Is the Ni's solution of the Tolman-Oppenheimer-Volkoff problem without the maximum-mass limit applicable to the real neutron stars? A discussion

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Abstract

In 2011, Jun Ni published solution of the equations in the classical Tolman-Oppenheimer-Volkoff modeling of spherically symmetric neutron star. The Ni's solution implies no upper-mass limit and the outer surface of modeled object always appears to be above the event horizon. In addition, it indicates that the collaps of a very massive object below its event horizon is forbidden because an infinite energy is required. In fact, the Ni's solutions are the super-class of the original Oppenheimer-Volkoff solutions. The proof of the maximum mass of stable object is valid only for the latter. Ni noted that the type of solution found by him cannot be obtained in Newtonian physics. However, general relativity may not obey the limitations sourcing from the Newtonian gravity and, thus, it seems that the neutron-star models based on the Ni's solution are still applicable on real compact objets. We discuss the relevance of main objections against this applicability. As well, we mention some observations to support or reject the applicability.

1 Introduction

After the discovery of new solution of Einstein field equations (EFEs, hereinafter) (Einstein, 1915; 1916) for the spherical symmetry (SS) by Chinese researcher Jun Ni in 2011, it appears that there are two variants of general relativity (GR): the pure geometrical GR (G-GR) characterized with inequality $|g_{rr}| > 0$ (g_{rr} is the radial component of metric tensor) and GR with the Newtonian-type potential (GR-NTP) characterized with $|g_{rr}| \geq 1$. The current astrophysics is built on the GR-NTP.

Ni solved the original Tolman-Oppenheimer-Volkoff (TOV) problem (Tolman, 1939; Oppenheimer and Volkoff, 1939). In contrast to Oppenheimer and Volkoff (O-V), he started the numerical integration of the EFEs in a finite star-centric distance and obtained stable solution for object of whatever large mass and with the outer surface always situated above the corresponding event horizon. In fact, the continuum of the Ni's solution is the super-class of the original O-V solutions.

We know, there was given the proof that there is no solution for the compact object without internal source of energy, which has its mass larger than a certain limit (Oppenheimer and Snyder, 1939; Rhoades and Ruffini, 1972; and others). The proof (calculation of maximum possible mass of stable object) has been several times improved. Analyzing all these proofs, one can however find that there is always adopted an assumption consistent with the O-V type object, but clearly inconsistent with the object constructed using the Ni's solution. Therefore, the proof is valid only for the O-V sub-class of solutions. After all, it cannot be valid for the Ni's super-class, because the Ni's solutions simply exist. There are many contra-examples.

In our contribution, we re-introduce the Ni's solution and describe the basic features of the object constructed by using it. Then, we ask and try to answer some questions, which should be answered in course to decide if the Ni's solution is applicable to the real objects or it is only a theoretical curiosity.

2 Basic features of the Ni's object

Ni (2011) started the numerical integration of field equations in a non-zero star-centric distance and inward proceeded part of the integration always ended with zero pressure and energy density also in

