_g}{\delta \bar{N}} = \bar{L} + \bar{N} L_N - 3H\mathcal{F} = \rho_m$$

$$\frac{1}{3a^2\bar{N}}\frac{\delta \bar{S}_g}{\delta a} = \bar{L} - 3H\mathcal{F} - \frac{\dot{\mathcal{F}}}{\bar{N}} = -p_m$$

Matter

- For GR: $\bar{L}_{\text{GR}} = -3M_P^2 H^2\,, \quad \mathcal{F}_{\text{GR}} = -2M_P^2 H$

Linear perturbations

- **Perturbations**

$$\delta N \equiv N - \bar{N}\,, \quad \delta K^i_j \equiv K^i_j - H\delta^i_j\,, \quad \delta R^i_j \equiv R^j_i$$

- Expand the Lagrangian

$$L(q_A) \quad \text{with} \quad q_A \equiv \{N, K^i_j, R^i_j\}$$

yields

$$L(q_A) = \bar{L} + \frac{\partial L}{\partial q_A}\delta q^A + \frac{1}{2}\frac{\partial^2 L}{\partial q_A \partial q_B}\delta q_A \delta q_B + \dots$$

- The **quadratic** action describes the **dynamics of linear perturbations**

Linear perturbations

- The coefficients are evaluated on the homogeneous background, e.g.

$$\frac{\partial^2 L}{\partial K^j_i \, \partial K^l_k} \equiv \hat{\mathcal{A}}_K \, \delta^i_j \, \delta^k_l + \mathcal{A}_K \left(\delta^i_l \, \delta^k_j + \delta^{ik} \delta_{jl} \right)$$

$$\frac{\partial^2 L}{\partial R^j_i \, \partial R^l_k} \to \ (\hat{\mathcal{A}}_R, \mathcal{A}_R) \qquad \frac{\partial^2 L}{\partial K^j_i \, \partial R^l_k} \to \ (\hat{\mathcal{C}}, \mathcal{C}) \quad \ldots$$

- For simplicity, we assume the three conditions

$$\hat{\mathcal{A}}_K + 2\mathcal{A}_K = 0 \, , \quad \hat{\mathcal{C}} + \frac{1}{2}\mathcal{C} = 0 \, , \quad 4\hat{\mathcal{A}}_R + 3\mathcal{A}_R = 0$$

so that the EOM are 2nd order in spatial gradients.

Linear perturbations

- Quadratic action in terms of **5 functions of time**

$$S^{(2)} = \int dx^3 dt\, a^3 \frac{M^2}{2} \Big[\delta K^i_j \delta K^j_i - \delta K^2 + \alpha_K H^2 \delta N^2 + 4\, \alpha_B H\, \delta K\, \delta N + (1+\alpha_T)\delta_2\left(\frac{\sqrt{h}}{a^3} R\right) + (1+\alpha_H) R\, \delta N \Big]$$

Gleyzes, DL, Piazza & Vernizzi ’13,
[notation from Bellini & Sawicki ‘14]

- Includes many models
  - GR: $M = M_P, \quad \alpha_i = 0$
  - Quintessence, K-essence: $\alpha_K \neq 0$
  - Kinetic braiding, DGP: $\alpha_B \neq 0$
  - Brans-Dicke, F(R): $M = M(t)$
  - Horndeski: $\alpha_T \neq 0$
  - beyond Horndeski: $\alpha_H \neq 0$

Scalar degree of freedom

- Scalar perturbations: $\delta N\,, \quad N_i \equiv \partial_i \psi\,, \quad h_{ij} = a^2(t) e^{2\zeta} \delta_{ij}$

- Quadratic action for the **physical degree of freedom**:

$$S^{(2)} = \frac{1}{2} \int dx^3 dt\, a^3 \left[\mathcal{K}_t\, \dot{\zeta}^2 + \mathcal{K}_s \frac{(\partial_i \zeta)^2}{a^2} \right]$$

$$\mathcal{K}_t \equiv \frac{\alpha_K + 6\alpha_B^2}{(1+\alpha_B)^2}\,, \quad \mathcal{K}_s \equiv 2M^2 \left\{ 1 + \alpha_T - \frac{1+\alpha_H}{1+\alpha_B} \left(1 + \alpha_M - \frac{\dot{H}}{H^2} \right) - \frac{1}{H} \frac{d}{dt} \left(\frac{1+\alpha_H}{1+\alpha_B} \right) \right\}$$

- Stability
  - No ghost: $\mathcal{K}_t > 0$
  - No gradient instability: $c_s^2 \equiv -\frac{\mathcal{K}_s}{\mathcal{K}_s} > 0$

Tensor degrees of freedom

- Quadratic action for the **tensor modes**:

$$S_\gamma^{(2)} = \frac{1}{2}\int dt\, d^3x\, a^3 \left[\frac{M^2}{4}\dot{\gamma}_{ij}^2 - \frac{M^2}{4}(1+\alpha_T)\frac{(\partial_k \gamma_{ij})^2}{a^2}\right]$$

- Stability
  - No ghost: $M^2 > 0$
  - No gradient instability: $c_T^2 \equiv 1 + \alpha_T > 0$

Example: Horndeski theories

- Most general scalar-tensor action leading to at most second order equations of motion for the scalar field and metric. Horndeski 74

- **Generalized galileons** coupled to gravity Nicolis et al. 08; Deffayet et al. 09 & 11

- Combination of the following four Lagrangians

$$L_2^H = G_2(\phi, X)$$
$$L_3^H = G_3(\phi, X)\,\Box\phi$$
$$L_4^H = G_4(\phi, X)\,{}^{(4)}R - 2G_{4X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$
$$L_5^H = G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}G_{5X}(\phi, X)(\Box\phi^3 - 3\,\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})$$

with
$$X \equiv \nabla_\mu\phi\nabla^\mu\phi$$
$$\phi_{\mu\nu} \equiv \nabla_\nu\nabla_\mu\phi$$

- **Higher order** derivatives in the Lagrangian

Beyond Horndeski

- 2nd order time derivatives in the **Lagrangian** usually lead to an extra DOF, which is unstable (**Ostrogradski)**

$$\text{e.g.} \quad L(q, \dot{q}, \ddot{q})$$

- 2nd order **EOMs** were believed to be necessary to avoid Ostrogradski's ghost but higher order equations of motion are in fact possible.

- Two extensions beyond Horndeski [Gleyzes, DL, Piazza & Vernizzi '14]

$$L_4^{\rm bH} \equiv F_4(\phi, X)\, \epsilon^{\mu\nu\rho}{}_{\sigma}\, \epsilon^{\mu'\nu'\rho'\sigma} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'}$$

$$L_5^{\rm bH} \equiv F_5(\phi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \phi_\mu \phi_{\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} \phi_{\sigma\sigma'}$$

- Crucial ingredient: $\mathcal{L}[\phi, g_{\mu\nu}]$ must be **degenerate** [DL & Noui '15]

Horndeski & beyond in ADM form

- One obtains combinations of the Lagrangians

$$L_2 = A_2 \qquad\qquad L_3 = A_3\, K$$

$$L_4 = A_4\, (K^2 - K_{ij}K^{ij}) + B_4\, R$$

$$L_5 = A_5\, (K^3 - 3KK_{ij}K^{ij} + 2K_{ij}K^{ik}K^j{}_k) + B_5\, K^{ij}\left[R_{ij} - h_{ij}R/2\right]$$

where the *A's* and *B's* depend on the functions *G's* & *F's.*

- Horndeski theories (only four *G's*) satisfy the relations

$$A_4 = -B_4 + 2XB_{4X}$$

$$A_5 = -XB_{5X}/3$$

- One can then use the results of the general formalism.