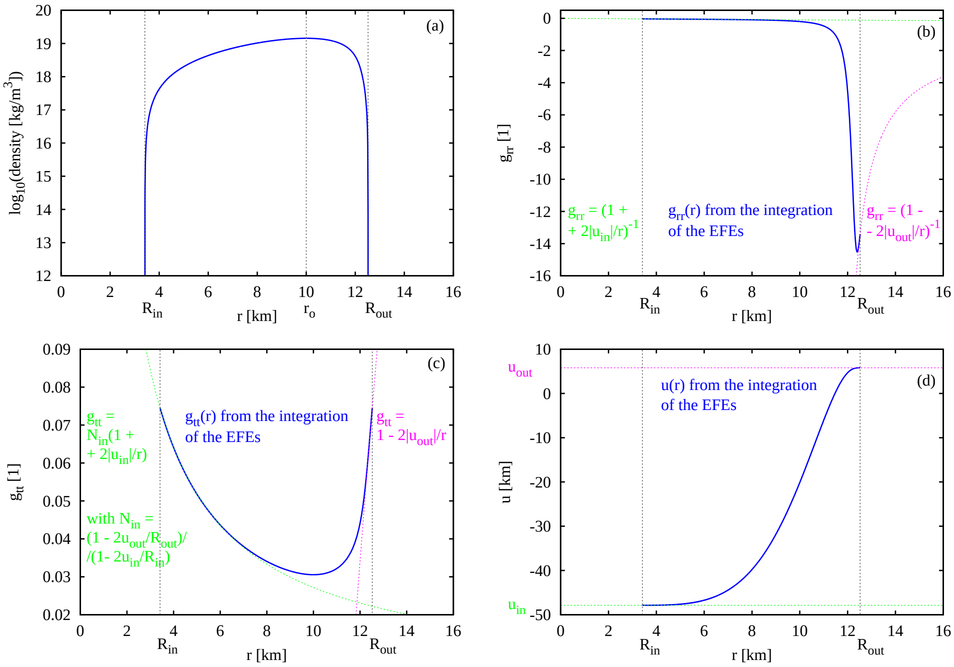

Figure 1: The density (plot a), components g_{rr} and g_{tt} of metric tensor (b and c), and auxiliary quantity u (d) as the functions of radial distance, r, in an example of RCO. Its mass equals 3.923 $M_\odot$.

a non-zero star-centric distance. So, the relativistic compact object (RCO) described by using the Ni's solution is a hollow sphere bordered not only with the outer, but also with the inner physical surface. The behaviors of the density, components g_{rr} and g_{tt} of metric tensor, and auxiliary quantity u established by Oppenheimer and Volkoff as $u = (r/2)(1+1/g_{rr})$ (r is the radial, star-centric distance), are shown in Fig. 1 for an example of the Ni's RCO.

In Fig. 1c, g_{tt} component is not monotonous function of r. Its derivative is zero in $r = r_o$ implying the zero gravity, here. In the interval from 0 to r_o, function $g_{tt}(r)$ is decreasing. It means that the gravitational acceleration of a test particle is oriented outward in this interval. This outward oriented gravitational attraction (there is not any reason to assume a repulsive gravity, despite of its orientation outward) is balanced with the gradient of pressure, which is always oriented against the gravity, according to the EFEs. The inner physical surface of RCO occurs due to the same mechanism as the outer surface. A more detailed description of the Ni's RCO can be found in our recent paper (Neslušan, 2015).

One can construct a series of RCO models having the same distance r_o of zero gravity and, at the same time, maximum pressure and energy density. The mass of the RCO as the function of the extent, from inner to outer radius, of RCO is shown in Fig. 2 for two such series. We can see an indication that the outer radius likely approaches, from outside, the gravitational radius (dashed black straight line) in the limit of infinite mass. The infinite mass implies the infinite internal energy of the object. If this is actually confirmed, once, then one would have to deliver an infinite amount of energy to a RCO to force its collapse to event horizon. In other words, the black holes would be energetically forbidden.

3 Interpretation of O-V auxiliary quantity u

Oppenheimer and Volkoff (1939) established the auxiliary function u, which is defined as $u = (r/2)(1+1/g_{rr})$. In the limit of weak field, this quantity occurs, in the appropriate GR formulas, in the same position as the mass of object in the corresponding Newtonian formulas. This circumstance was utilized to determine the vacuum value of u outside a SS object. Later, in the theory of neutron stars, u was generalized to be the mass inside the radius r.

Taking the Ni's solution into account, this representation of u is incorrect. As seen in Fig. 1d, there

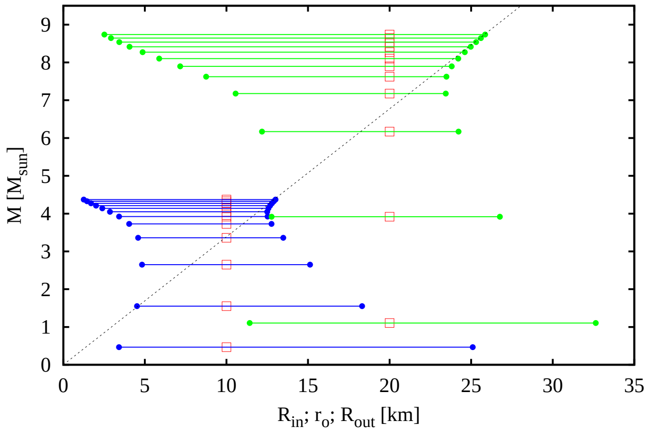

Figure 2: The mass of the RCO, M, as the function of the extent, from inner, R_{in}, to outer, R_{out}, radius, of RCO (full circles connected with an abscissa). The mass is shown for two series of RCO models, whereby each series has the same distance r_o of zero gravity (empty square within the corresponding abscissa). The dotted straight line shows the behavior of gravitational radius.

is a region with the negative values of u inside the object. However, the energy density is positive everywhere in the object, therefore its integral through the volume of negative u, i.e. the energy, must be positive. According to the well-known Einstein formula, energy is related to mass, m, as mc^2 (c is the speed of light). Since energy is positive, the corresponding mass must be positive as well. If we insisted on the representation that m and u are identical functions, then the GR would be an internally inconsistent theory, because the same quantity cannot be negative according to some arguments and, at the same time, positive according to another argumentation. Considering the G-GR platform, we suggest that u is simply an alternative form of g_{rr} componet of metric tensor.