Generalized couplings to matter

Gleyzes, DL, Mancarella & Vernizzi '15

- Minimal coupling: $S_m = S_m[\psi_m, g_{\mu\nu}]$

- Conformal coupling: $S_m = S_m[\psi_m, C(\phi)\, g_{\mu\nu}]$

- **Conformal-disformal couplings**

$$S_m^{(I)} = S_m^{(I)}[\psi_m, \check{g}_{\mu\nu}^{(I)}]$$

with $\check{g}_{\mu\nu}^{(I)} = C_I(\phi)\, g_{\mu\nu} + D_I(\phi)\, \partial_\mu\phi\partial_\nu\phi$ [Bekenstein '93]

$$\alpha_{C,I} \equiv \frac{1}{2}\frac{d\ln C_I}{d\ln a}, \quad \alpha_{D,I} \equiv \frac{D_I}{C_I - D_I}$$

"Frame" transformation

- Gravity can be described by a different metric

$$g_{\mu\nu} \longrightarrow \tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi) \partial_\mu \phi \, \partial_\nu \phi$$

- Horndeski's structure is invariant [Bettoni & Liberati '12]

The quadratic action (with $\alpha_H = 0$ here)

$$S^{(2)} = \int dx^3 dt \, a^3 \, \frac{M^2}{2} \Big[\delta K^i_j \delta K^j_i - \delta K^2 + \alpha_K H^2 \delta N^2 + 4\, \alpha_B H \, \delta K \, \delta N + (1 + \alpha_T) \delta_2 \left(\frac{\sqrt{h}}{a^3} R \right) + R \, \delta N \Big]$$

gets transformed into a similar action

$$\{M, \alpha_K, \alpha_B, \alpha_T\} \underset{\{C, D\}}{\longrightarrow} \{\tilde{M}, \tilde{\alpha}_K, \tilde{\alpha}_B, \tilde{\alpha}_T\}$$

“Frame” transformation

- Metric transformation

$$\alpha_C \equiv \frac{1}{2}\frac{d\ln C}{d\ln a}\,, \qquad \alpha_D \equiv \frac{D}{C-D}$$

- **New gravitational coefficients**

$$\tilde{M}^2 = \frac{M^2}{C\sqrt{1+\alpha_{\rm D}}}\,, \quad \tilde{\alpha}_{\rm T} = (1+\alpha_{\rm T})(1+\alpha_{\rm D}) - 1 \quad \tilde{\alpha}_{\rm B} = \frac{1+\alpha_{\rm B}}{(1+\alpha_{\rm C})(1+\alpha_{\rm D})} - 1$$

$$\tilde{\alpha}_{\rm K} = \frac{\alpha_{\rm K} + 12\alpha_{\rm B}[\alpha_{\rm C} + (1+\alpha_{\rm D})\alpha_{\rm D}] - 6[\alpha_{\rm C} + (1+\alpha_{\rm D})\alpha_{\rm D}]^2 + 3\Omega_{\rm m}\alpha_{\rm D}}{(1+\alpha_{\rm C})^2(1+\alpha_{\rm D})^2}\,,$$

- **New matter couplings** $\{\alpha_{C,I}, \alpha_{D,I}\} \longrightarrow \{\tilde{\alpha}_{C,I}, \tilde{\alpha}_{D,I}\}$

$$\tilde{\alpha}_{{\rm C},I} = \frac{\alpha_{{\rm C},I} - \alpha_{\rm C}}{1+\alpha_{\rm C}} \qquad \tilde{\alpha}_{{\rm D},I} = \frac{\alpha_{{\rm D},I} - \alpha_{\rm D}}{1+\alpha_{\rm D}}$$

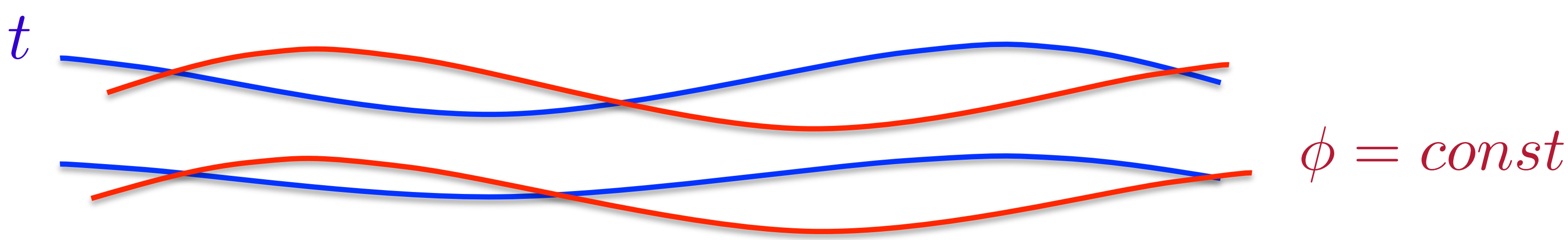
- N_S species: $4+2N_S-2=2(N_S+1)$ independent parameters

Confrontation with observations

- Use a traditional gauge, e.g. Newtonian gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)\,(1-2\Psi)\,\delta_{ij}\,dx^i dx^j$$

- Description in an arbitrary slicing ?