4 Gravity inside a spherically symmetric material shell

It appears that the outward oriented gravitational attraction and existence of the Ni's hollow sphere itself is closely related to to the question of what is the gravity inside a SS material shell. In the Euclidean space of Newtonian physics, the net gravitational force of the shell can be analytically calculated and is exactly zero in its internal cavity.

In the relativistic SS shell, the metrics in the cavity was *postulated* to be the Minkowski metrics, which implies zero gravity. This postulate was likely motivated by our old experience originating in the Newtonian physics that the gravity is always oriented inward throughout the whole body of such objects as stars or planets. In the center of object, the theory (GR as well as Newtonian) implies a singularity, which is, from the point of view of the GR, the naked singularity. This type of singularity is forbidden by the cosmic censorship theorem (Penrose, 1969). The singularity can be eliminated, if the metrics in the cavity of the shell is postulated to be the Minkowski metrics. This used to be applied to the material shell of every radius inside the object.

However, if we let the GR work, the metrics inside the SS shell is, in agreement with the Birkhoff theorem (Birkhoff and Langer, 1923), described by the outer Schwarschild (1916) solution of the EFEs. The Ni's solution for the RCO body then implies that the integration constant in the latter has the opposite sign in the cavity in comparison to its counterpart for outer empty space. Consequently, the gravity in the cavity is oriented outward, therefore the central singularity is the singularity of Big-Bang type, which is not in any conflict with the cosmic censorship.

The singularity is a vacuum point, where no particle can ever enter. Every particle in the cavity is attracted away from this point by the circumambient matter of hollow sphere. This matter is the "active agent" acting on any test particle in the cavity. There is no reason for the postulating of the Minkowski metrics. On contrary, there is the argument against the postulate: it faces the problem of the continuity of metrics in the radius of the shell (in its inner radius if the shell is thick).

5 Conclusion remarks

The Ni's solution of the field equations is the smoking gun of astrophysics of the RCOs. If accepted for the description of real RCOs, it implies a new concept of the latter. The remnant of star of whatever a high mass is, then, always the object with a visible surface. As well, the central objects of galaxies and quasars are the RCOs with a visible surface. And, using the Ni's strategy, we can construct a model of radiation sphere (equation of state: $E = 3P$; E − energy density, P − pressure) having the essential amount of internal energy concentrated within a sphere of radius not much larger than the corresponding gravitational radius. The (formally calculated) mass of such sphere can range from a fraction of solar mass up to $10^{10}\,\mathrm{M}_{\odot}$ or more. Maybe, the radiation fluid was torn into some chunks at the end of cosmological radiation era. These chunks then acquired a spherical or quasi-spherical shape due to self-gravity. They would resemble the quasars still surviving in the universe.

The acceptability of the Ni's solution, i.e., in fact, the acceptability of (i) the outward oriented gravitational attraction inside the object, (ii) purely geometric representation of auxiliary quantity u, and (iii) abolition of the postulate of Minkowski metrics inside a SS shell, will be decided by observations. There should be discovered the pulsars with a mass considerably exceeding $\sim 2\,\mathrm{M}_{\odot}$. Actually, pulsars PSR B1957+20 and J1311-3430 with masses $(2.4 \pm 0.12)\,\mathrm{M}_{\odot}$ (van Kirkwijk et al., 2011) and (2.68 ± 0.14), (2.15 ± 0.11), and $(2.92 \pm 0.16)\,\mathrm{M}_{\odot}$ (by various methods; Romani et al., 2012) were already discovered. Unfortunately, these mass determinations are suspected of a systematic error.

If the massive object in the true center of the Milky Way is not a black hole, but the Ni's RCO, then improving resolution of the telescope to observe its shadow will not result in a permanently decreasing observed size of the object, but this size will become fixed when the resulution will become good enough to recognize the visible outer physical radius.

The nature of quasars is hardly discernable. However, some discoveries of these objects with the red-shift much larger than 10 would support the concept of quasar as the Ni's radiation sphere.

All the above-mentioned new predictions can be regarded as another tests of the GR. We will see if this theory is correct when we "let it to work in its full extent" or is correct only when constrained by the additional postulate and its consequences.

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