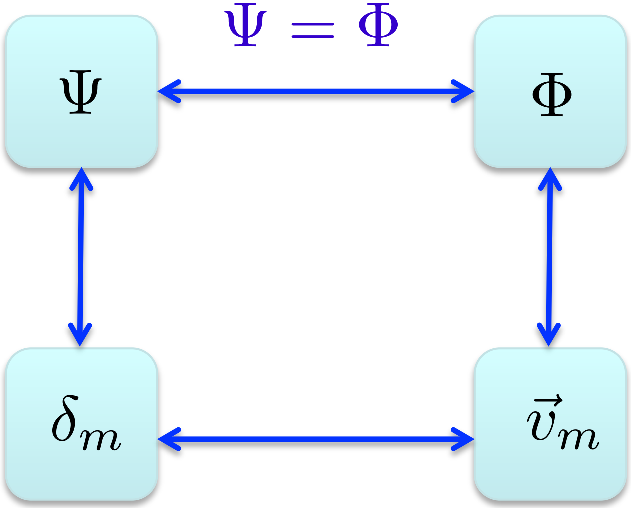
- Coordinate change $t \to t + \pi(t, \vec{x})$

- Perturbations: $\Phi\,,\;\Psi\,,\;\pi\,,\delta_m\,,\;\vec{v}_m$

Cosmological perturbations

- **Standard equations** (in GR)

$$\Psi = \Phi$$



$$\nabla^2 \Psi = 4\pi G a^2 \rho_m \delta_m$$

(relativistic Poisson)

$$\dot{\vec{v}}_m + H\,\vec{v}_m = -\vec{\nabla}\Phi$$

(Euler)

$$\dot{\delta}_m + \vec{\nabla} \cdot \vec{v}_m = 0$$

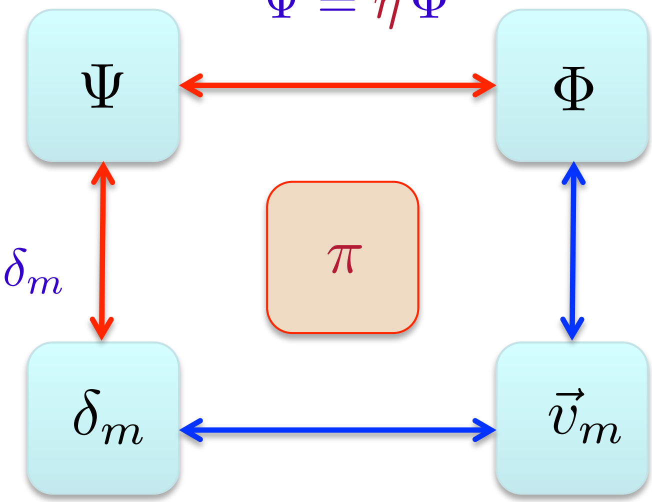
(continuity)

Cosmological perturbations

- **Modified equations** (minimal coupling)

$$\Psi = \eta\,\Phi$$

$$\nabla^2\Psi = 4\pi G(1+\Upsilon_G)a^2\rho_m\delta_m$$



Quasi-static approximation (valid on scales $kc_s \gg aH$ [Sawicki & Bellini '15])

$$G_{\rm eff} = G_{\rm eff}(\alpha_i)\,, \quad \eta = \eta(\alpha_i)$$

which can be confronted to observations (RSD, weak lensing, …).

Cosmological perturbations

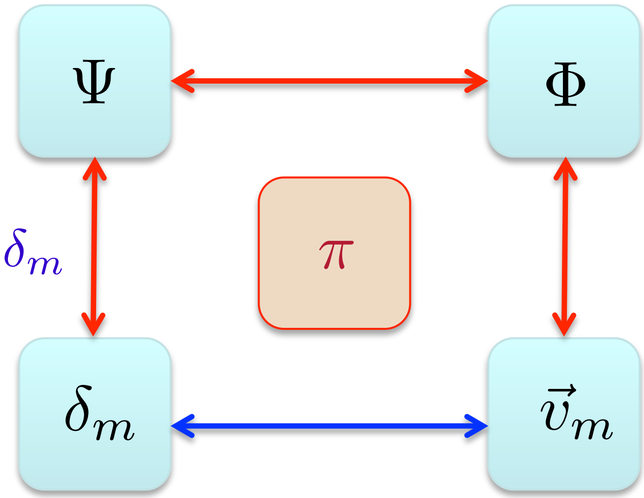
- **Modified equations** (non minimal coupling)

Gleyzes, DL, Mancarella & Vernizzi ’15

Quasi-static approximation

$$\nabla^2(\Phi+\Psi) = 4\pi G\,(2+\Upsilon_{\rm lens})\,a^2\rho_m\delta_m$$

$$\nabla^2\Psi = 4\pi G(1+\Upsilon_G)a^2\rho_m\delta_m$$



$$\dot{\vec{v}}_I + (H+3\gamma_I)\,\vec{v}_m = -\vec{\nabla}\Phi - 3H\gamma_I\vec{\nabla}\pi$$

- Generalization of coupled quintessence [Amendola ‘00]

On smaller scales

- Deviations from GR on cosmological scales should be compatible with small-scale observations (solar system, binary systems)

- **Screening mechanism**

$$Z(\phi_0)\,\nabla^2\delta\phi - m^2(\phi_0)\delta\phi = -\beta(\phi_0)\,\frac{\delta T}{M_P}$$

  - Chameleon: $m(\phi_0)$ is large
  - Dilaton & symmetron: $\beta(\phi_0) \ll 1$
  - Vainshtein: $Z(\phi_0) \gg \beta^2(\phi_0)$

Example: beyond Horndeski

Saito, Yamauchi, Mizuno, Gleyzes & DL '15
(see also Koyama & Sakstein '15)

- Partial breaking of Vainshtein mechanism inside matter

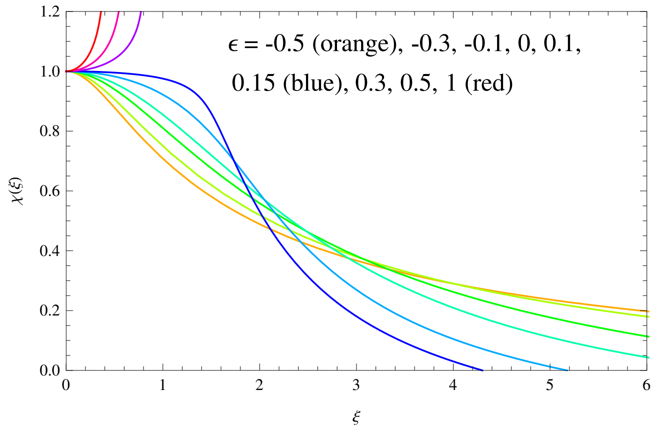
Kobayashi, Watanabe & Yamauchi '14

- Spherical symmetry & nonrelativistic limit:

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = G_{\mathrm{N}}\left(\frac{\mathcal{M}}{r^2} - \epsilon\frac{\mathrm{d}^2\mathcal{M}}{\mathrm{d}r^2}\right), \qquad \mathcal{M}(r) = 4\pi\int_0^r {r'}^2\rho(r')\mathrm{d}r'$$

- Modified **Lane-Emden equation**
  (for $P = K\rho^{1+\frac{1}{n}}$)
  - Universal bound $\epsilon < 1/6$
  - Astrophysical constraints [Sakstein '15]

$$\epsilon > -0.0068$$



Conclusions

- **Unifying description** of dark energy and modified gravity models
  - Easy comparisons between models
  - Identification of degeneracies
  - Observational data can constrain many models simultaneously
  - Explore unchartered territories (e.g. theories beyond Horndeski)

- Very general and efficient way to describe linear perturbations in scalar-tensor theories with **only five time-dependent functions.**

- Extension to include non-universal couplings